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A L G E B R A

ULUWMA BILIM BERETUG'I'N MEKTEPLERDIN'
9-KLASI' USHI'N SABAQLI'Q

Wo'zbekshe 3-basi'li'mi'na sa'ykes qaraqalpaqsha basi'li'm

O'zbekistan Respublikasi' Xali'q bilimlendiriw
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! — biliw a'hmiyetli ha'm yeslep qali'w paydali' (yadlaw sha'rt yemes) tekst

△ — ma'seleni sheshiw baslandi'

▲ — ma'seleni sheshiw tamamlandi'

○ — matematikali'q tasti'yi'qlawdi' tiykarlaw yaki formulani'keltirip shi'g'ari'w baslandi'

● — tiykarlaw yaki formulani' keltirip shi'g'ari'w tamamlandi'

— sheshiliwi ma'jbu'riy bolg'an ma'selelerdi aji'rati'p turi'wshi' belgi

— quramali'raq ma'sele

— tiykarg'i' materialdi' aji'rati'w

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8-KLASTA U'YRENILGEN TEMALARDI' TA'KIRARLAW

A'ziz woqi'wshi'! Siz 8-klasta si'zi'qli' funkciya ha'm woni'n' grafigi, yeki belgisizli yeki si'zi'qli' ten'lemeler sistemasi', ten'sizlikler, kvadrat korenler, kvadrat ten'lemeler juwi'q yesaplawlarg'a tiyisli mi'sal ha'm ma'selelerdi sheshkensiz. 8-klasta alg'an bilimlerin'izdi yeske tu'siriw maqsetinde Sizge bir qatar shi'ni'g'i'wlar usi'ni's yetemiz.

1. 1) $y = 2x + 3$; 2) $y = -3x + 4$; 3) $y = 4x - 1$; 4) $y = -2x - 5$ funkciyalari'ni'n' grafigin si'zi'n'. Grafik qaysi' shereklerde jatadi'? Grafiktin' Ox ha'm Oy ko'sherleri menen kesilisiw noqatlari'ni'n' koordinatalari'n ayti'n'.

2. $y = kx + b$ funkciya grafigi $A(0;\ -7)$, $B(2;\ 3)$ noqtalari'nan wo'tedi. k ha'm b ni' tabi'n'.

3. Tuwri' si'zi'q $A(0;\ 5)$, $B(1;\ 2)$ noqatlari'nan wo'tedi. Tuwri' si'zi'qti'n' ten'lemesin jazi'n'.

4. Ten'lemeler sistemasi'n sheshin':

1) $\begin{cases} 7x + 4y = 29; \\ 5x + 2y = 19; \end{cases}$ 2) $\begin{cases} 5x - 4y = 13; \\ 2x - y = 4; \end{cases}$ 3) $\begin{cases} x^2 - y^2 = 7; \\ x + y = 7. \end{cases}$

5. 3 at ha'm 4 si'yi'rg'a bir ku'nde 27 kg jem beriledi. Bir ku'nde 9 atqa berilgen jem 5 si'yi'rg'a berilgen jemnen 30 kg ko'p. Bir atqa ha'm bir si'yi'rg'a 1 ku'nde qansha jem beriledi?

6. Kitap ha'm da'pter birgelikte 5 800 swm turadi'. Kitap bahasi'ni'n' 10% i da'pter bahasi'ni'n' 35% inen 220 swm g'a qi'mbat. Kitap ha'm da'pterdin' ha'r qaysi'si' qansha swm turadi'?

7. Ten'sizlikti sheshin':

1) $3(x - 4) + 5x < 2x + 3$; | 2) $|5 - 2x| \le 1$; | 3) $|3x - 4| \ge 2$.

8. Ten'sizlikler sistemasi'n sheshin':

1) $\begin{cases} 4(2 - x) > 7 - 5x, \\ 15 - 4x < 3; \end{cases}$ 2) $\begin{cases} 2(3 - 2x) > 8 - 5x, \\ 10 - x > 2. \end{cases}$

9. $\frac{3x+4}{2}-\frac{1-x}{3}<\frac{7x-3}{2}-\frac{3-x}{3}$ ten'sizliktin' yen' kishi pu'tin sheshimin tabi'n'.

10. Yesaplan':

1) $\sqrt{121\cdot 0{,}04\cdot 289}$; 2) $\sqrt{5\frac{1}{7}\cdot 3\frac{4}{7}}$; 3) $(\sqrt{32}+\sqrt{8})^2$.

11. A'piwayi'lasti'ri'n':

1) $(8\sqrt{63}+3\sqrt{28}-5\sqrt{112}):2\sqrt{7}$;

2) $(15\sqrt{1{,}2}+\frac{1}{3}\sqrt{270}-2\sqrt{30}$;

3) $\frac{2}{\sqrt{11}+3}+\frac{7}{\sqrt{11}-2}$;

4) $\frac{4}{3-\sqrt{5}}+\frac{1}{2-\sqrt{5}}+\frac{3\sqrt{5}}{4}$.

Ten'lemeni sheshin' **(12–14):**

12. 1) $|7-x|=x-7$; | 2) $|x+6|=x+10$; | 3) $\sqrt{(x-9)^2}=x-9$.

13. 1) $x^2-12x+11=0$; 2) $x^2-15x+56=0$;
3) $6x^2+7x-3=0$; 4) $16x^2+8x+1=0$.

14. 1) $x^4-10x^2+9=0$; 2) $10x^4+7x^2+1=0$.

15. 240 km arali'qti' bir avtomobil yekinshisine qarag'anda 1 saat tezirek basi'p wo'tedi. Yeger birinshi avtomobildin' tezligi yekinshisinin' tezliginen 20 km/saat arti'q bolsa, ha'rbir avtomobildin' tezligin tabi'n'.

16. 1) Yeki sanni'n' ayi'rmasi' 2,5 ke, kvadratlari'ni'n' ayi'rmasi' bolsa 10 g'a ten'. Bul sanlardi' tabi'n'.
2) Qosi'ndi'si' 1,4 ke, kvadratlari'ni'n' qosi'ndi'si' 1 ge ten' bolg'an yeki sandi' tabi'n'.

17. $x^2-8x+3=0$ ten'lemenin' korenleri x_1 ha'm x_2 bolsa, 1) $x_1^2+x_2^2$; 2) $x_1^3+x_2^3$; 3) $x_1^2x_2+x_1x_2^2$; 4) $x_1^2-x_2^2$ tabi'n'.

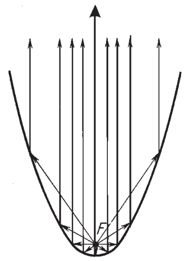
18. Sandi' ju'zden birge deyin do'n'geleklen'. Do'n'geleklewdin' sali'sti'rmali' qa'teligin tabi'n':

1) 6,7893; 2) 5,6409; 3) 0,9871; 4) 0,8245.

19. Sandi' standart tu'rinde jazi'n':

1) 437,105; | 2) 91,352; | 3) 0,000 000 85; | 4) 0,000 079.

I BAP. KVADRAT FUNKCIYA



1- §. KVADRAT FUNKCIYANI'N' ANI'QLAMASI'

Siz VIII klasta $y = kx + b$ si'zi'qli' funkciya ha'm woni'n' grafigi menen tani'sqansi'z.

Ilim ha'm texnikani'n' tu'rli tarawlari'nda *kvadrat funkciyalar* dep atalatug'i'n funkciyalar ushi'raydi'. Mi'sallar keltiremiz.

1) Ta'repi x bolg'an kvadratti'n' maydani' $y = x^2$formulasi' boyi'nsha yesaplanadi'.

2) Yeger dene joqari'g'a v tezlik penen ati'lg'an bolsa, wonda t waqi'tta wonnan Jer betine shekemgi arali'q $s = -\frac{gt^2}{2} + vt + s_0$ formulasi' menen ani'qlanadi', bunda s_0 – waqi'tti'n' $t = 0$ baslang'i'sh waqti'ndag'i' deneden Jer betine shekemgi bolg'an arali'q.

Bul mi'sallarda $y = ax^2 + bx + c$ tu'rindegi funkciyalar qaraldi'. Birinshi mi'salda $a = 1$, $b = c = 0$, wo'zgeriwshileri bolsa x ha'm y ler boladi'. Yekinshi mi'salda $a = -\frac{g}{2}$, $b = v$, $c = s_0$, wo'zgeriwshileri bolsa t ha'm s ha'ripleri menen belgilengen.

! ***Ani'qlama.*** ***$y = ax^2 + bx + c$ funkciyasi' kvadrat funkciya dep ataladi', bunda a, b ha'm c — berilgen haqi'yqi'y sanlar, $a \neq 0$, x – haqi'yqi'y wo'zgeriwshi.***

Mi'sali', to'mendegi funkciyalar kvadrat funkciyalar boladi':

$y = x^2$, $\quad y = -2x^2$, $\quad y = x^2 - x$,

$y = x^2 - 5x + 6$, $\quad y = -3x^2 + \frac{1}{2}x$.

1-ma'sele. $x = -2$, $x = 0$, $x = 3$ bolg'anda, $y(x) = x^2 - 5x + 6$ funkciyani'n' ma'nisin tabi'n':

△ $y(-2) = (-2)^2 - 5 \cdot (-2) + 6 = 20;$
$y(0) = 0^2 - 5 \cdot 0 + 6 = 6;$
$y(3) = 3^2 - 5 \cdot 3 + 6 = 0.$ ▲

2-ma'sele. x ti'n' qanday ma'nislerinde $y = x^2 + 4x - 5$ kvadrat funkciyasi': 1) 7 ge; 2) -9 g'a; 3) -8 ge; 4) 0 ge ten' bolg'an ma'nisti qabi'l yetedi?

△ 1) Sha'rt boyi'nsha $x^2 + 4x - 5 = 7$. Bul ten'lemeni sheship, to'mendegige iye bolami'z:

$$x^2 + 4x - 12 = 0,$$

$$x_{1,2} = -2 \pm \sqrt{4+12} = -2 \pm 4, \quad x_1 = 2, \quad x_2 = -6.$$

Demek, $y(2) = 7$ ha'm $y(-6) = 7$.

2) Sha'rt boyi'nsha $x^2 + 4x - 5 = -9$, bunnan

$$x^2 + 4x + 4 = 0, \quad (x + 2)^2 = 0, \quad x = -2.$$

3) Sha'rt boyi'nsha $x^2 + 4x - 5 = -8$, bunnan $x^2 + 4x + 3 = 0$. Bul ten'lemeni sheship, $x_1 = -3$, $x_2 = -1$ yekenin tabami'z.

4) Sha'rt boyi'nsha $x^2 + 4x - 5 = 0$, bunnan $x_1 = 1$, $x_2 = -5$. ▲

Son'g'i' jag'dayda x ti'n' $y = x^2 + 4x - 5$ funkciyasi' 0 ge ten', yag'ni'y $y(1) = 0$ ha'm $y(-5) = 0$ bolg'an ma'nisleri tabi'ldi'. x ti'n' bunday ma'nisleri *kvadrat funkciyani'n' nolleri* dep ataladi'.

3-ma'sele. $y = x^2 - 3x$ funkciyasi'ni'n' nollerin tabi'n'.

△ $x^2 - 3x = 0$ ten'lemeni sheship, $x_1 = 0$, $x_2 = 3$ yekenligin tabami'z. ▲

Shi'ni'g'i'wlar

1. (Awi'zeki.) To'mendegi berilgen funkciyalardi'n' qaysi'lari' kvadrat funkciya boladi':
   1) $y = 2x^2 + x + 3;$ 2) $y = 3x^2 - 1;$ 3) $y = 5x + 1;$
   4) $y = x^3 + 7x - 1;$ 5) $y = 4x^2;$ 6) $y = -3x^2 + 2x$?
2. x ti'n' sonday haqi'yqi'y ma'nislerin tabi'n', $y = x^2 - x - 3$ kvadrat funkciyasi': 1) -1 ge; 2) -3 ke; 3) $-\frac{13}{4}$ ge; 4) -5 ke ten' bolg'an ma'nisti qabi'l yetsin.

3. x ti'n' qanday haqi'yqi'y ma'nislerinde $y = -4x^2 + 3x - 1$ kvadrat funkciyasi': 1) -2; 2) -8; 3) $-0{,}5$; 4) -1 ge ten' bolg'an ma'nisti qabi'l yetedi.

4. -2; 0; 1; $\sqrt{3}$ sanlari'nan qaysi'lari' to'mendegi kvadrat funkciyani'n' nolleri boladi':

1) $y = x^2 + 2x$; 2) $y = x^2 + x$; 3) $y = x^2 - 3$;
4) $y = 5x^2 - 4x - 1$; 5) $y = x^2 - x$; 6) $y = x^2 + x - 2$?

5. Kvadrat funkciyani'n' nollerin tabi'n':

1) $y = x^2 - x$; 2) $y = x^2 + 3$;
3) $y = 12x^2 - 17x + 6$; 4) $y = -6x^2 + 7x - 2$;
5) $y = 3x^2 - 5x + 8$; 6) $y = 2x^2 - 7x + 9$;
7) $y = 8x^2 + 8x + 2$; 8) $y = \frac{1}{2}x^2 - x + \frac{1}{2}$;
9) $y = 2x^2 + x - 1$; 10) $y = 3x^2 + 5x - 2$.

6. Yeger $y = x^2 + px + q$ kvadrat funkciyasi'ni'n' x_1 ha'm x_2 nolleri belgili bolsa, p ha'm q koefficientlerin tabi'n':

1) $x_1 = 2,\ x_2 = 3$; 2) $x_1 = -4,\ x_2 = 1$;
3) $x_1 = -1,\ x_2 = -2$; 4) $x_1 = 5,\ x_2 = -3$.

7. $y = x^2 + 2x - 3$ ha'm $y = 2x + 1$ funkciyalari' ten'dey ma'nislerdi qabi'l yetetug'i'n x ti'n' ma'nislerin tabi'n'.

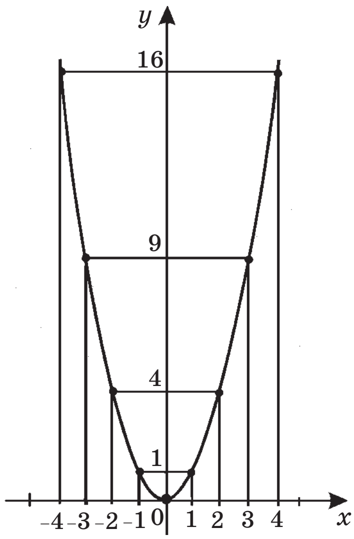
2- §. $y = x^2$ FUNKCIYASI'

$y = x^2$ funkciyasi'n, yag'ni'y $a = 1$, $b = c = 0$ bolg'andag'i' $y = ax^2 + bx + c$ kvadrat funkciyasi'n qaraymi'z. Bul funkciyani'n' grafigin jasaw ushi'n woni'n' ma'nislerinin' kestesin du'zemiz:

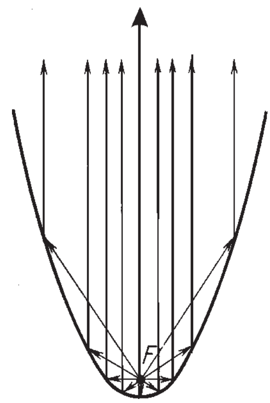
x	-4	-3	-2	-1	0	1	2	3	4
$y = x^2$	16	9	4	1	0	1	4	9	16

Kestede ko'rsetilgen noqatlardi' jasap ha'm wolardi' bir tegis iymek si'zi'q penen tutasti'ri'p, $y = x^2$ funkciyasi'ni'n' grafigin payda yetemiz (1-su'wret).

! **$y = x^2$ funkciyasi'ni'n' grafigi bolg'an iymek si'zi'q *parabola* dep ataladi'.**



1- su'wret.



2- su'wret.

$y = x^2$ *funkciyasi'ni'n' qa'siyetlerin* qaraymi'z.

1) $y = x^2$ funkciyasi'ni'n' ma'nisi $x \neq 0$ bolg'anda *won'* ha'm $x \neq 0$ bolg'anda *nolge* ten'. Demek, $y = x^2$ parabolasi' koordinatalar basi'nan wo'tedi, parabolani'n' qalg'an noqatlari' bolsa abcissalar ko'sherinen joqari'da jatadi'. $y = x^2$ *parabola abcissalar* ko'sherine (0; 0) ***noqati'nda uri'nadi'*** delinedi.

2) $y = x^2$ funkciyasi'ni'n' grafigi *ordinatalar ko'sherine sali'sti'r-g'anda simmetriyali',* sebebi $(-x)^2 = x^2$. Mi'sali', $y(-3) = y(3) = 9$ (1-su'wret). Solay yetip, ordinatalar ko'sheri *parabolani'n' sim-metriya ko'sheri* boladi'. Parabolani'n' wo'z simmetriya ko'sheri menen kesilisiw noqati' *parabolani'n' to'besi* delinedi. $y = x^2$ parabolasi' ushi'n koordinatalar basi' woni'n' to'besi boladi'.

3) $x \geq 0$ bolg'anda x ti'n' u'lken ma'nisine y ti'n' u'lken ma'nisi sa'ykes keledi. Mi'sali', $y(3) > y(2)$. $y = x^2$ funkciyasi' $x \geq 0$ *arali'g'i'n-da wo'siwshi* delinedi (1-su'wret).

$x \leq 0$ bolg'anda x ti'n' u'lken ma'nisine y ti'n' kishi ma'nisi sa'ykes keledi. Mi'sali', $y(-2) < y(-4)$. $y = x^2$ funkciyasi' $x \leq 0$ *arali'g'i'nda kemeyiwshi* delinedi (1-su'wret).

Ma'sele. $y = x^2$ parabolasi' menen $y = x + 6$ tuwri' si'zi'g'i'ni'n' kesilisiw noqatlari'ni'n' koordinatalari'n tabi'n'.

△ Kesilisiw noqatlari' $\begin{cases} y = x^2, \\ y = x + 6 \end{cases}$ sistemasi'ni'n' sheshimleri boladi'.

Bul sistemadan $x^2 = x + 6$, yag'ni'y $x^2 - x - 6 = 0$ di payda yetemiz, bunnan $x_1 = 3$, $x_2 = -2$. x_1 ha'm x_2 ni'n' ma'nislerin sistemani'n' ten'lemelerinin' birine qoyi'p, $y_1 = 9$, $y_2 = 4$ ti tabami'z.

Juwabi': (3; 9), (−2; 4). ▲

Parabola texnikada ken' ko'lemde paydalanatug'i'n ko'plegen qa'siyetlerge iye. Mi'sali', parabolani'n' simmetriya ko'sherinde *parabolani'n' fokusi'* dep atalatug'i'n F noqati' bar (2-su'wret). Yeger bul noqatta jaqti'li'q deregi jaylasqan bolsa, wonda paraboladan sa'wlelengen barli'q jaqti'li'q nurlari' parallel boladi'. Bul qa'siyetten projektor, lokator ha'm basqa a'sbaplar tayarlawda paydalani'ladi'.

$y = x^2$ parabolasi'ni'n' fokusi' $\left(0; \frac{1}{4}\right)$ noqati' boladi'.

Shi'ni'g'i'wlar

8. $y = x^2$ funkciyasi'ni'n' grafigin millimetrli qag'azda jasan'. Grafik boyi'nsha:

1) $x = 0{,}8$; $x = 1{,}5$; $x = 1{,}9$; $x = -2{,}3$; $x = -1{,}5$ bolg'anda y ti'n' ma'nisin juwi'q tabi'n';

2) yeger $y = 2$; $y = 3$; $y = 4{,}5$; $y = 6{,}5$ bolsa, x ti'n' ma'nisin juwi'q tabi'n'.

9. $y = x^2$ funkciyasi'ni'n' grafigin jasamastan: $A(2; 6)$, $B(-1; 1)$, $C(12; 144)$, $D(-3; -9)$ noqatlari'nan qaysi'lari' parabolag'a tiyisli yekenligin ani'qlan'.

10. (Awi'zeki.) Ordinata ko'sherine sali'sti'rg'anda $A(3; 9)$, $B(-5; 25)$, $C(4; 15)$, $D(\sqrt{3}; 3)$ noqatlari'na simmetriyali' bolg'an noqat-lardi' tabi'n'. Bul noqatlar $y = x^2$ funkciyasi'ni'n' grafigine tiyisli bolama?

11. (Awi'zeki.) $y=x^2$ funkciyasi'ni'n' ma'nislerin,

1) $x = 2{,}5$ ha'm $x = 3\frac{1}{3}$; 2) $x = 0{,}4$ ha'm $x = 0{,}3$;

3) $x = -0{,}2$ ha'm $x = -0{,}1$; 4) $x = 4{,}1$ ha'm $x = -5{,}2$

bolg'anda sali'sti'ri'n'.

12. $y = x^2$ parabolani'n':

1) $y = 25$; 2) $y = 5$; 3) $y = -x$;
4) $y = 2x$; 5) $y = 3 - 2x$; 6) $y = 2x - 1$

tuwri' si'zi'q penen kesilisiw noqati'ni'n' koordinatasi'n tabi'n'.

13. A noqati' $y = x^2$ parabola menen

1) $y = -x - 6$, $A(-3; 9)$; 2) $y = 5x - 6$, $A(2; 4)$

tuwri' si'zi'qti'n' kesilisiw noqati' bola ma?

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14. Tasti'yi'qlaw duri's pa: $y = x^2$ funkciyasi':

1) $[1; 4]$ kesindisinde; 2) $(2; 5)$ intervalda;
3) $x > 3$ intervalda; 4) $[-3; 4]$ kesindisinde wo'sedi.

15. Bir koordinata tegisliginde $y = x^2$ parabola menen $y = 3$ tuwri' si'zi'q si'zi'n'. x ti'n' qanday ma'nislerinde parabolani'n' noqatlari' tuwri' si'zi'qtan joqari'da boladi'; to'mende boladi'?

16. x ti'n' qanday ma'nislerinde $y = x^2$ funkciyasi'ni'n' ma'nisi: 1) 9 dan u'lken; 2) 25 ten u'lken yemes; 3) 16 dan kishi yemes; 4) 36 dan kishi boladi'?

3- §. $y = ax^2$ FUNKCIYASI'

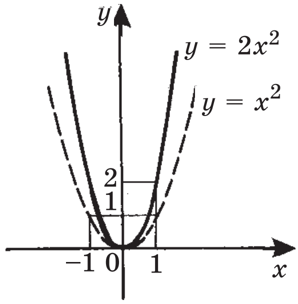
1-ma'sele. $y = 2x^2$ funkciyasi'ni'n' grafigin si'zi'n'.

△ $y = 2x^2$ funkciyasi'ni'n' ma'nislerinin' kestesin du'zemiz:

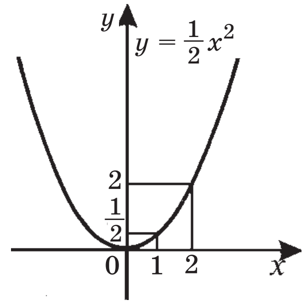
x	–3	–2	–1	0	1	2	3
$y = 2x^2$	18	8	2	0	2	8	18

Tabi'lg'an noqatlardi' jasaymi'z ha'm wolar arqali' bir tegis iymek si'zi'q ju'rgizemiz (3-su'wret). ▲

$y = 2x^2$ ha'm $y = x^2$ funkciyalari'ni'n' grafiklerin sali'sti'rami'z (3-su'wret). x ti'n' ani'q bir ma'nisinde $y = 2x^2$ funkciyasi'ni'n' ma'nisi $y = x^2$ funkciyasi'ni'n' ma'nisinen 2 ma'rte arti'q. Bul $y = 2x^2$ funkciyasi' grafiginin' ha'rbir noqati'n $y = x^2$ funkciyasi' grafiginin' da'l sonday abcissali' noqati'ni'n' ordinatasi'n 2 ma'rte artti'ri'w menen payda yetiw mu'mkin yekenligin bildiredi.



3- su'wret.



4- su'wret.

$y = 2x^2$ funkciyasi'ni'n' grafigi $y = x^2$ funkciyasi' grafigin Ox ko'sherinen Oy ko'sheri boyi'nsha 2 ma'rte *sozi'w* menen payda boladi', delinedi.

2-ma'sele. $y = \frac{1}{2}x^2$ funkciyasi'ni'n' grafigin jasan'.

△ $y = \frac{1}{2}x^2$ funkciyasi'ni'n' ma'nislerinin' kestesin du'zemiz:

x	–3	–2	–1	0	1	2	3
$y = \frac{1}{2}x^2$	4,5	2	0,5	0	0,5	2	4,5

Tabi'lg'an noqatlardi' jasap, wolar arqali' bir tegis iymek si'zi'q ju'rgizemiz (4-su'wret).▲

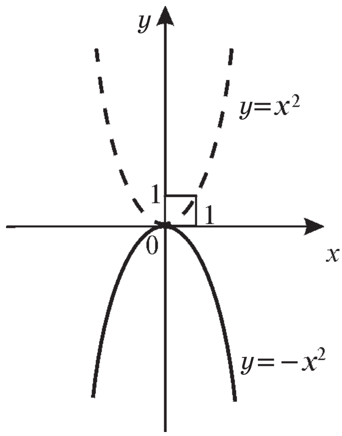
$y = \frac{1}{2}x^2$ ha'm $y = x^2$ funkciyalari'ni'n' grafiklerin sali'sti'rami'z.

$y = \frac{1}{2}x^2$ funkciyasi' grafiginin' ha'rbir noqati'n $y = x^2$ funkciyasi' grafiginin' da'l sonday abcissali' noqati'ni'n' ordinatasi'n 2 ma'rte kemeytiw arqali' payda yetiw mu'mkin.

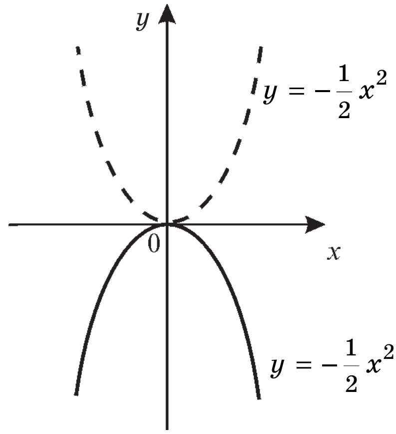
$y = \frac{1}{2}x^2$ funkciyasi'ni'n' grafigi $y=x^2$ funkciyasi' grafigin Ox ko'sherin Oy ko'sheri boyi'nsha 2 ma'rte *qi'si'w* joli' menen payda yetiledi, delinedi.

3-ma'sele. $y = -x^2$ funkciyasi'ni'n' grafigin jasan'.

△ $y = -x^2$ ha'm $y = x^2$ funkciyalari'ni'n' grafiklerin sali'sti'rami'z. x ti'n' sonday bir ma'nisinde bul funkciyalardi'n'



5- *su'wret.*



6- *su'wret.*

ma'nislerinin' modulleri boyi'nsha ten' ha'm qarama-qarsi' belgige iye. Demek, $y = -x^2$ funkciyasi'ni'n' grafigin $y = x^2$ funkciyasi' grafigi Ox ko'sherine sali'sti'rg'anda simmetriyali' ko'rinisinde payda yetiw mu'mkin (5-su'wret).

Usi'g'an uqsas, $y = -\frac{1}{2}x^2$ funkciyasi'ni'n' grafigi Ox ko'sherine sali'sti'rg'anda $y = \frac{1}{2}x^2$ funkciyasi' grafigine simmetriyali' (6-su'wret).

$y = ax^2$ funkciyasi'ni'n' grafigi qa'legen $a \neq 0$ de de *parabola* dep ataladi'. $a > 0$ de parabolani'n' shaqalari' *joqari'g'a,* al $a < 0$ de bolsa *to'menge* bag'darlang'an boladi'.

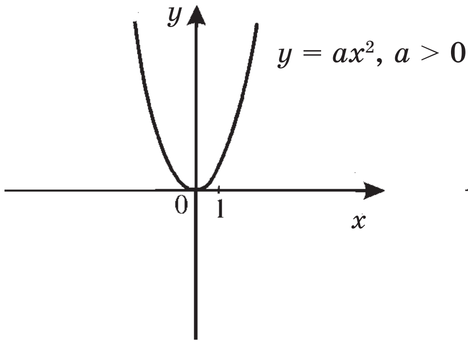
$y = ax^2$ parabolani'n' fokusi' $\left(0;\ \frac{1}{4a}\right)$ noqati'nda jaylasqani'n an'laymi'z.

$y = ax^2$ funkciyani'n' *tiykarg'i'* qa'siyetlerin atap wo'temiz, bunda $a \neq 0$:

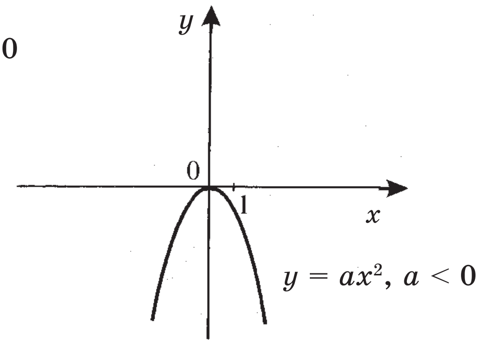
1) yeger $a > 0$ bolsa, wonda $y = ax^2$ funkciyasi' $x \neq 0$ bolg'anda won' ma'nislerdi qabi'l yetedi;

yeger $a < 0$ bolsa, wonda $y = ax^2$ funkciyasi' $x \neq 0$ bolg'anda teris ma'nislerdi qabi'l yetedi;

$y = ax^2$ funkciyasi'ni'n' ma'nisi tek $x = 0$ bolg'anda g'ana 0 ge ten' boladi'.



7- su'wret.



8- su'wret.

2) $y = ax^2$ parabolasi' ordinatalar ko'sherine sali'sti'rg'anda simmetriyali' boladi'.

3) yeger $a > 0$ bolsa, wonda $y = ax^2$ funkciyasi' $x \geq 0$ bolg'anda wo'siwshi ha'm $x \leq 0$ bolg'anda kemeyiwshi;

yeger $a < 0$ bolsa, wonda $y = ax^2$ funkciyasi' $x \geq 0$ bolg'anda kemeyiwshi ha'm $x \leq 0$ bolg'anda wo'siwshi boladi'.

Bul barli'q qa'siyetler grafikten ani'q ko'rinip tur (7-, 8-su'wretler).

Shi'ni'g'i'wlar

17. Millimetrli qag'azg'a $y = 3x^2$ funkciyasi'ni'n' grafigin jasan'. Grafik boyi'nsha:
1) $x = -2{,}8;\ -1{,}2;\ 1{,}5;\ 2{,}5$ bolg'anda y ti'n' ma'nisin tabi'n'.
2) yeger $y = 9;\ 6;\ 2;\ 8;\ 1{,}3$ bolsa, x ti'n' ma'nisin juwi'q yesaplan'.

18. (Awi'zeki.) Parabola tarmaqlari'ni'n' bag'dari'n ani'qlan':
1) $y = 3x^2$; 2) $y = \frac{1}{3}x^2$; 3) $y = -4x^2$; 4) $y = -\frac{1}{3}x^2$.

19. To'mendegi funkciyalardi'n' grafiklerin bir koordinata tegisliginde jasan':
1) $y = x^2$ ha'm $y = 3x^2$; 2) $y = -x^2$ ha'm $y = -3x^2$;
3) $y = 3x^2$ ha'm $y = -3x^2$; 4) $y = \frac{1}{3}x^2$ ha'm $y = -\frac{1}{3}x^2$.

Grafiklerden paydalani'p, bul funkciyalardan qaysi'lari' $x \geq 0$ arali'g'i'nda wo'siwshi yekenligin ani'qlan'.

20. To'mendegi funkciyalardi'n' grafiklerinin' kesilisiw noqatlari'ni'n' koordinatalari'n tabi'n':

1) $y = 2x^2$ ha'm $y = 3x + 2$; 2) $y = -\frac{1}{2}x^2$ ha'm $y = \frac{1}{2}x - 3$.

21. Funkciya $x \le 0$ arali'g'i'nda kemeyiwshi funkciya bola ma:

l) $y = 4x^2$; | 2) $y = \frac{1}{4}x^2$; | 3) $y = -5x^2$; | 4) $y = -\frac{1}{5}x^2$?

22. $y = -2x^2$ funkciyasi':

1) $[-4; -2]$ kesindisinde; 3) $(3; 5)$ intervali'nda;
2) $[-5; 0]$ kesindisinde; 4) $(-3; 2)$ intervali'nda

wo'siwshi yaki kemeyiwshi bolatug'i'ni'n ani'qlan'.

23. Denenin' turaqli' tezleniwshi qozg'ali'sta basi'p wo'tken joli' $s = \frac{at^2}{2}$ formula menen yesaplanadi', bunda s — jol, metrde; a – tezleniw, m/s^2 ta; t – waqi't, sekunda wo'lshenedi. Yeger dene 8 s ta 96 m joldi' basi'p wo'tken bolsa, a tezleniwin tabi'n'.

4-§. $y = ax^2 + bx + c$ FUNKCIYASI'

1-ma'sele. $y=x^2 - 2x + 3$ funkciyasi'ni'n' grafigin jasan' ha'm woni' $y = x^2$ funkciyasi' grafigi menen sali'sti'ri'n'.

△ $y = x^2 - 2x + 3$ ti'n' ma'nislerinin' kestesin du'zemiz:

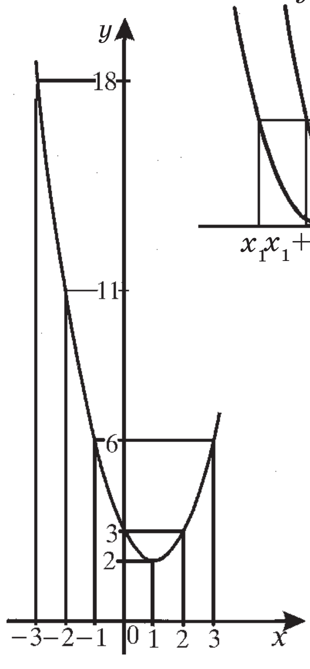
x	–3	–2	0	0	1	2	3
$y = x^2 - 2x + 3$	18	11	6	3	2	3	6

Tabi'lg'an noqatlardi' jasaymi'z ha'm wolar arqali' bir tegis iymek si'zi'q ju'rgizemiz (9-su'wret).

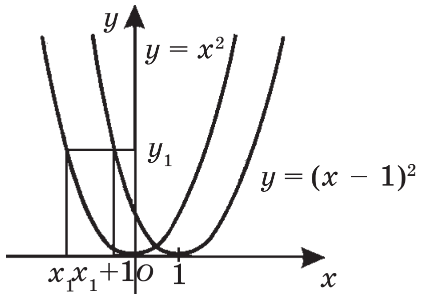
Grafiklerdi sali'sti'ri'w ushi'n toli'q kvadratti' jiklew usi'li'nan paydalani'p, $y = x^2 - 2x + 3$ formulasi'n tu'rlendiremiz:

$$y = x^2 - 2x + 1 + 2 = (x - 1)^2 + 2.$$

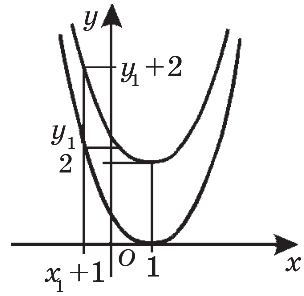
Da'slep $y = x^2$ ha'm $y = (x - 1)^2$ funkciyasi'ni'n' grafiklerin sali'sti'rami'z. Yeger $(x_1; y_1)$ noqati' $y = x^2$ parabolasi'ni'n' noqati', yag'ni'y $y_1 = x_1^2$ bolsa, wonda $(x_1 + 1; y_1)$ noqati' $y = (x—1)^2$



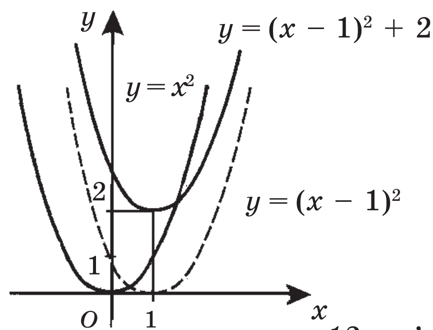
9- su'wret.



10- su'wret.



11- su'wret.



12- su'wret.

funkciyasi'ni'n' grafigine tiyisli, sebebi, $((x_1 + 1) - 1)^2 = x_1^2 = y_1$. Demek, $y = (x - 1)^2$ funkciyasi'ni'n' grafigi $y = x^2$ paraboladan woni' bir birlik won'g'a *ko'shiriw* arqali' payda yetilgen parabola (10-su'wret).

Yendi $y = (x - 1)^2$ ha'm $y = (x - 1)^2 + 2$ funkciyasi'ni'n' grafiklerin sali'sti'rami'z. x ti'n' ha'rbir ma'nisinde $y = (x - 1)^2 + 2$ funkciyasi'ni'n' ma'nisi $y = (x - 1)^2$ funkciyasi'ni'n' ma'nisinen 2 ge arti'q. Demek, $y = (x - 1)^2 + 2$ funkciyasi'ni'n' grafigi $y = (x - 1)^2$ parabolani' yeki birlik joqari'g'a ko'shiriw arqali' payda yetilgen parabola boladi' (11-su'wret).

Solay yetip, $y = x^2 - 2x + 3$ funkciyasi'ni'n' grafigi $y = x^2$ parabolani' bir birlik won'g'a ha'm yeki birlik joqari'g'a ko'shiriw arqali' payda yetilgen parabola (12-su'wret). $y = x^2 - 2x + 3$ parabolani'n' simmetriya ko'sheri ordinatalar ko'sherine parallel ha'm parabolani'n' to'besi bolg'an (1; 2) noqati'nan wo'tiwshi tuwri' si'zi'q boladi'. ▲

$y = a(x - x_0)^2 + y_0$ funkciyasi'ni'n' grafigi $y = ax^2$ parabolani':

yeger $x_0 > 0$ bolsa, abcissalar ko'sheri boyi'nsha won'g'a qaray x_0 ge, yeger $x_0 < 0$ bolsa, shepke $|x_0|$ g'a ko'shiriw;

yeger $y_0 > 0$ bolsa, ordinatalar ko'sheri boyi'nsha joqari'g'a y_0 g'a, yeger $y_0 < 0$ bolsa, to'menge $|y_0|$ g'a ko'shiriw joli' menen payda yetiletug'i'n parabola boli'wi' usi'g'an uqsas da'lillenedi.

Qa'legen $y = ax^2 + bx + c$ *kvadrat funkciyani'* wonnan toli'q kvadratti' jiklew formulasi' ja'rdeminde

$$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a},$$

yag'ni'y, $y = a(x - x_0)^2 + y_0$ *tu'rinde jazi'w mu'mkin*, bunda

$$x_0 = -\frac{b}{2a},\ y_0 = y(x_0) = \frac{-(b^2 - 4ac)}{4a}.$$

Solay yetip, $y = ax^2 + bx + c$ funkciyasi'ni'n' grafigi $y = ax^2$ parabolani' koordinatalar ko'sherleri boylap ko'shiriw na'tiyjesinde payda bolatug'i'n parabola boladi'. $y = ax^2 + bx + c$ ten'ligi parabolani'n' ten'lemesi delinedi. $y = ax^2 + bx + c$ parabola to'besinin' $(x_0; y_0)$ koordinatalari'n to'mendegi formula boyi'nsha tabi'w mu'mkin:

$$x_0 = -\frac{b}{2a},\ y_0 = y(x_0) = ax_0^2 + bx_0 + c.$$

$y = ax^2 + bx + c$ parabolasi'ni'n' simmetriya ko'sheri ordinatalar ko'sherine parallel ha'm parabolani'n' to'besinen wo'tiwshi tuwri' si'zi'q boladi'.

$y = ax^2 + bx + c$ parabolani'n' shaqalari', yeger $a > 0$ bolsa, joqari'g'a bag'darlang'an, yeger $a < 0$ bolsa, to'menge bag'darlang'an boladi'.

2-ma'sele. $y = 2x^2 - x - 3$ parabola to'besinin' koordinatalari'n tabi'n'.

△ Parabola to'besinin' abcissasi': $x_0 = -\frac{b}{2a} = \frac{1}{4}$.

Parabola to'besinin' ordinatasi':

$$y_0 = ax_0^2 + bx_0 + c = 2 \cdot \frac{1}{16} - \frac{1}{4} - 3 = -3\frac{1}{8}.$$

Juwabi': $\left(\frac{1}{4};\ -3\frac{1}{8}\right)$. ▲

3-ma'sele. Yeger parabolani'n' (−2; 5) noqati' arqali' wo'tetug'i'n ha'm to'besi (−1; 2) noqati'nda bolatug'i'ni' belgili bolsa, parabolani'n' ten'lemesin jazi'n'.

△ Parabolani'n' to'besi (−1; 2) noqati'nda bolg'anli'qtan ushi'n parabolani'n' ten'lemesin to'mendegishe jazi'w mu'mkin.

$$y = a(x + 1)^2 + 2.$$

Sha'rt boyi'nsha (−2; 5) noqati' parabolag'a tiyisli, demek,

$$5 = a(-2 + 1)^2 + 2,$$

bunnan $a = 3$.

Solay yetip, parabola

$$y = 3(x + 1)^2 + 2 \quad \text{yamasa} \quad y = 3x^2 + 6x + 5$$

ten'lemesi menen beriledi. ▲

Shi'ni'g'i'wlar

Parabola to'besinin' koordinatalari'n tabi'n' **(24–26)**:

24. (Awi'zeki.)
1) $y = (x - 3)^2 - 2$; 2) $y = (x + 4)^2 + 3$;
3) $y = 5(x + 2)^2 - 7$; 4) $y = -4(x - 1)^2 + 5$.

25. 1) $y = x^2 + 4x + 1$; 2) $y = x^2 - 6x - 7$;
3) $y = 2x^2 - 6x + 11$; 4) $y = -3x^2 + 18x - 7$.

26. 1) $y = x^2 + 2$; 2) $y = -x^2 - 5$; 3) $y = 3x^2 + 2x$;
4) $y = -4x^2 + x$; 5) $y = -3x^2 + x$. 6) $y = 2x^2 - x$.

27. Ox ko'sherinen sonday noqatti' tabi'n', wonnan parabolani'n' simmetriya ko'sheri wo'tetug'i'n bolsi'n:
1) $y = x^2 + 3$; 2) $y = (x + 2)^2$;
3) $y = -3(x + 2)^2 + 2$; 4) $y = (x - 2)^2 + 2$;
5) $y = x^2 + x + 1$; 6) $y = 2x^2 - 3x + 5$.

28. $y = x^2 - 10x$ parabolani'n' simmetriya ko'sheri: 1) (5; 10); 2) (3; –8); 3) (5; 0); 4) (–5; 1) noqati'nan wo'teme?

29. Parabolani'n' koordinatalar ko'sherleri menen kesilisiw noqatlari'ni'n' koordinatalari'n tabi'n':
1) $y = x^2 - 3x + 2$; 2) $y = -2x^2 + 3x - 1$;
3) $y = 3x^2 - 7x + 12$; 4) $y = 3x^2 - 4x$.

30. Yeger parabolani'n' $(-1;\ 6)$ noqati' arqali' wo'tetug'i'nli'g'i' ha'm woni'n' to'besi $(1;\ 2)$ noqati' yekenligi belgili bolsa, parabolani'n' ten'lemesin jazi'n'.

31. (Awi'zeki.) $(1;\ -6)$ noqati' $y = -3x^2 + 4x - 7$ parabolag'a tiyisli bola ma? $(-1;8)$ noqati' she?

32. Yeger $(-1;\ 2)$ noqati':1) $y = kx^2 + 3x - 4$; 2) $y = -2x^2 + kx - 6$ parabolasi'na tiyisli bolsa, k ni'n' ma'nisin tabi'n'.

33. $y = x^2$ parabola wo'lshemi ja'rdeminde funkciyani'n' grafigin jasan':

1) $y = (x + 2)^2$; | 2) $y = (x - 3)^2$; | 3) $y = x^2 - 2$;
4) $y = -x^2 + 1$; | 5) $y = -(x - 1)^2 - 3$; | 6) $y = (x + 2)^2 + 1$.

34. $y = 2x^2$ parabolasi'nan woni':

1) Ox ko'sheri boyi'nsha 3 birlik won' ta'repke ko'shiriw;
2) Oy ko'sheri boyi'nsha 4 birlik joqari'g'a ko'shiriw;
3) Ox ko'sheri boyi'nsha 2 birlik shep ta'repke ha'm keyin Oy ko'sheri boyi'nsha bir birlik to'menge ko'shiriw;
4) Ox ko'sheri boyi'nsha 1,5 birlik won' ta'repke ha'm keyin Oy ko'sheri boyi'nsha 3,5 birlik joqari'g'a ko'shiriw na'tiyjesinde payda bolg'an parabolani'n' ten'lemesin jazi'n'.

5- §. KVADRAT FUNKCIYANI'N' GRAFIGIN JASAW

1-ma'sele. $y = x^2 - 4x + 3$ funkciyasi'ni'n' grafigin jasan'.

△ 1. Parabola to'besinin' koordinatalari'n yesaplaymi'z:

$$x_0 = -\frac{-4}{2} = 2,$$

$$y_0 = 2^2 - 4 \cdot 2 + 3 = -1.$$

$(2;\ -1)$ noqati'n jasaymi'z.

2. $(2;\ -1)$ noqatlari' arqali' ordinatalar ko'sherine parallel tuwri' si'zi'q, yag'ni'y parabolani'n' simmetriya ko'sherin ju'rgizemiz (13-*a*, su'wret).

3. Mi'na, $x^2 - 4x + 3 = 0$ ten'lemeni sheship, funkciyasi'ni'n' nollerin tabami'z: $x_1 = 1$, $x_2 = 3$. $(1;\ 0)$ ha'm $(3;\ 0)$ noqatlari'n jasaymi'z (13-*b*, su'wret).

4. Ox ko'sherinde $x = 2$ noqati'na sali'sti'rg'anda simmetriyali' bolg'an yeki noqatti', mi'sali', $x = 0$ ha'm $x = 4$ noqatlari'n alami'z. Funkciyani'n' usi' noqatlardag'i' ma'nislerin yesaplaymi'z: $y(0) = y(4) = 3$. (0; 3) ha'm (4; 3) noqatlari'n jasaymi'z (13-*b*, su'wret).

5. Jasalg'an noqatlar arqali' parabolani' ju'rgizemiz (13-*d*, su'wret).▲

Bunday sxema boyi'nsha qa'legen $y = ax^2 + bx + c$ *kvadrat funkciyani'n' grafigin jasaw mu'mkin:*

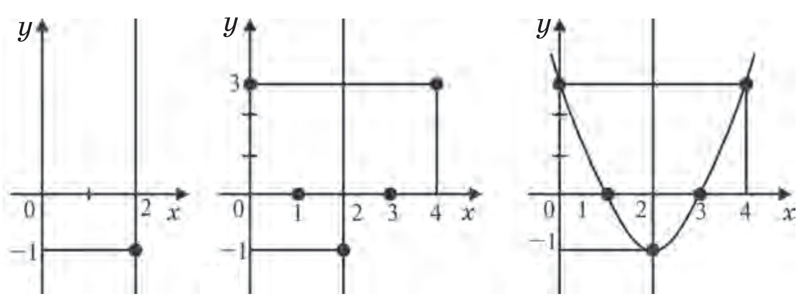
1. x_0, y_0 lerdi $x_0 = -\frac{b}{2a}$, $y_0 = y(x_0)$ formulalari'nan paydalani'p yesaplap, parabolani'n' $(x_0; y_0)$ to'besi jasaladi'.

2. Parabolani'n' to'besinen ordinatalar ko'sherine parallel tuwri' si'zi'q — parabolani'n' simmetriya ko'sheri ju'rgiziledi.

3. Funkciyani'n' nolleri (wolar bar bolsa) tabi'ladi' ha'm abcissalar ko'sherinde parabolani'n' sa'ykes noqatlari' jasaladi'.

4. Parabolani'n' woni'n' ko'sherine sali'sti'rg'anda simmetriyali' bolg'an qanday da yeki noqati' jasaladi'. Buni'n' ushi'n Ox ko'sherinde x_0 $(x_0 \neq 0)$ noqati'na sali'sti'rmali' simmetriyali' bolg'an yeki noqatti' ali'p ha'm funkciyani'n' sa'ykes ma'nislerin (ma'nisler birdey) yesaplaw kerek. Mi'sali', parabolani'n' abcissalari' $x = 0$ ha'm $x = 2x_0$ bolg'an noqatlari'n (noqatlardi'n' ordinatalari' c g'a ten') jasaw mu'mkin.

5. Jasalg'an noqatlar arqali' parabola ju'rgiziledi. Grafikti ja'ne de ani'g'i'raq jasaw ushi'n parabolani'n' tag'i'da birneshe noqati'n tabi'w paydali'.



a) *b)* *d)* *13- su'wret.*

2-ma’sele. $y = -2x^2 + 12x - 19$ funkciyani’n’ grafigin jasan’.

△ 1. Parabola to’besinin’ koordinatalari’n yesaplaymi’z:

$$x_0 = -\frac{12}{-4} = 3,\ y_0 = -2 \cdot 3^2 + 12 \cdot 3 - 19 = -1.$$

(3; −1) noqati’n – parabolani’n’ to’besin jasaymi’z (14-su’wret).

2. (3; −1) noqati’ arqali’ parabolani’n’ simmetriyali’q ko’sherin wo’tkeremiz (14-su’wret).

3. $-2x^2 + 12x - 19 = 0$ ten’lemesin sheship haqi’yqi’y korenler joqli’g’i’na ha’m soni’n’ ushi’n parabola Ox ko’sherin kespewine isenim payda yetemiz.

4. Ox ko’sherinde $x = 3$ noqatqa sali’sti’rg’anda simmetriyali’q bolg’an yeki noqatti’, ma’selen $x = 2$ ha’m $x = 4$ noqatlari’n alami’z. Funkciyani’n’ bul noqatlardag’i’ ma’nislerin yesaplaymi’z:

$$y(2) = y(4) = -3.$$

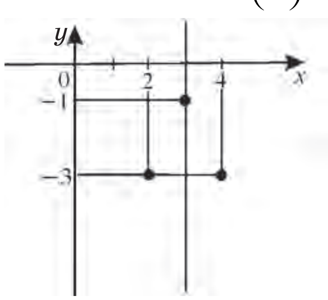
(2; −3) ha’m (4; −3) noqatlari’n jasaymi’z (14-su’wret).

5. Jasalg’an noqatlar arqali’ parabola ju’rgizemiz (15-su’wret).▲

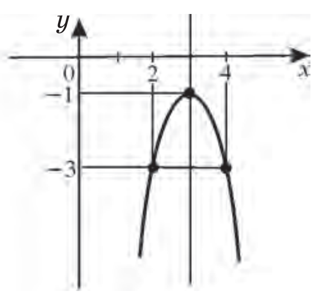
3-ma’sele. $y = -x^2 + x + 6$ funkciyasi’ni’n’ grafigin jasan’ ha’m bul funkciya qanday qa’siyetke iye yekenligin ani’qlan’.

△ Funkciyani’n’ grafigin jasaw ushi’n woni’n’ nollerin tabami’z: $-x^2 + x + 6 = 0$, bunnan $x_1 = -2$, $x_1 = 3$. Parabola to’besinin’ koordinatalari’n to’mendegishe tabi’wg’a boladi’:

$$x_0 = \frac{x_1 + x_2}{2} = \frac{-2+3}{2} = \frac{1}{2},$$
$$y_0 = y\left(\frac{1}{2}\right) = -\frac{1}{4} + \frac{1}{2} + 6 = 6\frac{1}{4}.$$



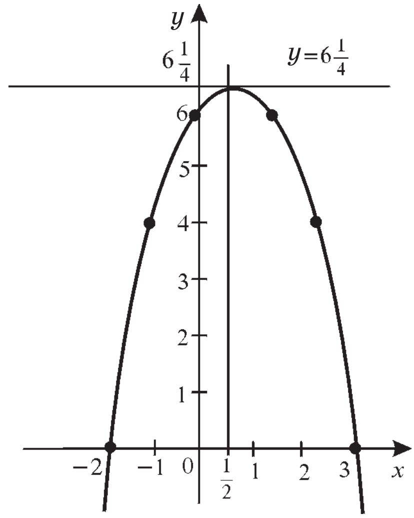
14- su’wret.



15- su’wret.

$a = -1 < 0$ bolg'ani' ushi'n parabolani'n' tarmaqlari' to'menge bag'dar–lang'an.

Parabolani'n' ja'ne birneshe noqati'n tabami'z: $y(-1) = 4$, $y(0) = 6$, $y(1) = 6$, $y(2) = 4$. Parabolani' jasaymi'z (16-su'wret).



16-su'wret.

Grafik ja'rdeminde $y = -x^2 + x + 6$ *funkciyasi'ni'n' to'mendegi qa'siyetlerin* payda yetemiz:

1) x ti'n' qa'legen ma'nislerinde funkciyani'n' ma'nisleri $6\frac{1}{4}$ ge ten' yaki wonnan kishi;

2) $-2 < x < 3$ te funkciyani'n' ma'nis-leri won', $x < -2$ de ha'm $x > 3$ te teris, $x = -2$ ha'm $x = 3$ te nolge ten';

3) funkciya $x \leq \frac{1}{2}$ arali'qta wo'siwshi, $x \geq \frac{1}{2}$ arali'qta kemeyiwshi;

4) $x = \frac{1}{2}$ bolg'anda funkciya $6\frac{1}{4}$ ge ten' bolg'an yen' u'lken ma'nisti qabi'l yetedi;

5) funciyani'n' grafigi $x = \frac{1}{2}$ tuwri' si'zi'qqa sali'sti'rg'anda simmetriyali'.

$y = ax^2 + bx + c$ funkciya $x_0 = -\frac{b}{2a}$ noqati'nda *yen' kishi* yamasa *yen' u'lken ma'nislerdi* qabi'l yetedi; bul x_0 noqati' parabola to'besinin' abcissasi' boladi'.

Funkciyani'n' x_0 noqati'ndag'i' ma'nisin $y_0 = y(x_0)$ formulasi' boyi'nsha tabi'w mu'mkin. Yeger $a > 0$ *bolsa, wonda funkciya yen' kishi ma'niske iye boladi', yeger* $a < 0$ *bolsa, wonda funkciya yen' u'lken ma'niske iye boladi'.*

Mi'sali', $y = x^2 - 4x + 3$ funkciyasi' $x = 2$ bolg'anda -1 ge ten' bolg'an yen' kishi ma'nisin qabi'l yetedi (13-*d*, su'wret); $y = -2x^2 + 12x - 9$ funkciyasi' $x = 3$ bolg'anda -1 ge ten' bolg'an yen' u'lken ma'nisin qabi'l yetedi (15-su'wret).

4-ma’sele. Yeki won’ sanni’n’ qosi’ndi’si’ 6 g’a ten’. Yeger wolardi’n’ kvadratlari’ni’n’ qosi’ndi’si’ yen’ kishi san bolsa, usi’ sanlardi’ tabi’n’. Bul sanlardi’n’ kvadratlari’ni’n’ qosi’ndi’si’ni’n’ yen’ kishi ma’nisi qanday boladi’?

△ Birinshi sandi’ x ha’ribi menen belgileymiz, wonda yekinshi san $6 - x$, wolardi’n’ kvadratlari’ni’n’ qosi’ndi’si’ bolsa $x^2 + (6 - x)^2$ boladi’. Bul an’latpani’n’ jazi’li’wi’n tu’rlendiremiz:

$$x^2 + (6 - x)^2 = x^2 + 36 - 12x + x^2 = 2x^2 - 12x + 36.$$

Mi’sali’, $y = 2x^2 - 12x + 36$ funkciyasi’ni’n’ yen’ kishi ma’nisin tabi’wg’a keltiriledi. Parabola to’besinin’ koordinatasi’n tabami’z:

$$x_0 = -\frac{b}{2a} = -\frac{-12}{2 \cdot 2} = 3, \qquad y_0 = y(3) = 2 \cdot 9 - 12 \cdot 3 + 36 = 18.$$

Demek, $x = 3$ bolg’anda funkciya 18 ge ten’ yen’ kishi ma’nisti qabi’l yetedi. Solay yetip, birinshi san 3 ke ten’, yekinshi san da $6 - 3 = 3$ ke ten’. Bul sanlardi’n’ kvadratlari’ni’n’ qosi’ndi’si’ni’n’ ma’nisi 18 ge ten’. ▲

Shi’ni’g’i’wlar

35. Parabola to’besinin’ koordinatalari’n tabi’n’:

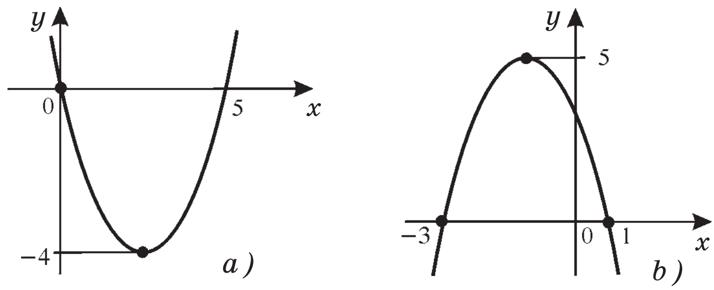
1) $y = x^2 - 4x - 5$; 2) $y = x^2 + 3x + 5$;
3) $y = -x^2 - 2x + 5$; 4) $y = -x^2 + 5x - 1$.

36. Parabolani’n’ koordinata ko’sherleri menen kesilisiw noqatlari’ni’n’ koordinatalari’n tabi’n’:

1) $y = x^2 - 3x + 5$; 2) $y = -2x^2 - 8x + 10$;
3) $y = -2x^2 + 6$; 4) $y = 7x^2 + 14$.

Funkciyani’n’ grafigin jasan’ ha’m grafik boyi’nsha: 1) x ti’n’ ma’nisleri won’, teris bolatug’i’n ma’nislerin tabi’n’; 2) funkciyani’n’ wo’siw ha’m kemeyiw arali’qlari’n tabi’n’; 3) x ti’n’ qanday ma’nislerinde funkciya yen’ u’lken yaki yen’ kishi ma’nislerdi qabi’l yetetug’i’ni’n ani’qlan’ ha’m tabi’n’ **(37–38):**

37. 1) $y = x^2 - 7x + 10$; 2) $y = -x^2 + x + 2$;
3) $y = -x^2 + 6x - 9$; 4) $y = x^2 + 4x + 5$.



17- su'wret.

38. 1) $y = 4x^2 + 4x - 3$; 2) $y = -3x^2 - 2x + 1$;
3) $y = -2x^2 + 3x + 2$; 4) $y = 3x^2 - 8x + 4$;
5) $y = 4x^2 + 12x + 9$; 6) $y = -4x^2 + 4x - 1$;
7) $y = 2x^2 - 4x + 5$; 8) $y = -3x^2 - 6x - 4$.

39. Kvadrat funkciyani'n' berilgen grafigi (17-su'wret) boyi'nsha woni'n' qa'siyetlerin ani'qlan'.

40. 15 sani'n yeki sanni'n' qosi'ndi'si' tu'rinde sonday yetip ko'rsetin', bul sanlardi'n' ko'beymesi yen' u'lkeni bolsi'n.

41. Yeki sanni'n' qosi'ndi'si' 10 g'a ten'. Yeger bul sanlardi'n' kublari'ni'n' qosi'ndi'si' yen' kishi san bolsa, wonda usi' sanlardi' tabi'n'.

42. U'y diywallari' qaptali'ndag'i' tuwri'mu'yeshlik forma-si'ndag'i' maydandi' u'sh ta'repinen 12 m li pa'njere menen worap ali'w talap yetiledi. Maydanni'n' wo'lshemleri qanday bolg'anda woni'n' maydani' yen' u'lken boladi'?

43. U'shmu'yeshlikte ultani' menen usi' ultang'a tu'sirilgen biyik-liktin' qosi'ndi'si' 14 sm ge ten'. Bunday u'shmu'yeshlik 25 sm^2 qa ten' maydang'a iye boli'wi' mu'mkin be?

44. Grafikti jasamay turi'p, x ti'n' qanday ma'nisinde funkciya yen' u'lken (yen' kishi) ma'niske iye bolatug'i'ni'n ani'q-lan'; usi' ma'nisti tabi'n':
1) $y = x^2 - 6x + 13$; 2) $y = x^2 - 2x - 4$;
3) $y = -x^2 + 4x + 3$; 4) $y = 3x^2 - 6x + 1$;
5) $y = -x^2 + 2x$; 6) $y = -x^2 - 3x$.

45. Yeger: 1) parabolani'n' shaqalari' joqari'g'a bag'darlang'an, woni'n' to'besinin' abcissasi' teris, ordinatasi' won' bolsa;

2) parabolani'n' shaqalari' to'menge bag'darlang'an, woni'n' to'besinin' abcissasi' ha'm ordinatasi' teris bolsa, $y = ax^2 + bx + c$ parabola ten'lemesinin' koefficientlerinin' belgilerin ani'qlan'.

46. 5 m biyiklikten woq jaydan 50 m/s tezlik penen joqari'g'a vertikal ta'rizde nayza ati'ldi'. Nayzani'n' t sekundtan keyin ko'terilgen biyikligi, metrlerde $h = h(t) = 5 + 50t - \frac{gt^2}{2}$ formulasi' menen yesaplanadi', bunda $g \approx 10$ m/s^2. Nayza neshe sekundtan keyin: 1) yen' u'lken biyiklikke yerisedi ha'm wol qanday biyiklik boladi'? 2) Jerge tu'sedi?

I bapqa tiyisli shi'ni'g'i'wlar

47. x ti'n' $y = 2x^2 - 5x + 3$ kvadrat funkciyasi': 1) 0; 2) 1; 3) 10; 4) –1 ge ten' ma'nislerin qabi'l yetetug'i'n ma'nisin tabi'n'.

48. Funkciyalardi'n' grafikleri kesilisiw noqatlari'ni'n' koordinatalari'n tabi'n':

1) $y = x^2 - 4$ ha'm $y = 2x - 4$;
2) $y = x^2$ ha'm $y = 3x - 2$;
3) $y = x^2 - 2x - 5$ ha'm $y = 2x^2 + 3x + 1$;
4) $y = x^2 + x - 2$ ha'm $y = (x + 3)(x - 4)$.

49. Ten'sizliklerdi sheshin':

1) $x^2 \le 5$; 2) $x^2 > 36$; 3) $x^2 \ge 9$; 4) $x^2 < 8$.

50. Parabolani'n' koordinata ko'sherleri menen kesilisiw noqatlari'ni'n' koordinatalari'n tabi'n':

1) $y = x^2 + x - 12$; 2) $y = -x^2 + 3x + 10$;
3) $y = -8x^2 - 2x + 1$; 4) $y = 7x^2 + 4x - 11$;
5) $y = 5x^2 + x - 1$; 6) $y = 5x^2 + 3x - 2$;
7) $y = 4x^2 - 11x + 6$; 8) $y = 3x^2 + 13x - 10$.

51. Parabola to'besinin' koordinatalari'n tabi'n':

1) $y = x^2 - 4x - 5$; 2) $y = -x^2 - 2x + 3$;
3) $y = x^2 - 6x + 10$; 4) $y = x^2 + x + \frac{5}{4}$;
5) $y = -2x(x + 2)$; 6) $y = (x - 2)(x + 3)$.

52. Funkciyani'n' grafigin jasan' ha'm grafik boyi'nsha woni'n' qa'siyetlerin ani'qlan':

1) $y = x^2 - 5x + 6$; 2) $y = x^2 + 10x + 30$;
3) $y = -x^2 - 6x - 8$; 4) $y = 2x^2 - 5x + 2$;
5) $y = -3x^2 - 3x + 1$; 6) $y = -2x^2 - 3x - 3$.

WO'ZIN'IZDI TEKSERIP KO'RIN'!

1. $y = x^2 - 6x + 5$ funkciyasi'ni'n' grafigin jasan' ha'm woni'n' yen' kishi ma'nisin tabi'n'.

2. $y = -x^2 + 2x + 3$ funkciyasi'ni'n' grafigi ja'rdeminde x ti'n' qanday ma'nisinde funkciyani'n' ma'nisi 3 ke ten' bolatug'i'ni'n tabi'n'.

3. $y = 1 - x^2$ funkciyasi'ni'n' grafigi boyi'nsha x ti'n' funkciya won': teris ma'nislerdi qabi'l yetetug'i'n ma'nislerin tabi'n'.

4. 1) $y = 2x^2$; 2) $y = -3x^2$ funkciyasi' qanday arali'qlarda wo'sedi? Kemiydi. Bul funkciyani'n' grafigin jasan'.

5. 1) $y = (x - 3)^2$; 2) $y = (x + 1)^2$ parabola to'besinin' koordinatalari'n tabi'n' ha'm woni'n' grafigin jasan'.

53. Funkciyani'n' grafigin jasamay turi'p, woni'n' yen' u'lken yaki yen' kishi ma'nisin tabi'n':

1) $y = x^2 + 2x + 3$; 2) $y = -x^2 + 2x + 3$;
3) $y = -3x^2 + 7x$; 4) $y = 3x^2 + 4x + 5$.

54. Tuwri'mu'yeshliktin' perimetri 600 m. Tuwri'mu'yeshliktin' maydani' yen' u'lken boli'wi' ushi'n woni'n' ultani' menen biyikligi qanday boli'wi' kerek?

55. Tuwri'mu'yeshlik woni'n' ta'reperinen birine parallel bolg'an yeki kesindi menen u'sh bo'lekke bo'lingen. Tuwri'mu'yeshlik perimetri menen kesindilerdin' uzi'nli'qlari'ni'n' qosi'ndi'si' 1600 m ge ten'. Yeger tuwri'mu'yeshliktin' maydani' yen' u'lken bolsa, woni'n' ta'replerin tabi'n'.

56. Yeger $y = x^2 + px + q$ kvadrat funkciyasi':

1) $x = 0$ bolg'anda 2 ge ten' ma'nisti, $x = 1$ bolg'anda bolsa 3 ke ten' ma'nisti qabi'l yetse, wonda, p, q koefficientin tabi'n';

2) $x = 0$ bolg'anda 0 ge ten' ma'nisti, $x = 2$ bolg'anda bolsa 6 g'a ten' ma'nisti qabi'l yetse, wonda p, q koefficientin tabi'n'.

57. Yeger $y = x^2 + px + q$ parabolasi':
1) abcissalar ko'sherin $x = 2$ ha'm $x = 3$ noqatlari'nda kesip wo'tse; 2) abcissalar ko'sherin $x = 1$ noqati'nda ha'm ordinatalar ko'sherin $y = 3$ noqati'nda kesip wo'tse; 3) abcissalar ko'sherine $x = 2$ noqati'nda uri'nba jasasa, wonda, p ha'm q lardi' tabi'n'.

58. x ti'n' qanday ma'nislerinde funkciya ten'dey ma'nislerdi qabi'l yetedi.
1) $y = x^2 + 3x + 2$ ha'm $y = |7 - x|$;
2) $y = 3x^2 - 6x + 3$ ha'm $y = |3x - 3|$?

59. Yeger: 1) parabolani'n' (0; 0), (2; 0), (3; 3) koordinatali' noqatlari'nan wo'tiwi; 2) (1; 3) noqati' parabolani'n' to'besi boli'wi', (−1; 7) noqati'ni'n' bolsa parabolag'a tiyisli boli'wi'; 3) $y = ax^2 + bx + c$ funkciyasi'ni'n' nolleri $x_1 = 1$ ha'm $x_2 = 3$ sanlari' yekenligi, funkciyani'n' yen' u'lken ma'nisleri bolsa 2 ge ten' yekenligi belgili bolsa, wonda $y = ax^2 + bx + c$ parabolasi'n jasan'.

I bapqa tiyisli si'naq (test) shi'ni'g'i'wlari'

Si'naq shi'ni'g'i'wlardi'n' ha'r birine 4 ten «juwap» berilgen. To'rt «juwap» ti'n' tek birewi duri's, qalg'anlari' bolsa naduri's. Woqi'w-shi'lardan si'naq shi'ni'g'i'wlardi' wori'nlaw yaki basqa talqi'lawlar ja'rdeminde usi' duri's juwapti' tabi'w (woni' belgilew) talap yetiledi.

1. a ni'n' sonday ma'nisin tabi'n', wol $y = ax^2$ parabola menen $y = 5x + 1$ tuwri' si'zi'qti'n' kesilisiw noqatlari'nan birinin' abcissasi' $x = 1$ bolsi'n.
A) $a = 6$; B) $a = -6$; C) $a = 4$; D) $a = -4$.

2. k ni'n' sonday ma'nisin tabi'n', wol $y = -x^2$ parabola menen $y = kx - 6$ tuwri' si'zi'qti'n' kesilisiw noqatlari'nan birinin' abcissasi' $x = 2$ bolsi'n.
A) $k = -1$; B) $k = 1$; C) $k = 2$; D) $k = -2$.

3. b ni'n' sonday ma'nisin tabi'n', wol $y = 3x^2$ parabola menen $y = 2x + b$ tuwri' si'zi'qti'n' kesilisiw noqatlari'nan birinin' abcissasi' abssissasi $x = 1$ bolsi'n.
A) $b = 2$; B) $b = -1$; C) $b = 1$; D) $b = -2$.

Parabolani'n' koordinata ko'sherleri menen kesilisiw noqatlari'ni'n' koordinatalari'n tabi'n' **(4–7):**

4. $y = x^2 - 2x + 4$.

A) $(-1;\ 3)$; B) $(3;\ 1)$; C) $(1;\ 3)$; D) $(0;\ 4)$.

5. $y = -x^2 - 4x - 5$.

A) $(0;\ -5)$; B) $(2;\ -1)$; C) $(5;\ 0)$; D) $(-5;\ 0)$.

6. $y = 6x^2 - 5x + 1$.

A) $(\frac{1}{3};\ 0)$, $(\frac{1}{2};\ 0)$, $(0;\ 1)$; B) $(-\frac{1}{3};\ 0)$, $(-\frac{1}{2};\ 0)$, $(1;\ 0)$;

C) $(0;\ \frac{1}{3})$, $(0;\ \frac{1}{2})$, $(0;\ 1)$; D) $(\frac{1}{3};\ 0)$, $(-\frac{1}{2};\ 0)$, $(0;\ -1)$.

7. $y = -x^2 + 6x + 7$.

A) $(-1;\ 0)$, $(-7;\ 0)$, $(0;\ -7)$; B) $(-1;\ 0)$, $(7;\ 0)$, $(0;\ 7)$;
C) $(1;\ 0)$, $(7;\ 0)$, $(0;\ -7)$; D) $(-1;\ 2)$, $(7;\ -1)$, $(7;\ 0)$.

Parabola to'besinin' koordinatalari'n tabi'n' **(8–11):**

8. $y = x^2 - 4x$.

A) $(0;\ 4)$; B) $(4;\ 2)$; C) $(2;\ -4)$; D) $(-4;\ 2)$.

9. $y = -x^2 + 2x$.

A) $(-1;\ -1)$; B) $(1;\ -2)$; C) $(0;\ 2)$; D) $(1;\ 1)$.

10. $y = x^2 + 6x + 5$.

A) $(-3;\ -4)$; B) $(-5;\ -1)$; C) $(-1;\ -5)$; D) $(3;\ 4)$.

11. $y = -5x^2 + 4x + 1$.

A) $(\frac{2}{5};\ \frac{9}{5})$; B) $(-\frac{2}{5};\ \frac{9}{5})$; C) $(-\frac{9}{5};\ \frac{2}{5})$; D) $(2;\ 9)$.

12. Abcissa ko'sherin $x = 1$ ha'm $x = 2$ noqati'nda, ordinata ko'sherin bolsa $y = \frac{1}{2}$ noqati'nda kesip wo'tiwshi parabolani'n' ten'lemesin jazi'n'.

A) $y = \frac{1}{2}x^2 - \frac{3}{4}x + \frac{1}{2}$; B) $y = \frac{1}{4}x^2 - \frac{3}{4}x + \frac{1}{2}$;

C) $y = x^2 - 3x + 2$; D) duri's juwap berilmegen.

13. Abcissalar ko'sherin $x = -1$ ha'm $x = 3$ noqatlarda, ordinatalar ko'sherin bolsa $y = 1$ noqati'nda kesip wo'tiwshi parabolani'n' ten'lemesin jazi'n'.

A) $y = -x^2 + 2x + 3$; B) $y = -\frac{x^2}{3} + 2x + 1$;

C) $y = -\frac{x^2}{3} + \frac{2}{3}x + 1$; D) $y = \frac{x^2}{3} - \frac{2}{3}x - 1$.

Parabola qaysi' shereklerde jaylasqan? **(14–18):**

14. $y = 3x^2 + 5x - 2$.

A) I, II, III; B) II, III, IV; C) I, III, IV; D) I, II, III, IV;

15. $y = x^2 - 4x + 6$.

A) I, II; B) II, III; C) I, II, III, IV; D) II, III, IV.

16. $y = -x^2 - 6x - 11$.

A) III, IV; B) I, II, III; C) II, III, IV; D) I, II.

17. $y = -x^2 + 5x$.

A) I, II, III; C) I, II, III, IV;

B) I, III, IV; D) duri's juwap berilmegen.

18. $y = x^2 - 4x$.

A) I, II, III; B) II, III, IV; C) I, II, IV; D) III, IV.

19. Yeki won' sanni'n' qosi'ndi'si' 160. Yeger sanlardi'n' kubları'ni'n' qosi'ndi'si' yen' kishisi bolsa, usi' sanlardi' tabi'n'.

A) 95; 65; B) 155; 5; C) 75; 85; D) 80; 80.

20. Yeki won' sanni'n' qosi'ndi'si' a g'a ten'. Yeger usi' sanlardi'n' kvadratlari'ni'n' qosi'ndi'si' yen' kishisi bolsa, usi' sanlardi' tabi'n'.

A) $\frac{a}{2}, \frac{a}{2}$; B) $a^3, a^3 - a$; C) $\frac{3a}{4}, \frac{a}{4}$; D) a^2; $a - a^2$.

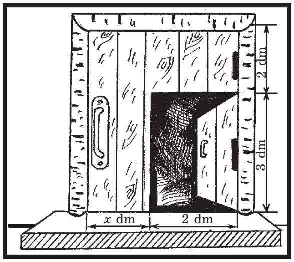
21. $y = x^2 - 4x + 3$ funkciyani'n' yen' kishi ma'nisin tabi'n'.

A) -1; B) 1; C) 7; D) -8.

22. $y = -x^2 + 5x - 6$ funkciyani'n' yen' u'lken ma'nisin tabi'n'.

A) $-\frac{5}{2}$; B) $\frac{1}{4}$; C) $-\frac{1}{6}$; D) $\frac{5}{6}$.

II B A P. KVADRAT TEN'SIZLIKLER



6-§. KVADRAT TEN'SIZLIK HA'M WONI'N' SHESHIMI

1-ma'sele. Tuwri'mu'yeshliktin' ta'repleri 2 ha'm 3 dm ge ten'. Woni'n' ha'rbir ta'repi bir qi'yli' sandag'i' decimetrlerge sonday yetip artti'ri'ldi', na'tiyjede tu'wri'mu'yeshliktin' maydani' 12 dm^2 tan arti'q boldi'. Ha'rbir ta'rep qanday wo'zgergen?

△ Tuwri'mu'yeshliktin' ha'rbir ta'repi x decimetrge artti'ri'lg'an bolsi'n. Wonda jan'a tuwri'mu'yeshliktin' ta'repleri $(2 + x)$ ha'm $(3 + x)$ decimetrge, woni'n' maydani' bolsa $(2 + x)(3 + x)$ kvadrat decimetrge ten' boladi'. Ma'selenin' sha'rti boyi'nsha $(2 + x)(3 + x) > 12$, bunnan $x^2 + 5x + 6 > 12$ yaki $x^2 + 5x - 6 > 0$.

Bul ten'sizliktin' shep bo'limin ko'beytiwshilerge jikleymiz:

$$(x + 6)(x - 1) > 0.$$

Ma'selenin' sha'rti boyi'nsha, $x > 0$ bolg'ani' ushi'n $x + 6 > 0$.

Ten'sizliktin' yeki bo'limin $x + 6$ won' sang'a bo'lip, $x - 1 > 0$, yag'ni'y $x > 1$ di payda yetemiz.

Juwabi'. Tuwri'mu'yeshliktin' ha'rbir ta'repi 1 dm den ko'birek artti'ri'lg'an. ▲

$x^2 + 5x - 6 > 0$ ten'sizliginde x penen belgisiz san belgilengen. Bul — kvadrat ten'sizlikke mi'sal bola aladi'.

! ***Yeger ten'sizliktin' shep bo'liminde kvadrat funkciya, al won' bo'liminde bolsa nol tursa, bunday ten'sizlikke kvadrat ten'sizlik dep ataladi'.***

Mi'sali',

$$2x^2 - 3x + 1 \geq 0, \qquad -3x^2 + 4x + 5 < 0$$

ten'sizlikleri kvadrat ten'sizlikler boladi'.

Bir belgisizli *ten'sizliktin' sheshimi* dep, belgisizdin' usi' ten'sizlikti duri's sanli' ten'sizlikke aylandi'ri'wshi' ma'nisine ayti'latug'i'n yesletip wo'temiz.

Ten'sizlikti sheshiw — woni'n' barli'q sheshimlerin tabi'w yamasa wolardi'n' joq yekenligin ko'rsetiw degen so'z.

2-ma'sele. Ten'sizlikti sheshin':

$$x^2 - 5x + 6 > 0.$$

△ $x^2 - 5x + 6 = 0$ kvadrat ten'lemesi yeki tu'rli $x_1 = 2$, $x_2 = 3$ korenge iye. Demek, $x^2 - 5x + 6$ kvadrat u'sh ag'zali'ni' ko'beytiwshilerge jiklewge boladi':

$$x^2 - 5x + 6 = (x - 2)(x - 3).$$

Soni'n' ushi'n berilgen ten'sizlikti to'mendegishe jazi'w mu'mkin:

$$(x - 2)(x - 3) > 0.$$

Yeger yeki ko'beytiwshi bir qi'yli' belgige iye bolsa, wolardi'n' ko'beymesi won' boladi'.

1) Yeki ko'beytiwshi won', yag'ni'y $x - 2 > 0$ ha'm $x - 3 > 0$ bolg'an jag'daydi' qaraymi'z.

Bul yeki ten'sizlik to'mendegi sistemani' du'zedi:

$$\begin{cases} x - 2 > 0, \\ x - 3 > 0. \end{cases}$$

Sistemani' sheship, $\begin{cases} x > 2, \\ x > 3 \end{cases}$ ti payda yetemiz, bunnan $x > 3$.

Demek, barli'q $x > 3$ sanlari' $(x - 2)(x - 3) > 0$ ten'sizliginin' sheshimleri boladi'.

2) Yendi yeki ko'beytiwshi teris, yag'ni'y $x - 2 < 0$ ha'm $x - 3 < 0$ bolg'an jag'daydi' qaraymi'z.

Bul yeki ten'sizlik to'mendegi sistemani' du'zedi:

$$\begin{cases} x - 2 < 0, \\ x - 3 < 0. \end{cases}$$

Sistemani' sheship, $\begin{cases} x < 2, \\ x < 3 \end{cases}$ ti payda yetemiz, bunnan $x < 2$.

Demek, barli'q $x < 2$ sanlari' da $(x-2)(x-3) > 0$ ten'sizliginin' sheshimleri boladi'.

Solay yetip, $(x-2)(x-3) > 0$ ten'sizliginin', demek, berilgen $x^2 - 5x + 6 > 0$ ten'sizliginin' de, sheshimleri $x < 2$, sonday-aq, $x > 3$ sanlari' boladi'.

Juwabi': $x < 2$, $x > 3$. ▲

! **Uluwma, yeger $ax^2 + bx + c = 0$ kvadrat ten'lemesi yeki korenge iye bolsa, wonda $ax^2 + bx + c > 0$ ha'm $ax^2 + bx + c < 0$ kvadrat ten'sizliklerin sheshiwdi, kvadrat ten'sizliktin' shep bo'limin ko'beytiwshilerge jiklep, birinshi da'rejeli ten'sizlikler sistemasi'n sheshiwge keltiriw mu'mkin.**

3-ma'sele. $-3x^2 - 5x + 2 > 0$ ten'sizligin sheshin'.

△ Yesaplawlardi' qolayli'raq ali'p bari'w ushi'n berilgen ten'sizliktin' birinshi koefficienti won' bolg'an kvadrat ten'sizlikler tu'rinde ko'rsetemiz. Buni'n' ushi'n woni'n' yeki bo'limin -1 ge ko'beytemiz:

$$3x^2 + 5x - 2 < 0.$$

$3x^2+5x-2=0$ ten'lemesinin' korenlerin tabami'z:

$$x_{1,2} = \frac{-5 \pm \sqrt{25+24}}{6} = \frac{-5 \pm 7}{6},$$

$$x_1 = \frac{1}{3}, \quad x_2 = -2.$$

Kvadrat u'sh ag'zali'ni' ko'beytiwshilerge jiklep, to'mendegini payda yetemiz:

$$3\left(x - \frac{1}{3}\right)(x+2) < 0.$$

Bunnan yeki sistemani' payda yetemiz:

$$\begin{cases} x - \frac{1}{3} > 0, \\ x + 2 < 0; \end{cases} \quad \begin{cases} x - \frac{1}{3} < 0, \\ x + 2 > 0. \end{cases}$$

Birinshi sistemani' to'mendegishe jazi'w mu'mkin:

$$\begin{cases} x > \frac{1}{3}, \\ x < -2, \end{cases}$$

bunnan sistema sheshimlerge iye yemesligi ko'rinip turi'pti'.

Yekinshi sistemani' sheship to'mendegini tabami'z:

$$\begin{cases} x < \dfrac{1}{3}, \\ x > -2, \end{cases}$$

bunnan $-2 < x < \dfrac{1}{3}$.

Demek, $3\left(x - \dfrac{1}{3}\right)(x + 2) < 0$ ten'sizliginin', yag'ni'y $-3x^2 - 5x + 2 > 0$ ten'sizliginin' sheshimleri $\left(-2;\ \dfrac{1}{3}\right)$ intervaldag'i' barli'q sanlar boladi'.

Juwabi': $-2 < x < \dfrac{1}{3}$. ▲

Shi'ni'g'i'wlar

60. (Awi'zeki.) To'mendegi ten'sizliklerden qaysi'lari' kvadrat ten'sizlik yekenligin ko'rsetin':

1) $x^2 - 4 > 0$; 2) $x^2 - 3x - 5 \le 0$; 3) $3x + 4 > 0$;
4) $4x - 5 < 0$; 5) $x^2 - 1 \le 0$; 6) $x^4 - 16 > 0$.

61. To'mendegi ten'sizlikti kvadrat ten'sizlikke keltirin':

1) $x^2 < 3x + 4$; 2) $3x^2 - 1 > x$;
3) $3x^2 < x^2 - 5x + 6$; 4) $2x(x + 1) < x + 5$.

62. (Awi'zeki.) 0; −1; 2 sanlari'nan qaysi'lari'

1) $x^2 + 3x + 2 > 0$; 2) $-x^2 + 3{,}5x + 2 \ge 0$;
3) $x^2 - x - 2 \le 0$; 4) $-x^2 + x + \dfrac{3}{4} < 0$

ten'sizliginin' sheshimleri boladi'?

Ten'sizlikti sheshin' **(63—65)**:

63. 1) $(x - 2)(x + 4) > 0$; 2) $(x - 11)(x - 3) < 0$;
3) $(x - 3)(x + 5) < 0$; 4) $(x + 7)(x + 1) > 0$.

64. 1) $x^2 - 4 < 0$; 2) $x^2 - 9 > 0$;
3) $x^2 + 3x < 0$; 4) $x^2 - 2x > 0$.

65. 1) $x^2 - 3x + 2 < 0$; 4) $x^2 + 2x - 3 > 0$;
2) $x^2 + x - 2 < 0$; 5) $2x^2 + 3x - 2 > 0$;
3) $x^2 - 2x - 3 > 0$; 6) $3x^2 + 2x - 1 > 0$.

66. Ten'sizlikti sheshin':

1) $2\cdot\left(x-\frac{1}{3}\right)^2>0;$ 2) $7\cdot\left(\frac{1}{6}-x\right)^2\leq 0;$ 3) $3x^2-3<x^2-x;$

4) $(x-1)(x+3)>5;$ 5) $3\cdot\left(\frac{1}{4}-x\right)^2\leq 0;$ 6) $5\cdot(x-1)^2\geq 0.$

67. Funkciyani'n' grafigin jasan'. Grafik boyi'nsha funkciya won' ma'nisler; teris ma'nisler; nolge ten' ma'nisti qabi'l yetetug'i'n x ti'n' barli'q ma'nislerin tabi'n':

1) $y=2x^2;$ 2) $y=-(x+1{,}5)^2;$
3) $y=2x^2-x+2;$ 4) $y=-3x^2-x-2;$
5) $y=-(3-x)^2;$ 6) $y=-x^2-1.$

68. x_1 ha'm x_2 sanlari' (bunda $x_1<x_2$) $y=ax^2+bx+c$ funkciyasi'ni'n' nolleri yekenligi belgili. Yeger x_0 sani' x_1 ha'm x_2 arasi'nda jatsa, yag'ni'y $x_1<x_0<x_2$ bolsa, wonda $a(ax_0^2+bx_0+c)<0$ ten'sizligi wori'nlanatug'i'ni'n da'lillen'.

7-§. KVADRAT TEN'SIZLIKTI KVADRAT FUNKCIYA GRAFIGI JA'RDEMINDE SHESHIW

Kvadrat funkciya $y=ax^2+bx+c$ (bunda $a\neq 0$) formulasi' menen beriletug'i'ni'n yesletip wo'temiz. Soni'n' ushi'n kvadrat ten'sizlikti sheshiw kvadrat funkciyani'n' nollerin ha'm kvadrat funkciya won' yaki teris ma'nislerdi qabi'l yetetug'i'n arali'qlardi' tabi'wg'a keltiriledi.

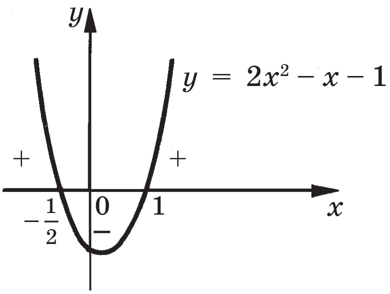
1-ma'sele. Ten'sizlikti grafik ja'rdeminde sheshin':

$$2x^2-x-1\leq 0.$$

△ $y=2x^2-x-1$ kvadrat funkciyasi'ni'n' grafigi — shaqalari' joqari'g'a bag'darlang'an parabola.

Bul parabolani'n' Ox ko'sheri menen kesilisiw noqatlari'n tabami'z. Buni'n' ushi'n $2x^2-x-1=0$ kvadrat ten'lemeni sheshemiz. Bul ten'lemenin' korenleri:

$$x_{1,2}=\frac{1\pm\sqrt{1+8}}{4}=\frac{1\pm 3}{4};\ x_1=1,\ x_2=-\frac{1}{2}.$$



18-su'wret.

Demek, parabola Ox ko'sherin $x = -\frac{1}{2}$ ha'm $x = 1$ noqatlari'nda kesip wo'tedi (18-su'wret).

$2x^2 - x - 1 \le 0$ ten'sizlikti x ti'n' funkciya nolge ten' bolg'an yaki funkciyani'n' ma'nisleri teris bolg'an ma'nisleri qanaatlandi'radi', yag'ni'y x ti'n' sonday ma'nisleri boli'p, bul ma'nislerde parabolani'n' noqatlari' Ox ko'sherinde yaki usi' ko'sherden to'men-de jatadi'. 18-su'wretten, bul ma'nisler $\left[-\frac{1}{2}; 1\right]$ kesindidegi barli'q sanlar bolatug'i'ni' ko'rinip tur.

Juwabi': $-\frac{1}{2} \le x \le 1$. ▲

Bul funkciyani'n' grafiginen berilgen ten'sizlikten tek belgisi menen wo'zgeshelenetug'i'n basqa ten'sizliklerdi sheshiwde de paydalani'w mu'mkin. 18-su'wrette ko'rinip turi'pti':

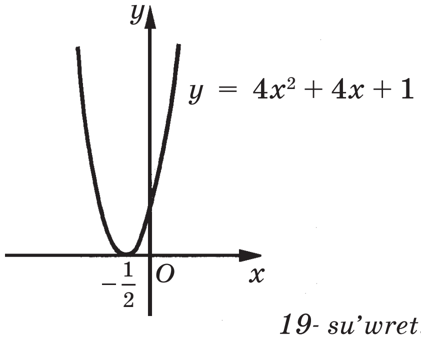
1) $2x^2 - x - 1 < 0$ ten'sizliginin' sheshimleri $-\frac{1}{2} < x < 1$, yag'ni'y $\left(-\frac{1}{2}; 1\right)$ intervali'ndag'i' barli'q sanlar boladi';

2) $2x^2 - x - 1 > 0$ ten'sizliginin' sheshimleri $x < -\frac{1}{2}$ ha'm $x > 1$ arali'qlari'ndag'i' barli'q sanlar boladi'.

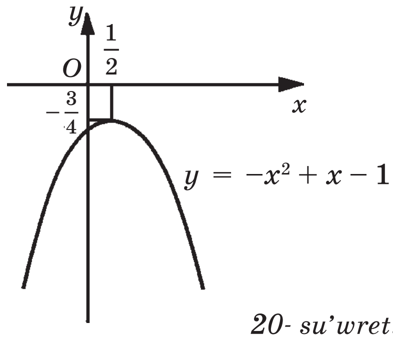
3) $2x^2 - x - 1 \ge 0$ ten'sizliginin' sheshimleri $x \le -\frac{1}{2}$ ha'm $x \ge 1$ arali'qlari'ndag'i' barli'q sanlar boladi'.

2-ma'sele. Ten'sizlikti sheshin': $4x^2 + 4x + 1 > 0$.

△ $y = 4x^2 + 4x + 1$ funkciyasi' grafiginin' eskizin si'zami'z. Bul parabolani'n' shaqalari' joqari'g'a bag'darlang'an. $4x^2 + 4x + 1 = 0$ ten'lemesi bir $x = -\frac{1}{2}$ korenge iye, soni'n' ushi'n parabola Ox ko'sherine $\left(-\frac{1}{2}; 0\right)$ noqati'nda uri'nadi'. Bul funkciyani'n' grafigi 19-su'wrette ko'rsetilgen. Berilgen ten'sizlikti sheshiw ushi'n x ti'n' qanday ma'nislerinde funkciyani'n' ma'nisleri won' bolatug'i'ni'n



19- su'wret.



20- su'wret.

ani'qlaw kerek. Solay yetip, $4x^2 + 4x + 1 > 0$ ten'sizlikti parabolani'n' noqatlari' Ox ko'sherinen joqari'da jati'wshi' x ti'n' ma'nisleri qanaatlandi'radi' 19-su'wretten, bunday ma'nisler $x = -0{,}5$ ten basqa barli'q haqi'yqi'y sanlar bolatug'i'nli'g'i' belgili.

J u w a b i ' : $x \neq -0{,}5$. ▲

19-su'wrette ko'rinip turg'ani'nday:

1) $4x^2 + 4x + 1 \geq 0$ ten'sizliginin' sheshimi barli'q haqi'yqi'y sanlar boladi'; 2) $4x^2 + 4x + 1 \leq 0$ ten'sizligi bir $x = -\frac{1}{2}$ sheshimge iye; 3) $4x^2 + 4x + 1 < 0$ ten'sizligi sheshimlerge iye yemes yekenligi ko'rinip tur.

Yeger $4x^2 + 4x + 1 = (2x + 1)^2$ yekenligi itibarg'a ali'nsa, bul ten'sizliklerdi awi'zeki sheshiw mu'mkin.

3 - m a ' s e l e . $-x^2 + x - 1 < 0$ ten'sizligin sheshin'.

△ $y = -x^2 + x - 1$ funkciyasi' grafiginin' eskizin si'zami'z. Bul parabolani'n' shaqalari' to'menge bag'darlang'an. $-x^2 + x - 1 = 0$ ten'lemesinin' haqi'yqi'y korenleri joq, soni'n' ushi'n parabola Ox ko'sherin kesip wo'tpeydi. Demek, bul parabola Ox ko'sherinen to'mende jaylasqan (20-su'wret). Bul barli'q x larda kvadrat funkciyani'n' ma'nisleri teris, yag'ni'y $-x^2 + x - 1 < 0$ ten'sizligi x ti'n' barli'q haqi'yqi'y ma'nisinde wori'nlanatug'i'ni'n an'latadi'. ▲

20-su'wretten de $-x^2 + x - 1 \leq 0$ ten'sizliginin' sheshimleri x ti'n' barli'q haqi'yqi'y ma'nisleri boli'wi'nan, $-x^2 + x - 1 > 0$ ha'm $-x^2 + x - 1 \geq 0$ ten'sizligi sheshimlerge iye yemesligi ko'rinip tur.

Solay yetip, *kvadrat ten'sizlikti grafik ja'rdeminde sheshiw ushi'n:*

1) kvadrat funkciyani'n' birinshi koefficientinin' belgisi boyi'nsha parabola shaqalari'ni'n' bag'dari'n ani'qlaw;

2) tiyisli kvadrat ten'lemenin' haqi'yqi'y korenlerin tabi'w yaki wolardi'n' joq yekenligin ani'qlaw;

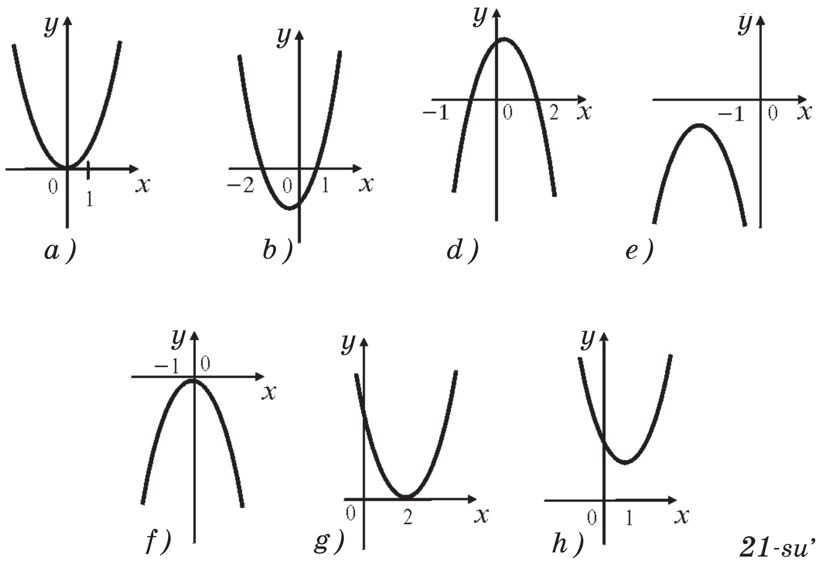
3) kvadrat funkciyani'n' Ox ko'sheri menen kesilisiw noqatlari' (yeger wolar bar bolsa) yaki uri'nbadan paydalani'p, kvadrat funkciya grafiginin' eskizin jasaw;

4) grafik boyi'nsha funkciya kerekli ma'nislerdi qabi'l yetetug'i'n arali'qlari'n ani'qlaw kerek.

Shi'ni'g'i'wlar

69. $y = x^2 + x - 6$ funkciyasi'ni'n' grafigin jasan'. Grafik boyi'nsha funkciya won' ma'nislerdi: teris ma'nislerdi qabi'l yetetug'i'n x ti'n' ma'nislerin tabi'n'.

70. (Awi'zeki.) $y = ax^2 + bx + c$ funkciyasi'ni'n' grafiginen paydalani'p (21-su'wret), x ti'n' qanday ma'nislerinde bul funkciya won' ma'nislerge, teris ma'nislerge, nolge ten' ma'nisti qabi'l yetetug'i'ni'n ko'rsetin'.



21-su'wret.

Kvadrat ten'sizlikti sheshin' **(71–75)**:

71. 1) $x^2 - 3x + 2 \le 0$; 2) $x^2 - 3x - 4 \ge 0$;
3) $-x^2 + 3x - 2 < 0$; 4) $-x^2 + 3x + 4 > 0$.

72. 1) $2x^2 + 7x - 4 < 0$; 2) $3x^2 - 5x - 2 > 0$;
3) $-2x^2 + x + 1 \ge 0$; 4) $-4x^2 + 3x + 1 \le 0$.

73. 1) $x^2 - 6x + 9 > 0$; 2) $x^2 - 14x + 49 \le 0$;
3) $4x^2 - 4x + 1 \ge 0$; 4) $4x^2 - 20x + 25 < 0$;
5) $-9x^2 - 6x - 1 < 0$; 6) $-2x^2 + 6x - 4{,}5 \le 0$.

74. 1) $x^2 - 4x + 6 > 0$; 2) $x^2 + 6x + 10 < 0$;
3) $x^2 + x + 2 > 0$; 4) $x^2 + 3x + 5 < 0$;
5) $2x^2 - 3x + 7 < 0$; 6) $4x^2 - 8x + 9 > 0$.

75. 1) $5 - x^2 \ge 0$; 2) $-x^2 + 7 < 0$;
3) $-2{,}1x^2 + 10{,}5x < 0$; 4) $-3{,}6x^2 - 7{,}2x < 0$;
5) $-6x^2 - x + 12 > 0$; 6) $-3x^2 - 6x + 45 < 0$;
7) $-\frac{1}{2}x^2 + 4{,}5x - 4 > 0$; 8) $-x^2 - 3x - 2 > 0$.

76. (Awi'zeki.) Ten'sizlikti sheshin':
1) $x^2 + 10 > 0$; 2) $x^2 + 9 < 0$;
3) $(x - 1)^2 + 1 > 0$; 4) $(x + 5)^2 + 3 < 0$;
5) $-(x + 1)^2 - 2 < 0$; 6) $-(x - 2)^2 - 4 > 0$;
7) $0{,}5x^2 + 8 \le 0$; 8) $\left(x - \frac{3}{4}\right)^2 + 21 \ge 0$.

Kvadrat ten'sizlikti sheshin' **(77–79)**:

77. 1) $4x^2 - 9 > 0$; 2) $9x^2 - 25 > 0$;
3) $x^2 - 3x + 2 > 0$; 4) $x^2 - 3x - 4 < 0$;
5) $2x^2 - 4x + 9 \le 0$; 6) $3x^2 + 2x + 4 \ge 0$;
7) $\frac{1}{2}x^2 - 4x \ge -8$; 8) $\frac{1}{3}x^2 + 2x \le -3$.

78. 1) $2x^2 - 8x \le -8$; 2) $x^2 + 12x \ge -36$;
3) $9x^2 + 25 < 30x$; 4) $16x^2 + 1 > 8x$;
5) $2x^2 - x \ge 0$; 6) $3x^2 + x \le 0$;
7) $0{,}4x^2 - 1{,}1x + 1 \ge 0$; 8) $x^2 - x + 0{,}26 \le 0$.

79. 1) $x(x + 1) < 2(1 - 2x - x^2)$; 2) $x^2 + 2 < 3x - \frac{1}{8}x^2$;

3) $6x^2 + 1 \le 5x - \frac{1}{4}x^2$; 4) $2x(x - 1) < 3(x + 1)$;

5) $\frac{5}{3}x - \frac{1}{6}x^2 \le x + 1$; 6) $\frac{1}{6}x^2 + \frac{2}{3} \ge x - 1$.

80. Funkciya nolden u'lken bolmag'an ma'nislerin qabi'l yetetug'i'n x ti'n' barli'q ma'nislerin tabi'n':

1) $y = -x^2 + 6x - 9$; 2) $y = x^2 - 2x + 1$;

3) $y = -\frac{1}{2}x^2 - 3x - 4\frac{1}{2}$; 4) $y = -\frac{1}{3}x^2 - 4x - 12$.

81. 1) $x^2 - 2x + q > 0$ ten'sizliginin' $q > 1$ bolg'andag'i' sheshimleri x ti'n' barli'q haqi'yqi'y ma'nisleri bolatug'i'ni'n ko'rsetin'; 2) $x^2 + 2x + q \le 0$ ten'sizliginin' $q > 1$ bolg'anda haqi'yqi'y sheshimlerge iye yemesligin ko'rsetin'.

82. r di'n', $x^2 - (2 + r)x + 4 > 0$ ten'sizligi x ti'n' barli'q haqi'yqi'y ma'nislerinde wori'nlanatug'i'n barli'q ma'nislerin tabi'n'.

8- §. INTERVALLAR USI'LI'

Ten'sizliklerdi sheshkende ko'binese intervallar usi'li' qollani'ladi'. Bul usi'ldi' mi'sallarda tu'sindiremiz.

1-ma'sele. x ti'n' qanday ma'nislerinde $x^2 - 4x + 3$ kvadrat u'sh ag'zali' won' ma'nislerdi, qanday ma'nislerinde bolsa teris ma'nislerdi qabi'l yetetug'i'ni'n ani'qlan'.

△ $x^2 - 4x + 3 = 0$ ten'lemenin' korenlerin tabami'z:

$$x_1 = 1, \quad x_2 = 3.$$

Sonli'qtan, $x^2 - 4x + 3 = (x - 1)(x - 3)$.

$x = 1$ ha'm $x = 3$ noqatlari' (22-su'wret) san ko'sherin u'sh arali'qqa bo'ledi:

$$x < 1, \; 1 < x < 3, \; x > 3.$$

$1 < x < 3$ arali'q si'yaqli' $x < 1$, $x > 3$ arali'qlar da *intervallar* delinedi.

San ko'sheri boyi'nsha won'nan shepke ji'lji'ti'p $x > 3$ intervalda $x^2 - 4x + 3 = (x - 1)(x - 3)$ u'sh ag'zali' won' ma'nislerin qabi'l

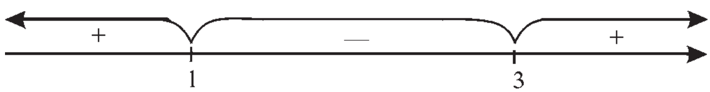
22-su'wret.

yetetug'i'ni'n ko'remiz, sebebi bul jag'dayda yeki $x - 1$ ha'm $x - 3$ ko'beytiwshisi de won'.

Keyingi $1 < x < 3$ intervalda bul u'shag'zali' teris ma'nislerdi qabi'l yetedi, solay yetip, $x = 3$ noqati' arqali' wo'tkende belgisin wo'zgertedi. Bul jag'day sonı'n' ushi'n da boli'p o'tedi, $(x - 1)(x - 3)$ ko'beymede $x = 3$ noqati' arqali' o'tkende $x - 1$ ko'beytiwshisi belgisin o'zgertpeydi, ekinshi $x - 3$ ko'beytiwshisi bolsa belgisin o'zgertedi. $x = 1$ noqati' arqali' wo'tkende u'sh ag'zali' ja'ne belgisin wo'zgertedi, sebebi $(x - 1)(x - 3)$ ko'beymede birinshi $x - 1$ ko'beytiwshi belgisin wo'zgertedi, yekinshi $x - 3$ ko'beytiwshisi belgilerin wo'zgertpeydi. Demek, san ko'sheri boyi'nsha won'nan shepke qarap ji'lji'ti'p bir intervaldan qon'si'las intervalg'a wo'tip barg'anda $(x - 1)(x - 3)$ ko'beymenin' belgileri almasi'p baradi'.

Solay yetip, $x^2 - 4x + 3$ kvadrat u'sh ag'zali'ni'n' belgisi haqqi'ndag'i' ma'seleni to'mendegi usi'l menen sheshiw mu'mkin.

$x^2 - 4x + 3 = 0$ ten'lemesinin' korenlerin san ko'sherinde belgileymiz: $x_1 = 1$, $x_2 = 3$. Wolar san ko'sherin u'sh intervalg'a aji'ratadi' (22-su'wret). $x > 3$ intervalda $x^2 - 4x + 3$ u'sh ag'zali'ni'n' won' bolatug'i'nli'g'i'n ani'qlap, u'sh ag'zali'ni'n' qalg'an intervaldag'i' belgilerin almasi'p baratug'i'n ta'rtipte belgileymiz (23-su'wret). 23-su'wrettegidey, $x < 1$ ha'm $x > 3$ bolg'anda $x^2 - 4x + 3 > 0$, $1 < x < 3$ bolg'anda $x^2 - 4x + 3 < 0$. ▲



23- su'wret.

Qarap wo'tilgen usi'l *intervallar usi'li'* delinedi. Bul usi'ldan kvadrat ten'sizliklerdi ha'm basqa ten'sizliklerdi sheshiwde paydalani'ladi'.

Mi'sali', 1-ma'seleni sheshiwde biz tiykari'nda $x^2 - 4x + 3 > 0$ ha'm $x^2 - 4x + 3 < 0$ ten'sizliklerin intervallar usi'li' menen sheshtik.

2-ma'sele. $x^3 - x < 0$ ten'sizligin sheshin'.

△ $x^3 - x$ ko'p ag'zali'ni' ko'beytiwshilerge jikleymiz:

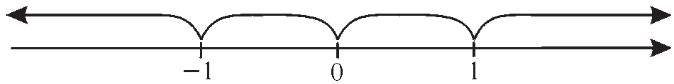
$$x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1).$$

Demek, ten'sizlikti to'mendegishe jazi'wg'a boladi':

$$(x + 1)x(x - 1) < 0.$$

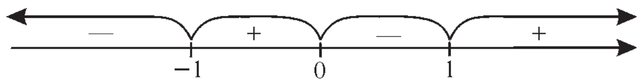
San ko'sherinde -1, 0 ha'm 1 noqatlari'n belgileymiz. Bul noqatlar san ko'sherin to'rt intervalg'a bo'ledi (24-su'wret):

$$x < -1,\ -1 < x < 0,\ 0 < x < 1,\ x > 1.$$



24- su'wret.

$x > 1$ bolg'anda $(x + 1)x(x - 1)$ ko'beymenin' barli'q ko'beytiwshileri won', soni'n' ushi'n $x > 1$ intervalda $(x + 1)x(x - 1) > 0$ boladi'. Qon'si' intervalg'a wo'tiwde ko'beyme belgisinin' almasi'wi'n itibarg'a ali'p, ha'rbir interval ushi'n $(x + 1)x(x - 1)$ ko'beymenin' belgisin tabami'z (25-su'wret).



25- su'wret.

Solay yetip, ten'sizliktin' sheshimleri x ti'n' $x < -1$ ha'm $0 < x < 1$ intervallardag'i' barli'q ma'nisleri boladi'.

Juwabi': $x < -1,\ 0 < x < 1$. ▲

3-ma'sele. $(x^2 - 9)(x + 3)(x - 2) > 0$ ten'sizligin sheshin'.

△ Berilgen ten'sizlikti to'mendegi tu'rde jazi'w mu'mkin:

$$(x + 3)^2(x - 2)(x - 3) > 0. \qquad (1)$$

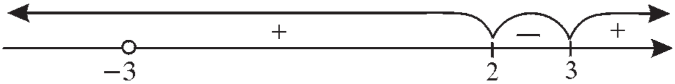
Barli'q $x \neq -3$ da $(x + 3)^2 > 0$ bolg'ani' ushi'n $x \neq -3$ de (1) ten'sizliktin' sheshimlerinin' ko'pligi,

$$(x - 2)(x - 3) > 0 \qquad (2)$$

ten'sizlik sheshimlerinin' ko'pligi menen u'stpe-u'st tu'sedi.

$x = -3$ ma'nisi (1) ten'sizliktin' sheshimi bolmaydi', sebebi $x= -3$ bolg'anda ten'sizliktin' shep bo'limi 0 ge ten'.

(2) ten'sizlikti intervallar usi'li' menen sheship, $x < 2$, $x > 3$ ti payda yetemiz (26-su'wret).



26-su'wret.

$x = -3$ berilgen ten'sizliktin' sheshimi bola almaytug'i'ni'n itibarg'a ali'p, son'i'nda na'tiyjeni to'mendegishe jazami'z:

$$x < -3, \quad -3 < x < 2, \quad x > 3. \blacktriangle$$

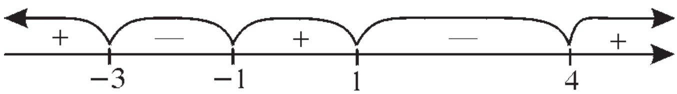
4-ma'sele. To'mendegi ten'sizlikti sheshin':

$$\frac{x^2+2x-3}{x^2-3x-4} \geq 0.$$

$\triangle$ Bo'lshektin' ali'mi'n ha'm bo'limin ko'beytiwshilerge jiklep to'mendegini payda yetemiz:

$$\frac{(x+3)(x-1)}{(x+1)(x-4)} \geq 0. \qquad (3)$$

San ko'sherinde bo'lshektin' ali'mi' yaki bo'limi nolge aylanatug'i'n -3; -1; 1; 4 noqatlari'n belgileymiz. Bul noqatlar san tuwri'si'n bes intervalg'a bo'ledi (27-su'wret). $x > 4$ bolg'anda bo'lshektin' ali'mi' ha'm bo'limindegi barli'q ko'beytiwshileri won' ha'm soni'n' ushi'n bo'lshek won' boladi'.



27-su'wret.

Bir intervaldan keyingisine wo'tiwde bo'lshek belgisin wo'zgertedi, soni'n' ushi'n bo'lshektin' belgilerin 27-su'wrettegidey yetip qoyi'w mu'mkin. $x = -3$ ha'm $x = 1$ ma'nisleri (3) ten'sizlikti qanaatlandi'radi', $x = -1$ ha'm $x = 4$ bolg'anda bolsa bo'lshek ma'niske iye yemes. Solay yetip, berilgen ten'sizlik to'mendegi sheshimlerge iye:

$$x \leq -3, \quad -1 < x \leq 1, \quad x > 4. \blacktriangle$$

Shi'ni'g'i'wlar

83. (Awi'zeki.) $x = 5$ ma'nisi ten'sizliktin' sheshimi bolatug'i'ni'n ko'rsetin':

1) $(x - 1)(x - 3) > 0$; 2) $(x + 2)(x + 5) > 0$;
3) $(x - 7)(x - 10) > 0$; 4) $(x + 1)(x - 4) > 0$.

Ten'sizlikti intervallar usi'li' menen sheshin' **(84–90)**:

84. 1) $(x + 2)(x - 7) > 0$; 2) $(x + 5)(x - 8) < 0$;

3) $(x-2)\left(x+\frac{1}{2}\right) < 0$; 4) $(x+5)\left(x-3\frac{1}{2}\right) > 0$.

85. 1) $x^2 + 5x > 0$; | 2) $x^2 - 9x > 0$; | 3) $2x^2 - x < 0$;
4) $x^2 + 3x < 0$; | 5) $x^2 + x - 12 < 0$; | 6) $x^2 - 2x - 3 > 0$.

86. 1) $x^3 - 16x < 0$; 2) $4x^3 - x > 0$;
3) $(x^2 - 1)(x + 3) < 0$; 4) $(x^2 - 4)(x - 5) > 0$.

87. 1) $(x - 5)^2(x^2 - 25) > 0$; | 2) $(x + 7)^2(x^2 - 49) < 0$;
3) $(x - 3)(x^2 - 9) < 0$; | 4) $(x - 4)(x^2 - 16) > 0$;
5) $(x - 8)(x - 1)(x^2 - 1) \geq 0$; | 6) $(x - 5)(x + 2)(x^2 - 4) \leq 0$.

88. 1) $\frac{x-2}{x+5} > 0$; 2) $\frac{x-4}{x+3} < 0$; 3) $\frac{1{,}5-x}{3+x} \geq 0$;
4) $\frac{3{,}5+x}{x-7} \leq 0$; 5) $\frac{(2x+1)(x+2)}{x-3} < 0$; 6) $\frac{(x-3)(2x+4)}{x+1} \geq 0$.

89. 1) $\frac{x^2+2x+3}{(x-2)^2} \leq 0$; | 2) $\frac{(x+4)^2}{2x^2-3x+1} \geq 0$; | 3) $\frac{x^2-x}{x^2-4} > 0$; | 4) $\frac{9x^2-4}{x-2x^2} < 0$.

90. 1) $(x^2 - 5x + 6)(x^2 - 1) > 0$;
2) $(x + 2)(x^2 + x - 12) > 0$;
3) $(x^2 - 7x + 12)(x^2 - x + 2) \leq 0$;
4) $(x^2 - 3x - 4)(x^2 - 2x - 15) \leq 0$.

Ten'sizlikti sheshin' **(91–93)**:

91. 1) $\frac{x^2-x-12}{x-1} > 0$; 2) $\frac{x^2-4x-12}{x-2} < 0$; 3) $\frac{x^2+3x-10}{x^2+x-2} \leq 0$;
4) $\frac{x^2-3x-4}{x^2+x-6} \geq 0$; 5) $\frac{x^2+5x+6}{x+3} \geq 0$; 6) $\frac{x^2-8x+7}{x-1} \leq 0$.

92. 1) $\frac{x}{x-2} + \frac{3}{x} > \frac{3}{x-2}$; 2) $\frac{x^2}{x^2+3x} + \frac{2-x}{x+3} < \frac{5-x}{x}$.

93. 1) $\frac{x^2-7x-8}{x^2-64} < 0$; 2) $\frac{x^2+7x+10}{x^2-4} > 0$; 3) $\frac{5x^2-3x-2}{1-x^2} \geq 0$;
4) $\frac{x^2-16}{2x^2+5x-12} > 0$; 5) $\frac{x^2+3x+2}{x^2-1} \geq 0$; 6) $\frac{x^2-9}{2x^2-5x-3} < 0$.

II bapqa tiyisli shi'ni'g'i'wlar

Ten'sizlikti sheshin' (**94–100**):

94. 1) $(x - 5{,}7)(x - 7{,}2) > 0$; 2) $(x - 2)(x - 4) > 0$;
3) $(x - 2{,}5)(3 - x) < 0$; 4) $(x - 3)(4 - x) < 0$.

95. 1) $x^2 > x$; 2) $x^2 > 36$; 3) $4 > x^2$; 4) $\frac{9}{16} \geq x^2$.

96. 1) $-9x^2 + 1 \leq 0$; 2) $-4x^2 + 1 \geq 0$;
3) $-5x^2 - x \geq 0$; 4) $-3x^2 + x \leq 0$.

97. 1) $-2x^2 + 4x + 30 < 0$; 2) $-2x^2 + 9x - 4 > 0$;
3) $4x^2 + 3x - 1 < 0$; 4) $2x^2 + 3x - 2 < 0$;
5) $6x^2 + x - 1 > 0$; 6) $5x^2 - 9x + 4 > 0$.

98. 1) $x^2 - 2x + 1 \geq 0$; 2) $x^2 + 10x + 25 > 0$;
3) $-x^2 + 6x - 9 < 0$; 4) $-4x^2 - 12x - 9 < 0$;
5) $\frac{1}{9}x^2 - \frac{4}{3}x + 4 > 0$; 6) $-x^2 + x - \frac{1}{4} < 0$.

99. 1) $x^2 - 3x + 8 > 0$; 2) $x^2 - 5x + 10 < 0$;
3) $2x^2 - 3x + 5 \geq 0$; 4) $3x^2 - 4x + 5 \leq 0$;
5) $-x^2 + 2x + 4 \leq 0$; 6) $-4x^2 + 7x - 5 \geq 0$.

100. 1) $(x - 2)(x^2 - 9) > 0$; 2) $(x^2 - 1)(x - 4) < 0$;
3) $\frac{(x+3)(x-5)}{x+1} \leq 0$; 4) $\frac{x-7}{(4-x)(2x+1)} \geq 0$; 5) $\frac{4x^2-4x-3}{x+3} \geq 0$;
6) $\frac{2x^2-3x-2}{x-1} < 0$; 7) $\frac{(x+1)(x-4)}{x^2-1} \geq 0$; 8) $\frac{x+1}{6x^2-7x-3} \leq 0$.

Ten'sizlikti sheshin' **(101–105):**

101. 1) $x^2 > 2 - x$; 2) $x^2 - 5 < 4x$; 3) $x + 8 < 3x^2 - 9$;
4) $x^2 \leq 10 - 3x$; 5) $10x - 12 < 2x^2$; 6) $3 - 7x \leq 6x^2$.

102. 1) $x^2 + 4 < x$; 2) $x^2 + 3 > 2x$; 3) $-x^2 + 3x \leq 4$;
4) $-x^2 - 5x \geq 8$; 5) $3x^2 - 5 > 2x$; 6) $2x^2 + 1 < 3x$;
7) $\frac{x^2}{10} + 2 \leq \frac{7x}{10}$; 8) $\frac{x^2}{3} - \frac{2x}{3} > \frac{3x-10}{4}$; 9) $\frac{x^2}{6} - \frac{5}{6}x + 1 \geq 0$.

103. 1) $\frac{1}{3}x - \frac{4}{9}x^2 \geq 1 - x$; 2) $\frac{1}{3}x(x+1) \leq (x+1)^2$;
3) $x(1-x) > 1{,}5-x$; 4) $\frac{1}{3}x - \frac{4}{9} \geq x(x-1)$;
5) $x\left(\frac{x}{4} - 1\right) \leq x^2 + x + 1$; 6) $2x - 2{,}5 > x(x - 1)$.

104. 1) $\frac{2}{x-\sqrt{2}} > \frac{3}{x+\sqrt{2}}$; 2) $\frac{\sqrt{3}}{3-x^2} < \frac{2}{\sqrt{3}-x}$; 3) $\frac{9}{2x+2} + \frac{x}{x-1} \geq \frac{1-3x}{2-2x}$;
4) $\frac{3}{x^2-1} - \frac{1}{2} < \frac{3}{2x-2}$; 5) $\frac{6}{5-x^2} > \frac{\sqrt{5}}{\sqrt{5}+x}$; 6) $\frac{1}{x^2-4} - 1 \leq \frac{1}{x+2}$.

WO'ZIN'IZDI TEKSERIP KO'RIN'!

1. Ten'sizlikti sheshin':

1) $x^2 - 3x - 4 < 0$; 2) $3x^2 - 4x + 8 \geq 0$;
3) $-x^2 + 3x - 5 > 0$; 4) $x^2 + 20x + 100 \leq 0$.

2. Ten'sizlikti intervallar usi'li' menen sheshin':

1) $x(x-1)(x+2) \geq 0$; 2) $(x+1)(2-x)(x-3) \leq 0$.

105. 1) $\frac{3x^2-5x-8}{2x^2-5x-3} > 0$; 2) $\frac{4x^2+x-3}{5x^2-9x-2} < 0$; 3) $\frac{2+7x-4x^2}{3x^2+2x-1} \leq 0$;

4) $\frac{2+9x-5x^2}{3x^2-2x-1} \geq 0$; 5) $\frac{x^2-5x+6}{x^2+5x+6} > 0$; 6) $\frac{x^2+8x+7}{x^2+x-2} \leq 0$.

106. Kater 4 saattan ko'p bolmag'an waqi't dawami'nda da'rya ag'i'si' boyi'nsha 22,5 km ju'ziwi ha'm keyinge qayti'wi' kerek. Yeger da'rya ag'i'si'ni'n' tezligi 3 km/saat bolsa, kater suwg'a sali'sti'rg'anda qanday tezlik penen ju'ziwi kerek?

107. Funkciyalardi'n' grafiklerin bir koordinata sistemasi'nda jasan' ha'm x ti'n' qanday ma'nislerinde bir funkciyani'n' ma'nisi yekinshisinen u'lken (kishi) bolatug'i'ni'n ani'qlan', na'tiyjeni, tiyisli ten'sizlikti sheship, tekserin':

1) $y = 2x^2$, $y = 2 - 3x$;
2) $y = x^2 - 2$, $y = 1 - 2x$;
3) $y = x^2-5x+4$, $y = 7 - 3x$;
4) $y = 3x^2 - 2x + 5$, $y = 5x + 3$;
5) $y = x^2-2x$, $y = -x^2 + x + 5$;
6) $y = 2x^2 - 3x + 5$, $y = x^2 + 4x - 5$.

<u>108</u>. Ten'sizlikti sheshin':

1) $\frac{x^4-5x^2-36}{x^2+x-2} \geq 0$; 2) $\frac{x^4+4x^2-5}{x^2+5x+6} \leq 0$; 3) $\frac{x^4-x^2-2}{x^4+x^2-2} < 0$;

4) $\frac{x^4-2x^2-8}{x^4-2x^2-3} \geq 0$; 5) $\frac{x^4-4x^2+3}{x^4-5x^2+6} \geq 0$; 6) $\frac{x^4-6x^2+5}{x^4-3x^2+2} \leq 0$.

II bapqa tiyisli si'naq (test) shi'ni'g'i'wlari'

Ten'sizlikti sheshin' **(1–12)**:

1. $2x^2 - 8 \le 0$.

A) $-2 \le x \le 2$; B) $-2 \le x$; C) $x \ge 2$; D) $0 \le x \le 4$.

2. $-3x^2 + 27 \ge 0$.

A) $x \le 3$; B) $|x| \le 3$; C) $x \ge 3$; D) $0 \le x \le 9$.

3. $3x^2 - 9 \ge 0$.

A) $x < \sqrt{3}$; B) $x > \sqrt{3}$; C) $x < -\sqrt{3},\ x > \sqrt{3}$; D) $x \ge 3$.

4. $x^2 + 7x \ge 0$.

A) $x > 0$; B) $x > 7$; C) $0 < x < 7$; D) $x \le -7,\ x \ge 0$.

5. $-x^2 + 3x \le 0$.

A) $x \le 0,\ x \ge 3$; B) $x \ge 0$;
C) $0 < x < 3$; D) $-3 < x < 3$.

6. $(x + 3)(x - 4) > 0$.

A) $x < -3,\ x > 4$; B) $-3 < x < 4$;
C) $x > 4$; D) $x < -3$.

7. $(x - 1)(x + 7) < 0$.

A) $x > -7$; B) $-7 < x < 1$;
C) $x > 1$; D) $x < -7,\ x > 1$.

8. $6x^2 + 5x - 6 > 0$.

A) $x > \frac{2}{3}$; B) $x < \frac{3}{2}$;
C) $x < -\frac{3}{2},\ x > \frac{2}{3}$; D) $-\frac{3}{2} < x < \frac{2}{3}$.

9. $-4x^2 + 8x - 3 > 0$.

A) $x > \frac{3}{2}$; B) $x < \frac{1}{2}$;
C) $x < -\frac{1}{2}$; D) $\frac{1}{2} < x < \frac{3}{2}$.

10. $\dfrac{x^2 - 7x + 10}{x^2 - 3x - 10} \le 0$.

A) $-2 < x \leq 2$; B) $-2 < x < 5$;
C) $x \neq -2,\ x \neq 5$; D) $-2 < x < 0$.

11. $\frac{x^2+x}{-x^2+6x-8} \geq 0$.

A) $-1 \leq x \leq 0,\ 2 < x < 4$; B) $-2 < x < 4$;
C) $0 \leq x \leq 1$; D) $-1 \leq x < 4$.

12. $\frac{x^2-1}{x^2-x-6} \geq 0$.

A) $-2 < x < 3$; B) $x < -2;\ -1 \leq x \leq 1,\ x > 3$;
C) $-1 \leq x < 3$; D) $x \neq -2,\ x \neq 3$.

13. $x^2 + 6x + 5 < 0$ ten'sizliginin' barli'q pu'tin sheshimlerinin' qosi'ndi'si'n tabi'n'.

A) 10; B) 9; C) -9; D) -10.

14. $\frac{x^2-6x-7}{x^2+4x+4} \leq 0$ ten'sizliginin' barli'q natural sheshimlerinin' qosi'ndi'si'n tabi'n'.

A) 29; B) 24; C) 25; D) 28.

15. p ni'n' neshe pu'tin ma'nisinde $x^2 + px + 9 = 0$ ten'lemesi haqi'yqi'y korenge iye yemes?

A) 11; B) 8; C) 13; D) 12.

Sheshiliwi: $D = b^2 - 4ac = p^2 - 4 \cdot 1 \cdot 9 = p^2 - 36 < 0$ boli'wi' kerek. $p^2 - 36 < 0$ ten'sizliktin' sheshimi $|p| < 6$, yag'ni'y $-6 < p < 6$ arali'qtan ibarat, $(-6;\ 6)$ arali'qta bolsa 11 pu'tin san bar. *Juwabi'*: A) 11.

16. a ni'n' qanday ma'nislerinde $ax^2 + 4x + 9a < 0$ ten'sizligi x ti'n' barli'q ma'nislerinde wori'nli' boladi'?

A) $a < -\frac{2}{3}$; B) $a > \frac{2}{3}$; C) $a < -1$; D) $a > 1$.

17. k ni'n' qanday yen' kishi pu'tin ma'nisinde $x^2 - 2(k+3)x + 20 + k^2 = 0$ ten'lemesi yeki tu'rli haqi'yqi'y korenlerge iye boladi'?

A) $k = 3$; B) $k = 2$; C) $k = 1$; D) $k = -2$.

18. k ni'n' qanday ma'nislerinde $\frac{4x-3}{x+2} = k+1$ ten'leme teris korenge iye?

A) $\frac{3}{4} < k < 2$; B) $\frac{5}{2} < k < 3$; C) $k < -\frac{5}{2}$, $k > 3$; D) $k > 3$.

19. a ni'n' qanday ma'nisinde $ax^2 - 8x - 2 < 0$ ten'sizligi x ti'n' barli'q ma'nislerinde wori'nli' boladi'?

A) $-8 < a < 8$; B) $a \geq 8$; C) $a < 8$; D) $a < -8$.

20. a ni'n' qanday ma'nislerinde $4(x + 2) = 5 - ax$ ten'lemenin' koreni -2 den u'lken boladi'?

A) $a < -4, a > -\frac{5}{2}$; B) $-\frac{5}{2} < a < 4$;

C) $-4 < a < \frac{5}{2}$; D) $a \geq -4$.

21. Ten'sizlikti sheshin': $\frac{1}{x} \geq x$.

A) $x \leq -1$, $0 < x \leq 1$; B) $x \leq -1$;
C) $0 < x < 1$; D) $-1 \leq x \leq 1$.

22. Ten'sizlikti sheshin': $\frac{2x-1}{x} < 2$.

A) $x < 0$; B) $x > 0$; C) $\frac{1}{2} < x < 2$; D) $x < 2$.

23. $\frac{x-3}{x+2} \leq 0$ ten'sizliginin' barli'q pu'tin sheshimlerinin' qosi'ndi'si'n tabi'n'.

A) -3; B) 6; C) 3; D) 4.

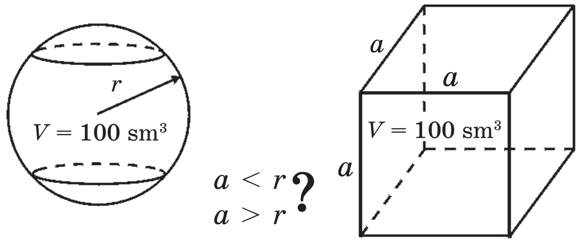
24. $\frac{x^2-x-20}{x^2+11x+24} \geq 0$ ten'sizligin sheshin'.

A) $x < -8$, $x \geq 5$; B) $-4 \leq x < -3$; C) $-4 \leq x \leq 5$;
D) $x < -8, -4 \leq x < -3, x \geq 5$.

25. $\frac{-x^2-5x+6}{x^2+7x+10} \leq 0$ ten'sizliginin' barli'q pu'tin sheshimlerinin' ko'beymesin tabi'n'.

A) 0; B) -1; C) -6; D) 2.

III BAP. RACIONAL KO'RSETKISHLI DA'REJE



9- §. PU'TIN KO'RSETKISHLI DA'REJE

Natural ko'rsetkishli da'rejenin' qa'siyetleri qaralg'anda da're-jelerdi bo'liwdin'

$$a^n : a^m = a^{n-m} \qquad (1)$$

qa'siyeti $n > m$ ha'm $a \neq 0$ bolg'anda duri's bolatug'i'nli'g'i' ayti'p wo'tilgen yedi.

Yeger $n \leq m$ bolsa, wonda **(1)** ten'liktin' won' bo'limindegi $n - m$ da'reje ko'rsetkishi teris san yaki nolge ten' boladi'.

Teris ha'm nol ko'rsetkishli da'reje sonday ani'qlanadi', wol **(1)** ten'lik tek $n > m$ bolg'anda g'ana yemes, ba'lki $n \leq m$ bolg'anda da duri's boladi'. Mi'sali', $n = 2$, $m = 5$ bolg'anda **(1)** formula boyi'nsha to'mendegini payda yetemiz:

$$a^2 : a^5 = a^{2-5} = a^{-3}.$$

Yekinshi ta'repten,

$$a^2 : a^5 = \frac{a^2}{a^5} = \frac{a^2}{a^2a^3} = \frac{1}{a^3}.$$

Soni'n' ushi'n $a^{-3} = \dfrac{1}{a^3}$ dep yesaplanadi'.

1-ani'qlama. ***Yeger $a \neq 0$ ha'm n — natural san bolsa, wonda***

$$a^{-n} = \frac{1}{a^n}$$

boladi'.

M i ' s a l l a r :

1) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$; 2) $(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{81}$;

3) $(-1)^{-7} = \frac{1}{(-1)^7} = \frac{1}{-1} = -1$; 4) $(-0,5)^{-3} = \frac{1}{(-0,5)^3} = -\frac{1}{0,125} = -8$;

5) $(-0,2)^{-5} = \frac{1}{(-0,2)^5} = -5^5 = -3125$.

Yeger $n = m$ bolsa, wonda (1) formula boyi'nsha to'mendegini payda yetemiz:
$$a^n : a^n = a^{n-n} = a^0.$$

Yekinshi ta'repten, $a^n : a^n = \frac{a^n}{a^n} = 1$. Soni'n' ushi'n $a^0 = 1$ dep yesaplanadi'.

2 - a n i ' q l a m a . ***Yeger $a \neq 0$ bolsa, wonda $a^0 = 1$ boladi'.***

Mi'sali', $3^0 = 1$, $\left(\frac{2}{5}\right)^{\cdot} = 1$; $(0,75)^0 = 1$; $(-3)^0 = 1$; $\left(-\frac{1}{7}\right)^0 = 1$.

Teris ko'rsetkishli da'rejelerden sandi' *standart tu'rde jazi'wda* paydalani'lg'an. Mi'sali',

$$0,00027 = 2,7 \cdot \frac{1}{10^4} = 2,7 \cdot 10^{-4}; \quad 0,000016 = 1,6 \cdot \frac{1}{10^5} = 1,6 \cdot 10^{-5}.$$

Natural ko'rsetkishli da'rejelerdin' barli'q qa'siyetleri qa'legen pu'tin ko'rsetkishli da'rejeler ushi'n da duri's boladi'.

Qa'legen $a \neq 0$, $b \neq 0$ ha'm qa'legen n ha'm m ler ushi'n to'mendegi ten'likler duri's boladi':

1. $a^n a^m = a^{n+m}$. 2. $(a^n)^m = a^{nm}$. 3. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

4. $a^n : a^m = a^{n-m}$. 5. $(ab)^n = a^n b^n$.

Mi'sali', $n < 0$ bolg'anda $(ab)^n = a^n b^n$ ten'liginin' duri's yekenligin da'lilleymiz.

n –pu'tin teris san bolsi'n. Wonda $n = -k$ (bunda k – natural san). Teris ko'rsetkishli da'rejenin' ani'qlamasi'nan ha'm natural ko'rsetkishli da'rejenin' qa'siyetlerinen paydalani'p, to'mendegini payda yetemiz:

$$(ab)^n = (ab)^{-k} = \frac{1}{(ab)^k} = \frac{1}{a^k b^k} = \frac{1}{a^k} \cdot \frac{1}{b^k} = a^{-k} \cdot b^{-k} = a^n b^n.$$

Pu'tin ko'rsetkishli da'rejelerdin' barli'q qa'siyetleri de usi'g'an uqsas da'lillenedi.

Pu'tin ko'rsetkishli da'rejelerdin' qa'siyetlerin qollani'wg'a mi'sallar keltiremiz:

1) $4^{-3}\cdot 4^{11}\cdot 4^{-6}=4^{-3+11-6}=4^{2}=16;$

2) $\left(\dfrac{p^{-3}}{3q^{2}}\right)^{-2}=\dfrac{p^{-3\cdot(-2)}}{3^{-2}\cdot q^{2\cdot(-2)}}=\dfrac{3^{2}p^{6}}{q^{-4}}=9p^{6}q^{4}.$

M a ' s e l e . $a^{6}(a^{-2}-a^{-4})(a^{2}+a^{3})^{-1}$ an'latpasi'n a'piwayi'lasti'ri'n'.

$$\triangle\ a^{6}(a^{-2}-a^{-4})(a^{2}+a^{3})^{-1}=a^{6}\left(\frac{1}{a^{2}}-\frac{1}{a^{4}}\right)\cdot\frac{1}{a^{2}+a^{3}}=$$

$$=a^{6}\cdot\frac{a^{2}-1}{a^{4}}\cdot\frac{1}{a^{2}(1+a)}=a-1.\ \blacktriangle$$

S h i ' n i ' g ' i ' w l a r

109. Yesaplan':

1) $2^{3}+(-3)^{3}-(-2)^{2}+(-1)^{5};$ 2) $(-7)^{2}-(-4)^{3}-3^{4};$

3) $13\cdot 2^{3}-9\cdot 2^{3}+2^{3};$ 4) $6(-2)^{3}-5(-2)^{3}-(-2)^{3}.$

110. An'latpani' natural ko'rsetkishli da'reje tu'rinde ko'rsetin':

1) $\dfrac{7^{2}\cdot 7^{15}}{7^{13}};$ 2) $\dfrac{5^{3}\cdot 5^{10}\cdot 5}{5^{4}\cdot 5^{15}};$ 3) $\dfrac{a^{2}a^{8}b^{3}}{a^{9}b^{2}};$ 4) $\dfrac{c^{3}d^{5}c^{9}}{c^{10}d^{7}}.$

111. (Awi'zeki.) Yesaplan':

1) $1^{-5};$ 2) $4^{-3};$ 3) $(-10)^{0};$ 4) $(-5)^{-2};$ 5) $\left(\dfrac{1}{2}\right)^{-4};$ 6) $\left(\dfrac{3}{4}\right)^{-1}.$

112. Teris ko'rsetkishli da'reje tu'rinde jazi'n':

1) $\dfrac{1}{4^{5}};$ 2) $\dfrac{1}{21^{3}};$ 3) $\dfrac{1}{x^{7}};$ 4) $\dfrac{1}{a^{9}};$ 5) $\dfrac{1}{z^{11}}.$

Yesaplan' **(113–114)**:

113. 1) $\left(\dfrac{10}{3}\right)^{-3};$ 2) $\left(-\dfrac{9}{11}\right)^{-2};$ 3) $(0{,}2)^{-4};$ 4) $(0{,}5)^{-5};$

5) $-(-17)^{-1};$ 6) $-(-13)^{-2};$ 7) $(-0{,}7)^{-2}.$

114. 1) $3^{-1}+(-2)^{-2}$; | 2) $\left(\frac{2}{3}\right)^{-3}-4^{-2}$; | 3) $(0,2)^{-2}+(0,5)^{-5}$;

4) $(-0,1)^{-3}-(-0,2)^{-3}$; | 5) $5^{-2}+(-3)^{-2}$; | 6) $-(-0,1)^{-4}+(-0,2)^{-2}$.

115. (Awi'zeki.) Bir menen sali'sti'ri'n':

1) 12^{-3}; 2) 21^{0}; 3) $(0,6)^{-5}$; 4) $\left(\frac{5}{19}\right)^{-4}$; 5) $\left(\frac{6}{7}\right)^{-2}$.

116. An'latpani' teris ko'rsetkishsiz da'reje tu'rinde jazi'n':

1) $(x-y)^{-2}$; 2) $(x+y)^{-3}$; 3) $3^{-5}c^{8}$; 4) $9a^{3}b^{-4}$;

5) $a^{-1}b^{2}c^{-3}$; 6) $a^{2}b^{-1}c^{-4}$; 7) $4^{-7}\cdot a^{-3}$; 8) $5^{-2}\cdot a^{4}b^{-2}$.

Yesaplan' (**117–118**):

117. 1) $\left(\frac{1}{7}\right)^{-3}\cdot\left(\frac{1}{7}\right)$; 2) $\left(-\frac{1}{5}\right)\cdot\left(-\frac{1}{5}\right)^{-4}$;

3) $0,3^{7}\cdot 0,3^{-10}$; 4) $17^{-5}\cdot 17^{3}\cdot 17$.

118. 1) $9^{7}:9^{10}$; 2) $(0,2)^{2}:(0,2)^{-2}$;

3) $\left(\frac{2}{13}\right)^{12}:\left(\frac{2}{13}\right)^{-10}$; 4) $\left(\frac{2}{5}\right)^{3}:\left(\frac{2}{5}\right)^{-1}$.

119. Da'rejeni da'rejege ko'terin':

1) $(a^{3})^{-5}$; 2) $(b^{-2})^{-4}$; 3) $(a^{-3})^{7}$; 4) $(b^{7})^{-4}$; 5) $(a^{-2})^{-5}$.

120. Ko'beymeni da'rejege ko'terin':

1) $(ab^{-2})^{3}$; 2) $(a^{2}b^{-1})^{4}$; 3) $(2a^{2})^{-6}$; 4) $(3a^{3})^{-4}$; 5) $(a^{-2})^{-3}$.

121. A'mellerdi wori'nlan'.

1) $\left(\frac{a^{8}}{b^{7}}\right)^{-2}$; 2) $\left(\frac{m^{-4}}{n^{-5}}\right)^{-3}$; 3) $\left(\frac{2x^{6}}{3y^{-4}}\right)^{2}$; 4) $\left(\frac{-4x^{-5}y}{z^{3}}\right)^{3}$.

122. 1) $x=5,\ y=6,7$ bolg'anda, $(x^{2}y^{-2}-4y^{-2})\cdot\left(\frac{1}{y}\right)^{-2}$ nin' ma'nisin yesaplan'; 2) $a=2,\ b=-3$ bolg'anda $((a^{2}b^{-1})^{4}-a^{0}b^{4}):\frac{a^{4}-b^{4}}{b^{2}}$ ti'n' ma'nisin yesaplan'.

Standart tu'rinde jazi'n' **(123–124)**:

123. 1) $200\,000^4$; 2) $0{,}003^3$; 3) 4000^{-2}; 4) $0{,}002^{-3}$.

124. 1) 0,0000087; 2) 0,00000005086; 3) $\frac{1}{125}$; 4) $\frac{1}{625}$.

125. Aynani' tegislew bari'si'nda woni'n' betindegi woyi'qli'qlar shuqi'rli'g'i' $3 \cdot 10^{-3}$ mm den artpaytug'i'n bolg'anda toqtati'ladi'. Bul sandi' wonli'q bo'lshek tu'rinde jazi'n'.

126. Wortasha awi'rli'qtag'i' vodorod 0, 00 000 000 001 sekund g'ana «jasaydi'» (bar boladi'). Bul sandi' teris ko'rsetkishli da'reje tu'rinde jazi'n'.

127. Gripp virusi'ni'n' wo'lshemleri shama menen 10^{-4} mm di quraydi'. Usi' sandi' wonli'q bo'lshek tu'rinde jazi'n'.

128. Bo'lshekti da'reje tu'rinde ko'rsetin' ha'm woni'n' ma'nisin a ni'n' berilgen ma'nisinde tabi'n':

1) $\frac{a^8a^{-7}}{a^{-2}}$, $a = 0{,}8$; 2) $\frac{a^{15}a^3}{a^{13}}$, $a = \frac{1}{2}$; 3) $\frac{a^{10} \cdot a^2}{a^{14}}$, $a = 0{,}2$.

129. Yesaplan':

1) $((-20)^7)^{-7} : ((-20)^{-6})^8 + 2^{-2}$; | 2) $((-17)^{-4})^{-6} : ((-17)^{-13})^{-2} - \left(\frac{1}{17}\right)^2$.

130. A'piwayi'lasti'ri'n':

1) $(a^{-3} + b^{-3}) \cdot (a^{-2} - b^{-2})^{-1} \cdot (a^{-2} - a^{-1}b^{-1} + b^{-2})^{-1}$;

2) $(a^{-2}b - ab^{-2}) \cdot (a^{-2} + a^{-1}b^{-1} + b^{-2})^{-1}$.

10- §. NATURAL KO'RSETKISHLI DA'REJENIN' ARIFMETIKALI'Q KORENI

Worta Aziyali' ataqli' matematik ha'm astronom **Jamshid ibn Masud ibn Mahmud G'iyosiddin al-Koshiy** (shama menen 1430-ji'li' qayti's bolg'an) sanlardan qa'legen n-da'rejeli koren shi'g'ari'w a'melin ashqan. Woni'n' «Yesap gilti» atamali' miynetinin' besinshi babi' «da'rejenin' tiykari'n ani'qlaw» dep atalg'an.

To'mendegi ma'seleni qarayi'q.

1-ma'sele. Ten'lemeni sheshin': $x^4 = 81$.

△ Ten'lemeni $x^4 - 81 = 0$ yaki $(x^2 - 9)(x^2 + 9) = 0$ tu'rinde jazi'p alami'z. $x^2 + 9 \neq 0$ bolg'ani' ushi'n $x^2 - 9 = 0$ boladi', bunnan, $x_1 = 3$, $x_2 = -3$. ▲

Solay yetip, $x^4 = 81$ ten'lemesi yeki haqi'yqi'y korenge iye: $x_1 = 3$, $x_2 = -3$. Wolardi' 81 sani'ni'n' 4- *da'rejeli korenleri*, won' korendi (3 sani'n) bolsa 81 sani'ni'n' 4- *da'rejeli arifmetikali'q koreni* delinedi ha'm bi'layi'nsha belgilenedi: $\sqrt[4]{81}$. Solay yetip, $\sqrt[4]{81} = 3$.

$x^n = a$ ten'lemesi (bunda n — natural san, a — teris yemes san) jalg'i'z teris yemes korenge iye yekenligin da'lillew mu'mkin. Bul korendi *a sani'ni'n' n — da'rejeli arifmetikali'q koreni* delinedi.

! **Ani'qlama.** ***a teris yemes sani'ni'n' $n \geq 2$ natural ko'rsetkishli arifmetikali'q koreni dep, n - da'rejesi a g'a ten' bolg'an teris yemes sang'a ayti'ladi'.***

a sani'ni'n' n- da'rejeli arifmetikali'q koreni bi'lay belgilenedi: $\sqrt[n]{a}$. a sani' *koren asti'ndag'i' an'latpa* delinedi. Yeger $n = 2$ bolsa, wonda $\sqrt[2]{a}$ worni'na $\sqrt{a}$ tu'rinde jazi'ladi'.

Yekinshi da'rejeli arifmetikali'q koren *kvadrat koren* dep te ataladi', 3-da'rejeli koren bolsa *kub koren* delinedi.

So'z n-da'rejeli arifmetikali'q koren haqqi'nda bolg'anli'g'i' ani'q bolg'an jag'dayda qi'sqasha «n - da'rejeli koren» delinedi.

! **Ani'qlamadan paydalani'p, $\sqrt[n]{a}$ ni'n' b g'a ten' yekenligin da'lillew ushi'n: 1) $b \geq 0$; 2) $b^n = a$ yekenligin ko'rsetiw kerek.**

Mi'sali', $\sqrt[3]{64} = 4$, sebebi $4 > 0$ ha'm $4^3 = 64$.

Arifmetikali'q korennin' ani'qlamasi'nan, yeger $a \geq 0$ bolsa, wonda

$$\left(\sqrt[n]{a}\right)^n = a, \quad \sqrt[n]{a^n} = a$$

bolatug'i'nli'g'i' kelip shi'g'adi'.

Mi'sali', $\left(\sqrt[5]{7}\right)^5 = 7$, $\sqrt[6]{13^6} = 13$.

n - da'rejeli koren izleniwshi a'mel *n - da'rejeli koren shi'g'ari'w a'meli* delinedi. Wol n - da'rejege ko'teriw a'meline keri a'mel boladi'.

2-ma'sele. $x^3 = -8$ ten'lemesin sheshin'.

△ Bul ten'lemeni $-x^3 = 8$ yaki $(-x)^3 = 8$ tu'rinde jazi'w mu'mkin. $-x = y$ dep belgileymiz, wonda $y^3 = 8$ boladi'.

Bul ten'leme bir korenge iye: $y = \sqrt[3]{8} = 2$. $y^3 = 8$ ten'leme teris korenge iye yemes, sebebi $y < 0$ bolg'anda $y^3 < 0$ boladi'. $y = 0$ sani' da bul ten'lemenin' koreni bola almaydi'.

Solay yetip, $y^3 = 8$ ten'lemesi tek g'ana bir $y = 2$ korenge iye, demek, $x^3 = -8$ ten'lemesi de tek g'ana bir korenge iye: $x = -y = -2$.

Juwabi': $x = -2$. ▲

$x^3 = -8$ ten'lemesinin' sheshimin qi'sqasha to'mendegishe jazi'w mu'mkin:

$$x = -\sqrt[3]{8} = -2.$$

Uluwma, qa'legen taq $2k + 1$ natural sani' ushi'n $a < 0$ bolg'anda $x^{2k+1} = a$ ten'lemesi tek g'ana birew, buni'n' u'stine wol teris korenge iye. Bul koren arifmetikali'q koren si'yaqli' belgilenedi: $\sqrt[2k+1]{a}$. Bul *teris sanni'n' taq da'rejeli koreni* dep ataladi'.

Mi'sali', $\sqrt[3]{-27} = -3$, $\sqrt[5]{-32} = -2$.

Teris a sani'ni'n' taq da'rejeli koreni menen $-a = |a|$ sani'ni'n' arifmetikali'q koreni arasi'nda mi'na ten'lik bar:

$$\sqrt[2k+1]{a} = -\sqrt[2k+1]{-a} = -\sqrt[2k+1]{|a|}.$$

Mi'sali', $\sqrt[5]{-243} = -\sqrt[5]{243} = -3$.

Shi'ni'g'i'wlar

131. (Awi'zeki.) 1) Sanni'n' arifmetikali'q kvadrat korenin tabi'n':

$$1; \quad 0; \quad 16; \quad 0{,}81; \quad 169; \quad \frac{16}{121}; \quad \frac{49}{144}.$$

2) Sanni'n' arifmetikali'q kub korenin tabi'n':

$$1; \quad 0; \quad 5; \quad \frac{1}{27}; \quad 0{,}027; \quad 0{,}064; \quad 0{,}729; \quad \frac{1}{343}.$$

3) Sanni'n' to'rtinshi da'rejeli arifmetikali'q korenin tabi'n':

$$0; \quad 1; \quad 16; \quad \frac{16}{81}; \quad \frac{256}{625}; \quad 0{,}0016; \quad \frac{625}{1296}.$$

Yesaplan’ **(132–134)**:

132. 1) $\sqrt[6]{36^3}$; 2) $\sqrt[12]{64^2}$; 3) $\sqrt[4]{\left(\frac{1}{25}\right)^2}$; 4) $\sqrt[8]{225^4}$; 5) $\sqrt[7]{2\cdot 4^3}$.

133. 1) $\sqrt[3]{10^6}$; 2) $\sqrt[3]{3^{12}}$; 3) $\sqrt[4]{\left(\frac{1}{2}\right)^{12}}$; 4) $\sqrt[4]{\left(\frac{1}{3}\right)^{16}}$; 5) $\sqrt[5]{32^2}$.

134. 1) $\sqrt[3]{-8}$; | 2) $\sqrt[15]{-1}$; | 3) $\sqrt[3]{-\frac{1}{27}}$; | 4) $\sqrt[5]{-1024}$; | 5) $\sqrt[3]{-34^3}$; | 6) $\sqrt[7]{-8^7}$.

135. Ten’lemeni sheshin’:

1) $x^4=81$; 2) $x^5=-\frac{1}{32}$; 3) $5x^5=-160$; 4) $2x^6=128$.

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136. x ti’n’ qanday ma’nislerinde an’latpa ma’niske iye boladi’:

1) $\sqrt[6]{2x-3}$; 2) $\sqrt[3]{x+3}$; 3) $\sqrt[3]{2x^2-x-1}$; 4) $\sqrt[4]{\frac{2-3x}{2x-4}}$?

Yesaplan’ **(137–138)**:

137. 1) $\sqrt[3]{-125}+\frac{1}{8}\sqrt[6]{64}$; 2) $\sqrt[5]{32}-0{,}5\sqrt[3]{-216}$;

3) $-\frac{1}{3}\sqrt[4]{81}+\sqrt[4]{625}$; 4) $\sqrt[3]{-1000}-\frac{1}{4}\sqrt[4]{256}$;

5) $\sqrt[4]{0{,}0001}-2\sqrt{0{,}25}+\sqrt[5]{\frac{1}{32}}$; 6) $\sqrt[5]{\frac{1}{243}}+\sqrt[3]{-0{,}001}-\sqrt[4]{0{,}0016}$.

138. 1) $\sqrt{9+\sqrt{17}}\cdot\sqrt{9-\sqrt{17}}$; 2) $\left(\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}}\right)^2$;

3) $\left(\sqrt{5+\sqrt{21}}+\sqrt{5-\sqrt{21}}\right)^2$; 4) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$.

139. 1) a) $x\ge 2$; b) $x<2$ bolg’anda $\sqrt[3]{(x-2)^3}$ ti’ a’piwayi’lasti’ri’n’;

2) a) $x\le 3$; b) $x>3$ bolg’anda $\sqrt{(3-x)^6}$ ni’ a’piwayi’lasti’ri’n’.

140. $1987<\sqrt{n}<1988$ bolatug’i’n neshe n natural sani’ bar?

11- §. ARIFMETIKALI'Q KORENNIN' QA'SIYETLERI

n-da'rejeli arifmetikali'q koren to'mendegi qa'siyetlerge iye:

Yeger $a \ge 0$, $b > 0$, n, m n, m natural sanlar boli'p, $n \ge 2$, $m \ge 2$ bolsa, wonda to'mendegi ten'likler duri's boladi':

!

1. $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$.
2. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$.
3. $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$.
4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a}$.

1-qa'siyette b sani' 0 ge ten' boli'wi' da mu'mkin. 3-qa'siyette m sani', yeger $a > 0$ bolsa, qa'legen pu'tin san boli'wi' mu'mkin.

Mi'sali',

$$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$$

yekenligin da'lilleymiz.

○ Arifmetikali'q korennin' ani'qlamasi'nan paydalanami'z:

1) $\sqrt[n]{a}\sqrt[n]{b} \ge 0$, sebebi $a \ge 0$ ha'm $b \ge 0$.

2) $\left(\sqrt[n]{a}\sqrt[n]{b}\right)^n = ab$, sebebi $\left(\sqrt[n]{a}\sqrt[n]{b}\right)^n = \left(\sqrt[n]{a}\right)^n\left(\sqrt[n]{b}\right)^n = ab$. ●

Qalg'an qa'siyetleri de usi'g'an uqsas da'lillenedi.

Arifmetikali'q korennin' qa'siyetlerin qollani'wg'a mi'sallar keltiremiz.

1) $\sqrt[4]{27}\cdot\sqrt[4]{3} = \sqrt[4]{27\cdot 3} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$.

2) $\sqrt[3]{\dfrac{256}{625}} : \sqrt[3]{\dfrac{4}{5}} = \sqrt[3]{\dfrac{256}{625} : \dfrac{4}{5}} = \dfrac{\sqrt[3]{64}}{\sqrt[3]{125}} = \dfrac{4}{5}$;

3) $\sqrt[7]{5^{21}} = (\sqrt[7]{5^7})^3 = 5^3 = 125$;

4) $\sqrt[3]{\sqrt[4]{4096}} = \sqrt[12]{4096} = \sqrt[12]{2^{12}} = 2$;

5) $\left(\sqrt[4]{9}\right)^{-2} = \sqrt[4]{9^{-2}} = \sqrt[4]{\dfrac{1}{81}} = \dfrac{1}{3}$.

M a ' s e l e . An'latpani' a'piwayi'lasti'ri'n':

$$\frac{\left(\sqrt[4]{a^3b^2}\right)^4}{\sqrt[3]{\sqrt{a^{12}b^6}}},$$

bunda $a > 0$, $b > 0$.

△ Arifmetikali'q korennin' qa'siyetlerinen paydalani'p, to'mendegige iye bolami'z:

$$\frac{\left(\sqrt[4]{a^3b^2}\right)^4}{\sqrt[3]{\sqrt{a^{12}b^6}}} = \frac{a^3b^2}{\sqrt[6]{a^{12}b^6}} = \frac{a^3b^2}{a^2b} = ab. \blacktriangle$$

S h i ' n i ' g ' i ' w l a r[1]

Yesaplan' (**141–144**):

141. 1) $\sqrt[3]{343 \cdot 0,125}$; 2) $\sqrt[3]{864 \cdot 216}$; 3) $\sqrt[4]{256 \cdot 0,0081}$;
4) $\sqrt[5]{32 \cdot 100000}$; 5) $\sqrt[3]{125 \cdot 27}$; 6) $\sqrt[4]{0,0016 \cdot 625}$.

142. 1) $\sqrt[3]{5^3 \cdot 7^3}$; | 2) $\sqrt[4]{11^4 \cdot 3^4}$; | 3) $\sqrt[5]{(0,2)^5 \cdot 8^5}$; | 4) $\sqrt[7]{\left(\frac{1}{3}\right)^7 \cdot 21^7}$.

143. 1) $\sqrt[3]{2} \cdot \sqrt[3]{500}$; | 2) $\sqrt[3]{0,2} \cdot \sqrt[3]{0,04}$; | 3) $\sqrt[4]{324} \cdot \sqrt[4]{4}$; | 4) $\sqrt[5]{2} \cdot \sqrt[5]{16}$.

144. 1) $\sqrt[5]{3^{10} \cdot 2^{15}}$; 2) $\sqrt[3]{2^3 \cdot 5^6}$; 3) $\sqrt[4]{3^{12} \cdot \left(\frac{1}{3}\right)^8}$; 4) $\sqrt[10]{4^{30} \cdot \left(\frac{1}{2}\right)^{20}}$.

145. Korenin tabi'n':

1) $\sqrt[3]{64x^3z^6}$; 2) $\sqrt[4]{a^8b^{12}}$; 3) $\sqrt[5]{32x^{10}y^{20}}$; 4) $\sqrt[6]{a^{12}b^{18}}$.

146. An'latpani' a'piwayi'lasti'ri'n':

1) $\sqrt[3]{2ab^2} \cdot \sqrt[3]{4a^2b}$; 2) $\sqrt[4]{3a^2b^3} \cdot \sqrt[4]{27a^2b}$; 3) $\sqrt[4]{\frac{ab}{c}} \cdot \sqrt[4]{\frac{a^3c}{b}}$;

4) $\sqrt[3]{\frac{16a}{b^2}} \cdot \sqrt[3]{\frac{1}{2ab}}$; 5) $\sqrt[4]{2a^2b} \cdot \sqrt[4]{8a^2b^2}$; 6) $\sqrt[3]{\frac{250}{a^4 \cdot b}} \cdot \sqrt[3]{\frac{a}{2b^2}}$.

Yesaplan' (**147–148**):

147. 1) $\sqrt[3]{\frac{64}{125}}$; 2) $\sqrt[4]{\frac{16}{81}}$; 3) $\sqrt[3]{3\frac{3}{8}}$; 4) $\sqrt[5]{7\frac{19}{32}}$; 5) $\sqrt[4]{2\frac{113}{256}}$.

148. 1) $\sqrt[4]{324} : \sqrt[4]{4}$; 2) $\sqrt[3]{128} : \sqrt[3]{2000}$; 3) $\frac{\sqrt[3]{16}}{\sqrt[3]{2}}$; 4) $\frac{\sqrt[5]{256}}{\sqrt[5]{8}}$;

5) $\left(\sqrt{20} - \sqrt{45}\right) : \sqrt{5}$; | 6) $\left(\sqrt[3]{625} - \sqrt[3]{5}\right) : \sqrt[3]{5}$; | 7) $\left(\sqrt{27} - \sqrt{12}\right) : \sqrt{3}$.

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[1] Bul jerde ha'm bunnan keyin, yeger qosi'msha sha'rtler bolmasa, ha'ripler menen won' sanlar belgilengen dep yesaplaymi'z.

149. An'latpani' a'piwayi'lasti'ri'n':

1) $\sqrt[5]{a^6b^7} : \sqrt[5]{ab^2}$; 2) $\sqrt[3]{81x^4y} : \sqrt[3]{3xy}$; 3) $\sqrt[3]{\frac{3x}{y^2}} : \sqrt[3]{\frac{y}{9x^2}}$;

4) $\sqrt[4]{\frac{2b}{a^3}} : \sqrt[4]{\frac{a}{8b^3}}$; 5) $\sqrt[4]{32a^4b} : \sqrt[4]{2b^5}$; 6) $\sqrt[3]{\frac{4a}{b^2}} : \sqrt[3]{\frac{b}{2a^2}}$.

Yesaplan' **(150–151):**

150. 1) $\left(\sqrt[6]{7^3}\right)^2$; 2) $\left(\sqrt[6]{9}\right)^{-3}$; 3) $\left(\sqrt[10]{32}\right)^2$; 4) $\left(\sqrt[8]{16}\right)^{-4}$.

151. 1) $\sqrt{\sqrt[3]{729}}$; 2) $\sqrt{\sqrt{1024}}$; 3) $\sqrt[3]{\sqrt[3]{9}} \cdot \sqrt[9]{3^7}$; 4) $\sqrt[4]{\sqrt[3]{25}} \cdot \sqrt[6]{5^5}$.

152. An'latpani' a'piwayi'lasti'ri'n':

1) $\left(\sqrt[3]{x}\right)^6$; 2) $\left(\sqrt[3]{y^2}\right)^3$; 3) $\left(\sqrt{a} \cdot \sqrt[3]{b}\right)^6$; 4) $\left(\sqrt[3]{a^2} \cdot \sqrt[4]{b^3}\right)^{12}$;

5) $\left(\sqrt{\sqrt[3]{a^2b}}\right)^6$; | 6) $\left(\sqrt[3]{\sqrt[4]{27a^3}}\right)^4$; | 7) $\left(\sqrt[4]{a^3} \cdot \sqrt{a}\right)^8$; | 8) $\left(\sqrt[5]{\sqrt[3]{a^2}}\right)^{15}$.

Yesaplan' **(153–155):**

153. 1) $\sqrt[3]{\frac{3}{2}} \cdot \sqrt[3]{2\frac{1}{4}}$; 2) $\sqrt[4]{\frac{3}{4}} \cdot \sqrt[4]{6\frac{3}{4}}$; 3) $\sqrt[4]{15\frac{5}{8}} : \sqrt[4]{\frac{2}{5}}$;

4) $\sqrt[3]{22\frac{1}{2}} \cdot \sqrt[3]{6\frac{2}{3}}$; 5) $\left(\sqrt[3]{\sqrt{27}}\right)^2$; 6) $\left(\sqrt{\sqrt[3]{16}}\right)^3$.

154. 1) $\sqrt[3]{\frac{ab^2}{c}} \cdot \sqrt[3]{\frac{a^5b}{c^2}}$; 2) $\sqrt[5]{\frac{8a^3}{b^2}} \cdot \sqrt[5]{\frac{4a^7}{b^3}}$; 3) $\frac{\sqrt[4]{a^2b^2c} \cdot \sqrt[4]{a^3b^3c^2}}{\sqrt[4]{abc^3}}$;

4) $\frac{\sqrt[3]{2a^4b} \cdot \sqrt[3]{4ab}}{2b\sqrt[3]{a^2b^2}}$; 5) $\left(\sqrt[5]{a^3}\right)^5 \cdot \left(\sqrt[3]{b^2}\right)^3$; 6) $\left(\sqrt[4]{a^3b^3}\right)^4 : \left(\sqrt[3]{ab^2}\right)^3$.

155. 1) $\frac{\sqrt[3]{49} \cdot \sqrt[3]{112}}{\sqrt[3]{250}}$; 2) $\frac{\sqrt[4]{54} \cdot \sqrt[4]{120}}{\sqrt[4]{5}}$;

3) $\frac{\sqrt[4]{32}}{\sqrt[4]{2}} + \sqrt[6]{27} - \sqrt{\sqrt[3]{64}}$; 4) $\sqrt[3]{3\frac{3}{8}} + \sqrt[4]{18}\sqrt[4]{4\frac{1}{2}} - \sqrt{\sqrt{256}}$;

5) $\sqrt[3]{11-\sqrt{57}} \cdot \sqrt[3]{11+\sqrt{57}}$; 6) $\sqrt[4]{17-\sqrt{33}} \cdot \sqrt[4]{17+\sqrt{33}}$.

An'latpani' a'piwayi'lasti'ri'n' **(156–157)**:

156. 1) $\sqrt[3]{2ab}\cdot\sqrt[3]{4a^2b}\cdot\sqrt[3]{27b}$; 2) $\sqrt[4]{abc}\cdot\sqrt[4]{a^3b^2c}\cdot\sqrt[4]{b^5c^2}$;

3) $\dfrac{\sqrt[5]{a^3b^2}\cdot\sqrt[5]{3a^2b^3}}{\sqrt[5]{3ab}}$; 4) $\dfrac{\sqrt[4]{8x^2y^5}\cdot\sqrt[4]{4x^3y}}{\sqrt[4]{2xy^2}}$.

157. 1) $\sqrt[3]{\sqrt[3]{a^{18}}}+\left(\sqrt{\sqrt[3]{a^4}}\right)^3$; 2) $\left(\sqrt{\sqrt[3]{x^2}}\right)^3+2\left(\sqrt[4]{\sqrt{x}}\right)^8$;

3) $2\sqrt{\sqrt{a^4b^8}}-\left(\sqrt[3]{\sqrt{a^3b^6}}\right)^2$; 4) $\sqrt[3]{\sqrt{x^6y^{12}}}-\left(\sqrt[5]{xy^2}\right)^5$;

5) $\left(\sqrt{\sqrt[4]{x^8y^2}}\right)^4-\left(\sqrt[4]{x^2y^8}\right)^2$; 6) $\left(\left(\sqrt[5]{a\sqrt[5]{a}}\right)^5-\sqrt[5]{a}\right):\sqrt[10]{a^2}$.

158. Yesaplan':

1) $\dfrac{\sqrt{3}\cdot\sqrt[3]{9}}{\sqrt[6]{3}}$; 2) $\dfrac{\sqrt[3]{7}\cdot\sqrt[4]{343}}{\sqrt[12]{7}}$; 3) $\dfrac{\sqrt[3]{16}\cdot\sqrt[5]{3072}}{\sqrt[3]{2}\cdot\sqrt[5]{3}}$;

4) $(\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4})(\sqrt[3]{3}-\sqrt[3]{2})$; 5) $(\sqrt[3]{4}-\sqrt[3]{10}+\sqrt[3]{25})(\sqrt[3]{2}+\sqrt[3]{5})$.

159. Da'lillen': 1) $\sqrt{4+2\sqrt{3}}-\sqrt{4-2\sqrt{3}}=2$;

2) $10+4\sqrt{6}-\sqrt{10-4\sqrt{6}}=4$.

12- §. RACIONAL KO'RSETKISHLI DA'REJE

1-ma'sele. Yesaplan'. $\sqrt[4]{5^{12}}$.

△ $5^{12}=(5^3)^4$ bolg'ani' ushi'n $\sqrt[4]{5^{12}}=\sqrt[4]{(5^3)^4}=5^3=125$. ▲

Solay yetip, $\sqrt[4]{5^{12}}=5^{\frac{12}{4}}$.

Usi'g'an uqsas, $\sqrt[5]{7^{-15}}=7^{-\frac{15}{5}}$ yekenligin ko'rsetiw mu'mkin.

Uluwma, yeger n – natural san, $n\geq 2$, m – pu'tin san ha'm $\frac{m}{n}$ pu'tin san bolsa, wonda $a>0$ bolg'anda to'mendegi ten'lik duri's boladi':

$$\sqrt[n]{a^m}=a^{\frac{m}{n}}. \qquad (1)$$

○ Sha'rt boyi'nsha $\frac{m}{n}$ — pu'tin san, yag'ni'y m di n ge bo'lgende k pu'tin san payda boladi'. Bul jag'dayda $\frac{m}{n}=k$ ten'liginen $m=kn$ yekenligi kelip shi'g'adi'. Da'rejenin' ha'm arifmetikali'q korennin' qa'siyetlerin qollani'p, to'mendegini payda yetemiz:

$$\sqrt[n]{a^m}=\sqrt[n]{a^{kn}}=\sqrt[n]{\left(a^k\right)^n}=a^k=a^{\frac{m}{n}}.$$ ●

Yeger de $\frac{m}{n}$ pu'tin san bolmasa, wonda $a^{\frac{m}{n}}$ (bunda $a>0$) da'rejesi (1) formula duri'sli'g'i'nsha qalatug'i'nday yetip ta'riyiplenedi, yag'ni'y bul jag'dayda

$$a^{\frac{m}{n}}=\sqrt[n]{a^m} \tag{2}$$

dep yesaplanadi'.

Solay yetip, (2) formula qa'legen pu'tin m ha'm qa'legen natural $n \geq 2$ ha'm $a>0$ sani' ushi'n duri's boladi'. Mi'sali',

$$16^{\frac{3}{4}}=\sqrt[4]{16^3}=\sqrt[4]{2^{12}}=2^3=8;$$

$$7^{\frac{5}{4}}=\sqrt[4]{7^5}=\sqrt[4]{7^4\cdot 7}=7\sqrt[4]{7};$$

$$27^{-\frac{2}{3}}=\sqrt[3]{27^{-2}}=\sqrt[3]{\frac{1}{27^2}}=\frac{\sqrt[3]{1}}{\sqrt[3]{3^6}}=\frac{1}{3^2}=\frac{1}{9}.$$

r racional san — bul $\frac{m}{n}$ ko'rinisindegi san yekenligin, bunda m — pu'tin san, n — natural san, yag'ni'y $r=\frac{m}{n}$ bolatug'i'ni'n yesletip wo'temiz. Bul jag'dayda (2) formula boyi'nsha $a^r=a^{\frac{m}{n}}=\sqrt[n]{a^m}$ di payda yetemiz. Solay yetip, da'reje qa'legen racional ko'rsetkish ha'm qa'legen won' tiykar ushi'n ani'qlandi'. Yeger $r=\frac{m}{n}>0$ bolsa, wonda $\sqrt[n]{a^m}$ an'latpasi' tek $a>0$ bolg'anda g'ana yemes, ba'lki $a=0$ bolg'anda da ma'niske iye boladi'. $a=0$ bolsa, $\sqrt[n]{0^m}=0$. Soni'n' ushi'n $r>0$ bolg'anda $0^r=0$ ten'ligi wori'nli' dep yesaplanadi'.

(1) ha'm (2) formulalardan paydalani'p, racional ko'rsetkishli da'rejeni koren tu'rinde ha'm kerisinshe ko'rsetiw mu'mkin.

(2) formuladan ha'm korennin' qa'siyetlerinen

$$a^{\frac{m}{n}} = a^{\frac{mk}{nk}}$$

ten'ligi kelip shi'g'atug'i'ni'n atap wo'temiz, bunda $a > 0$, m — pu'tin san ha'm n, k — natural sanlar.

Mi'sali', $7^{\frac{3}{4}} = 7^{\frac{6}{8}} = 7^{\frac{9}{12}}$.

***Natural ko'rsetkishli da'rejenin' barli'q qa'siyetleri qa'legen racional ko'rsetkishli ha'm won' tiykarg'a iye da'rejeler ushi'n duri's bolatug'i'ni'n ko'rsetiw mu'mkin.* Atap aytqanda, qa'legen racional p ha'm q sanlari' ha'm qa'legen $a > 0$ ha'm $b > 0$ ushi'n to'mendegi ten'likler duri's boladi':**

1) $a^p \cdot a^q = a^{p+q}$; **4)** $(ab)^p = a^p b^p$;

2) $a^p : a^q = a^{p-q}$; **5)** $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$.

3) $(a^p)^q = a^{pq}$;

Bul qa'siyetler korenlerdin' qa'siyetlerinen kelip shi'g'adi'. Mi'sali', $a^p \cdot a^q = a^{p+q}$ qa'siyetin da'lilleyik.

○ Meyli, $p = \frac{m}{n}$, $q = \frac{k}{l}$ (bunda n ha'm l — natural sanlar, m ha'm k — pu'tin sanlar) bolsi'n.

$$a^{\frac{m}{n}} \cdot a^{\frac{k}{l}} = a^{\frac{m}{n}+\frac{k}{l}} \qquad (3)$$

yekenligin da'lillew kerek.

$\frac{m}{n}$ *ha'm* $\frac{k}{l}$ bo'lsheklerin uluwma bo'limge keltirip, (3) ten'liktin' shep bo'limin

$$a^{\frac{m}{n}} \cdot a^{\frac{k}{l}} = a^{\frac{ml}{nl}} \cdot a^{\frac{kn}{nl}}$$

ko'rinisinde jazami'z.

Racional ko'rsetkishli da'rejenin' ani'qlamasi'nan, korennin' ha'm pu'tin ko'rsetkishli da'rejenin' qa'siyetlerinen paydalani'p, to'mendegini payda yetemiz:

$$a^{\frac{m}{n}} \cdot a^{\frac{k}{l}} = a^{\frac{ml}{nl}} \cdot a^{\frac{kn}{nl}} = \sqrt[nl]{a^{ml}} \cdot \sqrt[nl]{a^{kn}} =$$

$$= \sqrt[nl]{a^{ml} \cdot a^{kn}} = \sqrt[nl]{a^{ml+kn}} = a^{\frac{ml+kn}{nl}} = a^{\frac{m}{n}+\frac{k}{l}}.$$

Racional ko'rsetkishli da'rejenin' qalg'an qa'siyetleri de usi'g'an uqsas da'lillenedi.

Da'rejenin' qa'siyetlerin qollani'wg'a mi'sallar keltiremiz.

1) $7^{\frac{1}{4}} \cdot 7^{\frac{3}{4}} = 7^{\frac{1}{4}+\frac{3}{4}} = 7;$ $\qquad 5^{\frac{1}{3}} \cdot 5^{\frac{2}{3}} = 5^{\frac{1}{3}+\frac{2}{3}} = 5^{1} = 5;$

2) $9^{\frac{2}{3}} : 9^{\frac{1}{6}} = 9^{\frac{2}{3}-\frac{1}{6}} = 9^{\frac{1}{2}} = \sqrt{9} = 3;$ $\qquad 8^{\frac{2}{3}} : 8 = 8^{\frac{2}{3}-1} = 8^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2};$

3) $\left(16^{\frac{1}{3}}\right)^{\frac{9}{4}} = 16^{\frac{1}{3}\cdot\frac{9}{4}} = 16^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^{4\cdot\frac{3}{4}} = 2^3 = 8;$

4) $24^{\frac{2}{3}} = (2^3 \cdot 3)^{\frac{2}{3}} = 2^{3\cdot\frac{2}{3}} \cdot 3^{\frac{2}{3}} = 4\sqrt[3]{3^2} = 4\sqrt[3]{9};$

5) $\left(\frac{8}{27}\right)^{\frac{1}{3}} = \frac{8^{\frac{1}{3}}}{27^{\frac{1}{3}}} = \frac{(2^3)^{\frac{1}{3}}}{(3^3)^{\frac{1}{3}}} = \frac{2}{3};$ $\qquad \left(\frac{16}{81}\right)^{\frac{1}{4}} = \left(\frac{2^4}{3^4}\right)^{\frac{1}{4}} = \left(\frac{2}{3}\right)^{4\cdot\frac{1}{4}} = \frac{2}{3}.$

2-ma'sele. Yesaplan': $25^{\frac{1}{5}} \cdot 125^{\frac{1}{5}}$.

$$\triangle\ 25^{\frac{1}{5}} \cdot 125^{\frac{1}{5}} = (25 \cdot 125)^{\frac{1}{5}} = (5^5)^{\frac{1}{5}} = 5. \ \blacktriangle$$

3-ma'sele. An'latpani' a'piwayi'lasti'ri'n': $\dfrac{a^{\frac{4}{3}}b + ab^{\frac{4}{3}}}{\sqrt[3]{a} + \sqrt[3]{b}}$.

$$\triangle\ \frac{a^{\frac{4}{3}}b + ab^{\frac{4}{3}}}{\sqrt[3]{a} + \sqrt[3]{b}} = \frac{ab\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\right)}{\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}\right)} = ab. \ \blacktriangle$$

4-ma'sele. An'latpani' a'piwayi'lasti'ri'n': $\dfrac{a^{\frac{1}{3}}-a^{\frac{7}{3}}}{a^{\frac{1}{3}}-a^{\frac{4}{3}}}-\dfrac{a^{-\frac{1}{3}}-a^{\frac{5}{3}}}{a^{\frac{2}{3}}+a^{-\frac{1}{3}}}$.

$$\triangle\ \frac{a^{\frac{1}{3}}-a^{\frac{7}{3}}}{a^{\frac{1}{3}}-a^{\frac{4}{3}}}-\frac{a^{-\frac{1}{3}}-a^{\frac{5}{3}}}{a^{\frac{2}{3}}+a^{-\frac{1}{3}}}=\frac{a^{\frac{1}{3}}(1-a^2)}{a^{\frac{1}{3}}(1-a)}-\frac{a^{-\frac{1}{3}}(1-a^2)}{a^{-\frac{1}{3}}(1+a)}=$$

$$=1+a-(1-a)=2a.\ \blacktriangle$$

$3^{\sqrt{2}}$ mi'sali'nda *irracional ko'rsetkishli da'rejeni* qalay kiritiw mu'mkin yekenligin ko'rsetemiz. $\sqrt{2}$ ni'n' juwi'q ma'nisleri 0,1; 0,01; 0,001; ... ha'm t.b. ani'qli'q penen izbe-iz jazi'p shi'g'ami'z. Wonda to'mendegi izbe-izlik payda boladi':

$$1{,}4;\ 1{,}41;\ 1{,}414;\ 1{,}4142;\ \ldots$$

3 sani'ni'n' da'reje ko'rsetkishlerinin' izbe-izligin usi' racional ko'rsetkishleri menen jazi'p shi'g'ami'z:

$$3^{1{,}4};\ 3^{1{,}41};\ 3^{1{,}414};\ 3^{1{,}4142};\ \ldots$$

Bul da'rejeler $3^{\sqrt{2}}$ si'yaqli' belgilenetug'i'n qanday da bir haqi'yqi'y sanni'n' izbe-iz juwi'q ma'nisleri yekenin ko'rsetiw mu'mkin:

$$\begin{aligned}3^{1{,}4} &= \underline{4}{,}6555355,\\ 3^{1{,}41} &= \underline{4{,}7}069644,\\ 3^{1{,}414} &= \underline{4{,}72}76942,\\ 3^{1{,}442} &= \underline{4{,}728}7329,\\ 3^{\sqrt{2}} &\approx \underline{4{,}7288033}.\end{aligned}$$

Won' a tiykari'na iye ha'm qa'legen irracional ko'rsetkishli a^b da'rejesi usi'g'an uqsas ani'qlanadi'. Solay yetip, yendi won' tiykari'na iye da'reje qa'legen haqi'yqi'y ko'rsetkish ushi'n ani'qlandi', soni'n' menen birge haqi'yqi'y ko'rsetkishli da'rejenin' qa'siyetleri racional ko'rsetkishli da'rejenin' qa'siyetleri si'yaqli' boladi'.

Shi'ni'g'i'wlar

160. (Awi'zeki.) Racional ko'rsetkishli da'reje tu'rinde ko'rsetin':

1) $\sqrt{x^3}$; 2) $\sqrt[3]{a^4}$; 3) $\sqrt[4]{b^3}$; 4) $\sqrt[5]{x^{-1}}$; 5) $\sqrt[6]{a}$; 6) $\sqrt[7]{b^{-3}}$.

161. (Awi'zeki.) Pu'tin ko'rsetkishli da'rejenin' koreni tu'rinde ko'rsetin'.

1) $x^{\frac{1}{4}}$; | 2) $y^{\frac{2}{5}}$; | 3) $a^{-\frac{5}{6}}$; | 4) $b^{-\frac{1}{3}}$; | 5) $(2x)^{\frac{1}{2}}$; | 6) $(3b)^{-\frac{2}{3}}$.

Yesaplan' **(162–165):**

162. 1) $64^{\frac{1}{2}}$; | 2) $27^{\frac{1}{3}}$; | 3) $8^{\frac{2}{3}}$; | 4) $81^{\frac{3}{4}}$; | 5) $16^{-0,75}$; | 6) $9^{-1,5}$.

163. 1) $2^{\frac{4}{5}} \cdot 2^{\frac{11}{5}}$; 2) $5^{\frac{2}{7}} \cdot 5^{\frac{5}{7}}$; 3) $9^{\frac{2}{3}} : 9^{\frac{1}{6}}$; 4) $4^{\frac{1}{3}} : 4^{\frac{5}{6}}$;

5) $(7^{-3})^{-\frac{2}{3}}$; 6) $\left(8^{\frac{1}{12}}\right)^{-4}$; 7) $8^{\frac{4}{5}} : 8^{\frac{7}{15}}$; 8) $(5^{-4})^{-\frac{3}{4}}$.

164. 1) $9^{\frac{2}{5}} \cdot 27^{\frac{2}{5}}$; 2) $7^{\frac{2}{3}} \cdot 49^{\frac{2}{3}}$; 3) $144^{\frac{3}{4}} : 9^{\frac{3}{4}}$; 4) $150^{\frac{3}{2}} : 6^{\frac{3}{2}}$.

165. 1) $\left(\frac{1}{16}\right)^{-0,75} + \left(\frac{1}{8}\right)^{-\frac{4}{3}}$; 2) $(0,04)^{-1,5} - (0,125)^{-\frac{2}{3}}$;

3) $8^{\frac{9}{7}} : 8^{\frac{2}{7}} - 3^{\frac{6}{5}} \cdot 3^{\frac{4}{5}}$; 4) $\left(5^{-\frac{2}{5}}\right)^{-5} + \left((0,2)^{\frac{3}{4}}\right)^{-4}$.

166. Yesaplan':

1) $a = 0{,}09$ bolg'anda $\sqrt[3]{a} \cdot \sqrt[6]{a}$ ni'n' ma'nisin;

2) $b = 27$ bolg'anda $\sqrt{b} : \sqrt[6]{b}$ ni'n' ma'nisin;

3) $b = 1{,}3$ bolg'anda $\frac{\sqrt{b} \cdot \sqrt[3]{b^2}}{\sqrt[6]{b}}$ ni'n' ma'nisin;

4) $a = 2{,}7$ bolg'anda $\sqrt[3]{a} \cdot \sqrt[4]{a} \cdot \sqrt[12]{a^5}$ ni'n' ma'nisin.

167. Racional ko'rsetkishli da'reje tu'rinde ko'rsetin':

1) $a^{\frac{1}{3}} \cdot \sqrt{a}$; 2) $b^{\frac{1}{2}} \cdot b^{\frac{1}{3}} \cdot \sqrt[6]{b}$; 3) $\sqrt[3]{b} : b^{\frac{1}{6}}$;

4) $a^{\frac{4}{3}} : \sqrt[3]{a}$; 5) $x^{1,7} \cdot x^{2,8} : \sqrt{x^5}$; 6) $y^{-3,8} : y^{-2,3} \cdot \sqrt{y^3}$.

An'latpani' a'piwayi'lasti'ri'n' **(168–169)**:

168. 1) $(a^4)^{-\frac{3}{4}} \cdot \left(b^{-\frac{2}{3}}\right)^{-6}$; 2) $\left(\left(\frac{a^6}{b^{-3}}\right)^4\right)^{\frac{1}{12}}$; 3) $(a^{-7})^{-\frac{5}{7}} \cdot \left(b^{-\frac{3}{4}}\right)^{-4}$.

169. 1) $\dfrac{a^{\frac{4}{3}}\left(a^{-\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{4}}\left(a^{\frac{3}{4}}+a^{-\frac{1}{4}}\right)}$; 2) $\dfrac{b^{\frac{1}{5}}\left(\sqrt[5]{b^4}-\sqrt[5]{b^{-1}}\right)}{b^{\frac{2}{3}}\left(\sqrt[3]{b}-\sqrt[3]{b^{-2}}\right)}$; 3) $\dfrac{a^{\frac{5}{3}}b^{-1}-ab^{-\frac{1}{3}}}{\sqrt[3]{a^2}-\sqrt[3]{b^2}}$;

4) $\dfrac{a^{\frac{1}{3}}\sqrt{b}+b^{\frac{1}{3}}\sqrt{a}}{\sqrt[6]{a}+\sqrt[6]{b}}$; 5) $\dfrac{a^{-\frac{1}{3}}\left(a^{\frac{1}{3}}+a^{\frac{4}{3}}\right)}{a^{\frac{2}{5}}\left(a^{\frac{8}{5}}-a^{-\frac{2}{5}}\right)}$; 6) $\dfrac{\sqrt[3]{a}-\sqrt[3]{b}}{\sqrt[6]{a}-\sqrt[6]{b}}$.

170. Yesaplan':

1) $\left(2^{\frac{5}{3}} \cdot 3^{-\frac{1}{3}} - 3^{\frac{5}{3}} \cdot 2^{-\frac{1}{3}}\right) \cdot \sqrt[3]{6}$; 2) $\left(5^{\frac{1}{4}} : 2^{\frac{3}{4}} - 2^{\frac{1}{4}} : 5^{\frac{3}{4}}\right) \cdot \sqrt[4]{1000}$.

171. An'latpani' a'piwayi'lasti'ri'n':

1) $a^{\frac{1}{9}}\sqrt[6]{a\sqrt[3]{a}}$; 2) $b^{\frac{1}{12}}\sqrt[3]{b\sqrt[4]{b}}$; 3) $\left(\sqrt[3]{ab^{-2}}+(ab)^{-\frac{1}{6}}\right)\sqrt[6]{ab^4}$;

4) $\left(\sqrt[3]{a}+\sqrt[3]{b}\right)\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}-\sqrt[3]{ab}\right)$; 5) $\dfrac{x-y}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}$; 6) $\dfrac{\sqrt{a}-\sqrt{b}}{a^{\frac{1}{4}}-b^{\frac{1}{4}}}$;

7) $\dfrac{m^{\frac{1}{2}}+n^{\frac{1}{2}}}{m+2\sqrt{mn}+n}$; 8) $\dfrac{c-2c^{\frac{1}{2}}+1}{\sqrt{c}-1}$; 9) $\left(\sqrt[3]{a}-\sqrt[3]{b}\right)\left(a^{\frac{2}{3}}+\sqrt[3]{ab}+b^{\frac{2}{3}}\right)$.

An'latpani' a'piwayi'lasti'ri'n' **(172–174)**:

172. 1) $\left(1-2\sqrt{\frac{b}{a}}+\frac{b}{a}\right) : \left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)^2$; 2) $\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\right) : \left(2+\sqrt[3]{\frac{a}{b}}+\sqrt[3]{\frac{b}{a}}\right)$;

3) $\dfrac{a^{\frac{1}{4}}-a^{\frac{9}{4}}}{a^{\frac{1}{4}}-a^{\frac{5}{4}}}-\dfrac{b^{-\frac{1}{2}}-b^{\frac{3}{2}}}{b^{\frac{1}{2}}-b^{-\frac{1}{2}}}$; 4) $\dfrac{\sqrt{a}-a^{-\frac{1}{2}}b}{1-\sqrt{a^{-1}b}}-\dfrac{\sqrt[3]{a^2}-a^{-\frac{1}{3}}b}{\sqrt[6]{a}+a^{-\frac{1}{3}}\sqrt{6}}$.

173. 1) $\dfrac{a^{\frac{3}{2}}}{\sqrt{a}+\sqrt{b}}-\dfrac{ab^{\frac{1}{2}}}{\sqrt{b}-\sqrt{a}}-\dfrac{2a^2-4ab}{a-b}$; 2) $\dfrac{3xy-y^2}{x-y}-\dfrac{y\sqrt{y}}{\sqrt{x}-\sqrt{y}}-\dfrac{y\sqrt{x}}{\sqrt{x}+\sqrt{y}}$;

3) $\dfrac{1}{\sqrt[3]{a}+\sqrt[3]{b}}-\dfrac{\sqrt[3]{a}+\sqrt[3]{b}}{a^{\frac{2}{3}}-\sqrt[3]{ab}+b^{\frac{2}{3}}}$; 4) $\dfrac{\sqrt[3]{a^2}-\sqrt[3]{b^2}}{\sqrt[3]{a}-\sqrt[3]{b}}-\dfrac{a-b}{a^{\frac{2}{3}}+\sqrt[3]{ab}+b^{\frac{2}{3}}}$.

174. 1) $\frac{a-b}{\sqrt[3]{a}-\sqrt[3]{b}}-\frac{a+b}{a^{\frac{1}{3}}+b^{\frac{1}{3}}}$; 2) $\frac{a+b}{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}}-\frac{a-b}{a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}}$;

3) $\frac{a^{\frac{2}{3}}+b^{\frac{2}{3}}}{a-b}-\frac{1}{a^{\frac{1}{3}}-b^{\frac{1}{3}}}$; 4) $\frac{a^{\frac{1}{3}}-b^{\frac{1}{3}}}{a+b}+\frac{1}{a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}}$.

13- §. SANLI' TEN'SIZLIKLERDI DA'REJEGE KO'TERIW

8-klass «Algebra» kursi'nda shep ha'm won' bo'limleri won' bolg'an bir qi'yli' belgige iye ten'sizliklerdi ag'zama-ag'za ko'beyttirilgende sol belgige iye ten'sizlik payda bolatug'i'nli'g'i' ko'rsetilgen yedi.

Bunnan, yeger $a > b > 0$ ha'm n natural san bolsa, wonda $a^n > b^n$ bolatug'i'ni' kelip shi'g'adi'.

○ Sha'rt boyi'nsha $a > 0$, $b > 0$. n bir qi'yli' $a > b$ ten'sizligin ag'zama-ag'za ko'beytip, payda yetemiz: $a^n > b^n$. ●

1-ma'sele. $(0{,}43)^5$ ha'm $\left(\frac{3}{7}\right)^5$ sanlari'n sali'sti'ri'n'.

△ 0,001 ge shekemgi da'llik penen $\frac{3}{7} \approx 0{,}428$ bolg'ani' ushi'n $0{,}43 > \frac{3}{7}$ boladi'. Soni'n' ushi'n $(0{,}43)^5 > \left(\frac{3}{7}\right)^5$. ▲

Shep ha'm won' bo'limleri won' bolg'an ten'sizlikti qa'legen racional da'rejege ko'teriw mu'mkin:

yeger $a > b > 0$, $r > 0$ bolsa, wonda

$$a^r > b^r \qquad (1)$$

boladi';

yeger $a > b > 0$, $r < 0$ bolsa, wonda

$$a^r < b^r \qquad (2)$$

boladi'.

1- qa'siyetti da'lilleymiz.

◯ Da'slep (1) qa'siyettin' $r=\frac{1}{n}$ bolg'anda duri's yekenligin, keyin bolsa uluwma jag'day ushi'n $r=\frac{m}{n}$ bolg'anda duri's yekenligin da'lilleymiz.

a) Aytayi'q, $r=\frac{1}{n}$ bolsi'n, bunda n — birden u'lken natural san, $a>0$, $b>0$. Sha'rt boyi'nsha $a>b$. $a^{\frac{1}{n}}>b^{\frac{1}{n}}$ yekenligin da'lillew kerek. Bul natuwri' dep woylayi'q, yag'ni'y $a^{\frac{1}{n}}\le b^{\frac{1}{n}}$ bolsi'n. Wonda bul ten'sizlikti n natural da'rejege ko'terip, $a\le b$ ni payda yetemiz, bul bolsa, $a>b$ sha'rtine qayshi' keledi. Demek, $a>b>0$ den $a^{\frac{1}{n}}>b^{\frac{1}{n}}$ yekenligi kelip shi'g'adi'.

b) Aytayi'q, $r=\frac{m}{n}$ bolsi'n, bunda m ha'm n — natural sanlar. Wonda $a>b>0$ sha'rtten, da'lillegenimiz boyi'nsha $a^{\frac{1}{n}}>b^{\frac{1}{n}}$ yekenligi kelip shi'g'adi'. Bul ten'sizlikti m natural da'rejege ko'terip, mi'nag'an iye bolami'z:

$$\left(a^{\frac{1}{n}}\right)^m>\left(b^{\frac{1}{n}}\right)^m,\ \text{yag'ni'y}\ a^{\frac{m}{n}}>b^{\frac{m}{n}}.$$ ●

Mi'sali', $5^{\frac{2}{7}}>3^{\frac{2}{7}}$, sebebi $5>3$; $2^{\frac{3}{4}}<4^{\frac{3}{4}}$, sebebi $2<4$; $\sqrt[5]{7^2}>\sqrt[5]{6^2}$ sebebi $7>6$.

Yendi (2) qa'siyetti da'lilleymiz.

◯ Yeger $r<0$ bolsa, wonda $-r>0$ boladi'. (1) qa'siyet boyi'nsha $a>b>0$ sha'rtten $a^{-r}>b^{-r}$ yekenligi kelip shi'g'adi'. Bul ten'sizliktin' yeki bo'limin won' $b^r>a^r$ sang'a ko'beytip, yag'ni'y $a^r<b^r$. ●

Mi'sali', $(0{,}7)^{-8}<(0{,}6)^{-8}$, sebebi $0{,}7>0{,}6$; $13^{-0{,}6}>15^{-0{,}6}$, sebebi $13<15$; $\sqrt[4]{8^{-3}}<\sqrt[4]{7^{-3}}$, sebebi $8>7$.

Joqari' matematika kursi'nda (1) qa'siyet qa'legen won' r haqi'yqi'y san ushi'n, (2) qa'siyet bolsa qa'legen teris r haqi'yqi'y san ushi'n duri's yekenligi da'lillenedi. Mi'sali',

$$\left(\frac{8}{9}\right)^{\sqrt{2}}>\left(\frac{7}{8}\right)^{\sqrt{2}},\ \text{sebebi}\ \frac{8}{9}>\frac{7}{8};\qquad \left(\frac{7}{8}\right)^{-\sqrt{3}}<\left(\frac{6}{7}\right)^{-\sqrt{3}},\ \text{sebebi}\ \frac{7}{8}>\frac{6}{7}.$$

Qatan' ten'sizliklerdi ($>$ yaki $<$ belgili) da'rejege ko'teriwdin' ko'rip wo'tilgen qa'siyetleri qatan' yemes ten'sizlikler ($\geq$ yaki $\leq$ belgili) ushi'n da duri's bolatug'i'ni'n ayti'p wo'temiz.

Solay yetip, ***yeger ten'sizliktin' yeki ta'repi won' san bolsa, wonda woni' won' da'rejege ko'tergende ten'sizlik belgisi saqlanadi', teris da'rejege ko'tergende bolsa ten'sizlik belgisi qarama-qarsi' belgige wo'zgeredi.***

Qatan' ten'sizlikler ushi'n $>$ ha'm $<$ belgileri, qatan' yemes ten'sizlikler ushi'n bolsa $\geq$ ha'm $\leq$ belgileri qarama-qarsi' belgiler bolatug'i'ni'n yesletip wo'temiz.

2-ma'sele. Sanlardi' sali'sti'ri'n':

1) $\left(\frac{17}{18}\right)^{-\frac{1}{3}}$ ha'm $\left(\frac{18}{17}\right)^{-\frac{1}{3}}$; 2) $\left(\frac{6}{7}\right)^{\sqrt{2}}$ ha'm $(0,86)^{\sqrt{2}}$.

△ 1. $\frac{17}{18} < 1$ ha'm $\frac{18}{17} > 1$ bolg'ani' ushi'n $\frac{17}{18} < \frac{18}{17}$ boladi'.

Bul ten'sizlikti teris $\left(-\frac{1}{3}\right)$ da'rejege ko'terip, to'mendegige iye bolami'z: $\left(\frac{17}{18}\right)^{-\frac{1}{3}} > \left(\frac{18}{17}\right)^{-\frac{1}{3}}$.

2. Da'rejenin' tiykarlari'n sali'sti'rami'z $\frac{6}{7} = 0,857...$ bolg'ani' ushi'n $\frac{6}{7} < 0,86$ boladi'. Bul ten'sizlikti won' $\sqrt{2}$ da'rejege ko'terip to'mendegini payda yetemiz:

$$\left(\frac{6}{7}\right)^{\sqrt{2}} < 0,86^{\sqrt{2}}.$$ ▲

3-ma'sele. Ten'lemeni sheshin': $10^x = 1$.

△ $x = 0$ sani' bul ten'lemenin' koreni boladi', sebebi $10^0 = 1$. Basqa korenleri joq yekenligin ko'rsetemiz.

Berilgen ten'lemeni $10^x = 1^x$ ko'rinisinde jazami'z.

Yeger $x > 0$ bolsa, wonda $10^x > 1^x$ ha'm, demek, ten'leme won' korenlerge iye yemes.

Yeger $x < 0$ bolsa, wonda $10^x < 1^x$ ha'm, demek, ten'leme teris korenlerge iye yemes.

Solay yetip, $x = 0$ berilgen $10^x = 1$ ten'lemesinin' jalg'i'z koreni yeken. ▲

Usi'g'an uqsas, $a^x = 1$ ($a > 0$, $a \neq 1$) ten'lemesi jalg'i'z $x = 0$ korenge iye yekenligi da'lillenedi. Bunnan,

$$a^x = a^y \qquad (3)$$

ten'ligi $x = y$ bolg'anda g'ana duri's bolatug'i'ni' kelip shi'g'adi', bul jerde $a > 0$, $a \neq 1$.

○ (3) ten'likti a^{-y} ke ko'beytip, $a^{x-y} = 1$ di payda yetemiz, bunnan $x = y$. ●

4-ma'sele. $3^{2x-1} = 9$ ten'lemesin sheshin'.

△ $3^{2x-1} = 3^2$, bunnan $2x - 1 = 2$, $x = 1{,}5$. ▲

$a^x = b$ ten'lemesin qaraymi'z, bunda $a > 0$, $a \neq 1$, $b > 0$.

Bul ten'leme jalg'i'z x_0 korenge iye yekenligin da'lillew mu'mkin. *x_0 san a tiykar boyi'nsha b sani'ni'n' logarifmi delinedi ha'm* $\log_a b$ tu'rinde belgilenedi. Mi'sali', $3^x = 9$ ten'lemesinin' koreni 2 sani' boladi', yag'ni'y $\log_3 9 = 2$. Sonday-aq, $\log_2 16 = 4$, sebebi $2^4 = 16$,

$\log_5 \frac{1}{5} = -1$, sebebi $5^{-1} = \frac{1}{5}$; $\log_{\frac{1}{3}} 27 = -3$, sebebi $\left(\frac{1}{3}\right)^{-3} = 27$.

b sani'ni'n' 10 tiykari' boyi'nsha logarifmi *wonli'q logarifm* delinedi ha'm $\lg b$ tu'rinde belgilenedi. Mi'sali', $\lg 100 = 2$, sebebi $10^2 = 100$; $\lg 0{,}001 = -3$, sebebi $10^{-3} = 0{,}001$.

Shi'ni'g'i'wlar

175. (Awi'zeki.) Sanlardi' sali'sti'ri'n':

1) $2^{\frac{1}{3}}$ ha'm $3^{\frac{1}{3}}$; 2) $5^{-\frac{4}{5}}$ ha'm $3^{-\frac{4}{5}}$; 3) $5^{\sqrt{3}}$ ha'm $7^{\sqrt{3}}$; 4) $21^{-\sqrt{2}}$ ha'm $31^{-\sqrt{2}}$.

176. Sanlardi' sali'sti'ri'n':

1) $(0{,}88)^{\frac{1}{6}}$ ha'm $\left(\frac{6}{11}\right)^{\frac{1}{6}}$; 2) $\left(\frac{5}{12}\right)^{-\frac{1}{4}}$ ha'm $(0{,}41)^{-\frac{1}{4}}$;

3) $(4{,}09)^{\sqrt[3]{2}}$ ha'm $\left(4\frac{3}{25}\right)^{\sqrt[3]{2}}$; 4) $\left(\frac{11}{12}\right)^{-\sqrt{5}}$ ha'm $\left(\frac{12}{13}\right)^{-\sqrt{5}}$.

177. Ten'lemelerdi sheshin':

1) $6^{2x} = 6^{\frac{1}{5}}$; 2) $3^x = 27$; 3) $7^{1-3x} = 7^{10}$;

4) $2^{2x+1} = 32$; 5) $4^{2+x} = 1$; 6) $\left(\frac{1}{5}\right)^{4x-3} = 5$.

178. Sanlardi' sali'sti'ri'n':

1) $\sqrt[7]{\left(\frac{1}{2}-\frac{1}{3}\right)^2}$ ha'm $\sqrt[7]{\left(\frac{1}{3}-\frac{1}{4}\right)^2}$; | 2) $\sqrt[5]{\left(1\frac{1}{4}-1\frac{1}{5}\right)^3}$ ha'm $\sqrt[5]{\left(1\frac{1}{6}-1\frac{1}{7}\right)^3}$.

Ten'lemeni sheshin' **(179–180):**

179. 1) $3^{2-y} = 27$; 2) $3^{5-2x} = 1$;

3) $9^{\frac{1}{2}x-1} - 3 = 0$; 4) $27^{3-\frac{1}{3}y} - 81 = 0$.

180. 1) $\left(\frac{1}{9}\right)^{2x-5} = 3^{5x-8}$; 2) $2^{4x-9} = \left(\frac{1}{2}\right)^{x-4}$; 3) $8^x 4^{x+13} = \frac{1}{16}$;

4) $\frac{25^{x-2}}{\sqrt{5}} = \left(\frac{1}{5}\right)^{x-7,5}$; 5) $\left(\frac{1}{4}\right)^{x-4} = 2^{x+2}$; 6) $3^x \cdot 9^{x-1} = \frac{1}{27}$.

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181. 1) $\left(\frac{1}{\sqrt{3}}\right)^{2x+1} = (3\sqrt{3})^x$; 2) $(\sqrt[3]{2})^{x-1} = \left(\frac{2}{\sqrt[3]{2}}\right)^{2x}$;

3) $9^{3x+4}\sqrt{3} = \frac{27^{x-1}}{\sqrt{3}}$; 4) $\frac{8}{(\sqrt{2})^x} = 4^{3x-2}\sqrt{2}$.

182. Yesaplan':

1) $\log_7 49$; | 2) $\log_2 64$; | 3) $\log_{\frac{1}{2}} 4$; | 4) $\log_3 \frac{1}{27}$; | 5) $\log_7 \frac{1}{7}$.

III bapqa tiyisli shi'ni'g'i'wlar

183. Yesaplan':

1) $(0,175)^0 + (0,36)^{-2} - 1^{\frac{4}{3}}$; 2) $1^{-0,43} - (0,008)^{\frac{1}{3}} + (15,1)^0$;

3) $\left(\frac{4}{5}\right)^{-2} - \left(\frac{1}{27}\right)^{\frac{1}{3}} + 4 \cdot 379^0$; 4) $(0,125)^{\frac{1}{3}} + \left(\frac{3}{4}\right)^2 - (1,85)^0$.

184. Yesaplan':

1) $9,3 \cdot 10^{-6} : (3,1 \cdot 10^{-5})$; 2) $1,7 \cdot 10^{-6} \cdot 3 \cdot 10^7$;

3) $8,1 \cdot 10^{16} \cdot 2 \cdot 10^{-14}$; 4) $6,4 \cdot 10^5 : (1,6 \cdot 10^7)$;

5) $2 \cdot 10^{-1} + \left(6^0 - \frac{1}{6}\right)^{-1} \cdot \left(\frac{1}{3}\right)^{-2} \cdot \left(\frac{1}{3}\right)^3 \cdot \left(-\frac{1}{4}\right)^{-1}$;

6) $3 \cdot 10^{-1} - \left(8^0 - \frac{1}{8}\right)^{-1} \cdot \left(\frac{1}{4}\right)^{-3} \cdot \left(\frac{1}{4}\right)^4 \cdot \left(\frac{5}{7}\right)^{-1}$.

185. An'latpani'n' ma'nisin tabi'n':

1) $\left(\dfrac{x^{\frac{1}{2}}\cdot x^{\frac{5}{6}}}{x^{\frac{1}{6}}}\right)^{-2}$, bunda $x=\dfrac{7}{9}$; 2) $\left(\dfrac{a^{\frac{2}{3}}\cdot a^{\frac{1}{9}}}{a^{-\frac{2}{9}}}\right)^{-3}$, bunda $a=0,1$.

186. An'latpani' a'piwayi'lasti'ri'n':

1) $(\sqrt[3]{125x}-\sqrt[3]{8x})-(\sqrt[3]{27x}-\sqrt[3]{64x})$;

2) $(\sqrt[4]{x}+\sqrt[4]{16x})+(\sqrt[4]{81x}-\sqrt[4]{625x})$;

3) $\left(\dfrac{3}{\sqrt{1+a}}+\sqrt{1-a}\right):\dfrac{3+\sqrt{1+a}}{\sqrt{1+a}}$;

4) $\left(1-\dfrac{x}{\sqrt{x^2-y^2}}\right):\left(\sqrt{x^2-y^2}-x\right)$.

187. Ten'lemeni sheshin':

1) $7^{5x-1}=49$; 2) $(0,2)^{1-x}=0,04$; 3) $\left(\dfrac{1}{3}\right)^{3x+3}=3^{2x}$;

4) $3^{5x-7}=\left(\dfrac{1}{3}\right)^{2x}$; 5) $(0,3)^{2-3x}=0,027$; 6) $\left(\dfrac{1}{6}\right)^{2x-3}=6^{x}$.

188. Yesaplan':

1) $\left(\dfrac{1}{16}\right)^{-0,75}+10000^{0,25}-\left(7\dfrac{19}{32}\right)^{\frac{1}{5}}$; 2) $(0,001)^{-\frac{1}{3}}-2^{-2}\cdot 64^{\frac{2}{3}}-8^{-1\frac{1}{3}}$;

3) $27^{\frac{2}{3}}-(-2)^{-2}+\left(3\dfrac{3}{8}\right)^{-\frac{1}{3}}$; 4) $(-0,5)^{-4}-625-\left(2\dfrac{1}{4}\right)^{-1\frac{1}{2}}$.

189. x ti'n' qanday ma'nislerinde an'latpa ma'niske iye boladi':

1) $\sqrt[4]{x^2-4}$; 2) $\sqrt[3]{x^2-5x+6}$; 3) $\sqrt[6]{\dfrac{x-2}{x+3}}$;

4) $\sqrt[4]{x^2-5x+6}$; 5) $\sqrt[8]{x^3-x}$; 6) $\sqrt[6]{x^3-5x^2+6x}$?

190. An'latpani' a'piwayi'lasti'ri'n':

1) $\dfrac{a^{\frac{1}{4}}-a^{-\frac{7}{4}}}{a^{\frac{1}{4}}-a^{-\frac{3}{4}}}$; 2) $\dfrac{a^{\frac{4}{3}}-a^{-\frac{2}{3}}}{a^{\frac{1}{3}}-a^{-\frac{2}{3}}}$; 3) $\dfrac{b^{\frac{5}{4}}+2b^{\frac{1}{4}}+b^{-\frac{3}{4}}}{b^{\frac{3}{4}}-b^{-\frac{1}{4}}}$;

4) $\dfrac{a^{-\frac{4}{3}}b^{-2}-a^{-2}b^{-\frac{4}{3}}}{a^{-\frac{5}{3}}b^{-2}-b^{-\frac{5}{3}}a^{-2}}$; 5) $\dfrac{\sqrt{a^3b^{-1}}-\sqrt{a^{-1}b^3}}{\sqrt{ab^{-1}}-\sqrt{a^{-1}b}}$; 6) $\dfrac{a^{\frac{3}{4}}b^{-\frac{1}{4}}-a^{-\frac{1}{4}}b^{\frac{3}{4}}}{a^{\frac{1}{4}}b^{-\frac{1}{4}}-a^{-\frac{1}{4}}b^{\frac{1}{4}}}$.

WO'ZIN'IZDI SI'NAP KO'RIN'!

1. Yesaplan':

1) $3^{-5}:3^{-7}-2^{-2}\cdot 2^4+\left(\left(\frac{2}{3}\right)^{-1}\right)^3$; 2) $\sqrt[5]{3^{10}\cdot 32}-\frac{\sqrt[3]{48}}{\sqrt[3]{2}\cdot\sqrt[3]{3}}$;

3) $25^{\frac{3}{2}}\cdot 25^{-1}+(5^3)^{\frac{2}{3}}:5^3-48^{\frac{2}{3}}:6^{\frac{2}{3}}$; 4) $4^{-7}:4^{-10}-3^{-2}\cdot 3^5+\left(\frac{1}{2}\right)^{-2}$.

2. 8600 ha'm 0,0078 sanlari'n standart tu'rinde jazi'n' ha'm de ko'beytin' ha'm bo'lin'.

3. An'latpalardi' a'piwayi'lasti'ri'n':

1) $\frac{3x^{-9}\cdot 2x^5}{x^{-4}}$; 2) $(x^{-1}+y^{-1})\left(\frac{1}{xy}\right)^{-2}$; 3) $\frac{2a^{-8}\cdot 4a^3}{16\cdot a^{-5}}$.

4. $\frac{a^{\frac{5}{3}}}{\sqrt[3]{a^5}\cdot a^{-\frac{3}{4}}}$ an'latpasi'n a'piwayi'lasti'ri'n' ha'm $a=81$ bolg'anda woni'n' san ma'nisin tabi'n'.

5. Sanlardi' sali'sti'ri'n':

$(0,78)^{\frac{2}{3}}$ ha'm $(0,67)^{\frac{2}{3}}$; $(3,09)^{-\frac{1}{3}}$ ha'm $(3,08)^{-\frac{1}{3}}$.

III bapqa tiyisli si'naq (test) shi'ni'g'i'wlari'

1. Yesaplan': $(-8)^2-(-5)^3-(12)^{-1}$.

A) $188\frac{11}{12}$; B) $-61\frac{1}{12}$; C) $189\frac{1}{12}$; D) $61\frac{1}{12}$.

2. Yesaplan': $(-0,2)^{-3}+(0,2)^{-2}-(-2)^{-2}$.

A) $-150\frac{1}{4}$; B) $-100\frac{1}{4}$; C) $99\frac{1}{4}$; D) 11,25.

3. Yesaplan': $\frac{\sqrt[3]{-16}+\sqrt[3]{54}+\sqrt[3]{128}}{\sqrt[3]{-250}}$.

A) $\sqrt[3]{2}$; B) 1; C) -1; D) $\frac{9}{5}$.

4. Yesaplan': $\sqrt[4]{\frac{(4{,}15)^3-(1{,}61)^3}{2{,}54}+4{,}15\cdot 1{,}61}$.

A) 3,4; B) 5,76; C) 24; D) 2,4.

5. Yesaplan': $\sqrt[3]{\frac{(2{,}08)^3+(2{,}016)^3}{4{,}096}-2{,}08\cdot 2{,}016}$.

A) 0,16; B) 4,096; C) 1,6; D) 0,8.

6. Yesaplan': $\sqrt{2\sqrt{2}+1}\cdot\sqrt[4]{9-4\sqrt{2}}$. *Ko'rsetpe:* $\sqrt{a}\cdot\sqrt[4]{b}=\sqrt[4]{a^2\cdot b}$.

A) $\sqrt{7}$; B) $2\sqrt{15}$; C) $3-2\sqrt{2}$; D) 7.

7. Yesaplan': $\sqrt[3]{2-\sqrt{3}}\cdot\sqrt[6]{7+4\sqrt{3}}$. *Ko'rsetpe:* $\sqrt[3]{a}\cdot\sqrt[6]{b}=\sqrt[6]{a^2\cdot b}$.

A) -1; B) 1; C) $3+2\sqrt{3}$; D) $5+3\sqrt{3}$.

8. Yesaplan': $\sqrt[3]{1+\sqrt{2}}\cdot\sqrt[6]{3-2\sqrt{2}}$.

A) $3-\sqrt{2}$; B) -1; C) 1; D) $2\sqrt{2}$.

9. Yesaplan': $\frac{\sqrt[3]{45-29\sqrt{2}}\cdot(3-\sqrt{2})}{11-6\sqrt{2}}$. *Ko'rsetpe:* $\sqrt[3]{a}\cdot b=\sqrt[3]{a\cdot b^3}$.

A) $5-\sqrt{2}$; B) $5\sqrt{2}$; C) -1; D) 1.

10. Yesaplan': $\sqrt{\sqrt[3]{64}}$.

A) 2; B) $\sqrt{2}$; C) $2\sqrt{2}$; D) -2.

11. Yesaplan': $\sqrt[4]{8\sqrt[4]{16}}$.

A) 2; B) -2; C) $4\sqrt{2}$; D) 8.

12. Yesaplan': $\sqrt[3]{-4\cdot\sqrt[3]{8}}$.

A) 2; B) -2; C) $\sqrt[3]{-4}$; D) $\sqrt[6]{32}$.

13. Yesaplan': $\dfrac{\sqrt[3]{98}\cdot\sqrt[3]{-112}}{\sqrt[3]{500}}$.

A) $-\sqrt[3]{4}$; B) 2,84; C) –2,8; D) –1,4.

14. $a = 125$ bolg'anda $\sqrt{a} : \sqrt[6]{a}$ an'latpasi'ni'n' san ma'nisin tabi'n':

A) –25; B) 15; C) –5; D) 5.

15. $a = 0{,}04$ bolg'anda $\sqrt[3]{a}\cdot\sqrt[6]{a}$ an'latpasi'ni'n' san ma'nisin tabi'n':

A) 0,2; B) $\sqrt[3]{0,4}$; C) 0,4; D) –0,2.

16. An'latpani' a'piwayi'lasti'ri'n': $\left(a^5\right)^{-\frac{4}{5}}\cdot\left(b^{-\frac{3}{4}}\right)^{-\frac{2}{3}}$.

A) $a^{-4}\cdot b^{\frac{1}{2}}$; B) $a^4\cdot b^{-\frac{1}{2}}$; C) $a^5\cdot b^2$; D) $a^{-5}\cdot b^{-2}$.

17. An'latpani' a'piwayi'lasti'ri'n': $\left(\sqrt[3]{a}-\sqrt[3]{b}\right)\cdot\left(a^{\frac{2}{3}}+\sqrt[3]{ab}+b^{\frac{2}{3}}\right)$.

A) $a + b$; B) $a - b$; C) $a^3 + b^3$; D) $a^3 - b^3$.

18. An'latpani' a'piwayi'lasti'ri'n': $\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}\right):\left(\sqrt[3]{\dfrac{a}{b}}+\sqrt[3]{\dfrac{b}{a}}-2\right)$.

A) $\sqrt[3]{ab}$; B) $\sqrt[3]{a}+\sqrt[3]{b}$; C) $\dfrac{\sqrt[3]{ab}}{\sqrt[3]{a}-\sqrt[3]{b}}$; D) $\dfrac{\sqrt[3]{a}-\sqrt[3]{b}}{\sqrt[3]{ab}}$.

19. Sanlardi' sali'sti'ri'n': $a=\left(\dfrac{7}{12}\right)^{-\frac{1}{4}}$ ha'm $b=(0,58)^{-\frac{1}{4}}$.

A) $b = a + 0,5$; B) $a = b + 0,8$; C) $b < a$; D) $b > a$.

20. Sanlardi' sali'sti'ri'n': $a=(3,09)^{\sqrt{2}}$ ha'm $b=\left(3\dfrac{10}{11}\right)^{\sqrt{2}}$.

A) $a < b$; B) $a = b - 0,09$; C) $a > b$; D) $a = b$.

21. Sanlardi' wo'siw ta'rtibinde jaylasti'ri'n': $a=\sqrt{2}$, $b=\sqrt[3]{3}$,

$c=\sqrt[6]{7}$.

A) $c < a < b$; B) $c < b < a$; C) $b < a < c$; D) $a < b < c$.

22. Sanlardi' kemeyiw ta'rtibinde jaylasti'ri'n': $a=\sqrt[3]{2}$, $b=\sqrt[4]{3}$,

A) $a > b > c$; B) $b > c > a$; C) $c > a > b$; D) $b > a > c$.

Racional ko'rsetkishli da'reje **I. Nyuton (1643—1727)** ta'repinen kiritilgen. Qa'legen α haqi'yqi'y sani' ushi'n a^{α}, $a>0$, da'reje tu'sinigi **L. Eyler (1707—1783)** din'. «Analizge kirisiw» miynetinde berilgen.

Abu Rayxan Beruniy wo'zinin' belgili «Qononiy Masudiy» miynetinde «shen'ber uzi'nli'g'i'ni'n' woni'n' diametrine sali'sti'rg'anda irracional san» yekenligin aytadi'. A'yyemgi Greciyada «yeger kvadratti'n' ta'repi wo'lshem birlik yetip ali'nsa, woni'n' diagonali'n racional san menen an'lati'w mu'mkin yemesligi» da'lillengen. Bizin' erami'zg'a shekemgi V–IV a'sirlerden-aq a'yyemgi grek ali'mlari' toli'q kvadrat bolmag'an qa'legen n natural sani' ushi'n $\sqrt{n}$ sani'ni'n' irracional yekenligin da'lillegen.

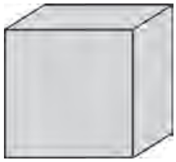
G'iyosiddin Jamshid al-Koshiydin'«Arifmetika gilti» miynetinde natural sannan koren shi'g'ari'wdi'n' uluwma usi'li' ayti'p wo'tiledi. $\sqrt[n]{a^n+r}$ korenin al-Koshiy juwi'q $\sqrt[n]{a^n+r}\approx a+\frac{r}{(a+1)^n-a^n}$ tu'rinde an'latadi', bunda a – natural san ha'm $r<(a+1)^n-a^n$.

Al-Koshiy korendi da'lirek yesaplaw ushi'n koren asti'ndag'i' sandi' 10 ni'n' sa'ykes da'rejesine ko'beytiwdi usi'ni's yetedi: $\sqrt[n]{N}=\frac{\sqrt[n]{10^{mn}\cdot N}}{10^m}$. Bo'lshekten koren shi'g'ari'wda paydalanadi': $\sqrt[n]{\frac{M}{N}}=\frac{\sqrt[n]{M\cdot N^{n-1}}}{N}$.

Soni'n' menen birge, Al-Koshiy korenlerdin' ko'beymesin uluwma ko'rsetkishke keltiriw qag'i'ydasi'n ayti'p wo'tken:

$$\sqrt[n]{a}\cdot\sqrt[k]{b}=\sqrt[kn]{a^k}\cdot\sqrt[kn]{b^n}=\sqrt[kn]{a^k\cdot b^n}.$$

IV BAP. DA’REJELI FUNKCIYA



$V=x^3$

$x=\sqrt[3]{V}$

$x=V^{\frac{1}{3}}$

$S=\pi r^2$

$r=\sqrt{\frac{S}{\pi}}$

14- §. FUNKCIYANI’N’ ANI’QLANI’W OBLASTI’

Siz 8-klasta funkciya tu’sinigi menen tani’sqansi’z. Usi’ tu’sinikti yesletip wo’temiz.

! **Yeger sanlardi’n’ bazi’ bir ko’pliginen ali’ng’an *x* ti’n’ ha’rbir ma’nisine *y* sani’ sa’ykes keltirilgen bolsa, usi’ ko’plikte *y(x) funkciyasi’* berilgen dep ayti’ladi’. Bunda *x yerkli wo’zgeriwshi* yaki *argument*, al *y yerksiz wo’zgeriwshi* yaki *funkciya* dep ataladi’.**

Siz $y=kx+b$ si’zi’qli’ funkciyasi’ ha’m de $y=ax^2+bx+c$ kvadrat funkciyasi’ menen tani’ssi’z. Bul funkciyalar ushi’n argumenttin’ ma’nisi qa’legen haqi’yqi’y san boli’wi’ mu’mkin.

Yendi ha’rbir teris yemes x sani’na $\sqrt{x}$ sani’ sa’ykes qoyi’latug’i’n funkciyani’, yag’ni’y $y=\sqrt{x}$ funkciyasi’n qaraymi’z. Bul funkciya ushi’n argument tek teris yemes ma’nislerdi qabi’l yetiwi mu’mkin: $x \geq 0$. Bul jag’dayda funkciya barli’q teris yemes sanlar ko’pliginde ani’qlang’an delinedi ha’m bul ko’plik $y=\sqrt{x}$ funkciyasi’ni’n’ *ani’qlani’w oblasti’* dep ataladi’.

Uluwma, funkciyani’n’ ani’qlani’w oblasti’ dep woni’n’ argumenti qabi’l yetiwi mu’mkin bolg’an barli’q ma’nislerinin’ ko’pligine ayti’ladi’.

Ma’selen, $y=\frac{1}{x}$ formulasi’ menen berilgen funkciya $x \neq 0$ de ani’qlang’an, yag’ni’y bul funkciyani’n’ ani’qlani’w oblasti’ — nolden wo’zge barli’q haqi’yqi’y sanlar ko’pligi.

Yeger funkciya formula menen berilgen bolsa, wol jag'dayda funkciya argumenttin' berilgen formula mag'anag'a iye bolatug'i'n (yag'ni'y formulani'n' won' bo'leginde turg'an an'latpada ko'rsetilgen barli'q a'meller wori'nlanatug'i'n) barli'q ma'nislerinde ani'qlang'an dep yesaplaw qabi'l yetilgen.

Formula menen berilgen funkciyani'n' ani'qlani'w oblasti'n tabi'w—argumenttin' formula mag'anag'a iye bolatug'i'n barli'q ma'nislerin tabi'w degendi an'latadi'.

1-ma'sele. Funkciyani'n' ani'qlani'w oblasti'n tabi'n':

1) $y(x)=2x^2+3x+5$; 2) $y(x)=\sqrt{x-1}$;

3) $y(x)=\frac{1}{x+2}$; 4) $y(x)=\sqrt[4]{\frac{x+2}{x-2}}$.

△ 1) $2x^2+3x+5$ an'latpasi' x ti'n' qa'legen ma'nisinde mag'anag'a iye bolg'ani' ushi'n, funkciya barli'q x larda ani'qlang'an.

Juwabi': x — qa'legen san.

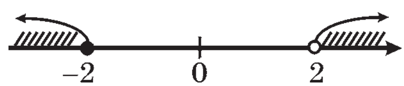
2) $\sqrt{x-1}$ an'latpasi' $x-1\geq 0$ bolg'anda mag'anag'a iye, yag'ni'y funkciya $x\geq 1$ bolg'anda ani'qlang'an.

Juwabi': $x\geq 1$.

3) $\frac{1}{x+2}$ an'latpasi' $x+2\neq 0$ bolg'anda mag'anag'a iye, yag'ni'y funkciya $x\neq -2$ bolg'anda ani'qlang'an.

Juwabi': $x\neq -2$.

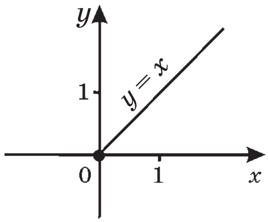
4) $\sqrt[4]{\frac{x+2}{x-2}}$ an'latpasi' $\frac{x+2}{x-2}\geq 0$ bolg'anda mag'anag'a iye. Bul ten'sizlikti sheship, mi'nag'an iye bolami'z (28-su'wret): $x\leq -2$ ha'm $x>2$, yag'ni'y funkciya $x\leq -2$ ha'm $x>2$ bolg'anda ani'qlang'an.



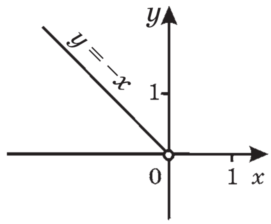
28-su'wret.

Juwabi': $x\leq -2$, $x>2$. ▲

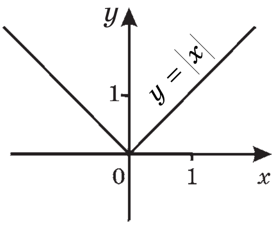
Funkciyani'n' grafigi dep, koordinatalar tegisliginin' abcissalari' usi' funkciyani'n' ani'qlani'w oblasti'nan ali'ng'an yerksiz wo'zgeriwshinin' ma'nislerine, al ordinatalari' funkciyani'n' sa'ykes ma'nislerine ten' bolg'an noqatlar ko'pligine ayti'latug'i'ni'n yesletip wo'temiz.



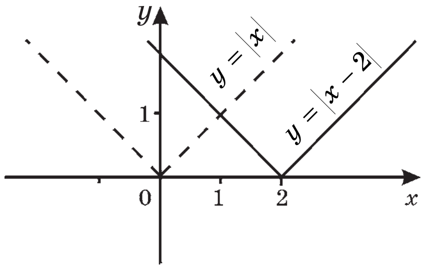
29- su'wret.



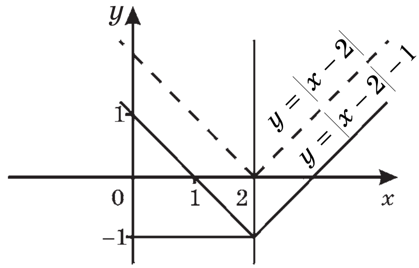
30- su'wret.



31- su'wret.



32- su'wret.



33- su'wret.

2-ma'sele. $y = |x|$ funkciyasi'ni'n' ani'qlani'w oblasti'n tabi'n' ha'm woni'n' grafigin jasan'.

△ Yesletip wo'temiz: $|x| = \begin{cases} x, \text{ yeger } x \geq 0 \text{ bolsa,} \\ -x, \text{ yeger } x < 0 \text{ bolsa.} \end{cases}$

Solay yetip, $|x|$ an'latpasi' qa'legen haqi'yqi'y x ta mag'anag'a iye, yag'ni'y $y = |x|$ funkciyasi'ni'n' ani'qlani'w oblasti' barli'q haqi'yqi'y sanlar ko'pliginen ibarat.

Yeger $x \geq 0$ bolsa, wonda $|x| = x$ boladi' ha'm sonli'qtanda $x \geq 0$ bolg'anda $y = |x|$ funkciyasi'ni'n' grafigi birinshi koordinata mu'yeshinin' bissektrisasi' boladi' (29-su'wret).

Yeger $x < 0$ bolsa, wonda $|x| = -x$ boladi', demek, teris x lar ushi'n $y = |x|$ funkciyasi'ni'n' grafigi yekinshi koordinata mu'yeshinin' bissektrisasi' boladi' (30-su'wret).

$y = |x|$ funkciyasi'ni'n' grafigi 31-su'wrette ko'rsetilgen. ▲

Qa'legen x ushi'n $|-x| = -x$. Sonli'qtan da $y = |x|$ funkciyasi'ni'n' grafigi ordinatalar ko'sherine qarata simmetriyali' jaylasqan.

3-ma'sele. $y = |x - 2| - 1$ funkciyasi'ni'n' grafigin jasan'.

△ $y = |x-2|$ funkciyasi'ni'n' grafigi $y = |x|$ funkciyasi'ni'n' grafiginen woni' Ox ko'sheri boyi'nsha 2 birlik won'g'a ji'li'sti'ri'w menen payda yetiledi (32-su'wret).

$y = |x-2|-1$ funkciyasi'ni'n' grafigin payda yetiw ushi'n $y = |x-2|$ funkciyasi'ni'n' grafigin bir birlik to'menge qaray ji'li'sti'ri'w jetkilikli (33-su'wret). ▲

Shi'ni'g'i'wlar

191. Funkciya $y(x) = x^2 - 4x + 5$ formulasi' menen berilgen:
1) $y(-3)$, $y(-1)$, $y(0)$, $y(2)$ ni tabi'n'; 2) yeger $y(x)=1$, $y(x)=5$, $y(x)=10$, $y(x)=17$ bolsa, x ti'n' ma'nisin tabi'n'.

192. Funkciya $y(x) = \frac{x+5}{x-1}$ formulasi' menen berilgen.
1) $y(-2)$, $y(0)$, $y(\frac{1}{2})$, $y(3)$ ti tabi'n'; 2) yeger $y(x)=-3$, $y(x)=-2$, $y(x)=13$, $y(x)=19$ bolsa, x ti'n' ma'nisin tabi'n'.

Funkciyani'n' ani'qlani'w oblasti'n tabi'n' **(193–194):**

193. (Awi'zeki.)
1) $y = 4x^2 - 5x + 1$; 2) $y = 2 - x - 3x^2$; 3) $y = \frac{2x-3}{x-3}$;
4) $y = \frac{3}{5-x^2}$; 5) $y = \sqrt[4]{6-x}$; 6) $y = \sqrt{\frac{1}{x+7}}$;
7) $y = -x^2 + 5x + 1$; 8) $y = \frac{3x-4}{x+1}$; 9) $y = \sqrt{1-x} + \sqrt{x-1}$.

194. 1) $y = \frac{2x}{x^2-2x-3}$; 2) $y = \sqrt[6]{x^2 - 7x + 10}$;
3) $y = \sqrt[3]{3x^2 - 2x + 5}$; 4) $y = \sqrt[6]{\frac{2x+4}{3-x}}$; 5) $y = \sqrt{\frac{3x-2}{4-x}}$.

---

195. Funktsiya $y(x) = |2-x| - 2$ formulası' menen berilgen.
1) $y(-3)$, $y(-1)$, $y(1)$, $y(3)$ ni tabı'n';
2) eger $y(x) = -2$, $y(x) = 0$, $y(x) = 2$, $y(x) = 4$ bolsa, x tı'n' ma'nisin tabı'n'.

196. Funkciyani'n' ani'qlani'w oblasti'n tabi'n':

1) $y = \sqrt{\frac{x-2}{x+3}}$; 2) $y = \sqrt[3]{\frac{1-x}{1+x}}$;

3) $y=\sqrt[4]{(x-1)(x-2)(x-3)}$;   4) $y=\sqrt{\frac{x^2-4}{x+1}}$;

5) $y=\sqrt{(x+1)(x-1)(x-4)}$;   6) $y=\sqrt[8]{\frac{x^2+4x-5}{x-2}}$;

7) $y=\sqrt[4]{-x}+\sqrt{x+2}$;   8) $y=\sqrt[6]{x}+\sqrt{1+x}$.

197. (–2; 1) noqati' funkciyani'n' grafigine tiyisli me:

1) $y=3x^2+2x+29$;   2) $y=|4-3x|-9$;

3) $y=\frac{x^2+3}{x-1}$;   4) $y=\left|\sqrt{2-x}-5\right|-2$?

198. Funkciyani'n' grafigin jasan':

1) $y=|x+3|+2$;   2) $y=-|x|$;   3) $y=2|x|+1$;

4) $y=1-|1-2x|$;   5) $y=|x|+|x-2|$;   6) $y=|x+1|-|x|$.

199. $y=ax^2+bx+c$ funkciyasi'ni'n' grafigi A (0; 1), B (1; 2), C ($\frac{5}{6}$; 1) noqatlari' arqali' wo'tedi. 1) a, b, c ni' tabi'n', 2) x ti'n' qanday ma'nislerinde $y=0$ boladi'? 3) Funkciyani'n' grafigin si'zi'n'.

15- §. FUNKCIYANI'N' WO'SIWI HA'M KEMEYIWI

Siz $y=x$ ha'm $y=x^2$ funkciyalari' menen tani'ssi'z. Bul funkciyalar *da'rejeli funkciyani'n'*, yag'ni'y

$$y=x^r \tag{1}$$

(bunda r – berilgen san) funkciyani'n' jeke hallari'nan ibarat.

r – natural san bolsi'n, $r=n=1, 2, 3, \ldots$ deyik. Bul jag'dayda natural ko'rsetkishli da'rejeli funkciya $y=x^n$ di payda yetemiz.

Bul funkciya barli'q haqi'yqi'y sanlar ko'pliginde, yag'ni'y san ko'sherinin' barli'q jerinde ani'qlang'an. A'dette, barli'q haqi'yqi'y sanlar ko'pligi $\boldsymbol{R}$ ha'ribi menen belgilenedi. Solay yetip, natural ko'rsetkishli da'rejeli funkciya $y=x^n$, $x\in \boldsymbol{R}$ ushi'n ani'qlang'an. Yeger (1) da $r=-2k$, $k\in \boldsymbol{N}$ bolsa, wonda $y=x^{-2k}=\frac{1}{x^{2k}}$ funkciyasi' payda boladi'. Bul funkciya x ti'n' nolden basqa barli'q ma'nislerinde ani'qlang'an. Woni'n' grafigi Oy ko'sherine sali'sti'rg'anda simmetriyali' boladi'. $r=-(2k-1)$, $k\in \boldsymbol{N}$ bolsa, wonda $y=x^{-(2k-1)}=\frac{1}{x^{2k-1}}$

funkciyasi' kelip shi'g'adi'. Woni'n' qa'siyetleri sizge tani's $y=\frac{1}{x}$ funkciyasi'ni'n' qa'siyetleri si'yaqli' boladi'. p ha'm q – natural sanlar ha'm $r=\frac{p}{q}$ – qi'sqarmaytug'i'n bo'lshek bolsi'n. $y=\sqrt[q]{x^p}$ funkciyasi'ni'n' ani'qlani'w oblasti' p ha'm q din' jup-taqli'g'i'na qarap ha'rqi'yli' boladi'. Ma'selen, $y=\sqrt[3]{x^2}$, $y=\sqrt[3]{x}$ funkciyalari' yerkli $x\in \boldsymbol{R}$ de ani'qlang'an. $y=\sqrt[4]{x^3}$ funkciyasi' bolsa, x ti'n' teris yemes, yag'ni'y $x\geq 0$ ma'nislerinde ani'qlang'an.

8-klass «Algebra» kursi'nan ma'lim bolg'ani'nday ha'rbir irracional sandi' shekli wonli'q bo'lshek penen, yag'ni'y racional san menen almasti'ri'w mu'mkin. A'meliy islerde irracional sanlar u'stinde a'meller wolardi'n' racional juwi'qlasi'wlari' ja'rdeminde wori'nlanadi'. Bul a'meller sonday yetip kiritiledi, a'mellerdin' ten'lik ha'm ten'sizliklerdin' racional sanlar ushi'n qa'siyetleri irracional sanlar ushi'n da toli'q saqlanadi'.

$r_1, r_2, \ldots, r_k, \ldots$ racional sanlari' r irracional sani'ni'n' racional juwi'qlasi'wlari' bolsi'n. Bul jag'dayda x won' san bolg'anda, x ti'n' racional da'rejeleri, yag'ni'y $x^{r_1}, x^{r_2}, \ldots, x^{r_k}, \ldots$ sanlari' x^r da'rejenin' (da'rejeli sanni'n') juwi'qlasi'wlari' boladi'. Bunday ani'qlang'an da'reje *irracional ko'rsetkishli da'reje* dep ataladi'. Demek, $x>0$ ushi'n da'reje ko'rsetkishi qa'legen r bolg'an $y=x^r$ funkciyasi'n ani'qlaw mu'mkin.

Da'rejeli funkciya x ti'n' (1) formula mag'anag'a iye bolatug'i'n ma'nisleri ushi'n ani'qlang'an. Ma'selen, $y=x$ ha'm $y=x^2$ ($r=1$ ha'm $r=2$) funkciyalari'ni'n' ani'qlani'w oblasti' haqi'yqi'y sanlar ko'pligi boladi'; $y=\frac{1}{x}$ ($r=-1$) funkciyani'n' ani'qlani'w oblasti' nolge ten' bolmag'an barli'q haqi'yqi'y sanlar ko'pligi boladi'; $y=\sqrt{x}$ ($r=\frac{1}{2}$) funkciyasi'ni'n' ani'qlani'w oblasti' barli'q teris yemes sanlar ko'pliginen ibarat.

! **Yeger argumenttin' bazi' bir arali'qtan ali'ng'an u'lken ma'nisine funkciyani'n' u'lken ma'nisi sa'ykes kelse, yag'ni'y usi' arali'qqa tiyisli qa'legen x_1, x_2 ushi'n $x_2>x_1$ ten'sizliginen $y(x_2)>y(x_1)$ ten'sizligi kelip shi'qsa, $y(x)$ funksiyasi' usi' arali'qta *wo'siwshi* funkciya delinedi.**

Yeger bazi' bir arali'qqa tiyisli qa'legen x_1, x_2 ushi'n $x_2 > x_1$ ten'sizliginen $y(x_2) < y(x_1)$ kelip shi'qsa, $y(x)$ funkciyasi' usi' arali'qta *kemeyiwshi* funkciya delinedi.

Ma'selen, $y = x$ funkciyasi' sanlar ko'sherinde wo'sedi, $y = x^2$ funkciyasi' $x \geq 0$ arali'g'i'nda wo'sedi, $x \leq 0$ arali'g'i'nda kemeyedi.

$y = x^r$ da'rejeli funkciyasi'ni'n' wo'siwi yaki kemeyiwi da'reje ko'rsetkishinin' belgisine baylani'sli'.

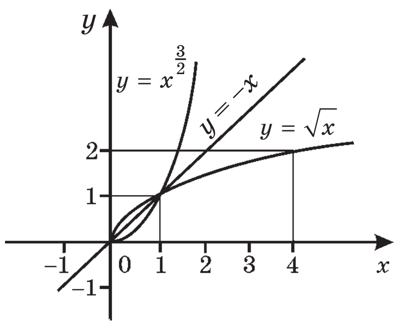
Yeger $r > 0$ bolsa, wonda $y = x^r$ da'rejeli funkciyasi' $x \geq 0$ arali'g'i'nda *wo'sedi*.

$x_2 > x_1 \geq 0$ bolsi'n. $x_2 > x_1$ ten'sizligin won' r da'rejege ko'terip, $x_2^r > x_1^r$ di, yag'ni'y $y(x_2) > y(x_1)$ di payda yetemiz.

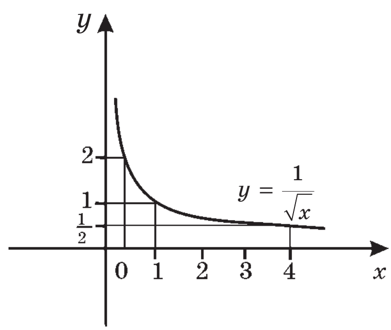
Ma'selen, $y = \sqrt{x}$ ha'm $y = x^{\frac{3}{2}}$ funkciyalari' $x \geq 0$ arali'g'i'nda wo'sedi. Bul funkciyalardi'n' grafikleri 34-su'wrette ko'rsetilgen. Usi' su'wretten $y = \sqrt{x}$ funkciyasi'ni'n' grafigi $0 < x < 1$ arali'g'i'nda $y = x$ funkciyasi'ni'n' grafiginen joqari'da, $x > 1$ al arali'qta $y = x$ funkciyasi'ni'n' grafiginen to'mende jatatug'i'nli'g'i' ko'rinip tur.

Yeger $0 < r < 1$ bolsa, $y = x^r$ funkciyasi'ni'n' grafigi tap usi'nday $y = x^{\frac{3}{2}}$ funkciyasi'ni'n' grafigi $0 < x < 1$ arali'g'i'nda $y = x$ funkciyasi'ni'n' grafiginen to'mende, $x > 1$ arali'qta $y = x$ funkciyasi'ni'n' grafiginen to'mende jatatug'i'nli'g'i' ko'rinip tur.

$r > 1$ bolsa, $y = x^r$ funkciyasi'ni'n' grafigi tap usi'nday qa'siyetke iye boladi'.



34- su'wret.



35- su'wret.

Yendi $r < 0$ bolg'an jag'daydi' qaraymi'z.

Yeger $r < 0$ bolsa, wonda $y = x^r$ da'rejeli funkciyasi' $x > 0$ arali'g'i'nda *kemeyedi*.

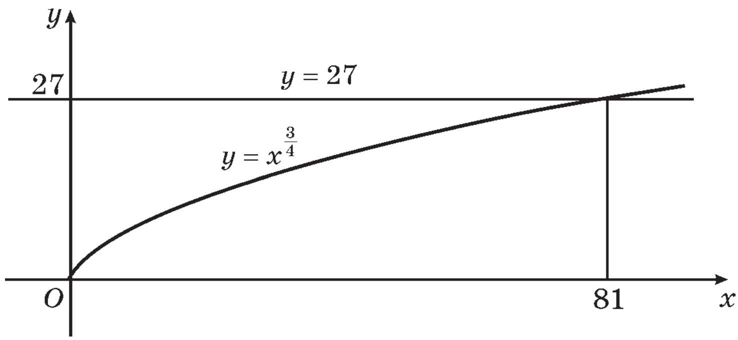
$x_2 > x_1 > 0$ bolsi'n. $x_2 > x_1$ ten'sizligin won' r da'rejege ko'terip, shep ha'm won' ta'repleri won' bolg'an ten'sizliklerdin' qa'siyetine muwapi'q $x_2^r < x_1^r$ di, yag'ni'y $y(x_2) < y(x_1)$ payda yetemiz.

Ma'selen, $y = \frac{1}{\sqrt{x}}$, yag'ni'y $y = x^{-\frac{1}{2}}$ funkciyasi' $x > 0$ arali'g'i'nda kemeyedi. Bul funkciyani'n' grafigi 35-su'wrette ko'rsetilgen.

1-ma'sele. $x^{\frac{3}{4}} = 27$ ten'lemesin sheshin'.

$y = x^{\frac{3}{4}}$ funkciyasi' $x \geq 0$ de ani'qlang'an. Sonli'qtan da, berilgen ten'leme tek teris yemes korenlerge g'ana iye boli'wi' mu'mkin. Bunday $x = 27^{\frac{4}{3}} = \left(\sqrt[3]{3^3}\right)^4 = 3^4 = 81$. Ten'lemenin' basqa koreni joq, sebebi $y = x^{\frac{3}{4}}$ funksiya $x \geq 0$ bolg'anda wo'sedi ha'm sonli'qtan da, yeger $x > 81$ bolsa, wonda $x^{\frac{3}{4}} > 27$, yeger $x < 81$ bolsa, wonda $x^{\frac{3}{4}} < 27$ boladi' (36-su'wret).

$x^r = b$ (bunda $r \neq 0$, $b > 0$) ten'lemesinin' ba'rqulla won' $x = b^{\frac{1}{r}}$ korenge iye yekenligi, soni'n' menen birge bul korennin' jalg'i'z bir g'ana yekenligi usi'g'an uqsas da'lillenedi. Demek, $y = x^r$ (bunda $r > 0$) funkciya $x > 0$ bolg'anda *barli'q won' ma'nislerdi qabi'l yetedi.*



36- su'wret.

Bul bolsa, ma'selen $y = x^{\frac{3}{4}}$ (36-su'wret) funkciyasi'ni'n' a'stelik penen wo'siwine qaramastan, woni'n' grafigi Ox ko'sherinen kem-kemnen uzaqlasi'wi' ha'm $y = b$ tuwri'si'n, b ni'n' qanday won' san boli'wi'na qaramastan kesip wo'tetug'i'ni'n bildiredi.

2-ma'sele. $y = x + \frac{1}{x}$ funkciyasi'ni'n' $x > 1$ arali'qta wo'se-tug'i'ni'n da'lillen'.

△ $x_2 > x_1 > 1$ bolsi'n. $y(x_2) > y(x_1)$ yekenligin ko'rsetemiz. $y(x_2) - y(x_1)$ ayi'rmasi'n qaraymi'z:

$$y(x_2) - y(x_1) = x_2 + \frac{1}{x_2} - (x_1 + \frac{1}{x_1}) = (x_2 - x_1)\frac{x_1 x_2 - 1}{x_1 x_2}.$$

$x_2 > x_1$, $x_1 > 1$, $x_2 > 1$ bolg'anli'qtan $x_2 - x_1 > 0$, $x_1 x_2 > 1$, $x_1 x_2 > 0$. Sonli'qtan da $y(x_2) - y(x_1) > 0$, yag'ni'y $y(x_2) > y(x_1)$. ▲

Shi'ni'g'i'wlar

200. Funkciyani'n' grafigin jasan' ha'm de woni'n' wo'siw ha'm kemeyiw arali'qlari'n tabi'n':

1) $y = 2x + 3$; 2) $y = 1 - 3x$; 3) $y = x^2 + 2$;
4) $y = 3 - x^2$; 5) $y = (1 - x)^2$; 6) $y = (2 + x)^2$.

201.(Awi'zeki.) Funkciya $x>0$ arali'qta wo'sedi me yaki kemeye me:

1) $y = x^{\frac{3}{7}}$; 2) $y = x^{-\frac{3}{4}}$; 3) $y = x^{-\sqrt{2}}$; 4) $y = x^{\sqrt{3}}$?

202. $x > 0$ bolg'anda:

1) $y = x^{\frac{3}{2}}$; 2) $y = x^{\frac{2}{3}}$; 3) $y = x^{-\frac{3}{2}}$; 4) $y = x^{-\frac{2}{3}}$

funkciyasi' grafiginin' eskizin si'zi'n'.

203. Ten'lemenin' won' korenin tabi'n':

1) $x^{\frac{1}{2}} = 3$; 2) $x^{\frac{1}{4}} = 2$; 3) $x^{-\frac{1}{2}} = 3$; 4) $x^{-\frac{1}{4}} = 2$;
5) $x^{\frac{5}{6}} = 32$; 6) $x^{-\frac{4}{5}} = 81$; 7) $x^{-\frac{1}{3}} = 8$; 8) $x^{\frac{4}{5}} = 16$.

204. Millimetrli qag'azg'a $y = \sqrt[4]{x}$ funkciyasi'ni'n' grafigin si'zi'n'. Grafik boyi'nsha:

1) $y = 0{,}5;\ 1;\ 4;\ 2{,}5$ bolg'anda x ti'n' ma'nislerin tabi'n';

2) $\sqrt[4]{1{,}5};\ \sqrt[4]{2};\ \sqrt[4]{2{,}5};\ \sqrt[4]{3}$ ma'nislerin juwi'q tu'rde tabi'n'.

205. Funkciyalardi'n' grafiklerinin' kesilisiw noqatlari'ni'n' koordinatalari'n tabi'n':

1) $y = x^{\frac{4}{3}}$ ha'm $y = 625$; 2) $y = x^{\frac{6}{5}}$ ha'm $y = 64$;

3) $y = x^{\frac{3}{2}}$ ha'm $y = 216$; 4) $y = x^{\frac{7}{3}}$ ha'm $y = 128$.

206. 1) $y = x + \frac{1}{x}$ funkciyasi'ni'n' $0 < x < 1$ arali'qta kemeyiwin da'lillen';

2) $y = \frac{1}{x^2+1}$ funkciyasi'ni'n' $x \geq 0$ arali'qlarda kemeyetug'i'ni'n ha'm $x \leq 0$ arali'qta wo'setug'i'ni'n da'lillen';

3) $y = x^3 - 3x$ funkciyasi'ni'n' $x \leq -1$ ha'm $x \geq 1$ arali'qlarda wo'setug'i'ni'n ha'm $-1 \leq x \leq 1$ kesindisinde kemeyetug'i'ni'n da'lillen';

4) $y = x - 2\sqrt{x}$ funkciyasi'ni'n' $x \geq 1$ arali'qlarda wo'setug'i'ni'n ha'm $0 \leq x \leq 1$ kesindisinde kemeyetug'i'ni'n da'lillen';

207. Funkciyasi'ni'n' grafigin jasan' ha'm de woni'n' wo'siw ha'm kemeyiw arali'qlari'n tabi'n':

1) $y = \begin{cases} x+2, & \text{yeger } x \leq -1 \text{ bolsa,} \\ x^2, & \text{yeger } x > -1 \text{ bolsa;} \end{cases}$

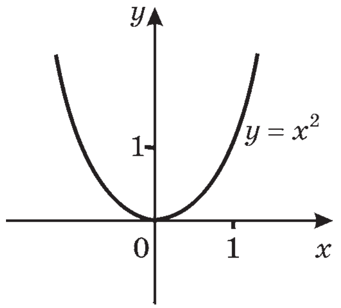
2) $y = \begin{cases} x^2, & \text{yeger } x \leq 1 \text{ bolsa,} \\ 2 - x^2, & \text{yeger } x > 1 \text{ bolsa;} \end{cases}$

3) $y = \begin{cases} -x-1, & \text{yeger } x < -1 \text{ bolsa,} \\ -x^2+1, & \text{yeger } x \geq -1 \text{ bolsa;} \end{cases}$

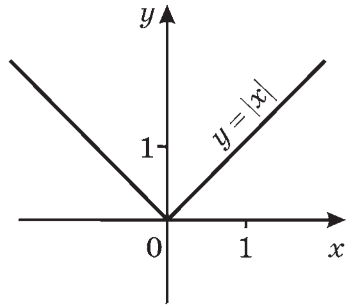
4) $y = \begin{cases} x^3, & \text{yeger } x \leq 1 \text{ bolsa,} \\ -x^2+2x, & \text{yeger } x \geq 1 \text{ bolsa.} \end{cases}$

16- §. FUNKCIYANI'N' JUPLI'LI'G'I' HA'M TAQLI'G'I'

Siz $y = x^2$ ha'm $y = |x|$ funkciyalari'ni'n' grafikleri ordinatalar ko'sherine qarata simmetriyali' (37 ha'm 38-su'wretler) yekenligin bilesiz. Bunday funkciyalar *jup funkciyalar* dep ataladi'.



37- su'wret.



38- su'wret.

Yeger $y(x)$ funkciyasi'ni'n' ani'qlani'w oblasti'nan ali'ng'an qa'legen x ushi'n $y(-x) = y(x)$ bolsa, bul funkciya *jup funkciya* dep ataladi'.

Ma'selen, $y = x^4$ ha'm $y = \frac{1}{x^2}$ funkciyalari' jup funkciyalar, sebebi qa'legen x ushi'n $(-x)^4 = x^4$ ha'm qa'legen $x \neq 0$ ushi'n $\frac{1}{(-x)^2} = \frac{1}{x^2}$.

1-ma'sele. $y=x^3$ funkciyasi'ni'n' grafigi koordinatalar basi'na sali'sti'rg'anda simmetriyali' yekenligin da'lillen' ha'm grafigin jasan'.

△ 1) $y = x^3$ funkciyasi'ni'n' ani'qlani'w oblasti' — barli'q haqi'yqi'y sanlar ko'pligi.

2) $y = x^3$ funkciyasi'ni'n' ma'nisleri $x > 0$ bolg'anda won', $x < 0$ bolg'anda teris, al $x = 0$ bolg'anda nolge ten'.

○ Meyli, $(x_0;\ y_0)$ noqati' $y = x^3$ funkciyasi'ni'n' grafigine tiyisli, yag'ni'y $y_0 = x_0^3$ bolsi'n deyik. $(x_0;\ y_0)$ noqati'na koordinatalar basi'na sali'sti'rg'anda simmetriyali' bolg'an noqat $(-x_0;\ -y_0)$ koordinatalari'na iye boladi'. Bul noqatta funkciyani'n' grafigine tiyisli boladi', sebebi $y = x^3$ funkciyasi'ni'n' grafigin jasawg'a imkaniyat beredi: da'slep grafik $y_0 = x_0^3$ duri's ten'liginin' yeki jag'i'n da -1 ge ko'beytip $-y_0 = -x_0^3$ ti, yaki $-y_0 = (-x_0)^3$ ti payda yetemiz. ●

Bul qa'siyet $y = x^3$ funkciyasi'ni'n' grafigin jasawg'a imkaniyat beredi: da'slep grafik $x \geq 0$ ushi'n jasaladi', al son'i'nan woni' koordinatalar basi'na qarata simmetriyali' sa'wlelendiriledi.

3) $y=x^3$ funkciyasi' pu'tkil ani'qlani'w oblasti' boyi'nsha wo'sedi. Bul won' ko'rsetkishli da'rejeli funkciyani'n' $x \geq 0$ bolg'anda wo'siw

qa'siyetinen ha'm grafiktin' koordinatalar basi'na qarata simmetriyali'li'g'i'nan kelip shi'g'adi'.

4) $x \geq 0$ din' bazi' bir ma'nisleri (ma'selen, $x=0$, 1, 2, 3) ushi'n $y=x^3$ funkciyasi'ni'n' ma'nislerinin' kestesin du'zemiz, $x \geq 0$ bolg'anda grafiktin' bir bo'legin jasaymi'z ha'm son'i'nan simmetriya ja'rdeminde grafiktin' x ti'n' teris ma'nislerine sa'ykes keliwshi bo'legin jasaymi'z (39-su'wret). ▲

Grafikleri koordinatalar basi'na qarata simmetriyali' bolg'an funkciyalar *taq* funkciyalar dep ataladi'. Solay yetip, $y = x^3$ – bul *taq* funkciya.

Yeger $y(x)$ funkciyasi'ni'n' ani'qlani'w oblasti'nan ali'ng'an qa'legen x ushi'n

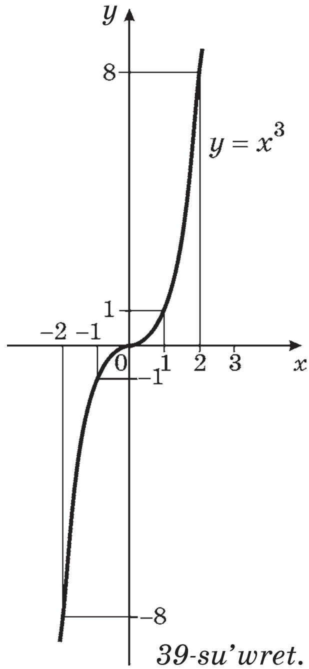
$$y(-x) = -y(x)$$

bolsa, bul funkciya *taq funkciya* delinedi.

Ma'selen, $y = x^5$, $y = \frac{1}{x^3}$ funkciyalari' taq funkciyalar, sebebi qa'legen x ushi'n $(-x)^5 = -x^5$ ha'm qa'legen $x \neq 0$ ushi'n $\frac{1}{(-x)^3} = -\frac{1}{x^3}$.

Jup ha'm taq funkciyalardi'n' *ani'qlani'w oblasti' koordinatalar basi'na qarata simmetriyali'* yekenligin yeskertip wo'temiz.

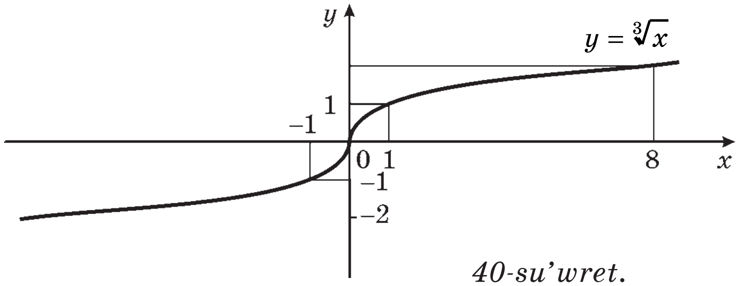
Jupli'q yaki taqli'q qa'siyetlerge iye bolmag'an funkciyalar da bar. Ma'selen, $y = 2x + 1$ funkciyasi'ni'n' jup ta, taq ta funkciya yemesligin ko'rsetemiz. Yeger bul funkciya jup bolg'ani'nda, wonda barli'q x ushi'n $2(-x) + 1 = 2x + 1$ ten'ligi wori'nlang'an bolar yedi, biraq, ma'selen $x=1$ bolg'anda bul ten'lik naduri's: $-1 \neq 3$. Yeger bul funkciya taq bolg'ani'nda, wonda barli'q x ushi'n $2(-x)+1=-(2x+1)$ ten'ligi wori'nlang'an bolar yedi; biraq, ma'selen, $x = 2$ bolg'anda bul ten'lik naduri's: $-3 \neq -5$.



39-su'wret.

2-ma'sele $y = \sqrt[3]{x}$ funkciyasi'ni'n' grafigin jasan'.

△ 1) Ani'qlani'w oblasti' — barli'q haqi'yqi'y sanlar ko'pligi.



40-su'wret.

2) funkciya — taq funkciya, sebebi qa'legen x ushi'n $\sqrt[3]{-x} = -\sqrt[3]{x}$.

3) $x \geq 0$ bolg'anda funkciya won' ko'rsetkishli da'rejeli funkciyani'n' qa'siyetine muwapi'q wo'sedi, sebebi $x \geq 0$ bolg'anda $\sqrt[3]{x} = x^{\frac{1}{3}}$.

4) $x > 0$ bolg'anda funkciyani'n' ma'nisi won': $y(0) = 0$.

5) grafikke tiyisli birneshe, ma'selen (0; 0), (1; 1), (8; 2) noqatlardi' tawi'p, $x \geq 0$ din' ma'nisleri ushi'n grafiktin' bir bo'legin jasaymi'z ha'm keyin simmetriya ja'rdeminde $x < 0$ ushi'n grafiktin' yekinshi bo'legin jasaymi'z (40-su'wret). ▲

$y = \sqrt[3]{x}$ funkciyasi' barli'q x lar ushi'n, al $y = x^{\frac{1}{3}}$ funkciyasi' tek $x \geq 0$ ushi'n ani'qlang'ani'n yeskertip wo'temiz.

Shi'ni'g'i'wlar

Funkciyani'n' taq yaki jup boli'wi'n ani'qlan' **(208–209):**

208. 1) $y = 2x^4$; | 2) $y = 3x^5$; | 3) $y = x^2 + 3$; | 4) $y = x^3 - 2$.

209. 1) $y = x^{-4}$; 2) $y = x^{-3}$; 3) $y = x^4 + x^2$;

4) $y = x^3 + x^5$; 5) $y = x^2 - x + 1$; 6) $y = \frac{1}{x+1}$.

210. Funkciya grafiginin' eskizin si'zi'n':

1) $y = x^4$; 2) $y = x^5$; 3) $y = -x^2 + 3$;

4) $y = \sqrt[5]{x}$; 5) $y = \sqrt[5]{-x}$.

211. Funkciyani'n' jup ta, taq ta yemesligin ko'rsetin':

1) $y = \frac{x+2}{x-3}$; 2) $y = \frac{x^2+x-1}{x+4}$; 3) $y = \frac{x-1}{x+1}$.

212. Funkciyani'n' jup yaki taq boli'wi'n ani'qlan':

1) $y = x^4 + 2x^2 + 3$; 2) $y = x^3 + 2x + 1$; 3) $y = \frac{3}{x^3} + \sqrt[3]{x}$;

4) $y = x^4 + |x|$; 5) $y = |x| + x^3$; 6) $y = \sqrt[3]{x - 1}$.

213. Simmetriyadan paydalani'p, jup funkciyani'n' grafigin jasan':

1) $y = x^2 - 2|x| + 1$; 2) $y = x^2 - 2x$.

214. Simmetriyadan paydalani'p, taq funkciyani'n' grafigin jasan':

1) $y = x|x| - 2x$; 2) $y = x|x| + 2x$.

215. Funkciyani'n' qa'siyetlerin ani'qlan', woni'n' grafigin jasan':

1) $y = \sqrt{x - 5}$; 2) $y = \sqrt{x} + 3$; 3) $y = x^4 + 2$;

4) $y = 1 - x^4$; 5) $y = (x + 1)^3$; 6) $y = x^3 - 2$.

216. Funkciyani'n' grafigin jasan':

1) $y = \begin{cases} x^2, & \text{yeger } x \geq 0 \text{ bolsa,} \\ x^3, & \text{yeger } x < 0 \text{ bolsa;} \end{cases}$ 2) $y = \begin{cases} x^3, & \text{yeger } x > 0 \text{ bolsa,} \\ x^2, & \text{yeger } x \leq 0 \text{ bolsa;} \end{cases}$

3) $y = \begin{cases} -x^3, & \text{yeger } x \leq 0 \text{ bolsa,} \\ -x^2, & \text{yeger } x \geq 0 \text{ bolsa;} \end{cases}$ 4) $y = \begin{cases} x^4, & \text{yeger } x \leq 1 \text{ bolsa,} \\ -x^2 + 2x, & \text{yeger } x \geq 1 \text{ bolsa.} \end{cases}$

Argumenttin' qanday ma'nislerinde funkciyani'n' ma'nisleri won' bolatug'i'ni'n ani'qlan'. Wo'siw ha'm kemeyiw arali'qlari'n ko'rsetin'.

217. y funkciyasi' berilgen:

1) $y = x$; 2) $y = x^2$; 3) $y = x^2 + x$; 4) $y = x^2 - x$.

$x > 0$ bolg'anda y funkciyasi'ni'n' grafigin jasan'. $x < 0$ ushi'n usi' funkciyalardi'n' ha'rbirinin' grafigin jasan', jasalg'an grafik: a) jup funkciyani'n'; b) taq funkciyani'n' grafigi bolsi'n. Payda yetilgen ha'rbir funkciyani' bir formula menen berin'.

218. Funkciya grafiginin' simmetriya ko'sherinin' ten'lemesin jazi'n':

1) $y = (x + 1)^6$; 2) $y = x^6 + 1$; 3) $y = (x - 1)^4$.

219. Funkciya grafiginin' simmetriya worayi'ni'n' koordinatalari'n ko'rsetin':

1) $y = x^3 + 1$; 2) $y = (x + 1)^3$; 3) $y = x^5 - 1$.

17- §. $y = \frac{k}{x}$ FUNKCIYA

1-ma'sele. $y = \frac{1}{x}$ funkciyasi'ni'n' grafigin jasan'.

△ 1) ani'qlani'w oblasti' — nolden wo'zge barli'q haqi'yqi'y sanlar.

2) funkciya — taq funkciya, sebebi $x \neq 0$ bolg'anda $\frac{1}{-x} = -\frac{1}{x}$.

3) funkciya $x > 0$ arali'qta teris ko'rsetkishli da'rejeli funkciyani'n' qa'siyetine muwapi'q kemeyedi, sebebi $\frac{1}{x} = x^{-1}$.

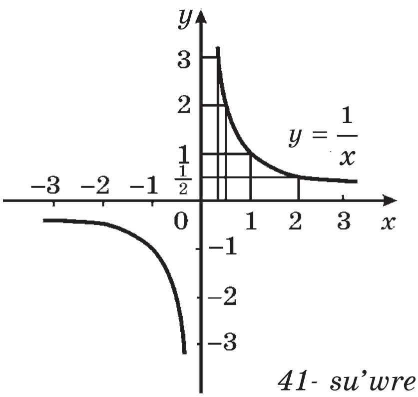
4) $x > 0$ bolg'anda funkciya won' ma'nislerdi qabi'l yetedi.

5) grafikke tiyisli birneshe, ma'selen $(\frac{1}{3}; 3)$, $(\frac{1}{2}; 2)$, $(1; 1)$, $(2; \frac{1}{2})$ noqatlari'n tawi'p, $x > 0$ ma'nisleri ushi'n grafiktin' bir bo'legin jasaymi'z ha'm son'i'nan simmetriya ja'rdeminde $x < 0$ ushi'n qalg'an bo'legin jasaymi'z (41-su'wret). ▲

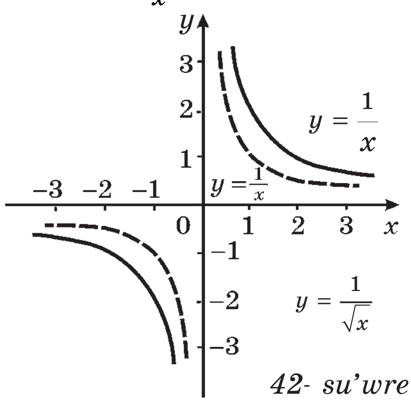
$y = \frac{1}{x}$ funkciyasi'ni'n' grafigi *giperbola* dep ataladi'. Wol *shaqalar* dep atali'wshi' yeki bo'lekten du'zilgen. Shaqalardan biri birinshi sherekte, al yekinshisi u'shinshi sherekte jaylasqan.

2-ma'sele. $k = 2$ ha'm $k = -2$ bolg'anda $y = \frac{k}{x}$ funkciyasi'ni'n' grafigin jasan'.

△ Argumenttin' da'l birdey ma'nislerinde $y = \frac{2}{x}$ funkciyasi'ni'n' ma'nisleri $y = \frac{1}{x}$ funkciyasi'ni'n' ma'nislerin 2 ge ko'beytiw menen payda yetiletug'i'ni'n yesletemiz Demek, $y = \frac{2}{x}$ funkciyasi'ni'n' grafigi



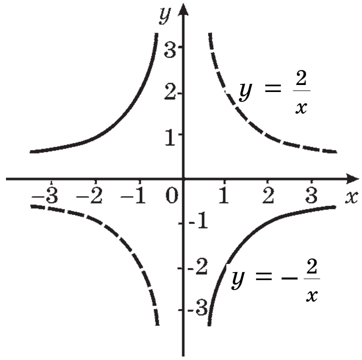
41- su'wret.



42- su'wret.

$y=\frac{1}{x}$ funkciyasi'ni'n' grafigin abcissa ko'sherinen ordinatalar ko'sherin boylap yeki ma'rte sozi'w menen payda yetiledi, degendi an'latadi' (42-su'wret).

$y=-\frac{2}{x}$ funkciyasi'ni'n' ma'nisleri $y=\frac{2}{x}$ funkciyasi'ni'n' ma'nislerinen tek belgisi menen g'ana pari'qlanadi'. Demek, $y=-\frac{2}{x}$ funkciyasi'ni'n' grafigi $y=\frac{2}{x}$ funkciyasi'ni'n' grafigine abcissalar ko'sherine qarata simmetriyali' (43-su'wret). ▲



43- su'wret.

Qa'legen $k \neq 0$ de $y=\frac{k}{x}$ funkciyasi'ni'n' grafigi de *giperbola* dep ataladi'. *Giperbola yeki tarmaqqa* iye. Wolar yeger $k>0$ bolsa, birinshi ha'm u'shinshi shereklerde, yeger $k<0$ bolsa, yekinshi ha'm to'rtinshi shereklerde jatadi'.

$y=\frac{k}{x}$ (bunda $k>0$) funkciya $y=\frac{1}{x}$ funkciyasi'ni'n' barli'q qa'siyetlerine iye, wo'ytkeni bul funkciya:

1) $x \neq 0$ *bolg'anda ani'qlang'an;*

2) *nolden wo'zge barli'q haqi'yqi'y ma'nislerdi qabi'l yetedi;*

3) *taq funkciya;*

4) $x>0$ bolg'anda *won'* ma'nislerdi ha'm $x<0$ bolg'anda *teris* ma'nislerdi qabi'l yetedi;

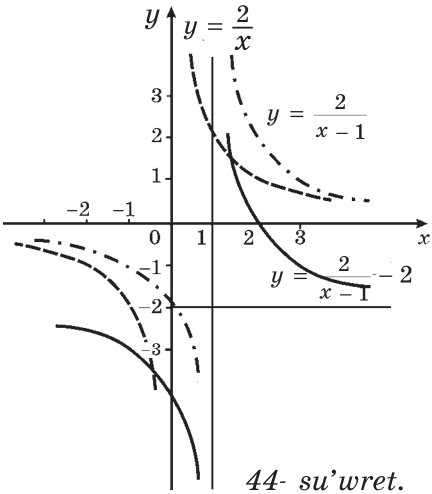
5) $x<0$ ha'm $x>0$ arali'qlarda *kemeyedi*.

Yeger $k<0$ bolsa, wonda $y=\frac{k}{x}$ funkciya 1–3-qa'siyetlerge iye boladi', al 4–5-qa'siyetler to'mendegishe an'lati'ladi':

4) $x<0$ bolg'anda funkciya won' ma'nislerdi ha'm $x>0$ bolg'anda *teris* ma'nislerdi qabi'l yetedi;

5) $x<0$ ha'm $x>0$ arali'qlarda *wo'sedi*.

$y=\frac{k}{x}$ funkciya $k>0$ bolg'anda x ha'm y arasi'ndag'i' *keri proporcional baylani'sti'* an'latadi'. Mug'darlar arasi'ndag'i' bunday baylani'slar ko'binese fizika, texnika ha'm basqa tarawlarda ushi'rasadi'.



44- su'wret.

Ma'selen, v dene turaqli' tezlik penen shen'ber boyi'nsha aylanbali' qozg'ali'sta bolg'ani'nda, wol $a = \frac{v^2}{r}$ ge ten' (bul jerde r – shen'berdin' radiusi') bolg'an worayg'a umti'li'wshi' tezleniw menen ha'reketlenedi, yag'ni'y bul jag'dayda tezleniw shen'berdin' radiu-si'na keri proporcional.

3-ma'sele. Ay Jerden $3{,}84 \cdot 10^8$ m qashi'qli'qta. Ay 27,3 sutka dawami'nda Jer a'tirapi'n bir ma'rte aylani'p shi'g'adi'. Aydi'n' worayg'a umti'li'wshi' tezleniwin yesaplan'.

△ a tezleniwdi $a = \frac{v^2}{r}$ formulasi' menen yesaplaymi'z, bunda $v = \frac{C}{t}$, $C = 2\pi r$, $t = 27{,}3 \cdot 24 \cdot 3600$ s, $r = 3{,}84 \cdot 10^8$. Wonda:

$$a = \frac{4 \cdot \pi^2 \cdot 3{,}84 \cdot 10^8}{(27{,}3 \cdot 24 \cdot 3600)^2} \approx 2{,}72 \cdot 10^{-3}.$$

Juwabi': $2{,}72 \cdot 10^{-3}$ m/s^2. ▲

4-ma'sele. $y = \frac{2}{x-1} - 2$ funkciyasi'ni'n' grafigin jasan'.

△ $y = \frac{2}{x}$ funkciyasi' grafigin (42-su'wret) Ox ko'sheri boylap won'g'a qaray bir birlik, Oy ko'sheri boylap yeki birlik to'menge ji'li'sti'ri'p $y = \frac{2}{x-1} - 2$ funkciyasi' grafigi jasadi' (44-su'wret).▲

Shi'ni'g'i'wlar

220. $y = \frac{2}{x}$ funkciyasi'ni'n' grafigin jasan'. x ti'n' qanday ma'nislerinde: 1) $y(x) = 4$; 2) $y(x) = -\frac{1}{2}$; 3) $y(x) > 1$; 4) $y(x) \le 1$ bolatug'i'ni'n ani'qlan'.

221. Bir koordinatalar tegisliginde $y = \frac{1}{x}$ ha'm $y = x$ funkciyalari'ni'n' grafigin jasan'. x ti'n' qanday ma'nislerinde:
1) bul funkciyalardi'n' grafikleri kesilisetug'i'ni'n;
2) birinshi funkciyani'n' grafigi yekinshi funkciya grafiginen joqari'da (to'mende) jatatug'i'ni'n ani'qlan'.

222. Funkciyalardi'n' grafiklerin jasamastan, wolardi'n' kesilisiw noqatlari'n tabi'n':

1) $y = \frac{12}{x}$, $y = 3x$; 2) $y = -\frac{8}{x}$, $y = -2x$;

3) $y = \frac{2}{x}$, $y = x - 1$; 4) $y = \frac{6}{x+1}$, $y = x + 2$.

223. Funkciyalardi'n' grafiklerin jasap, wolardi'n' kesilisiw noqatlari'n juwi'q ma'nisin tabi'n':

1) $y = \frac{3}{x}$, $y = x + 1$; 2) $y = -\frac{3}{x}$, $y = 1 - x$;

3) $y = \frac{2}{x}$, $y = x^2 + 2$; 4) $y = \frac{1}{x}$, $y = x^2 + 4x$.

224. Cilindrde porshen asti'ndag'i' gaz turaqli' temperaturada turi'pti'. Gazdi'n' V (litrlerde) ko'lemi p (atmosfera) basi'mi'nda $V = \frac{12}{p}$ formulasi' boyi'nsha yesaplanadi'.

1) Basi'm 4 atm; 5 atm; 10 atm bolg'anda gazdi'n' iyelegen ko'lemin tabi'n'; 2) qanday basi'mda gazdi'n' 3 l; 5 l; 15 l ko'lemdi iyeleytug'i'ni'n yesaplan': 3) gazdi'n' ko'leminin' woni'n' basi'mi'na baylani'sli'li'g'i'ni'n' grafigin jasan'.

225. Reostattag'i' I tok ku'shi (amperde) $I = \frac{U}{R}$ formulasi' menen ani'qlanadi', bunda U–kernew (voltda), R – qarsi'li'q (omda).

1) $U = 6$ bolg'anda $I(R)$ baylani'si'ni'n' grafigin jasan'.

2) Grafik boyi'nsha juwi'q tu'rde to'mendegilerdi tabi'n': a) R qarsi'li'q 6, 12, 20 Ω bolg'anda reostattag'i' tok ku'shin; b) tok ku'shi 10, 5, 1,2 A bolg'anda reostatti'n' qarsi'li'g'i'n.

226. Avtomobil joldi'n' radiusi' 150 m bolg'an aylanba bo'legi boyi'nsha 60 km/saat tezlik penen ha'reket yetpekte. Avtomobildin' worayg'a umti'li'wshi' tezleniwin tabi'n'. Yeger avtomobildin' tezligi wo'zgerissiz qali'p, joldi'n' aylanba bo'leginin' radiusi' artsa, worayg'a umti'li'wshi' tezleniw arta ma yaki kemeye me?

227. Funkciyani'n' grafigin jasan':

1) $y = \frac{3}{x} - 2$; | 2) $y = \frac{2}{x} + 1$; | 3) $y = \frac{2}{x+2} - 1$; | 4) $y = \frac{3}{1-x} + 1$.

18- §. DA'REJE QATNASQAN TEN'SIZLIK HA'M TEN'LEMELER

Da'rejeli funkciyani'n' qa'siyetlerinen ha'rqi'yli' ten'leme ha'm ten'sizliklerdi sheshiwde paydalani'ladi'.

1-ma'sele. $x^5 > 32$ ten'sizligin sheshin'.

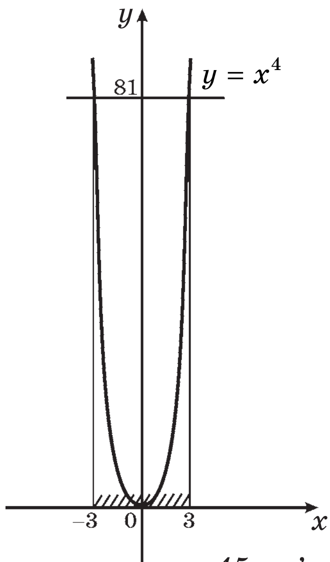
△ $y = x^5$ funkciyasi' x ti'n' barli'q haqi'yqi'y ma'nislerinde ani'qlang'an ha'm ani'qlani'w oblasti'nda wo'siwshi funkciya. $y(2) = 32$ bolg'anli'qtan $x > 2$ bolg'anda $y(x) > 32$ ha'm $x < 2$ bolg'anda $y(x) < 32$.

Juwabi': $x > 2$. ▲

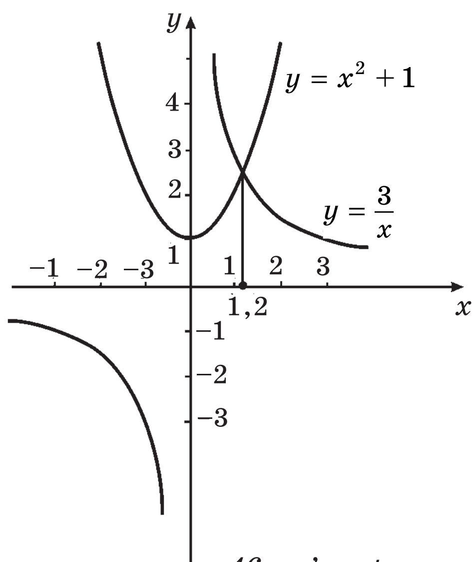
2-ma'sele. $x^4 \le 81$ ten'sizligin sheshin'.

△ $y = x^4$ funkciyasi' $x \le 0$ bolg'anda kemeyedi ha'm $x \ge 0$ bolg'anda wo'sedi. $x^4 = 81$ ten'lemesi yeki haqi'yqi'y korenge iye: $x_1 = -3$, $x_2 = 3$. Sonli'qtan da $x^4 \le 81$ ten'sizligi $x \le 0$ bolg'anda $-3 \le x \le 0$ sheshimlerine ha'm $x \ge 0$ bolg'anda $0 \le x \le 3$ sheshimlerine iye (45-su'wret).

Juwabi': $-3 \le x \le 3$. ▲



45- su'wret.



46- su'wret.

3-ma'sele. Funkciyalardi'n' grafikleri ja'rdeminde $\frac{3}{x} = x^2 + 1$ ten'lemesin sheshin'.

Bir koordinatalar tegisliginde $y = \frac{3}{x}$ ha'm $y = x^2 + 1$ funkciyalari'ni'n' grafigin jasaymi'z (46-su'wret).

△ $x < 0$ bolg'anda $\frac{3}{x} = x^2 + 1$ ten'lemesi korenlerge iye yemes, sebebi $\frac{3}{x} < 0$, biraq ta $x^2 + 1 > 0$. $x > 0$ bolg'anda bul ten'leme usi' funkciyalar grafiklerinin' kesilisiw noqati'ni'n' abcissasi'na ten' bolg'an bir g'ana korenge iye. 46-su'wretten bul korennin' $x_1 \approx 1,2$. yekenligi ko'rinip tur. Ten'leme basqa won' korenlerge iye yemes, sebebi $x > x_1$ bolg'anda $y = \frac{3}{x}$ funkciyasi' kemeyedi, al $y = x^2 + 1$ funkciyasi' wo'sedi ha'm demek, funkciyalardi'n' grafikleri $x > x_1$ bolg'anda kesilispeydi. Tap usi' sebepke baylani'sli' wolar $0 < x < x_1$ bolg'anda da kesilispeydi.

Juwabi': $x_1 \approx 1,2$. ▲

4-ma'sele. Ten'lemesin sheshin':

$$\sqrt{2 - x^2} = x. \qquad (1)$$

△ Meyli, x − berilgen ten'lemenin' koreni bolsi'n, yag'ni'y x − sonday san boli'p, (1) ten'lemeni duri's ten'likke aylandi'radi'. Ten'lemenin' yeki jag'i'nda kvadratqa ko'terip, mi'nag'an iye bolami'z:

$$2 - x^2 = x^2. \qquad (2)$$

Bunnan $x^2 = 1$, $x_{1,2} = \pm 1$.

Demek, (1) ten'leme korenlerge iye dep boljap, biz bul korenler tek 1 ha'm −1 boli'wi' mu'mkinligin bilip aldi'q, yendi bul sanlar (1) ten'lemenin' korenleri boli'w yaki bolmasli'g'i'n tekseremiz. $x = 1$ bolg'anda (1) ten'leme duri's ten'likke aylanadi': $\sqrt{2 - 1^2} = 1$. Sonli'qtan da $x = 1$ (1) ten'lemenin' koreni.

$x = -1$ bolg'anda (1) ten'lemenin' shep jag'i' $\sqrt{2 - (-1)^2} = \sqrt{1} = 1$ ge ten', al won' jag'i' −1 ge ten', yag'ni'y $x = -1$ (1) ten'lemenin' koreni bola almaydi'.

Juwabi': $x = 1$. ▲

Qaralg'an ma'selede (1) ten'leme woni'n' yeki jag'i'n da kvadratqa ko'teriw joli' menen sheshiledi. Bunda (2) ten'leme payda boladi'.

(1) ten'leme tek bir g'ana korenge iye: $x = 1$, (2) ten'leme yeki korenge iye: $x_{1,2} = \pm 1$, yag'ni'y (1) ten'lemeden (2) ten'lemege wo'tiwde *jat korenler* dep atali'wshi' korenler payda boldi'. Soni'n' ushi'n da (1) ten'leme $x = -1$ bolg'anda (1) ten'leme $1 = -1$ den ibarat naduri's ten'likke aylandi', al bul naduri's ten'liktin' yeki jag'i'n da kvadratqa ko'teriwde bolsa $1^2 = (-1)^2$ tan ibarat duri's ten'lik payda boldi'.

! **Ten'lemenin' yeki jag'i'n da kvadratqa ko'teriwde jat korenler payda boli'wi' mu'mkin.**

Ten'lemeni woni'n' yeki jag'i'n da kvadratqa ko'teriw menen sheshiwde tekseriw wo'tkeriw za'ru'r.

(1) ten'leme — *irracional ten'lemege* mi'sal.

Ja'ne irracional ten'lemelerge mi'sallar keltiremiz:

$$\sqrt{3-2x} = 1 - x; \ \sqrt{x+1} = 2 - \sqrt{x-3} \cdot$$

Birneshe irracional ten'lemelerdi sheshiwdi qaraymi'z.

5-ma'sele. Ten'lemesin sheshin': $\sqrt{5-2x} = 1 - x$.

△ Ten'lemenin' yeki jag'i'n da kvadratqa ko'teremiz:

$$5 - 2x = x^2 - 2x + 1$$

yaki $x^2 = 4$, bunnan $x_1 = 2, \ x_2 = -2$. Tabi'lg'an korenlerdi tekseremiz.

$x = 2$ bolg'anda berilgen ten'lemenin' shep jag'i' $\sqrt{5-2\cdot 2} = 1$ ge ten', won' jag'i' $1 - 2 = -1$ ge ten'. $1 \neq -1$ bolg'anli'qtan $x = 2$ berilgen ten'lemenin' koreni bola almaydi'. $x = -2$ bolg'anda berilgen ten'lemenin' shep jag'i' $\sqrt{5-2\cdot(-2)} = 3$ ke ten' won' jag'i' $1-(-2) = 3$ ke ten'. Demek, $x = -2$ berilgen ten'lemenin' koreni.

Juwabi': $x = -2$. ▲

6-ma'sele. Ten'lemeni sheshin': $\sqrt{x-2} + 3 = 0$.

△ Bul ten'lemeni $\sqrt{x-2} = -3$ tu'rinde jazi'p alami'z.

Arifmetikali'q koren teris boli'wi' mu'mkin yemes, demek, bul ten'leme korenlerge iye yemes.

Juwabi': Korenleri joq. ▲

7-ma'sele. Ten'lemeni sheshin': $\sqrt{x-1} + \sqrt{11-x} = 4$.

△ Ten'lemenin' yeki jag'i'n kvadratqa ko'terip, mi'nag'an iye bolami'z:

$$x-1+2\sqrt{x-1}\cdot\sqrt{11-x}+11-x=16.$$

Uqsas ag'zalari'n ji'ynap, ten'lemeni to'mendegi tu'rde jazami'z:

$$2\sqrt{x-1}\cdot\sqrt{11-x}=6 \text{ yaki } \sqrt{x-1}\cdot\sqrt{11-x}=3.$$

Aqi'rg'i' ten'lemenin' yeki jag'i'n da kvadratqa ko'tereyik:

$$(x-1)(11-x)=9 \quad \text{yaki} \quad x^2-12x+20=0,$$

bunnan $x_1=2,\ x_2=10$. Tekseriw 2 ha'm 10 sanlari'nan ha'rbiri berilgen ten'lemenin' koreni bolatug'i'ni'n ko'rsetedi.

J u w a b i ': $x_1=2,\ x_2=10$. ▲

8 -m a ' s e l e. Ten'sizlikti sheshin': $\sqrt{5-x}\le 7+x$.

△ Ten'sizlik x ti'n' $-7\le x\le 5$ ma'nislerinde sheshimge iye. Yeger ten'sizlik sheshimge iye bolsa, sheshim $[-7;\ 5]$ kesindige tiyisli boladi'. Ten'sizliktin' ha'r yeki bo'legin kvadratqa ko'teremiz ha'm a'piwayi'lasti'rg'annan son' $x^2+15x+44\ge 0$ ten'sizligine iye bolami'z. Woni'n' sheshimi $x\le -11,\ x\ge -4$ yekenligi belgili. Bul arali'qlardi'n' $[-7;\ 5]$ kesindi menen uluwma bo'legi $-4\le x\le 5$, yag'ni'y $[-4;\ 5]$ kesindisi boladi':

J u w a b i ': $-4\le x\le 5$. ▲

S h i ' n i ' g ' i ' w l a r

228. Ten'sizlikti sheshin':

1) $x^7>1$;	2) $x^3\le 27$;	3) $y^3\ge 64$;	4) $y^3<125$;
5) $x^4\le 16$;	6) $x^4>625$;	7) $x^5\le 243$;	8) $x^6\ge 64$.

229. 1) Kvadratti'n' maydani' 361 sm^2 tan u'lken yekenligi belgili bolsa, woni'n' ta'repinin' uzi'nli'g'i' qanday boli'wi' mu'mkin?
2) Kvadratti'n' maydani' 343 dm^3 tan u'lken yekenligi belgili bolsa, woni'n' mu'yeshi qanday boli'wi' mu'mkin?

230. (Awi'zeki.) 7 sani' ten'lemenin' koreni bolatug'i'ni'n ko'rsetin':

1) $\sqrt{x-3}=2$; 2) $\sqrt{x^2-13}-\sqrt{2x-5}=3$; 3) $\sqrt{2x+11}=5$.

231. (Awi'zeki.) Ten'lemeni sheshin':

1) $\sqrt{x}=3$; | 2) $\sqrt{x}=7$; | 3) $\sqrt{2x-1}=0$; | 4) $\sqrt{3x+2}=0$.

Ten'lemeni sheshin' **(232–233):**

232. 1) $\sqrt{x+1}=2$; 2) $\sqrt{x-1}=3$; 3) $\sqrt{1-2x}=4$;

4) $\sqrt{2x-1}=3$; 5) $\sqrt{3x+1}=10$; 6) $\sqrt{9-x}=4$.

233. 1) $\sqrt{x+1}=\sqrt{2x-3}$; 2) $\sqrt{x-2}=\sqrt{3x-6}$;

3) $\sqrt{x^2+24}=\sqrt{11x}$; 4) $\sqrt{x^2+4x}=\sqrt{14-x}$.

234. 1) $\sqrt{x+2}=x$; 2) $\sqrt{3x+4}=x$; 3) $\sqrt{20-x^2}=2x$;

4) $\sqrt{0,4-x^2}=3x$; 5) $\sqrt{4-x}=-\frac{x}{3}$; 6) $\sqrt{26-x^2}=5x$.

235. 1) $\sqrt{x^2-x-8}=x-2$; 2) $\sqrt{x^2+x-6}=x-1$.

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236. Ten'sizlikti sheshin':

1) $(x-1)^3>1$; 2) $(x+5)^3>8$; 3) $(2x-3)^7\geq 1$;

4) $(3x-5)^7<1$; 5) $(3-x)^4>256$; 6) $(4-x)^4>81$.

237. Berilgen ten'leme ne ushi'n korenlerge iye yemesligin tu'sindirin':

1) $\sqrt{x}=-8$; | 2) $\sqrt{x}+\sqrt{x-4}=-3$; | 3) $\sqrt{-2-x^2}=12$;

4) $\sqrt{7x-x^2-63}=5$; | 5) $\sqrt{x^2+7}=2$; | 6) $\sqrt{x-2}=x$.

Ten'lemeni sheshin' **(238–240):**

238. 1) $\sqrt{x^2-4x+9}=2x-5$; 2) $\sqrt{x^2+3x+6}=3x+8$;

3) $2x=1+\sqrt{x^2+5}$; 4) $x+\sqrt{13-4x}=4$.

239. 1) $\sqrt{x+12}=2+\sqrt{x}$; 2) $\sqrt{4+x}+\sqrt{x}=4$.

240. 1) $\sqrt{2x+1}+\sqrt{3x+4}=3$; 2) $\sqrt{4x-3}+\sqrt{5x+4}=4$;
3) $\sqrt{x-7}-\sqrt{x+17}=-4$; 4) $\sqrt{x+4}-\sqrt{x-1}=1$.

241. x ti'n' qanday ma'nislerinde funkciyalar bir qi'yli' ma'nislerdi qabi'l yetedi:

1) $y=\sqrt{4+\sqrt{x}},\ y=\sqrt{19-2\sqrt{x}}$; | 2) $y=\sqrt{7+\sqrt{x}},\ y=\sqrt{11-\sqrt{x}}$?

242. Ten'sizlikti sheshin':

1) $\sqrt{x-2}>3$; 2) $\sqrt{x-2}\le 1$; 3) $\sqrt{2-x}\ge x$;
4) $\sqrt{2-x}<x$; 5) $\sqrt{5x+11}>x+3$; 6) $\sqrt{x+3}\le x+1$.

IV bapqa tiyisli shi'ni'g'i'wlar

243. Funkciyani'n' ani'qlani'w oblasti'n tabi'n':

1) $y=\frac{1}{2x+1}$; 2) $y=(3-2x)^{-2}$;
3) $y=\sqrt{-5-3x}$; 4) $y=\sqrt[3]{7-3x}$.

244. Funkciyani'n' qa'siyetlerin ani'qlan' ha'm woni'n' grafiginin' eskizin si'zi'n'.

1) $y=-2x^4$; 2) $y=\frac{1}{2}x^5$; 3) $y=2\sqrt[4]{x}$; 4) $y=3\sqrt[3]{x}$.

245. Bir si'zi'lmada $y=x$ ha'm $y=x^3$ funkciyalari'ni'n' grafiklerin jasan'. Usi' grafiklerdin' kesilisiw noqatlari'ni'n' koordinatalari'n tabi'n'.

Funkciyalardi'n' grafiklerinin' kesilisiw noqatlari'ni'n' koordinatalari'n tabi'n' **(246—247):**

246. 1) $y=x^2,\ y=x^3$; 2) $y=\frac{1}{x},\ y=2x$; 3) $y=3x,\ y=\frac{3}{x}$.

247. 1) $y=\sqrt{x},\ y=|x|$; 2) $y=\sqrt[3]{x},\ y=\frac{1}{x}$; 3) $y=\sqrt{x},\ y=x$.

248. Ten'sizlikti sheshin':

1) $x^4\le 81$; 2) $x^5>32$; 3) $x^6>64$; 4) $x^5\le -32$.

Ten'lemeni sheshin' **(249—250):**

249. 1) $\sqrt{3-x}=2$; 2) $\sqrt{3x+1}=7$; 3) $\sqrt{3-11x}=2x$.

250. 1) $\sqrt{2x-1}=x-2$; 2) $\sqrt{5x-1+3x^2}=3x$; 3) $\sqrt{2-2x}=x+3$.

251. Funkciyani'n' ani'qlani'w oblasti'n tabi'n':

1) $y=\sqrt[5]{x^3+x-2}$; 2) $y=\sqrt[3]{x^2+2x-15}$;

3) $y=\sqrt[6]{6-x-x^2}$; 4) $y=\sqrt[4]{13x-22-x^2}$;

5) $y=\sqrt{\frac{x^2+6x+5}{x+7}}$; 6) $y=\sqrt{\frac{x^2-9}{x^2+8x+7}}$.

WO'ZIN'IZDI TEKSERIP KO'RIN'!

1. Funkciyani'n' ani'qlani'w oblasti'n tabi'n':

1) $y=\frac{8}{x-1}$; 2) $y=\sqrt{9-x^2}$; 3) $y=\sqrt{4-2x}$.

2. Funkciyani'n' grafigin jasan':

1) $y=\sqrt{x}$; 2) $y=\frac{6}{x}$; 3) $y=-\frac{5}{x}$; 4) $y=x^3$.

Ha'rbir funkciya ushi'n grafik boyi'nsha:

a) $y(2)$ ni tabi'n';

b) yeger $y(x)=3$ bolsa, x ti'n' ma'nisin tabi'n';

d) $y(x)>0$, $y(x)<0$ bolg'an arali'qlardi' tabi'n';

e) wo'siw ha'm kemeyiw arali'qlari'n tabi'n'.

3. Funkciyani'n' jup yaki taqli'g'i'n tekserin':

1) $y=3x^6+x^2$; 2) $y=8x^5-x$; 3) $y=2x-x^2$.

4. Ten'lemeni sheshin':

1) $\sqrt{x-3}=5$; 2) $\sqrt{3-x-x^2}=x$; 3) $y=\sqrt{32-x^2}=x$.

252. Funkciyani'n' ko'rsetilgen arali'qta wo'siwi yaki kemeyiwin ani'qlan':

1) $y=\frac{1}{(x-3)^2}$, $x>3$ arali'qta; 2) $y=\frac{1}{(x-2)^3}$, $x<2$ arali'qta;

3) $y=\sqrt[3]{x+1}$, $x\geq 0$ arali'qta; 4) $y=\frac{1}{\sqrt[3]{x+1}}$, $x<-1$ arali'qta;

253. Funkciyani'n' jup yaki taqli'g'i'n ani'qlan':

1) $y=x^6-3x^4+x^2-2$; 2) $y=x^5-x^3+x$;

3) $y=\frac{1}{(x-2)^2}+1$; 4) $y=x^7+x^5+1$.

254. Funkciyani'n' qa'siyetlerin ani'qlan' ha'm woni'n' grafigin jasan':

1) $y=\frac{1}{x^2}$; 2) $y=\frac{1}{x^3}$; 3) $y=\frac{1}{x^3}+2$;

4) $y=3-\frac{1}{x^2}$; 5) $y=\frac{1}{(3-x)^2}+1$; 6) $y=\frac{1}{(x-1)^3}-2$.

255. Ten'sizlikti sheshin':

1) $(3x+1)^4>625$; | 2) $(3x^2+5x)^5\le 32$; | 3) $(x^2-5x)^3>216$.

256. Ten'lemeni sheshin':

1) $\sqrt{2x^2+5x-3}=x+1$; 2) $\sqrt{3x^2-4x+2}=x+4$;

3) $\sqrt{x+11}=1+\sqrt{x}$; 4) $\sqrt{x+19}=1+\sqrt{x}$;

5) $\sqrt{x+3}+\sqrt{2x-3}=6$; 6) $\sqrt{7-x}+\sqrt{3x-5}=4$.

257. Ten'sizlikti sheshin':

1) $\sqrt{x^2-8x}>3$; | 2) $\sqrt{x^2-3x}<2$; | 3) $\sqrt{3x-2}>x-2$;

4) $\sqrt{2x+1}\le x-1$; | 5) $\sqrt{3-x}>1-x$; | 6) $\sqrt{4x-x^2}>4-x$.

IV bapqa tiyisli si'naq (test) shi'ni'g'i'wlari'

1. Funkciyani'n' ani'qlani'w oblasti'n tabi'n': $y=\sqrt{-x^2+3x-2}$.

A) $1\le x\le 2$; B) $1<x<2$; C) $x\ge 2,\ x\le 1$; D) $-2\le x\le -1$.

2. Funkciyani'n' ani'qlani'w oblasti'n tabi'n': $y=\sqrt[4]{\frac{3x+2}{4x-5}}$.

A) $-\frac{2}{3}\le x\le\frac{5}{4}$; B) $x\le -\frac{2}{3},\ x>\frac{5}{4}$; C) $x\ge\frac{5}{4}$; D) $x<-\frac{2}{3}$.

3. Funkciyani'n' ani'qlani'w oblasti'n tabi'n': $y=\sqrt{\frac{-x^2+13x-22}{x-2}}$.

A) $x<2$; B) $2<x<11$; C) $x<2,\ 2<x<11$; D) $x<-2$.

4. To'mendegi funkciyalardi'n' qaysi'lari' wo'siwshi?

1) $y=-x$; 2) $y=-\frac{2}{x}$; 3) $y=\sqrt{x}$; 4) $y=\sqrt{x-100}$.

A) ha'mmesi; B) 1, 2, 3; C) 1, 3, 4; D) 2, 3, 4.

5. To'mendegi funkciyalardi'n' qaysi'lari' wo'siwshi?

1) $y = \sqrt[3]{-x}$; | 2) $y = \sqrt[5]{x^2}$; | 3) $y = -2x + 7$; | 4) $y = -\sqrt{3 - x}$.

A) 2, 4; B) 3, 4; C) 2, 3; D) 1, 2.

6. To'mendegi funkciyalardi'n' qaysi'lari' kemeyiwshi?

1) $y = -\frac{1}{x^3}$; | 2) $y = -3x + 4$; | 3) $y = x^3 - 27$; | 4) $y = \sqrt[3]{8 - x}$.

A) 2, 4; B) 1, 2; C) 2, 3; D) 3, 4.

7. To'mendegi funkciyalardi'n' qaysi'lari' kemeyiwshi?

1) $y = \sqrt[5]{x^3}$; | 2) $y = \sqrt[3]{-x}$; | 3) $y = \frac{7}{\sqrt{3+2x}}$; | 4) $y = \sqrt[4]{x - 16}$.

A) 1, 2; B) 2, 3; C) 3, 4; D) 1, 3.

8. Funkciyani'n' qaysi'lari' jup funkciya?

1) $y = x + \frac{1}{x}$; | 2) $y = x^2 + |x|$; | 3) $y = -3 + \frac{5}{x^4}$; | 4) $y = x^2 - \frac{3}{x}$.

A) 1, 2; B) 3, 4; C) 2, 3; D) 1, 4.

9. Funkciyani'n' qaysi'lari' jup funkciya?

1) $y = 3x^6 - 7x^4 + 5x^2 + 9$; 2) $y = (x + 1)^4 + 3(x + 1)^2 - 6$;

3) $y = 1 + 4x^5 + 7x^7$; 4) $y = \frac{5x^4}{1+|x|}$.

A) 1, 2; B) 2, 3; C) 3, 4; D) 1, 4.

10. Funkciyani'n' qaysi'lari' taq funkciya?

1) $y = 6x$; 2) $y = \sqrt[3]{x}$; 3) $y = 4x + 7$; 4) $y = 2x^3 - 10$.

A) 1, 2; B) 2, 3; C) 3, 4; D) 1, 4.

11. Funkciyani'n' qaysi'lari' taq funkciya?

1) $y = \frac{1}{x^{2k-1}}$; | 2) $y = x^2 + x^5$; | 3) $y = x^3 + 7$; | 4) $y = x^{2n+1}$ $(k, n \in \boldsymbol{N})$.

A) 1, 4; B) 2, 3; C) 3, 4; D) 1, 2.

12. $y = ax^2$ ha'm $y = \frac{k}{x}$ si'zi'qlari' a ha'm k ni'n' qanday ma'nislerinde (3; 2) noqati'nda kesilisedi?

A) $a=-\frac{2}{9}$, $k=6$; B) $a=\frac{2}{9}$, $k=6$;

C) $a=6$, $k=\frac{2}{9}$; D) $a=-\frac{2}{9}$, $k=-6$.

13. k ni'n' qanday ma'nislerinde $y=\frac{k}{x}$ giperbola menen $y=2x+$ $+5$ tuwri'si' yeki noqatta kesilisedi:

A) $k<\frac{25}{8}$; B) $k<-\frac{25}{8}$; C) $k>-\frac{25}{8}$; D) $k>\frac{25}{8}$.

14. k ni'n' qanday ma'nislerinde $y=\frac{k}{x}$ giperbola menen $y=6-x$ tuwri'si' bir uluwma noqatqa iye boladi'?

A) 10; B) −9; C) 8; D) 9.

15. k ni'n' qanday ma'nislerinde $y=\frac{k}{x}$ giperbola menen $y=3-2x$ tuwri'si' kesilispeydi?

A) $k>\frac{9}{8}$; B) $k<\frac{9}{8}$; C) $k>-\frac{9}{8}$; D) $k<-\frac{9}{8}$.

16. $\sqrt{x-5}+\sqrt{10-x}=3$ ten'lemenin' $y=\sqrt{\frac{x^2-15x+50}{x^2-11x+24}}$ funkciyasi'-ni'n' ani'qlani'w oblasti'na tiyisli korenlerin tabi'n'.

A) 6; B) 9; C) −6; D) 3.

17. $\sqrt{x-50}\cdot\sqrt{100-x}>0$ ten'sizliginin' pu'tin sheshimlerinin' ko'pligin tabi'n'.

A) 3765; B) 3675; C) 49; D) 99.

18. $\sqrt{2x^2-8x+5}=x-2$ ten'lemesin sheshin'.

A) $4-\sqrt{3}$; B) $\sqrt{14}$; C) $2+\sqrt{3}$; D) $2-\sqrt{3}$.

19. $\sqrt{2x-3}=3-x$ ten'lemesin sheshin'.

A) 6; B) $\frac{3}{2}$; C) 3; D) 2.

20. $\sqrt[5]{3-x}\cdot\sqrt{-2x^2+9x+5}\geq 0$ ten'sizliginin' pu'tin sheshimlerinin' sani'n tabi'n'.

A) 4; B) 3; C) 5; D) 2.

Abu Rayxan Beruniy (973–1048)

«Funkciya» so'zi lati'nsha «*functio*» so'zinen ali'ng'an boli'p, wol «ju'zege keliw», «wori'nlaw» degen mag'anani' bildiredi. Funkciyani'n' da'slepki ta'riplemeleri **G. Leybnic** (1646—1716), **I. Bernulli** (1667—1748), **N. I. Lobachevskiy** (1792—1856) miynetlerinde berilgen.

Funkciyani'n' ha'zirgi ta'riypin bilmese de, buri'ng'i' zaman ilimpazlari' wo'zgeriwshi shamalar arasi'nda funkcional baylani'sti'n' boli'w za'ru'rligin tu'singen.

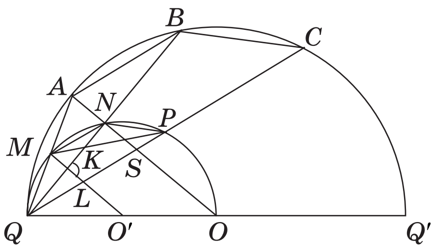
Bunnan to'rt mi'n' ji'l buri'ni'raq Bobil ilimpazlari' radiusi' r bolg'an do'n'gelektin' maydani' ushi'n — qa'teligi sezilerli bolsa da $S = 3r^2$ formulasi'n bergen.

Sanni'n' da'rejesi haqqi'ndag'i' yen' da'slepki mag'luwmatlar a'yyemgi bobilli'lardan bizge shekem jetip kelgen mag'luwmatlarda bar. Atap aytqanda, bul mag'luwmatlarda natural sanlardi'n' kvadratlari'ni'n' ha'm kublari'ni'n' kesteleri berilgen.

Sanlardi'n' kvadratlari'ni'n', kublari'ni'n' kestesi, logarifmler kestesi, trigonometriyali'q kesteler, kvadrat korenler kestesi mine bular shamalar arasi'ndag'i' baylani'sti'n' tek keste usi'li'nda beriliwi boladi'.

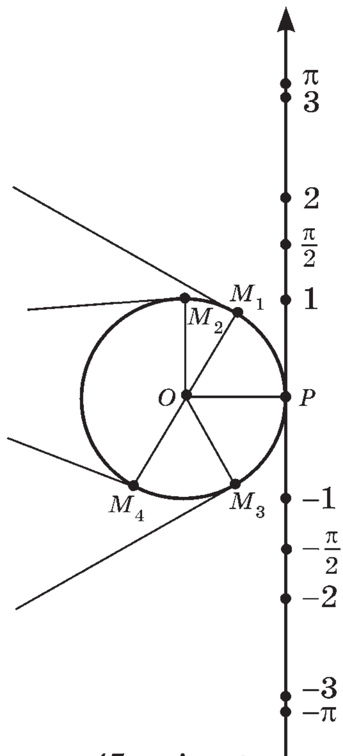
Ulli' enciklopedist dani'shpan ilimpaz **Abu Rayxan Beruniy** de wo'z miynetlerinde funkciya tu'siniginen ha'm woni'n' qa'siyetlerinen paydalang'an. Abu Rayxan Beruniy wo'zinin' «Qonuni Masudiy» atamasi'ndag'i' ataqli' miynetinin' 6-maqalasi'nda argument ha'm funkciyani'n' wo'zgeriw arali'qlari', funkciyani'n' belgileri ha'm de woni'n' yen' u'lken ha'm yen' kishi ma'nislerin ta'riyipleydi.

V BAP. TRIGONOMETRIYA ELEMENTLERI

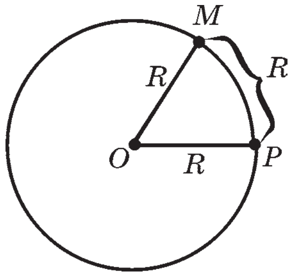


19- §. MU'YESHTIN' RADIAN WO'LSHEMI

Meyli, vertikal tuwri'si' worayi' O noqati'nda ha'm radiusi' 1 ge ten' bolg'an shen'berge P noqati'nda uri'nsi'n (47-su'wret). Bul tuwri'ni' basi' P noqati'nda bolg'an san ko'sheri dep, al joqari'g'a qarag'an bag'i'tti' won' bag'i't dep yesaplaymi'z. San ko'sherinde uzi'nli'q birligi retinde shen'berdin' radiusi'n alami'z. Tuwri'ni'n' boyi'nan birneshe noqatti' belgileyik: ± 1, $\pm \frac{\pi}{2}$, ± 3, $\pm \pi$ (π – shama menen 3,14 ke ten' bolg'an irracional san yekenin yesletip wo'temiz). Bul tuwri'ni' shen'berdin' P noqati'na bekitilgen sozi'lmaytug'i'n jip si'pati'nda ko'z aldi'g'a keltirip, woni' shen'berge woray baslaymi'z. Bunda san tuwri'si'ni'n' (ko'sherinin'), ma'selen 1, $\frac{\pi}{2}$, -1, -2 koordinatali' noqatlari', shen'berdin' sa'ykes tu'rde M_1, M_2, M_3, M_4 noqatlari'na wo'tip, PM_1 dog'asi'ni'n' uzi'nli'g'i' 1 ge ten', PM_2 dog'asi'ni'n' uzi'nli'g'i' $\frac{\pi}{2}$ ge ten' boladi' ha'm t.b.



47- su'wret.



48- su'wret.

Solay yetip, *tuwri'ni'n' ha'rbir noqati'na shen'berdin' qanday da bir noqati' sa'ykes keltiriledi.*

Tuwri'ni'n' koordinatasi' 1 ge ten' bolg'an noqati'na M_1 noqati' sa'ykes keltirilgenlikten, POM_1 mu'yeshin birlik mu'yesh dep yesaplaw ha'm usi' mu'yeshtin' wo'lshemi menen basqa mu'yeshlerdi wo'lshew ta'biyiy boladi'. Ma'selen, POM_2 mu'yeshin $\frac{\pi}{2}$ ge ten', POM_3 mu'yeshin –1 ge ten', POM_4 mu'yeshin –2 ge ten' dep yesaplaw kerek. Mu'yeshlerdi wo'lshewdin' bunday usi'li' matematika ha'm fizikada ken' qollani'ladi'. Mu'yeshlerdin' usi'layi'nsha wo'lsheniwine mu'yeshlerdin' *radianlarda wo'lsheniwi* dep ayti'ladi', POM_1 mu'yeshi 1 radiang'a (1 rad) ten' mu'yeshi dep ayti'ladi'. Shen'berdin' PM_1 dog'asi'ni'n' uzi'nli'g'i' radiusqa ten' yekenligin atap wo'temiz (47-su'wret). Yendi qa'legen R radiusli' shen'berdi qaraymi'z ha'm woni'n' uzi'nli'g'i' R ge ten' bolg'an PM dog'asi'n ha'm POM mu'yeshin belgileymiz (48-su'wret).

! **Uzi'nli'g'i' shen'berdin' radiusi'na ten' bolg'an dog'ag'a tirelgen mu'yesh 1 *radian mu'yesh* dep ataladi'.**

1 radian mu'yesh uzi'nli'gi' ***R*** ge ten' bolg'an dog'ani' tarti'p turadi' deymiz. 1 rad mu'yeshtin' gradusli'q wo'lshemin tabayi'q. 180° li'q worayli'q mu'yeshti uzi'nli'g'i' πR (yari'm shen'ber) ge ten' bolg'an dog'a kerip turg'anli'qtan uzi'nli'g'i' R bolg'an dog'ani' π ma'rte kishi bolg'an mu'yeshti kerip turadi', yag'ni'y

$$\boxed{1\ \text{rad} = \left(\frac{180}{\pi}\right)^{\circ}.}$$

$\pi \approx \mathbf{3{,}14}$ bolg'anli'qtan $\mathbf{1\ rad} \approx \mathbf{57{,}3}^{\circ}$ boladi'.

Yeger mu'yesh α radiannan ibarat bolsa, wonda woni'n' gradusli'q wo'lshemi to'mendegige ten' boladi':

$$\boxed{\alpha\ \text{rad} = \left(\frac{180}{\pi}\alpha\right)^{\circ}.} \qquad \textbf{(1)}$$

1-ma'sele. 1) π rad; 2) $\frac{\pi}{2}$ rad; 3) $\frac{3\pi}{4}$ rad g'a ten' mu'yeshtin' gradusli'q wo'lshemin tabi'n'.

△ (1) formula boyi'nsha to'mendegilerge iye bolami'z:

1) π rad $=180^\circ$; | 2) $\frac{\pi}{2}$ rad $=90^\circ$; | 3) $\frac{3\pi}{4}$ rad $=\left(\frac{180}{\pi}\cdot\frac{3\pi}{4}\right)^\circ=135^\circ$. ▲

1° li'q mu'yeshtin' radian wo'lshemin tabami'z. 180° li'q mu'yesh π rad g'a ten' bolg'anli'qtan,

$$\boxed{1^\circ=\frac{\pi}{180}\text{ rad}.}$$

Yeger mu'yesh α gradusqa ten' bolsa, wonda woni'n' radianli'q wo'lshemi,

$$\boxed{\alpha^\circ=\frac{\pi}{180}\alpha\text{ rad}} \tag{2}$$

g'a ten' boladi'.

2-ma'sele. 1) 45° qa ten' mu'yeshtin'; 2) 15° qa ten' mu'yeshtin' radian wo'lshemin tabi'n'.

△ (2) formula boyi'nsha to'mendegilerge iye bolami'z:

1) $45^\circ=\frac{\pi}{180}\cdot45\text{ rad}=\frac{\pi}{4}\text{ rad}$; | 2) $15^\circ=\frac{\pi}{180}\cdot15\text{ rad}=\frac{\pi}{12}\text{ rad}$. ▲

Ma'seleler sheshiwde ko'birek ushi'rasatug'i'n mu'yeshlerdin' gradusli'q wo'lshemin ha'm wolarg'a sa'ykes radianli'q wo'lshemin keltiremiz:

Gradus	0	30	45	60	90	180
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π

A'dette mu'yeshtin' wo'lshemi radianlarda berilse, «rad» atamasi' tu'sirip qaldi'ri'ladi'.

Mu'yeshtin' radianli'q wo'lshemi shen'ber dog'alari'ni'n' uzi'nli'qlari'n yesaplaw ushi'n qolayli'. 1 rad mu'yesh uzi'nli'g'i' R radiusqa ten' dog'ani' kerip turg'anli'qtan α radian mu'yesh

$$\boxed{l=\alpha R} \tag{3}$$

uzi'nli'qtag'i' dog'ani' kerip turadi'.

3-ma'sele. Qala kurantlari'ni'n' minutli'q tilinin' ushi' radiusi' $R\approx0{,}8$ m bolg'an shen'ber boylap ha'reket yetedi. Bul minutli'q tilinin' ushi' 15 min dawami'nda qanday joldi' basi'p wo'tedi?

△ Saatti'n' minutli'q tili 15 min dawami'nda $\frac{\pi}{2}$ radiang'a ten' mu'yeshke buri'ladi'. (3) formula boyi'nsha $\alpha=\frac{\pi}{2}$ bolg'anda mi'nag'an iye bolami'z: $l=\frac{\pi}{2}R\approx\frac{3{,}14}{2}\cdot 0{,}8\text{ m}\approx 1{,}3$ m.

Juwabi': 1,3 m. ▲

(3) formula shen'ber radiusi' $R=1$ bolg'anda ju'da' a'piwayi' ko'riniske iye. Bul jag'dayda dog'ani'n' uzi'nli'g'i' usi' dog'a menen tarti'li'p turg'an worayli'q mu'yeshtin' shamasi'na ten', yag'ni'y $l=\alpha$. Radian wo'lsheminin' matematika, fizika, mexanika ha'm basqa pa'nlerde qollani'li'wi'ni'n' qolayli'li'g'i' usi'ni'n' menen tu'sindiriledi.

4-ma'sele. Radiusi' R bolg'an do'n'gelek sektor α rad mu'yeshke iye. Usi' sektordi'n' maydani' $S=\frac{R^2}{2}\alpha$ ge ten' yekenligin da'lillen', bunda $0<\alpha<\pi$.

△ π rad li'q do'n'gelek sektordi'n' (yari'm do'n'gelek) maydani' $\frac{\pi R^2}{2}$ ge ten'. Sonli'qtan 1 rad sektordi'n' maydani' π ma'rte kishi, yag'ni'y $\frac{\pi R^2}{2}:\pi$. Demek, α rad sektordi'n' maydani' $\frac{R^2}{2}\alpha$ ge ten'. ▲

Shi'ni'g'i'wlar

258. Gradusta berilgen mu'yeshtin' radian wo'lshemin tabi'n':
1) $40°$; 2) $120°$; 3) $105°$; 4) $150°$;
5) $75°$; 6) $32°$; 7) $100°$; 8) $140°$.

259. Radianda berilgen mu'yeshtin' gradus wo'lshemin tabi'n':
1) $\frac{\pi}{6}$; 2) $\frac{\pi}{9}$; 3) $\frac{2}{3}\pi$; 4) $\frac{3}{4}\pi$; 5) 2;
6) 4; 7) 1,5; 8) 0,36; 9) $\frac{2\pi}{5}$; 10) 4,5.

260. Sanlardi' 0,01 ge shekemgi da'llik penen jazi'n':
1) $\frac{\pi}{2}$; 2) $\frac{3}{2}\pi$; 3) 2π; 4) $\frac{2}{3}\pi$; 5) $\frac{3\pi}{4}$.

261. Sanlardi' sali'sti'ri'n':
1) $\frac{\pi}{2}$ ha'm 2; 2) 2π ha'm 6,7; 3) π ha'm $3\frac{1}{5}$;
4) $\frac{3}{2}\pi$ ha'm 4,8; 5) $-\frac{\pi}{2}$ ha'm $-\frac{3}{2}$; 6) $-\frac{3}{2}\pi$ ha'm $-\sqrt{10}$.

262. (Awi'zeki.) a) ten' ta'repli u'shmu'yeshlik; b) ten' qaptalli' tuwri' mu'yeshli u'shmu'yeshlik; d) kvadrat; e) duri's alti'-mu'yeshliktin' mu'yeshlerinin' gradusli'q ha'm radian wo'lshemlerin ani'qlan'.

263. Yeger shen'berdin' 0,36 m uzi'nli'qtag'i' dog'asi' 0,9 rad worayli'q mu'yeshti tarti'p tursa, woni'n' radiusi'n yesaplan'.

264. Yeger shen'berdin' radiusi' 1,5 sm ge ten' bolsa, shen'berdin' uzi'nli'g'i' 3 sm bolg'an dog'asi' tarti'p turg'an mu'yeshtin' radian wo'lshemin tabi'n'.

265. Do'n'gelek sektor dog'asi' $\frac{3\pi}{4}$ rad mu'yeshti tarti'p turadi'. Yeger do'n'gelektin' radiusi' 1 sm ge ten' bolsa, sektordi'n' maydani'n tabi'n'.

266. Do'n'gelektin' radiusi' 2,5 sm ge ten', al do'n'gelek sektordi'n' maydani' 6,25 sm^2 qa ten'. Usi' do'n'gelek sektordi'n' dog'asi' tarti'p turg'an mu'yeshti tabi'n'.

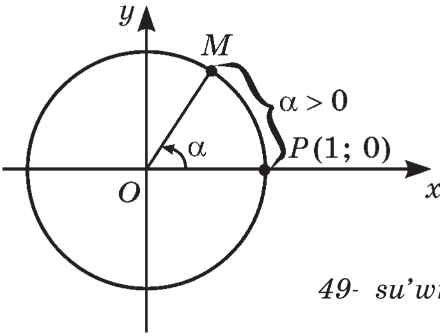
20- §. NOQATTI' KOORDINATALAR BASI' A'TIRAPI'NDA BURI'W

Da'slepki paragrafta san tuwri'si'ni'n' noqatlari' menen shen'ber noqatlari' wortasi'nda sa'ykeslik wornati'wdi'n' ko'rgizbeli usi'li'nan paydalandi'q. Yendi haqi'yqi'y sanlar menen shen'ber noqatlari' wortasi'nda shen'ber noqatlari'n buri'w ja'rdeminde sa'ykeslik wornati'w mu'mkinligin ko'rsetemiz.

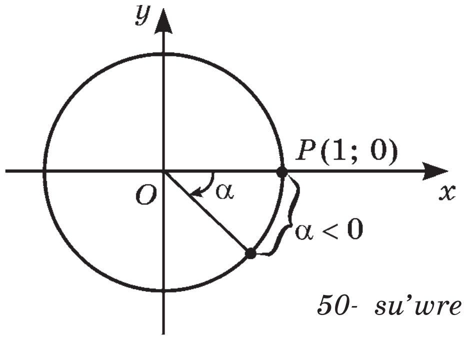
Koordinata tegisliginde radiusi' 1 ge ten' ha'm worayi' koordinata basi'nda jatqan shen'berdi qaraymi'z. Wol *birlik shen'ber* delinedi. Birlik shen'berdin' noqati'n koordinata basi' a'tirapi'nda α radian mu'yeshke *buri'w tu'sinigin* kiritemiz (bunda α –qa'legen haqi'yqi'y san).

1. M e y l i , $\alpha > 0$ b o l s i ' n . Noqat birlik shen'ber boylap *P* noqati'nan saat tili bag'ti'na qarama-qarsi' ha'reket yetip α uzi'nli'qtag'i' joldi' basi'p wo'tti dep aytayi'q (49-su'wret). Joldi'n' son'g'i' noqati'n *M* menen belgileymiz.

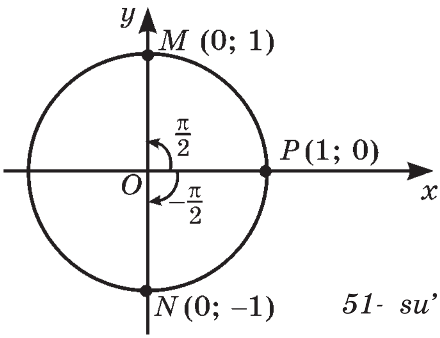
Bul jag'dayda *M* noqati' *P* noqati'n koordinata basi' a'tirapi'nda α radian mu'yeshke buri'w arqali' payda yetiledi, dep ataymi'z.



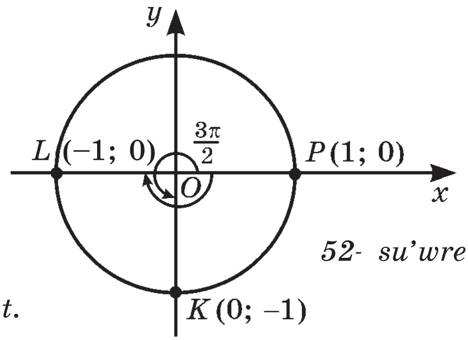
49- su'wret.



50- su'wret.



51- su'wret.



52- su'wret.

2. Meyli, $\alpha < 0$ bolsi'n. Bul jag'dayda α radian mu'yeshke buri'w noqatti'n' qozg'ali's ha'reketi saat tilinin' qozg'ali's bag'i'ti'nda ju'zege kelgenligin ha'm de noqat $|\alpha|$ uzi'nli'qtag'i' joldi' basi'p wo'tkenligin bildiredi (50-su'wret).

0 rad g'a buri'w noqatti'n' wo'z worni'nda qalg'anli'g'i'n an'latadi'.

Mi'sallar:

1) $P(1;\ 0)$ noqati'n $\frac{\pi}{2}$ rad mu'yeshke buri'wda $(0;\ 1)$ koordinatalari'na iye M noqati' payda yetiledi (51-su'wret).

2) $P(1;\ 0)$ noqati'n $-\frac{\pi}{2}$ rad mu'yeshke buri'wda $N(0;\ -1)$ noqati' payda yetiledi (51-su'wret).

3) $P(1;\ 0)$ noqati'n $\frac{3\pi}{2}$ rad mu'yeshke buri'wda $K(0;\ -1)$ noqati' payda yetiledi (52-su'wret).

4) $P(1;\ 0)$ noqati'n $-\pi$ rad mu'yeshke buri'wda $L(-1;\ 0)$ noqati' payda yetiledi (52-su'wret).

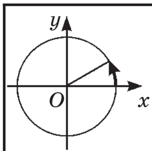
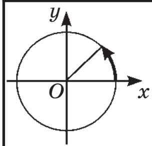
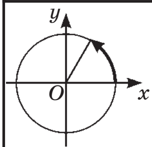
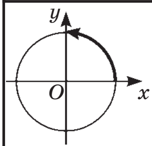
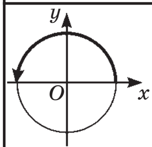
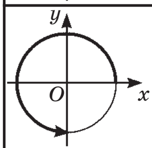
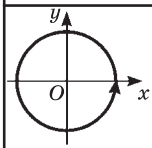
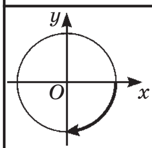
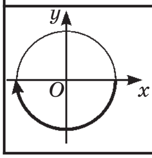
Geometriya kursi’nda $0°$ tan $180°$ qa shekemgi mu’yeshler qaralg’an. Birlik shen’berdin’ noqatlari’n koordinatalar basi’ a’tirapi’nda buri’wdan paydalani’p, $180°$ tan u’lken mu’yeshlerdi, sonday-aq teris mu’yeshlerdi de qaraw mu’mkin. Buri’w mu’yeshin graduslarda, radianlarda beriw mu’mkin. Mi’sali’ P (1; 0) noqati’n $\frac{3\pi}{2}$ mu’yeshke buri’w woni’ $270°$ qa buri’wdi’ an’latadi’; $-\frac{\pi}{2}$ mu’yeshke buri’w $-90°$ qa buri’wdan ibarat.

Bazi’ bir mu’yeshlerge buri’wdi’n’ radian ha’m graduslardag’i’ wo’lshemlerinin’ kestesin keltiremiz (53-su’wret).

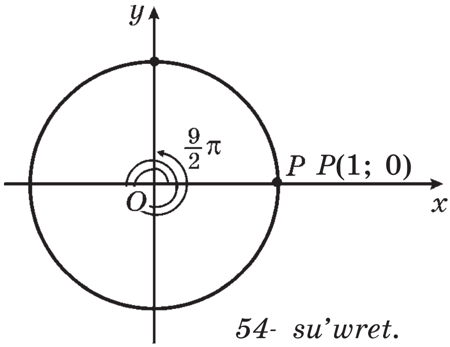
P(1; 0) noqati’n 2π ge, yag’ni’y $360°$ qa buri’wda noqat da’slepki jaylasqan worni’na qaytatug’i’ni’n atap wo’temiz (kes-tege qaran’). Usi’ noqatti’ -2π ge, yag’ni’y $-360°$ qa buri’wda wol ja’ne da’slepki hali’na qaytadi’.

Noqatti’ 2π den u’lken mu’yeshke ha’m -2π den kishi mu’yeshke buri’wg’a tiyisli mi’sallardi’ qaraymi’z. Mi’sali, $-\frac{9\pi}{2}=2\cdot 2\pi+\frac{\pi}{2}$ mu’yeshke buri’wda noqat saat tilinin’ qozg’ali’si’ bag’i’ti’nda yeki toli’q aynali’sti’ ha’m ja’ne $\frac{\pi}{2}$ joldi’ basi’p wo’tedi, (54-su’wret). $-\frac{9\pi}{2}=2\cdot 2\pi-\frac{\pi}{2}$ mu’yeshke buri’wda noqat saat tilinin’ qozg’ali’si’ bag’i’ti’na qarama-qarsi’ bag’i’tta yeki toli’q aynali’sti’ ha’m ja’ne usi’ bag’i’tta $\frac{\pi}{2}$ joldi’ wo’tedi (55-su’wret).

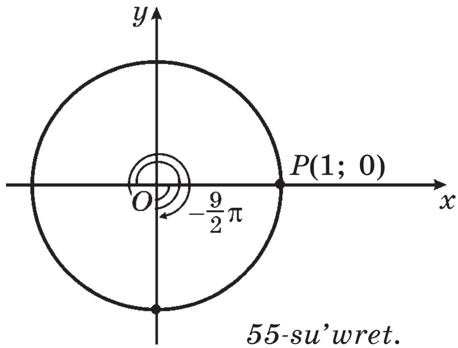
P(1; 0) noqati’n $\frac{9\pi}{2}$ mu’yeshke buri’wda $\frac{\pi}{2}$ mu’yeshke buri’wdag’i’ noqatti’n’

	$\frac{\pi}{6}$	$30°$
	$\frac{\pi}{4}$	$45°$
	$\frac{\pi}{3}$	$60°$
	$\frac{\pi}{2}$	$90°$
	π	$180°$
	$\frac{3\pi}{2}$	$270°$
	2π	$360°$
	$-\frac{\pi}{2}$	$-90°$
	$-\pi$	$-180°$

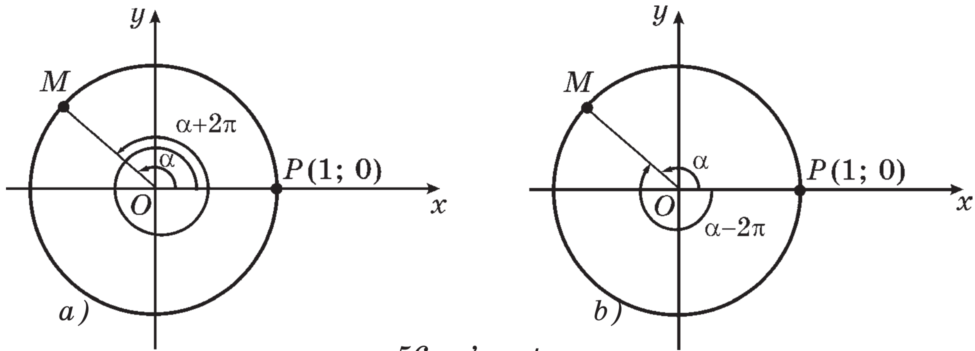
53- su’wret.



54- su'wret.



55-su'wret.



56-su'wret.

wo'zi payda bolatug'i'ni'n ko'rsetemiz (54-su'wret). $-\frac{9\pi}{2}$ mu'yeshke buri'wda $-\frac{\pi}{2}$ mu'yeshke buri'wdag'i' noqatti'n' wo'zi payda boladi' (55-su'wret).

Uluwma, yeger $\alpha = \alpha_0 + 2\pi k$ (bunda k – pu'tin san) bolsa, wonda α mu'yeshke buri'wda α_0 mu'yeshke buri'wdag'i' noqatti'n' wo'zi payda boladi'.

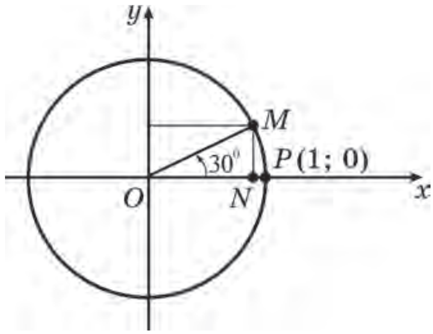
Solay yetip, ha'rbir α haqi'yqi'y sang'a birlik shen'berdin' (1; 0) noqati'n α rad mu'yeshke buri'w menen payda yetiletug'i'n bir g'ana noqati' sa'ykes keledi.

Biraq, birlik shen'berdin' jalg'i'z g'ana bir M noqati'na (P(1; 0) noqati'n buri'wda M noqati' payda yetiletug'i'n) sheksiz ko'p $\alpha + 2\pi k$ haqi'yqi'y sanlar sa'ykes keledi, k – pu'tin san (56- su'wret).

1-ma'sele. P (1; 0) noqati'n: 1) 7π; 2) $-\frac{5\pi}{2}$ mu'yeshke buri'wdan payda bolg'an noqatti'n' koordinatalari'n tabi'n'.

△ 1) $7\pi = \pi + 2\pi \cdot 3$ bolg'anli'qtan 7π ge buri'wda π ge buri'wdag'i' noqatti'n' wo'zi, yag'ni'y $(-1;\ 0)$ koordinatali' noqat payda boladi'.

2) $-\frac{5\pi}{2} = -\frac{\pi}{2} - 2\pi$ bolg'anli'qtan $-\frac{5\pi}{2}$ ge buri'wda $-\frac{\pi}{2}$ ge buri'wdag'i' noqatti'n' wo'zi, yag'ni'y $(0;\ -1)$ koordinatali' noqat payda boladi'. ▲



57- su'wret.

2-ma'sele. $\left(\frac{\sqrt{3}}{2};\ \frac{1}{2}\right)$ noqati'n payda yetiw ushi'n $(1;\ 0)$ noqati'n buri'w kerek bolg'an barli'q mu'yeshlerdi jazi'n'.

△ *NOM* tuwri' mu'yeshli u'shmu'yeshliginen (57-su'wret) *NOM* mu'yeshinin' $\frac{\pi}{6}$ g'a ten'ligi kelip shi'g'adi', yag'ni'y mu'mkin bolg'an buri'w mu'yeshlerinen biri $\frac{\pi}{6}$ g'a ten'. Sonli'qtan $\left(\frac{\sqrt{3}}{2};\ \frac{1}{2}\right)$ noqati'n payda yetiw ushi'n $(1;\ 0)$ noqati'n buri'w kerek bolg'an barli'q mu'yeshler bi'lay an'lati'ladi': $\frac{\pi}{6} + 2\pi k$, bundag'i' k – qa'legen pu'tin san, yag'ni'y $k = 0;\ \pm 1;\ \pm 2;\ \ldots$ ▲

Shi'ni'g'i'wlar

267. Birlik shen'berdin' $P\,(1;\ 0)$ noqati'n:

1) $90°$; 2) $-\pi$; 3) $180°$; 4) $-\frac{\pi}{2}$; 5) $270°$; 6) 2π

mu'yeshke buri'w na'tiyjesinde payda bolg'an noqatlardi'n' koordinatalari'n tabi'n'.

268. Birlik shen'berde $P\,(1;\ 0)$:

1) $\frac{\pi}{4}$; 2) $-\frac{\pi}{3}$; 3) $-\frac{2}{3}\pi$; 4) $\frac{3}{4}\pi$;

5) $\frac{\pi}{2} + 2\pi$; 6) $-\pi - 2\pi$; 7) $\frac{\pi}{4} - 4\pi$; 8) $-\frac{\pi}{3} + 6\pi$

mu'yeshke buri'w na'tiyjesinde payda bolg'an noqatti' belgilen'.

269. $P(1;\ 0)$ noqati'n:

1) $2{,}1\pi$; | 2) $2\frac{2}{3}\pi$; | 3) $-\frac{13}{3}\pi$; | 4) $-\frac{25}{4}\pi$; | 5) $727°$; | 6) $460°$

mu'yeshke buri'w na'tiyjesinde payda bolg'an noqat jaylasqan koordinatalar sheregin ani'qlan'.

270. $P(1;\ 0)$ noqati'n:

1) 3π; 2) $-\frac{7}{2}\pi$; 3) $-\frac{15}{2}\pi$; 4) 5π;

5) $540°$; 6) $810°$; 7) $-\frac{9}{2}\pi$; 8) $450°$

mu'yeshke buri'w na'tiyjesinde payda bolg'an noqatti'n' koordinatalari'n tabi'n'.

271. 1) $(-1;\ 0)$; 2) $(1;\ 0)$; 3) $(0;\ 1)$; 4) $(0;\ -1)$ noqatlari'n payda yetiw ushi'n $P(1;\ 0)$ noqati'n buri'w kerek bolg'an barli'q mu'yeshlerdi jazi'n'.

272. $P(1;\ 0)$ noqati' berilgen:

1) 1; 2) 2,75; 3) 3,16; 4) 4,95; 5) 1,8

mu'yeshke buri'w na'tiyjesinde payda bolg'an noqat jaylasqan koordinatalar sheregin ani'qlan'.

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273. Yeger:

1) $a = 6{,}7\pi$; 2) $a = 9{,}8\pi$; 3) $a = 4\frac{1}{2}\pi$;

4) $a = 7\frac{1}{3}\pi$; 5) $a = \frac{11}{2}\pi$; 6) $a = \frac{17}{3}\pi$

bolsa, $a = x + 2\pi k$ ten'ligi wori'nlanatug'i'n x sani'n (bul jerde $0 \le x < 2\pi$) ha'm k natural sandi' tabi'n'.

274. Birlik shen'berde $P(1;0)$ noqati'n:

1) $\frac{\pi}{4} \pm 2\pi$; 2) $-\frac{\pi}{3} \pm 2\pi$; 3) $\frac{2\pi}{3} \pm 6\pi$; 4) $-\frac{3\pi}{4} \pm 8\pi$;

5) $4{,}5\pi$; 6) $5{,}5\pi$; 7) -6π; 8) -7π

mu'yeshke buri'wdan payda bolg'an noqatti' jasan'.

275. $P(1;\ 0)$ noqati'n:

1) $-\frac{3\pi}{2} + 2\pi k$; 2) $\frac{5\pi}{2} + 2\pi k$; 3) $\frac{7\pi}{2} + 2\pi k$; 4) $-\frac{9\pi}{2} + 2\pi k$

mu'yeshke (bul jerde k – pu'tin san) buri'wdan payda bolg'an noqatti'n' koordinatalari'n tabi'n'.

276. (1; 0) noqati'n:

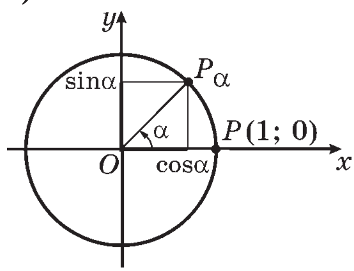
1) $\left(-\frac{1}{2};\ \frac{\sqrt{3}}{2}\right)$; | 2) $\left(\frac{\sqrt{3}}{2};\ -\frac{1}{2}\right)$; | 3) $\left(\frac{\sqrt{2}}{2};\ -\frac{\sqrt{2}}{2}\right)$; | 4) $\left(-\frac{\sqrt{2}}{2};\ \frac{\sqrt{2}}{2}\right)$

koordinatali' noqat payda yetiw ushi'n buri'w kerek bolg'an barli'q mu'yeshlerdi jazi'n'.

21- §. MU'YESHTIN' SINUSI', KOSINUSI', TANGENSI HA'M KOTANGENSININ' ANI'QLAMALARI'

Geometriya kursi'nda graduslarda an'lati'lg'an mu'yeshtin' sinusi', kosinusi' ha'm tangensi berilgen yedi. Bul mu'yesh 0° tan 180° qa shekemgi arali'qta qaralg'an. Qa'legen mu'yeshtin' sinusi' ha'm kosinusi' to'mendegishe ani'qlanadi':

1-ani'qlama. α ***mu'yeshinin' sinusi' dep (1; 0) noqati'n koordinatalar basi' a'tirapi'nda*** α ***mu'yeshke buri'w na'tiyjesinde payda bolg'an noqatti'n' ordinatasi'na ayti'ladi'*** **(sinα tu'rinde belgilenedi).**

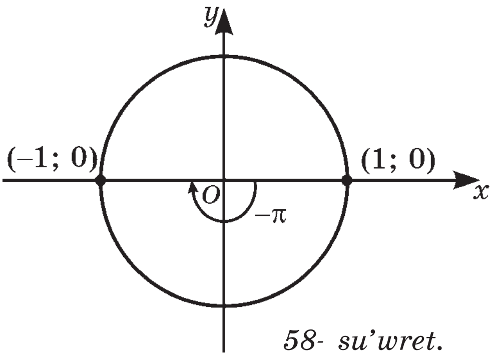


2-ani'qlama. α ***mu'yeshinin' kosinusi' dep (1; 0) noqati'n koordinatalar basi' a'tirapi'nda*** α ***mu'yeshke buri'w na'tiyjesinde payda bolg'an noqatti'n' abcissasi'na ayti'ladi'*** **(cosα tu'rinde belgilenedi).**

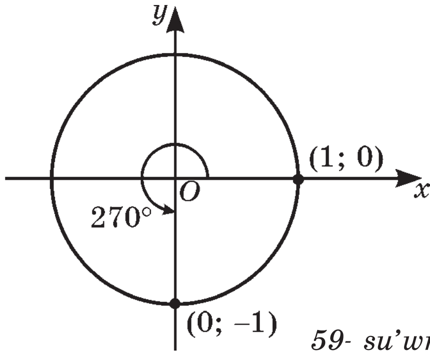
Bul ani'qlamalarda α mu'yeshi graduslarda, sonday- aq, radianlarda da an'lati'li'wi' mu'mkin.

Ma'selen, (1; 0) noqati'n $\frac{\pi}{2}$ mu'yeshke, yag'ni'y 90° qa buri'wda (0; 1) noqati' payda yetiledi. (0; 1) noqati'ni'n' ordinatasi' 1 ge ten', yag'ni'y

$$\sin\frac{\pi}{2} = \sin 90^\circ = 1;$$



58- su'wret.



59- su'wret.

bul noqatti'n' abcissasi' 0 ge ten', yag'ni'y

$$\cos\frac{\pi}{2}=\cos 90^\circ=0.$$

Mu'yesh 0° tan 180° qa shekemgi arali'qta bolg'an jag'dayda sinus ha'm kosinuslardi'n' ani'qlamalari' geometriya kursi'nan ma'lim bolg'an sinus ha'm kosinuslardi'n' ani'qlamalari' menen sa'ykes tu'setug'i'ni'n atap wo'temiz.

Mi'sali',

$$\sin\frac{\pi}{6}=\sin 30^\circ=\frac{1}{2},\ \cos\pi=\cos 180^\circ=-1.$$

1 -ma'sele sin(–π) ha'm cos(–π) di tabi'n'.

△ (1; 0) noqati'n –π mu'yeshke burg'anda wol (–1; 0) noqati'na wo'tedi (58-su'wret). Sonli'qtan $\sin(-\pi)=0$, $\cos(-\pi)=-1$. ▲

2 -ma'sele. $\sin 270^\circ$ ha'm $\cos 270^\circ$ ti' tabi'n'.

△ (1; 0) noqati'n 270° qa burg'anda wol (0; –1) noqati'na wo'tedi (59-su'wret). Sonli'qtan $\cos 270^\circ=0$, $\sin 270^\circ=-1$. ▲

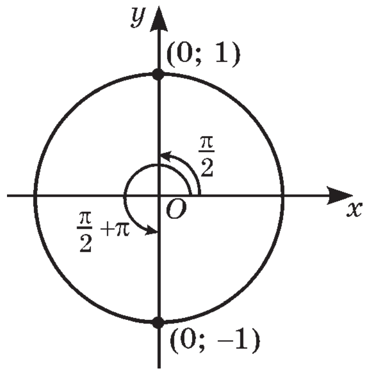
3 -ma'sele. $\sin t=0$ ten'lemesin sheshin'.

△ $\sin t=0$ ten'lemesin sheshiw — bul sinusi' nolge ten' bolg'an barli'q mu'yeshlerdi tabi'w degendi an'latadi'.

Birlik shen'berde ordinatasi' nolge ten' bolg'an yeki noqat bar: (1; 0) ha'm (–1; 0) (58-su'wret). Bul noqatlar (1; 0) noqati'n 0, π, 2π, 3π ha'm tag'i' basqa, sonday-aq, –π, –2π, –3π ha'm tag'i' basqa mu'yeshlerde buri'w menen payda yetiledi. Demek, $t=k\pi$ bolg'anda (bunda k – qa'legen pu'tin san) $\sin t=0$ boladi'. ▲

Pu'tin sanlar ko'pligi $\boldsymbol{Z}$ ha'ribi menen belgilenedi. k sani' $\boldsymbol{Z}$ ke tiyisli yekenin belgilew ushi'n $k \in \boldsymbol{Z}$ jazi'wi'nan paydalani'ladi' («k sani' Z ke tiyisli» dep woqi'ladi'). Sonli'qtan 3-ma'selenin' juwabi'n bi'lay jazi'w mu'mkin:

$$t = \pi k,\ k \in \boldsymbol{Z}.$$



60- su'wret.

4-ma'sele. $\cos t = 0$ ten'lemesin sheshin'.

△ Birlik shen'berde abcissasi' nolge ten' bolg'an yeki noqat bar: (0, 1) ha'm (0; –1) (60-su'wret).

Bul noqatlar (1; 0) noqati'n $\frac{\pi}{2}$, $\frac{\pi}{2}+\pi$, $\frac{\pi}{2}+2\pi$ ha'm tag'i' basqa, sonday-aq, $\frac{\pi}{2}-\pi$, $\frac{\pi}{2}-2\pi$ ha'm tag'i' basqa mu'yeshlerge, yag'ni'y $\frac{\pi}{2}+k\pi$ (bunda $k \in \boldsymbol{Z}$) mu'yeshlerge buri'w menen payda boladi'.

Juwabi': $t = \frac{\pi}{2} + \pi k$, $k \in \boldsymbol{Z}$. ▲

5-ma'sele. Ten'lemeni sheshin': 1) $\sin t = 1$; 2) $\cos t = 1$.

△ 1) Birlik shen'berdin' (0; 1) noqati' birge ten' ordinatag'a iye. Bul noqat (1; 0) noqatti' $\frac{\pi}{2} + 2\pi k$, $k \in \boldsymbol{Z}$ mu'yeshke buri'w menen payda yetiledi.

2) (1; 0) noqatti' $2k\pi$, $k \in \boldsymbol{Z}$ mu'yeshke buri'w menen payda yetilgen noqatti'n' abcissasi' birge ten' boladi'.

Juwabi': $t = \frac{\pi}{2} + 2\pi k$ bolg'anda $\sin t = 1$,

$t = 2\pi k$ bolg'anda $\cos t = 1$, $k \in \boldsymbol{Z}$. ▲

3-ani'qlama. α ***mu'yeshinin' tangensi dep*** α ***mu'yeshinin' sinusi'ni'n' woni'n' kosinusi'na qatnasi'na ayti'ladi'*** **(tgα tu'rinde belgilenedi).**

Solay yetip, $\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}$..

Ma'selen, $\operatorname{tg} 0^\circ = \frac{\sin 0^\circ}{\cos 0^\circ} = \frac{0}{1} = 0$, $\operatorname{tg}\frac{\pi}{4} = \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$.

Geyde α *mu'yeshinin' kotangensinen* de paydalani'ladi' ($\mathrm{ctg}\alpha$ tu'rinde jazi'li'p belgilenedi). Wol $\mathrm{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha}$ formulasi' menen ani'qlanadi'. Ma'selen,

$$\mathrm{ctg}270^\circ = \frac{\cos 270^\circ}{\sin 270^\circ} = \frac{0}{-1} = 0, \ \mathrm{ctg}\frac{\pi}{4} = \frac{1}{\mathrm{tg}\frac{\pi}{4}} = \frac{1}{1} = 1.$$

$\sin\alpha$ ha'm $\cos\alpha$ lar qa'legen mu'yeshler ushi'n ta'riyiplengenligin, al wolardi'n' ma'nisleri -1 den 1 ge shekemgi arali'qta yekenligin atap wo'temiz; $\mathrm{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}$ tek $\cos\alpha \neq 0$ bolg'an mu'yeshler ushi'n, yag'ni'y $\alpha = \frac{\pi}{2} + \pi k, \ k \in \boldsymbol{Z}$ ten basqa qa'legen mu'yeshler ushi'n ani'q-lang'an.

Sinus, kosinus, tangens ha'm kotangenslerdin' ko'birek ushi'rasatug'i'n ma'nislerinin' kestesin keltiremiz.

α	0 (0°)	$\frac{\pi}{6}$ (30°)	$\frac{\pi}{4}$ (45°)	$\frac{\pi}{3}$ (60°)	$\frac{\pi}{2}$ (90°)	π (180°)	$\frac{3}{2}\pi$ (270°)	2π (360°)
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\mathrm{tg}\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Ani'q yemes	0	Ani'q yemes	0
$\mathrm{ctg}\alpha$	Ani'q yemes	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Ani'q yemes	0	Ani'q yemes

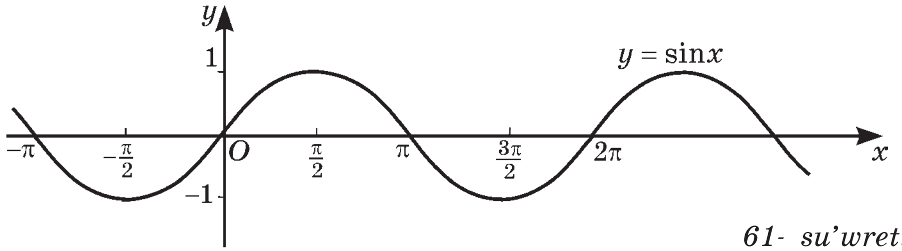
6-ma'sele. Yesaplan':

$$4\sin\frac{\pi}{6} + \sqrt{3}\cos\frac{\pi}{6} - \mathrm{tg}\frac{\pi}{4}.$$

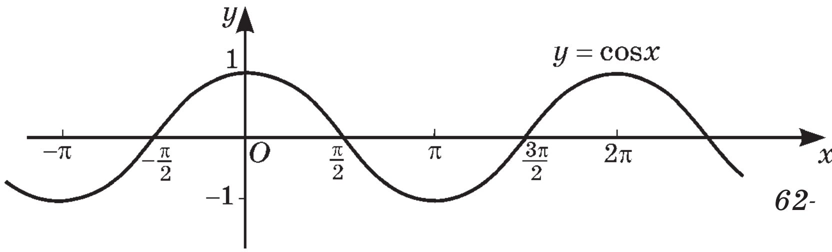
△ Kesteden paydalani'p to'mendegige iye bolami'z:

$$4\sin\frac{\pi}{6} + \sqrt{3}\cos\frac{\pi}{6} - \mathrm{tg}\frac{\pi}{4} = 4\cdot\frac{1}{2} + \sqrt{3}\cdot\frac{\sqrt{3}}{2} - 1 = 2{,}5. \ ▲$$

Sinus, kosinus, tangens ha'm kotangenslerdin' bul kestege kirmegen mu'yeshleri ushi'n ma'nislerin V.M. Bradistin' to'rt tan'bali' matematikali'q kestelerinen, sonday-aq, mikrokalkulyator ja'rdeminde tabi'w mu'mkin.



61- su'wret.



62- su'wret.

Yeger ha'rbir haqi'yqi'y x sani'na $\sin x$ sani' sa'ykes qoyi'lsa, wonda haqi'yqi'y sanlar ko'pliginde $y = \sin x$ funkciyasi' berilgen boladi'. $y = \cos x$, $y = \operatorname{tg} x$ ha'm $y = \operatorname{ctg} x$ funkciyalari' sog'an uqsas ani'qlanadi', $y = \cos x$ funkciya barli'q $x \in \boldsymbol{R}$ da ani'qlang'an, $y = \operatorname{tg} x$ funkciya $x \neq \frac{\pi}{2} + \pi k,\ k \in \boldsymbol{Z}$, $y = \operatorname{ctg} x$ bolsa $x \neq \pi k,\ k \in \boldsymbol{Z}$ bolg'anda ani'qlang'an. $y = \sin x$ ha'm $y = \cos x$ funkciyalari'ni'n' grafikleri 61-ha'm 62-su'wretlerde ko'rsetilgen.

$y = \sin x$, $y = \cos x$, $y = \operatorname{tg} x$, $y = \operatorname{ctg} x$ funkciyalari' *trigonometriyali'q* funkciyalar dep ataladi'.

Shi'ni'g'i'wlar

277. Yesaplan':

1) $\sin\frac{3\pi}{4}$;	2) $\cos\frac{2\pi}{3}$;	3) $\operatorname{tg}\frac{5\pi}{6}$;	4) $\sin(-90°)$;
5) $\cos(-180°)$;	6) $\operatorname{tg}\left(-\frac{\pi}{4}\right)$;	7) $\cos(-135°)$;	8) $\sin\left(-\frac{5\pi}{4}\right)$.

278. Yeger:

1) $\sin\alpha = \frac{1}{2}$; 2) $\sin\alpha = -\frac{\sqrt{2}}{2}$; 3) $\cos\alpha = \frac{\sqrt{3}}{2}$;

4) $\cos\alpha = -\frac{1}{2}$; 5) $\sin\alpha = -0,6$; 6) $\cos\alpha = \frac{1}{3}$

bolsa, birlik shen'berde α mu'yeshke sa'ykes keliwshi noqatti' ko'rsetin'.

Yesaplan' **(279–281):**

279. 1) $\sin\frac{\pi}{2}+\sin\frac{3\pi}{2}$; 2) $\sin\left(-\frac{\pi}{2}\right)+\cos\frac{\pi}{2}$; 3) $\sin\pi-\cos\pi$;

4) $\sin 0-\cos 2\pi$; 5) $\sin\pi+\sin 1{,}5\pi$; 6) $\cos 0-\cos\frac{3}{2}\pi$.

280. 1) $\mathrm{tg}\pi+\cos\pi$; 2) $\mathrm{tg}0^\circ-\mathrm{tg}180^\circ$; 3) $\mathrm{tg}\pi+\sin\pi$;

4) $\cos\pi-\mathrm{tg}2\pi$; 5) $\sin\frac{\pi}{4}-\cos\frac{\pi}{4}$; 6) $\mathrm{tg}\frac{\pi}{4}+\mathrm{ctg}\frac{\pi}{4}$.

281. 1) $3\sin\frac{\pi}{6}+2\cos\frac{\pi}{6}-\mathrm{tg}\frac{\pi}{3}$; 2) $5\sin\frac{\pi}{6}+3\mathrm{tg}\frac{\pi}{4}-\cos\frac{\pi}{4}-10\mathrm{tg}\frac{\pi}{4}$;

3) $\left(2\mathrm{tg}\frac{\pi}{6}-\mathrm{tg}\frac{\pi}{3}\right):\cos\frac{\pi}{6}$; 4) $\sin\frac{\pi}{4}\cos\frac{\pi}{6}-\mathrm{tg}\frac{\pi}{4}$.

282. Ten'lemeni sheshin':

1) $2\sin x=0$; | 2) $\frac{1}{2}\cos x=0$; | 3) $\cos x-1=0$; | 4) $1-\sin x=0$.

283. (Awi'zeki.) $\sin\alpha$ yaki $\cos\alpha$:

1) 0,49; 2) –0,875; 3) $-\sqrt{2}$; 4) $2-\sqrt{2}$; 5) $\sqrt{5}-1$

ge ten' boli'wi' mu'mkin be?

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284. α ni'n' berilgen ma'nisinde an'latpani'n' ma'nisin tabi'n':

1) $2\sin\alpha+\sqrt{2}\cos\alpha$, bunda $\alpha=\frac{\pi}{4}$;

2) $0{,}5\cos\alpha-\sqrt{3}\sin\alpha$, bunda $\alpha=60^\circ$;

3) $\sin 3\alpha-\cos 2\alpha$, bunda $\alpha=\frac{\pi}{6}$;

4) $\cos\frac{\alpha}{2}+\sin\frac{\alpha}{3}$, bunda $\alpha=\frac{\pi}{2}$.

285. Ten'lemeni sheshin':

1) $\sin x=-1$; 2) $\cos x=-1$; 3) $\sin 3x=0$;

4) $\cos 0{,}5x=0$; 5) $\cos 2x-1=0$; 6) $1-\cos 3x=0$.

286. Ten'lemeni sheshin':

1) $\sin(x+\pi)=-1$; | 2) $\sin\frac{1}{2}(x+1)=0$; | 3) $\cos(x+\pi)=-1$;

4) $\cos 2(x+1)-1=0$; | 5) $\sin 3(x-2)=0$; | 6) $1-\cos 3(x-1)=0$.

22- §. SINUS, KOSINUS HA'M TANGENSTIN' BELGILERI

1. Sinus ha'm kosinusti'n' belgileri

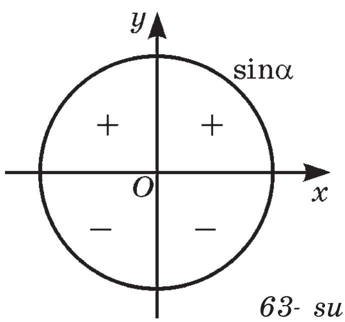
Meyli, (1; 0) noqati' birlik shen'ber boyi'nsha saat tilinin' qozg'ali'si'na qarama-qarsi' bag'i'tta qozg'ali'sta bolsi'n. Bul jag'dayda birinshi sherekte (kvadratta) jaylasqan noqatlardi'n' ordinatalari' ha'm abcissalari' won'. Sonli'qtan, yeger $0<\alpha<\frac{\pi}{2}$ bolsa, $\sin\alpha>0$ ha'm $\cos\alpha>0$ boladi' (63, 64-su'wretler).

Yekinshi sherekte jaylasqan noqatlar ushi'n ordinatalar won', al abcissalar bolsa teris. Sonli'qtan, yeger $\frac{\pi}{2}<\alpha<\pi$ bolsa, $\sin\alpha>0$, $\cos\alpha<0$ boladi' (63, 64-su'wretler). Sog'an uqsas, u'shinshi sherekte $\sin\alpha<0$, $\cos\alpha<0$, al to'rtinshi sherekte $\sin\alpha<0$, $\cos\alpha>0$ boladi' (63, 64-su'wretler). Noqatti'n' shen'ber boyi'nsha bunnan keyingi ha'reketinde sinus ha'm kosinuslardi'n' belgileri noqatti'n' qaysi' sherekte turg'anli'g'i' menen ani'qlanadi'.

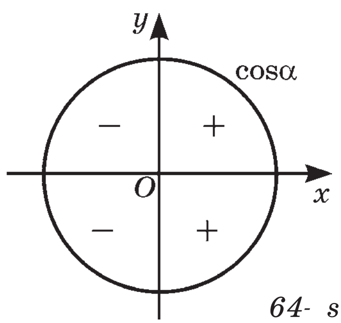
Sinusti'n' belgileri 63- su'wrette, kosinusti'n' belgileri bolsa 64-su'wrette ko'rsetilgen.

Yeger (1; 0) noqati' saat tilinin' qozg'ali'si' bag'i'ti'nda ha'reket yetse, *bul jag'dayda da sinus ha'm kosinusti'n' belgileri noqatti'n' qaysi' sherekte jaylasqanli'g'i'na qarap ani'qlanadi';* bul 63, 64-su'wretlerde ko'rsetilgen.

1-ma'sele. Mu'yeshtin' sinusi' ha'm kosinuslari'ni'n' belgilerin ani'qlan'. 1) $\frac{3\pi}{4}$; 2) $745°$; 3) $-\frac{5\pi}{7}$.



63- su'wret.



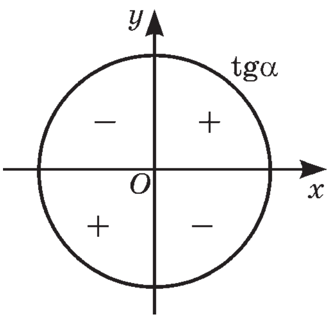
64- su'wret.

△ 1) $\frac{3\pi}{4}$ mu'yeshine birlik shen'berdin' yekinshi shereginde jaylasqan noqat sa'ykes keledi. Sonli'qtan da $\sin\frac{3\pi}{4} > 0$, $\cos\frac{3\pi}{4} < 0$.

2) $745^\circ = 2\cdot 360^\circ + 25^\circ$ bolg'anli'qtan (1; 0) noqati'n 745° qa buri'wda bul mu'yeshke birinshi sherekte jaylasqan noqat sa'ykes keledi. Sonli'qtan da $\sin 745^\circ > 0$, $\cos 745^\circ > 0$.

3) $-\pi < -\frac{5\pi}{7} < -\frac{\pi}{2}$ bolg'anli'g'i' sebepli (1; 0) noqati'n $-\frac{5\pi}{7}$ mu'yeshke burg'anda u'shinshi sherekte jaylasqan noqat payda yetiledi. $\sin\left(-\frac{5\pi}{7}\right) < 0$, $\cos\left(-\frac{5\pi}{7}\right) < 0$. ▲

2. Tangenstin' belgileri



65- su'wret.

Ani'qlamag'a muwapi'q $\mathrm{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}$. Sonli'qtan da, yeger $\sin\alpha$ ha'm $\cos\alpha$ bir qi'yli' belgige iye bolsa, $\mathrm{tg}\,\alpha > 0$, $\sin\alpha$ ha'm $\cos\alpha$ qarama-qarsi' belgilerge iye bolsa, $\mathrm{tg}\,\alpha < 0$ boladi'. Tangenstin' belgileri 65-su'wrette ko'rsetilgen.

$\mathrm{ctg}\alpha$ belgileri $\mathrm{tg}\alpha$ ni'n' belgileri menen bir qi'yli' boladi'.

2-ma'sele. Mu'yeshtin' tangensinin' belgilerin ani'qlan':

1) 260°; 2) 3.

△ 1) $180^\circ < 260^\circ < 270^\circ$ bolg'anli'g'i' sebepli $\mathrm{tg}260^\circ > 0$.

2) $\frac{\pi}{2} < 3 < \pi$ bolg'anli'qtan $\mathrm{tg}3 < 0$. ▲

Shi'ni'g'i'wlar

287. Yeger:

1) $\alpha = \frac{\pi}{6}$; 2) $\alpha = \frac{3\pi}{4}$; 3) $\alpha = 210^\circ$; 4) $\alpha = -210^\circ$;

5) $\alpha = 735^\circ$; 6) $\alpha = 848^\circ$; 7) $\alpha = -\frac{2\pi}{5}$; 8) $\alpha = \frac{5\pi}{8}$

bolsa, (1; 0) noqati'n α mu'yeshke buri'wda payda bolg'an noqat qaysi' sherekte jaylasatug'i'ni'n ani'qlan'.

288. Yeger:

1) $\alpha = \frac{5\pi}{4}$; 2) $\alpha = \frac{5\pi}{6}$; 3) $\alpha = -\frac{5}{8}\pi$; 4) $\alpha = -\frac{4}{3}\pi$;

5) $\alpha = 740^\circ$; 6) $\alpha = 510^\circ$; 7) $\alpha = -\frac{7\pi}{4}$; 8) $\alpha = 361^\circ$

bolsa, $\sin\alpha$ sani'ni'n' belgisin ani'qlan'.

289. Yeger:

1) $\alpha = \frac{2}{3}\pi$; | 2) $\alpha = \frac{7}{6}\pi$; | 3) $\alpha = -\frac{3\pi}{4}$; | 4) $\alpha = -\frac{2}{5}\pi$;

5) $\alpha = 290^\circ$; | 6) $\alpha = -150^\circ$; | 7) $\alpha = \frac{6\pi}{5}$; | 8) $\alpha = -100^\circ$

bolsa, $\cos\alpha$ sani'ni'n' belgisin ani'qlan'.

290. Yeger:

1) $\alpha = \frac{5}{6}\pi$; | 2) $\alpha = \frac{12}{5}\pi$; | 3) $\alpha = -\frac{3}{5}\pi$; | 4) $\alpha = -\frac{5}{4}\pi$;

5) $\alpha = 190^\circ$; | 6) $\alpha = 283^\circ$; | 7) $\alpha = 172^\circ$; | 8) $\alpha = 200^\circ$

bolsa, $\mathrm{tg}\alpha$ ha'm $\mathrm{ctg}\alpha$ sanlari'ni'n' belgilerin ani'qlan'.

291. Yeger:

1) $\pi < \alpha < \frac{3\pi}{2}$; 2) $\frac{3\pi}{2} < \alpha < \frac{7\pi}{4}$; 3) $\frac{7}{4}\pi < \alpha < 2\pi$;

4) $2\pi < \alpha < 2{,}5\pi$; 5) $\frac{3\pi}{4} < \alpha < \pi$; 6) $1{,}5\pi < \alpha \leq 1{,}8\pi$

bolsa, $\sin\alpha$, $\cos\alpha$, $\mathrm{tg}\alpha$, $\mathrm{ctg}\alpha$ sanlari'ni'n' belgilerin ani'qlan'.

292. Yeger:

1) $\alpha = 1$; | 2) $\alpha = 3$; | 3) $\alpha = -3{,}4$; | 4) $\alpha = -1{,}3$; | 5) $\alpha = 3{,}14$

bolsa, $\sin\alpha$, $\cos\alpha$, $\mathrm{tg}\alpha$ sanlari'ni'n' belgilerin ani'qlan'.

---

293. $0 < \alpha < \frac{\pi}{2}$ bolsi'n. Sanni'n' belgilerin ani'qlan':

1) $\sin\left(\frac{\pi}{2} - \alpha\right)$; | 2) $\cos\left(\frac{\pi}{2} + \alpha\right)$; | 3) $\mathrm{tg}\left(\frac{3}{2}\pi - \alpha\right)$; | 4) $\sin(\pi - \alpha)$;

5) $\cos(\alpha - \pi)$; | 6) $\mathrm{tg}(\alpha - \pi)$; | 7) $\cos\left(\alpha - \frac{\pi}{2}\right)$; | 8) $\mathrm{ctg}\left(\alpha - \frac{\pi}{2}\right)$.

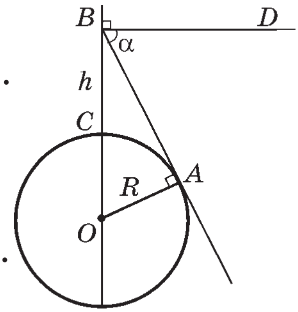
294. Sinus ha'm kosinuslardi'n' belgileri bir qi'yli' (ha'r qi'yli') bolatug'i'n α sanni'n' 0 den 2π ge shekemgi arali'qta jaylasqan barli'q ma'nislerin tabi'n'.

295. Sanni'n' belgisin ani'qlan':

1) $\sin\frac{2\pi}{3}\sin\frac{3\pi}{4}$; | 2) $\cos\frac{2\pi}{3}\cos\frac{\pi}{6}$; | 3) $\dfrac{\sin\frac{2\pi}{3}}{\cos\frac{3\pi}{4}}$; | 4) $\mathrm{tg}\frac{5\pi}{4} + \sin\frac{\pi}{4}$.

296. An'latpalardi'n' ma'nislerin sali'sti'ri'n':

1) sin 0,7 ha'm sin 4; 2) cos 1,3 ha'm cos 2,3.



66- su'wret.

297. Ten'lemeni sheshin':

1) $\sin(5\pi + x) = 1$; 2) $\cos(x + 3\pi) = 0$;

3) $\cos\left(\frac{5}{2}\pi + x\right) = -1$; 4) $\sin\left(\frac{9}{2}\pi + x\right) = -1$.

298. Yeger:

1) $\sin\alpha + \cos\alpha = -1{,}4$; 2) $\sin\alpha - \cos\alpha = 1{,}4$;

3) $\sin\alpha + \cos\alpha = 1{,}1$; 4) $\cos\alpha - \sin\alpha = 1{,}2$

bolsa, α sani'na sa'ykes keliwshi noqat qaysi' sherekte jaylasqan?

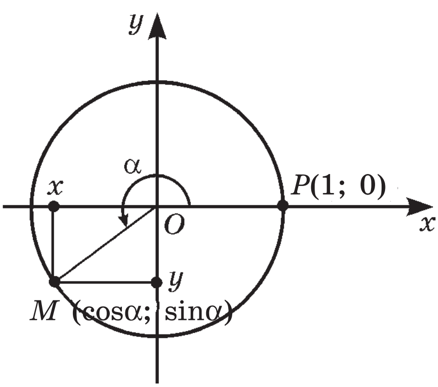
299. (*Beruniy ma'selesi*). Tawdi'n' biyikligi $h = BC$ ha'm $\alpha = \angle ABD$ mu'yeshi belgili bolsa, Jerdin' radiusi' R di tabi'n' (66-su'wret).

23- §. BERILGEN BIR MU'YESHTIN' SINUSI', KOSINUSI' HA'M TANGENSI ARASI'NDAG'I' QATNASLAR

Sinus penen kosinus arasi'ndag'i' qatnasti' ani'qlaymi'z.

Meyli birlik shen'berdin' $M(x;\ y)$ noqati' (1; 0) noqati'n α mu'yeshke buri'w na'tiyjesinde payda yetilgen bolsi'n (67-su'wret). Sonda sinus ha'm kosinusti'n' ani'qlamasi'na muwapi'q,

$$x = \cos\alpha,\ y = \sin\alpha.$$



67- su'wret.

M noqati' birlik shen'berge tiyisli, sonli'qtan da woni'n' $(x;\ y)$ koordinatalari' $x^2 + y^2 = 1$ ten'lemesin qanaatlandi'radi'. Demek,

$$\boxed{\sin^2\alpha + \cos^2\alpha = 1.} \qquad (1)$$

(1) ten'lik α ni'n' qa'legen ma'nisinde wori'nlanadi' ha'm *tiykarg'i' trigonometriyali'q birdeylik* dep ataladi'.

(1) ten'likten $\sin\alpha$ ni' $\cos\alpha$ arqali' ha'm kerisinshe, $\cos\alpha$ ni' $\sin\alpha$ arqali' an'lati'w mu'mkin:

$$\boxed{\begin{aligned} \sin\alpha &= \pm\sqrt{1-\cos^2\alpha}\,, \qquad (2) \\ \cos\alpha &= \pm\sqrt{1-\sin^2\alpha}\,. \qquad (3) \end{aligned}}$$

Bul formulalarda koren aldi'ndag'i' belgi formulani'n' shep jag'i'nda turg'an an'latpani'n' belgisi menen ani'qlanadi'.

1-ma'sele. $\cos\alpha = -\frac{3}{5}$ ha'm $\pi < \alpha < \frac{3\pi}{2}$ bolsa, $\sin\alpha$ ni' yesaplan'.

△ (2) formuladan paydalanami'z. $\pi < \alpha < \frac{3\pi}{2}$ bolg'anli'qtan $\sin\alpha < 0$ boladi', soni'n' ushi'n (2) formulada koren aldi'na «–» belgisin qoyi'w kerek:

$$\sin\alpha = -\sqrt{1-\cos^2\alpha} = -\sqrt{1-\frac{9}{25}} = -\frac{4}{5}\,. \ ▲$$

2-ma'sele. Yeger $\sin\alpha = \frac{1}{3}$ ha'm $-\frac{\pi}{2} < \alpha < 0$ bolsa, $\cos\alpha$ ni' yesaplan'.

△ $-\frac{\pi}{2} < \alpha < 0$ bolg'anli'qtan $\cos\alpha > 0$ boladi' ha'm sonli'qtan (3) formulada koren aldi'na «+» belgisin qoyi'w kerek:

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}\,. \ ▲$$

Yendi *tangens penen kotangens arasi'ndag'i' baylani'sti'* qaraymi'z.

Tangens ha'm kotangenstin' ani'qlamasi'na muwapi'q:

$$\mathrm{tg}\,\alpha = \frac{\sin\alpha}{\cos\alpha}, \quad \mathrm{ctg}\,\alpha = \frac{\cos\alpha}{\sin\alpha}\,.$$

Bul ten'liklerdi ko'beytip,

$$\boxed{\mathrm{tg}\,\alpha \cdot \mathrm{ctg}\,\alpha = 1} \qquad (4)$$

ten'ligin payda yetemiz. (4) ten'likten $\mathrm{tg}\,\alpha$ ni' $\mathrm{ctg}\,\alpha$ arqali', ha'm kerisinshe, $\mathrm{ctg}\,\alpha$ ni' $\mathrm{tg}\,\alpha$ arqali' an'lati'w mu'mkin:

$$\boxed{\begin{aligned} \mathrm{tg}\,\alpha &= \frac{1}{\mathrm{ctg}\,\alpha}\,, \qquad (5) \\ \mathrm{ctg}\,\alpha &= \frac{1}{\mathrm{tg}\,\alpha}\,. \qquad (6) \end{aligned}}$$

(4)–(6) ten'likler $\alpha \neq \frac{\pi}{2}k,\ k \in \boldsymbol{Z}$ bolg'anda wori'nli' boladi'.

3-ma'sele. Yeger $tg\alpha = 13$ bolsa, $ctg\alpha$ ni' yesaplan'.

△ (6) formula boyi'nsha tabami'z: $ctg\alpha = \frac{1}{tg\alpha} = \frac{1}{13}$. ▲

4-ma'sele. $\sin\alpha = 0{,}8$ ha'm $\frac{\pi}{2} < \alpha < \pi$ bolsa, $tg\alpha$ ni' yesaplan'.

△ (3) formula boyi'nsha $\cos\alpha$ ni' tabami'z. $\frac{\pi}{2} < \alpha < \pi$ bolg'ani' ushi'n $\cos\alpha < 0$ boladi'. Soni'n' ushi'n,

$$\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-0{,}64} = -0{,}6 .$$

Demek, $$tg\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{0{,}8}{-0{,}6} = -\frac{4}{3} . \ ▲$$

Tiykarg'i' trigonometriyali'q birdeylikten ha'm tangenstin' ani'qlamasi'nan paydalani'p, *tangens penen kosinus arasi'ndag'i' qatnasi'qti'* tabami'z.

△ $\cos\alpha \neq 0$ dep boljap, $\sin^2\alpha + \cos^2\alpha = 1$ ten'liginin' yeki ta'repin de $\cos^2\alpha$ g'a bo'lemiz: $\frac{\cos^2\alpha+\sin^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$, bunnan

$$\boxed{1 + tg^2\alpha = \frac{1}{\cos^2\alpha} .} \quad ▲ \tag{7}$$

Yeger $\cos\alpha \neq 0$ bolsa, yag'ni'y $\alpha \neq \frac{\pi}{2} + \pi k,\ k \in \boldsymbol{Z}$ bolsa, (7) formula duri's boladi'. (7) formuladan tangensti kosinus ha'm kosinusti' tangens arqali' an'lati'w mu'mkin.

5-ma'sele. $\cos\alpha = -\frac{3}{5}$ ha'm $\frac{\pi}{2} < \alpha < \pi$ bolsa, $tg\alpha$ ni' yesaplan'.

△ (7) formuladan paydalani'p, to'mendegige iye bolami'z:

$$tg^2\alpha = \frac{1}{\cos^2\alpha} - 1 = \frac{1}{\left(-\frac{3}{5}\right)^2} - 1 = \frac{16}{9} .$$

Tangens yekinshi sherekte teris, sonli'qtan $tg\alpha = -\frac{4}{3}$. ▲

6-ma'sele. $tg\alpha = 3$ ha'm $\pi < \alpha < \frac{3\pi}{2}$ bolsa, $\cos\alpha$ ni' yesaplan'.

△ (7) formuladan tabami'z:

$$\cos^2\alpha = \frac{1}{1+tg^2\alpha} = \frac{1}{10} \cdot$$

$\pi < \alpha < \frac{3\pi}{2}$ bolg'anli'qtan $\cos\alpha < 0$ ha'm sonli'qtan $\cos\alpha = -\sqrt{0{,}1}$. ▲

Shi'ni'g'i'wlar

300. Yeger:

1) $\cos\alpha = \frac{5}{13}$ ha'm $\frac{3\pi}{2} < \alpha < 2\pi$ bolsa, $\sin\alpha$ ha'm $\operatorname{tg}\alpha$ ni';

2) $\sin\alpha = 0{,}8$ ha'm $\frac{\pi}{2} < \alpha < \pi$ bolsa, $\cos\alpha$ ha'm $\operatorname{tg}\alpha$ ni';

3) $\cos\alpha = -\frac{3}{5}$ ha'm $\frac{\pi}{2} < \alpha < \pi$ bolsa, $\sin\alpha$, $\operatorname{tg}\alpha$ ha'm $\operatorname{ctg}\alpha$ ni';

4) $\sin\alpha = -\frac{2}{5}$ ha'm $\pi < \alpha < \frac{3\pi}{2}$ bolsa, $\cos\alpha$, $\operatorname{tg}\alpha$ ha'm $\operatorname{ctg}\alpha$ ni';

5) $\operatorname{tg}\alpha = \frac{15}{8}$ ha'm $\pi < \alpha < \frac{3\pi}{2}$ bolsa, $\sin\alpha$ ha'm $\cos\alpha$ ni';

6) $\operatorname{ctg}\alpha = -3$ ha'm $\frac{3\pi}{2} < \alpha < 2\pi$ bolsa, $\sin\alpha$ ha'm $\cos\alpha$ ni' yesaplan'.

301. Tiykarg'i' trigonometriyali'q birdeylik ja'rdeminde ten'likler bir waqi'tta wori'nlanatug'i'nli'g'i'n yaki wori'nlanbaytug'i'ni'n ani'qlan':

1) $\sin\alpha = 1$ ha'm $\cos\alpha = 1$; 2) $\sin\alpha = 0$ ha'm $\cos\alpha = -1$;

3) $\sin\alpha = -\frac{4}{5}$ ha'm $\cos\alpha = -\frac{3}{5}$; 4) $\sin\alpha = \frac{1}{3}$ ha'm $\cos\alpha = -\frac{1}{2}$.

302. Ten'likler bir waqi'tta wori'nlani'wi' mu'mkin be:

1) $\sin\alpha = \frac{1}{5}$ ha'm $\operatorname{tg}\alpha = \frac{1}{\sqrt{24}}$; 2) $\operatorname{ctg}\alpha = \frac{\sqrt{7}}{3}$ ha'm $\cos\alpha = \frac{3}{4}$?

303. Meyli, α tuwri' mu'yeshli u'shmu'yeshliktin' mu'yeshlerinen biri bolsi'n. $\sin\alpha = \frac{2\sqrt{10}}{11}$ bolsa, $\cos\alpha$ ha'm $\operatorname{tg}\alpha$ ni' tabi'n'.

304. Ten' qaptalli' u'shmu'yeshliktin' to'besindegi mu'yeshinin' tangensi $2\sqrt{2}$ ten'. Usi' u'shmu'yeshliktin' kosinusi'n tabi'n'.

305. Yeger $\cos^4\alpha - \sin^4\alpha = \frac{1}{8}$ bolsa, $\cos\alpha$ ni' tabi'n';

<u>306</u>. 1) $\sin\alpha = \frac{2\sqrt{3}}{5}$ bolsa, $\cos\alpha$ ni' tabi'n';

2) $\cos\alpha = -\frac{1}{\sqrt{5}}$ bolsa, $\sin\alpha$ ni' tabi'n'.

<u>307</u>. $\operatorname{tg}\alpha = 2$ yekenligi belgili. An'latpani'n' ma'nisin tabi'n':

1) $\frac{\operatorname{ctg}\alpha+\operatorname{tg}\alpha}{\operatorname{ctg}\alpha-\operatorname{tg}\alpha}$; 2) $\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}$; 3) $\frac{2\sin\alpha+3\cos\alpha}{3\sin\alpha-5\cos\alpha}$;

4) $\frac{\sin^2\alpha+2\cos^2\alpha}{\sin^2\alpha-\cos^2\alpha}$; 5) $\frac{3\sin\alpha-2\cos\alpha}{4\sin\alpha+\cos\alpha}$; 6) $\frac{3\sin^2\alpha+\cos^2\alpha}{2\sin^2\alpha-\cos^2\alpha}$.

308. $\sin\alpha+\cos\alpha=\frac{1}{2}$ yekenligi belgili. 1) $\sin\alpha\cos\alpha$; 2) $\sin^3\alpha+\cos^3\alpha$ an'latpalari'ni'n' ma'nisin tabi'n'.

309. Ten'lemeni sheshin':

1) $2\sin x+\sin^2 x+\cos^2 x=1$; 2) $\sin^2 x-2=\sin x-\cos^2 x$;
3) $2\cos^2 x-1=\cos x-2\sin^2 x$; 4) $3-\cos x=3\cos^2 x+3\sin^2 x$.

24- §. TRIGONOMETRIYALI'Q BIRDEYLIKLER

1-ma'sele. $\alpha\neq\pi k,\ k\in\boldsymbol{Z}$ bolg'anda

$$1+\mathrm{ctg}^2\alpha=\frac{1}{\sin^2\alpha} \tag{1}$$

ten'liginin' wori'nli' yekenligin da'lillen'.

△ Kotangenstin' ani'qlamasi'na muwapi'q $\mathrm{ctg}\alpha=\frac{\cos\alpha}{\sin\alpha}$ ha'm sonli'qtan,

$$1+\mathrm{ctg}^2\alpha=1+\frac{\cos^2\alpha}{\sin^2\alpha}=\frac{\sin^2\alpha+\cos^2\alpha}{\sin^2\alpha}=\frac{1}{\sin^2\alpha}. \tag{2}$$

Bul tu'rlendiriwler duri's, sebebi $\alpha\neq\pi k,\ k\in\boldsymbol{Z}$ bolg'anda $\sin\alpha\neq 0$.▲

(1) ten'lik α ni'n' mu'mkin bolg'an (barli'q ruqsat yetiletug'i'n) ma'nisleri ushi'n wori'nli', yag'ni'y woni'n' shep ha'm won' ta'repleri mag'anag'a iye bolatug'i'n barli'q ma'nisleri ushi'n duri's boladi'. Bunday ten'likler *birdeylikler* dep ataladi', bunday ten'liklerdi da'lillewge tiyisli ma'seleler birdeyliklerdi da'lillewge tiyisli ma'seleler dep ayti'ladi'. Bunnan bi'lay birdeyliklerdi da'lillewde, yeger ma'selenin' sha'rtinde talap yetilmegen bolsa, mu'yeshlerdin' ruqsat yetiletug'i'n ma'nislerin izlep woti'rmaymi'z.

2-ma'sele. Birdeylikti da'lillen': $\cos^2\alpha=(1-\sin\alpha)(1+\sin\alpha)$.

△ $(1-\sin\alpha)(1+\sin\alpha)=1-\sin^2\alpha=\cos^2\alpha$. ▲

3-ma'sele. Birdeylikti da'lillen': $\frac{\cos\alpha}{1-\sin\alpha}=\frac{1+\sin\alpha}{\cos\alpha}$.

△ Bul birdeylikti da'lillew ushi'n woni'n' shep ham won' ta'replerinin' ayi'rmasi' nolge ten' bolatug'i'ni'n ko'rsetiw jetkilikli:

$$\frac{\cos\alpha}{1-\sin\alpha}-\frac{1+\sin\alpha}{\cos\alpha}=\frac{\cos^2\alpha-(1-\sin^2\alpha)}{\cos\alpha(1-\sin\alpha)}=\frac{\cos^2\alpha-\cos^2\alpha}{\cos\alpha(1-\sin\alpha)}=0. \ ▲$$

1–3-ma'selelerdi sheshiwde *birdeyliklerdi da'lillewdin' to'mendegi usi'llari'nan* paydalani'ldi': won' ta'repin tu'rlendiriw menen woni'n' shep ta'repine ten'ligin ko'rsetiw; won' ha'm shep ta'replerinin' ayi'rmasi' nolge ten' bolatug'i'ni'n ko'rsetiw. Geyde birdeyliklerdi da'lillewde woni'n' won' ha'm shep ta'replerin tu'rlendiriw menen wolardi' bir qi'yli' an'latpag'a keltirgen qolayli'.

4-ma'sele. Birdeylikti da'lillen': $\frac{1-tg^2\alpha}{1+tg^2\alpha}=\cos^4\alpha-\sin^4\alpha$.

$$\triangle\ \frac{1-tg^2\alpha}{1+tg^2\alpha}=\frac{1-\frac{\sin^2\alpha}{\cos^2\alpha}}{1+\frac{\sin^2\alpha}{\cos^2\alpha}}=\frac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha+\sin^2\alpha}=\cos^2\alpha-\sin^2\alpha.$$

$$\cos^4\alpha-\sin^4\alpha=(\cos^2\alpha-\sin^2\alpha)(\cos^2\alpha+\sin^2\alpha)=\cos^2\alpha-\sin^2\alpha.$$

Birdeylik da'lillendi, sebebi woni'n' shep ha'm won' ta'repleri $\cos^2\alpha-\sin^2\alpha$ g'a ten'. ▲

5-ma'sele. An'latpani' a'piwayi'lasti'ri'n': $\frac{1}{tg\alpha+ctg\alpha}$.

$$\triangle\ \frac{1}{tg\alpha+ctg\alpha}=\frac{1}{\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}}=\frac{\sin\alpha\cos\alpha}{\sin^2\alpha+\cos^2\alpha}=\sin\alpha\cos\alpha.\ ▲$$

Trigonometriyali'q an'latpalardi' a'piwayi'lasti'ri'wg'a tiyisli ma'selelerdi sheshiwde, yeger ma'selenin' sha'rtinde talap yetilmegen bolsa, mu'yeshlerdin' qabi'l yetiwi mu'mkin bolg'an ruqsat yetiletug'i'n ma'nislerin tappaymi'z.

Shi'ni'g'i'wlar

310. Birdeylikti da'lillen':

1) $(1-\cos\alpha)(1+\cos\alpha)=\sin^2\alpha$; 2) $2-\sin^2\alpha-\cos^2\alpha=1$;

3) $\frac{\sin^2\alpha}{1-\sin^2\alpha}=tg^2\alpha$; 4) $\frac{\cos^2\alpha}{1-\cos^2\alpha}=ctg^2\alpha$;

5) $\frac{1}{1+tg^2\alpha}+\sin^2\alpha=1$; 6) $\frac{1}{1+ctg^2\alpha}+\cos^2\alpha=1$.

311. An'latpani' a'piwayi'lasti'ri'n':

1) $\cos\alpha\cdot tg\,\alpha-2\sin\alpha$; 2) $\cos\alpha-\sin\alpha\cdot ctg\,\alpha$;

3) $\frac{\sin^2\alpha}{1+\cos\alpha}$; 4) $\frac{\cos^2\alpha}{1-\sin\alpha}$; 5) $\frac{tg\alpha\cdot\cos\alpha}{\sin^2\alpha}$.

312. An'latpani' a'piwayi'lasti'ri'n' ha'm woni'n' san ma'nisin tabi'n':

1) $\frac{\sin^2\alpha-1}{1-\cos^2\alpha}$, bunda $\alpha=\frac{\pi}{6}$; 2) $\frac{1}{\cos^2\alpha}-1$, bunda $\alpha=\frac{\pi}{3}$;

3) $\cos^2\alpha+\mathrm{ctg}^2\alpha+\sin^2\alpha$, bunda $\alpha=\frac{\pi}{6}$;

4) $\cos^2\alpha+\mathrm{tg}^2\alpha+\sin^2\alpha$, bunda $\alpha=\frac{\pi}{3}$.

313. Birdeylikti da'lillen':

1) $(1-\sin^2\alpha)(1+\mathrm{tg}^2\alpha)=1$; 2) $\sin^2\alpha(1+\mathrm{ctg}^2\alpha)-\cos^2\alpha=\sin^2\alpha$.

314. α ni'n' barli'q mu'mkin bolg'an ma'nislerinde to'mendegi an'latpa birdey ma'nisti qabi'l yetetug'i'nli'g'i'n, yag'ni'y α g'a g'a'rezli yemesligin da'lillen':

1) $(1+\mathrm{tg}^2\alpha)\cos^2\alpha$; 2) $\sin^2\alpha(1+\mathrm{ctg}^2\alpha)$;

3) $\left(1+\mathrm{tg}^2\alpha+\frac{1}{\sin^2\alpha}\right)\sin^2\alpha\cos^2\alpha$; 4) $\frac{1+\mathrm{tg}^2\alpha}{1+\mathrm{ctg}^2\alpha}-\mathrm{tg}^2\alpha$.

315. Birdeylikti da'lillen':

1) $(1-\cos 2\alpha)(1+\cos 2\alpha)=\sin^2 2\alpha$; 2) $\frac{\sin\alpha-1}{\cos^2\alpha}=-\frac{1}{1+\sin\alpha}$;

3) $\cos^4\alpha-\sin^4\alpha=\cos^2\alpha-\sin^2\alpha$;

4) $(\sin^2\alpha-\cos^2\alpha)^2+2\cos^2\alpha\sin^2\alpha=\sin^2\alpha+\cos^2\alpha$;

5) $\frac{\sin\alpha}{1+\cos\alpha}+\frac{1+\cos\alpha}{\sin\alpha}=\frac{2}{\sin\alpha}$; 6) $\frac{\sin\alpha}{1-\cos\alpha}=\frac{1+\cos\alpha}{\sin\alpha}$;

7) $\frac{1}{1+\mathrm{tg}^2\alpha}+\frac{1}{1+\mathrm{ctg}^2\alpha}=1$; 8) $\mathrm{tg}^2\alpha-\sin^2\alpha=\mathrm{tg}^2\alpha\sin^2\alpha$.

316. An'latpani' a'piwayi'lasti'ri'n', woni'n' san ma'nisin tabi'n':

1) $\frac{(\sin\alpha+\cos\alpha)^2}{\sin^2\alpha}-(1+\mathrm{ctg}^2\alpha)$, bunda $\alpha=\frac{\pi}{3}$;

2) $(1+\mathrm{tg}^2\alpha)-\frac{(\sin\alpha-\cos\alpha)^2}{\cos^2\alpha}$, bunda $\alpha=\frac{\pi}{6}$.

317. Yeger $\sin\alpha-\cos\alpha=0{,}6$ bolsa, $\sin\alpha\cos\alpha$ ni'n' ma'nisin tabi'n'.

318. Yeger $\cos\alpha-\sin\alpha=0{,}2$ bolsa, $\cos^3\alpha-\sin^3\alpha$ ni'n' ma'nisin tabi'n'.

319. Ten'lemeni sheshin':

1) $3\cos^2 x-2\sin x=3-3\sin^2 x$;

2) $\cos^2 x-\sin^2 x=2\sin x-1-2\sin^2 x$.

25- §. α HA'M $-\alpha$ MU'YESHLERININ' SINUSI', KOSINUSI', TANGENSI HA'M KOTANGENSI

Meyli, birlik shen'berdin' M_1 ha'm M_2 noqatlari' $P(1;\ 0)$ noqati'n sa'ykes tu'rde α ha'm $-\alpha$ mu'yeshlerge buri'w na'tiyjesinde payda yetilgen bolsi'n (68-su'wret). Wonda Ox ko'sheri M_1OM_2 mu'yeshin ten' yekige bo'ledi ha'm sonli'qtan M_1 ha'm M_2 noqatlari' Ox ko'sherine sali'sti'rg'anda simmetriyali' jaylasqan. Bul noqatlardi'n' abcissalari' birdey, al ordinatalari' tek belgileri menen g'ana pari'qlanadi'. M_1 noqati' $(\cos\alpha;\ \sin\alpha)$ koordinatalari'na, M_2 noqati' $(\cos(-\alpha);\ \sin(-\alpha))$ koordinatalari'na iye. Sonli'qtan,

$$\boxed{\sin(-\alpha) = -\sin\alpha,\ \cos(-\alpha) = \cos\alpha.} \tag{1}$$

Tangenstin' ani'qlamasi'nan paydalani'p, to'mendegige iye bolami'z:

$$\mathrm{tg}(-\alpha) = \frac{\sin(-\alpha)}{\cos(-\alpha)} = \frac{-\sin\alpha}{\cos\alpha} = -\mathrm{tg}\alpha\,.$$

Demek,

$$\boxed{\mathrm{tg}(-\alpha) = -\mathrm{tg}\alpha.} \tag{2}$$

Sog'an uqsas,

$$\boxed{\mathrm{ctg}(-\alpha) = -\mathrm{ctg}\alpha.} \tag{3}$$

(1) formula α ni'n' qa'legen ma'nisinde wori'nli' boladi', al (2) formula $\alpha \neq \frac{\pi}{2} + \pi k,\ k \in \boldsymbol{Z}$ bolg'anda wori'nli'.

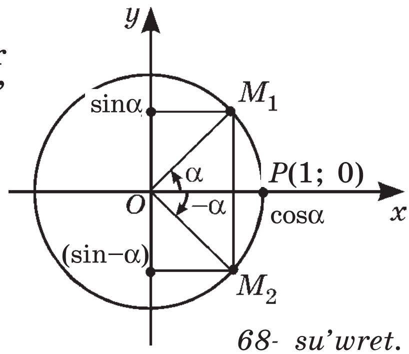
Yeger $\alpha \neq \pi k,\ k \in \boldsymbol{Z}$ bolsa, wonda $\mathrm{ctg}(-\alpha) = -\mathrm{ctg}\alpha$ bolatug'i'nli'g'i'n ko'rsetiw mu'mkin.

(1)–(2) formulalar teris mu'yeshler ushi'n sinus, kosinus ha'm tangenstin' ma'nislerin tabi'wg'a imkaniyat beredi.

Ma'selen: $\sin(-\frac{\pi}{6}) = -\sin\frac{\pi}{6} = -\frac{1}{2}$,

$$\cos(-\frac{\pi}{4}) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2},$$

$$\mathrm{tg}(-\frac{\pi}{3}) = -\mathrm{tg}\frac{\pi}{3} = -\sqrt{3}\,.$$



68- su'wret.

Shi'ni'g'i'wlar

320. Yesaplan':

1) $\cos(-\frac{\pi}{6})\sin(-\frac{\pi}{3})+\mathrm{tg}(-\frac{\pi}{4})$; 2) $\frac{1+\mathrm{tg}^2(-30^\circ)}{1+\mathrm{ctg}^2(-30^\circ)}$;

3) $2\sin(-\frac{\pi}{6})\cos(-\frac{\pi}{6})+\mathrm{tg}(-\frac{\pi}{3})+\sin^2(-\frac{\pi}{4})$;

4) $\cos(-\pi)+\mathrm{ctg}(-\frac{\pi}{2})-\sin(-\frac{3}{2}\pi)+\mathrm{ctg}(-\frac{\pi}{4})$.

321. An'latpani' a'piwayi'lasti'ri'n':

1) $\mathrm{tg}(-\alpha)\cos\alpha+\sin\alpha$; 2) $\cos\alpha-\mathrm{ctg}\alpha(-\sin\alpha)$;

3) $\frac{\cos(-\alpha)+\sin(-\alpha)}{\cos^2\alpha-\sin^2\alpha}$; 4) $\mathrm{tg}(-\alpha)\mathrm{ctg}(-\alpha)+\cos^2(-\alpha)+\sin^2\alpha$.

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322. Birdeylikti da'lillen': $\frac{\cos^2\alpha-\sin^2\alpha}{\cos\alpha+\sin(-\alpha)}+\mathrm{tg}(-\alpha)\cos(-\alpha)=\cos\alpha$.

323. Yesaplan':

1) $\frac{3-\sin^2\left(-\frac{\pi}{3}\right)-\cos^2\left(-\frac{\pi}{3}\right)}{2\cos\left(-\frac{\pi}{4}\right)}$;

2) $2\sin\left(-\frac{\pi}{6}\right)-3\mathrm{ctg}\left(-\frac{\pi}{4}\right)+7{,}5\mathrm{tg}\left(-\pi\right)+\frac{1}{8}\cos\left(-\frac{3}{2}\pi\right)$.

324. A'piwayi'lasti'ri'n':

1) $\frac{\sin^3(-\alpha)+\cos^3(-\alpha)}{1-\sin(-\alpha)\cos(-\alpha)}$; 2) $\frac{1-(\sin\alpha+\cos(-\alpha))^2}{-\sin(-\alpha)}$.

26- §. QOSI'W FORMULALARI'

Qosi'w formulalari' dep $\cos(\alpha\pm\beta)$ ha'm $\sin(\alpha\pm\beta)$ lardi' α ha'm β mu'yeshlerinin' sinus ha'm kosinuslari' arqali' an'lati'wshi' formulalarg'a ayti'ladi'.

T e o r e m a . ***Qa'legen α ha'm β ushi'n to'mendegi ten'lik duri's boladi':***

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta. \qquad (1)$$

○ M_0 (1; 0) noqati'n koordinatalar basi' a'tirapi'nda α, $-\beta$, $\alpha+\beta$ radian mu'yeshlerge buri'w na'tiyjesinde sa'ykes tu'rde M_α, $M_{-\beta}$ ha'm $M_{\alpha+\beta}$ noqatlari' payda bolatug'i'n bolsi'n (69-su'wret).

Sinus ha'm kosinusti'n' ani'qlamasi'na muwapi'q, bul noqatlar to'mendegi koordinatalarg'a iye:

$$M_\alpha\ (\cos\alpha;\ \sin\alpha), \quad M_{-\beta}\ (\cos(-\beta);\ \sin(-\beta)),$$
$$M_{\alpha+\beta}\ (\cos(\alpha+\beta);\ \sin(\alpha+\beta)).$$

$\angle M_0OM_{\alpha+\beta} = \angle M_{-\beta}OM_\alpha$ bolg'anli'qtan $M_0OM_{\alpha+\beta}$ ha'm $M_{-\beta}OM_\alpha$ ten' qaptalli' u'shmu'yeshlikleri ten' ha'm demek, wolardi'n' $M_0M_{\alpha+\beta}$ ha'm $M_{-\beta}M_\alpha$ ultanlari' da ten'. Sonli'qtan

$$(M_0M_{\alpha+\beta})^2 = (M_{-\beta}M_\alpha)^2.$$

Geometriya kursi'nan ma'lim bolg'an yeki noqat arasi'ndag'i' arali'q formulasi'nan paydalani'p, to'mendegige iye bolami'z:

$$(1-\cos(\alpha+\beta))^2 + (\sin(\alpha+\beta))^2 = (\cos(-\beta)-\cos\alpha)^2 + (\sin(-\beta)-\sin\alpha)^2.$$

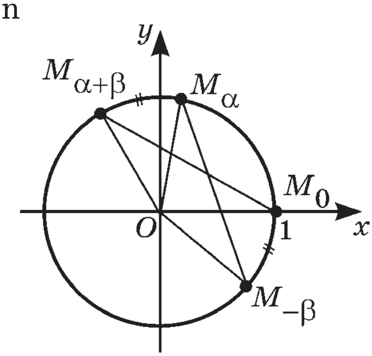
25- § degi (1) formuladan paydalani'p, bul ten'lemeni tu'rlendiremiz:

$$1 - 2\cos(\alpha+\beta) + \cos^2(\alpha+\beta) + \sin^2(\alpha+\beta) =$$
$$= \cos^2\beta - 2\cos\beta\cos\alpha + \cos^2\alpha + \sin^2\beta + 2\sin\beta\sin\alpha + \sin^2\alpha.$$

Tiykarg'i' trigonometriyali'q birdeylikten paydalani'p, to'mendegige iye bolami'z:

$$2 - 2\cos(\alpha+\beta) = 2 - 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta,$$

bunnan $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$. ●



69- su'wret.

1 - m a ' s e l e . $\cos75^\circ$ ti' yesaplan'.

△ (1) formula boyi'nsha tabami'z:

$$\cos75^\circ = \cos(45^\circ + 30^\circ) =$$
$$= \cos45^\circ\cos30^\circ - \sin45^\circ\sin30^\circ =$$
$$= \frac{\sqrt2}{2}\cdot\frac{\sqrt3}{2} - \frac{\sqrt2}{2}\cdot\frac{1}{2} = \frac{\sqrt6-\sqrt2}{4}.$$ ▲

(1) formulada β ni' –β g'a almasti'ri'p, to'mendegige iye bolami'z:

$$\cos(\alpha - \beta) = \cos\alpha\cos(-\beta) - \sin\alpha\sin(-\beta),$$

bunnan

$$\mathbf{\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta.} \qquad \textbf{(2)}$$

2-ma'sele. cos15° ti' yesaplan'.

△ (2) formulag'a muwapi'q, to'mendegige iye bolami'z:

$$\cos15^\circ = \cos(45^\circ - 30^\circ) = \cos45^\circ\cos30^\circ + \sin45^\circ\sin30^\circ =$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}. \ \blacktriangle$$

3-ma'sele. To'mendegi formulalardi' da'lillen':

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha, \ \sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha. \qquad (3)$$

△ $\alpha = \frac{\pi}{2}$ bolg'anda (2) formulag'a tiykarlani'p:

$$\cos\left(\frac{\pi}{2} - \beta\right) = \cos\frac{\pi}{2}\cos\beta + \sin\frac{\pi}{2}\sin\beta = \sin\beta,$$

ya'g'ni'y

$$\cos\left(\frac{\pi}{2} - \beta\right) = \sin\beta. \qquad (4)$$

Bul formulada β ni' α g'a almasti'ri'p, mi'nag'an iye bolami'z:

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha.$$

(4) formulada $\beta = \frac{\pi}{2} - \alpha$ dep woylasaq:

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha. \ \blacktriangle$$

(1)—(4) formulalardan paydalani'p, *sinus ushi'n qosi'w formulasi'n* keltirip shi'g'arami'z:

$$\sin(\alpha + \beta) = \cos\left(\frac{\pi}{2} - (\alpha + \beta)\right) = \cos\left(\left(\frac{\pi}{2} - \alpha\right) - \beta\right) =$$

$$= \cos\left(\frac{\pi}{2} - \alpha\right)\cos\beta + \sin\left(\frac{\pi}{2} - \alpha\right)\sin\beta = \sin\alpha\cos\beta + \cos\alpha\sin\beta.$$

Solay yetip,

$$\mathbf{\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta.} \qquad \textbf{(5)}$$

(5) formulada β ni' –β g'a almasti'ri'p, to'mendegige iye bolami'z:

$$\sin(\alpha-\beta)=\sin\alpha\cos(-\beta)+\cos\alpha\sin(-\beta),$$

bunnan

$$\mathbf{\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta.} \qquad \mathbf{(6)}$$

4-ma'sele. $\sin210^\circ$ ti' yesaplan'.

$$\triangle\ \sin210^\circ=\sin(180^\circ+30^\circ)=$$

$$=\sin180^\circ\cos30^\circ+\cos180^\circ\sin30^\circ=0\cdot\frac{\sqrt{3}}{2}+(-1)\cdot\frac{1}{2}=-\frac{1}{2}.\ \blacktriangle$$

5-ma'sele. Yesaplan':

$$\sin\frac{8\pi}{7}\cos\frac{\pi}{7}-\sin\frac{\pi}{7}\cos\frac{8\pi}{7}.$$

$$\triangle\ \sin\frac{8\pi}{7}\cos\frac{\pi}{7}-\sin\frac{\pi}{7}\cos\frac{8\pi}{7}=\sin\left(\frac{8\pi}{7}-\frac{\pi}{7}\right)=\sin\pi=0.\ \blacktriangle$$

6-ma'sele. Ten'likti da'lillen':

$$\mathrm{tg}(\alpha+\beta)=\frac{\mathrm{tg}\alpha+\mathrm{tg}\beta}{1-\mathrm{tg}\alpha\,\mathrm{tg}\beta}. \qquad \mathbf{(7)}$$

$$\triangle\ \mathrm{tg}(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}.$$

Bul bo'lshektin' ali'mi' ha'm bo'limin $\cos\alpha\cos\beta$ g'a bo'lip, (7) formulani' payda yetemiz. ▲

(7) formula yesaplawlarda paydali' boli'wi' mu'mkin.

Ma'selen, usi' formula boyi'nsha to'mendegini tabami'z:

$$\mathrm{tg}225^\circ=\mathrm{tg}(180^\circ+45^\circ)=\frac{\mathrm{tg}180^\circ+\mathrm{tg}45^\circ}{1-\mathrm{tg}180^\circ\,\mathrm{tg}45^\circ}=1.$$

Shi'ni'g'i'wlar

Qosi'w formulalari' ja'rdeminde yesaplan' **(325–326)**:

325. 1) $\cos135^\circ$; 2) $\cos120^\circ$; 3) $\cos150^\circ$; 4) $\cos240^\circ$.

326. 1) $\cos57^\circ30'\cos27^\circ30'+\sin57^\circ30'\sin27^\circ30'$;

2) $\cos19^\circ30'\cos25^\circ30'-\sin19^\circ30'\sin25^\circ30'$;

3) $\cos\frac{7\pi}{9}\cos\frac{11\pi}{9}-\sin\frac{7\pi}{9}\sin\frac{11\pi}{9}$; 4) $\cos\frac{8\pi}{7}\cos\frac{\pi}{7}+\sin\frac{8\pi}{7}\sin\frac{\pi}{7}$.

327. 1) $\cos\left(\frac{\pi}{3}+\alpha\right)$, bunda $\sin\alpha=\frac{1}{\sqrt{3}}$ ha'm $0<\alpha<\frac{\pi}{2}$;

2) $\cos\left(\alpha-\frac{\pi}{4}\right)$, bunda $\cos\alpha=-\frac{\pi}{3}$ ha'm $\frac{\pi}{2}<\alpha<\pi$.

An'latpani' a'piwayi'lasti'ri'n' **(328–329):**

328. 1) $\cos3\alpha\cos\alpha-\sin\alpha\sin3\alpha$; 2) $\cos5\beta\cos2\beta+\sin5\beta\sin2\beta$;

3) $\cos\left(\frac{\pi}{7}+\alpha\right)\cos\left(\frac{5\pi}{14}-\alpha\right)-\sin\left(\frac{\pi}{7}+\alpha\right)\sin\left(\frac{5\pi}{14}-\alpha\right)$;

4) $\cos\left(\frac{7\pi}{5}+\alpha\right)\cos\left(\frac{2\pi}{5}+\alpha\right)+\sin\left(\frac{7\pi}{5}+\alpha\right)\sin\left(\frac{2\pi}{5}+\alpha\right)$.

329. 1) $\cos(\alpha+\beta)+\cos\left(\frac{\pi}{2}-\alpha\right)\cos\left(\frac{\pi}{2}-\beta\right)$;

2) $\sin\left(\frac{\pi}{2}-\alpha\right)\sin\left(\frac{\pi}{2}-\beta\right)-\cos(\alpha-\beta)$.

Qosi'w formulalari' ja'rdeminde yesaplan' **(330–331):**

330. 1) $\sin73^\circ\cos17^\circ+\cos73^\circ\sin17^\circ$;

2) $\sin73^\circ\cos13^\circ-\cos73^\circ\sin13^\circ$;

3) $\sin\frac{5\pi}{12}\cos\frac{\pi}{12}+\sin\frac{\pi}{12}\cos\frac{5\pi}{12}$; 4) $\sin\frac{7\pi}{12}\cos\frac{\pi}{12}-\sin\frac{\pi}{12}\cos\frac{7\pi}{12}$.

331. 1) $\sin\left(\alpha+\frac{\pi}{6}\right)$, bunda $\cos\alpha=-\frac{3}{5}$ ha'm $\pi<\alpha<\frac{3\pi}{2}$;

2) $\sin\left(\frac{\pi}{4}-\alpha\right)$, bunda $\sin\alpha=\frac{\sqrt{2}}{3}$ ha'm $\frac{\pi}{2}<\alpha<\pi$.

332. An'latpani' a'piwayi'lasti'ri'n':

1) $\sin(\alpha+\beta)+\sin(-\alpha)\cos(-\beta)$; 2) $\cos(-\alpha)\sin(-\beta)-\sin(\alpha-\beta)$;

3) $\cos\left(\frac{\pi}{2}-\alpha\right)\sin\left(\frac{\pi}{2}-\beta\right)-\sin(\alpha-\beta)$;

4) $\sin(\alpha+\beta)+\sin\left(\frac{\pi}{2}-\alpha\right)\sin(-\beta)$.

333. Yeger $\sin\alpha=-\frac{3}{5}$, $\frac{3}{2}\pi<\alpha<2\pi$ ha'm $\sin\beta=\frac{8}{17}$, $0<\beta<\frac{\pi}{2}$ bolsa, $\cos(\alpha+\beta)$ ha'm $\cos(\alpha-\beta)$ ni' yesaplan'.

334. Yeger $\cos\alpha = -0{,}8$, $\frac{\pi}{2} < \alpha < \pi$ ha'm $\sin\beta = -\frac{12}{13}$, $\pi < \beta < \frac{3\pi}{2}$ bolsa, $\sin(\alpha - \beta)$ ni' yesaplan'.

335. An'latpani' a'piwayi'lasti'ri'n':

1) $\cos\left(\frac{2}{3}\pi - \alpha\right) + \cos\left(\alpha + \frac{\pi}{3}\right)$; 2) $\sin\left(\alpha + \frac{2}{3}\pi\right) - \sin\left(\frac{\pi}{3} - \alpha\right)$;

3) $\dfrac{2\cos\alpha\sin\beta + \sin(\alpha-\beta)}{2\cos\alpha\cos\beta - \cos(\alpha-\beta)}$; 4) $\dfrac{\cos\alpha\cos\beta - \cos(\alpha+\beta)}{\cos(\alpha-\beta) - \sin\alpha\sin\beta}$.

336. Birdeylikti da'lillen':

1) $\sin(\alpha - \beta)\sin(\alpha + \beta) = \sin^2\alpha - \sin^2\beta$;

2) $\cos(\alpha - \beta)\cos(\alpha + \beta) = \cos^2\alpha - \sin^2\beta$;

3) $\dfrac{\sqrt{2}\cos\alpha - 2\cos\left(\frac{\pi}{4} - \alpha\right)}{2\sin\left(\frac{\pi}{4} + \alpha\right) - \sqrt{3}\sin\alpha} = -\sqrt{2}\,\mathrm{tg}\,\alpha$; 4) $\dfrac{\cos\alpha - 2\cos\left(\frac{\pi}{3} + \alpha\right)}{2\sin\left(\alpha - \frac{\pi}{6}\right) - \sqrt{3}\sin\alpha} = -\sqrt{3}\,\mathrm{tg}\,\alpha$.

337. An'latpani' a'piwayi'lasti'ri'n': 1) $\dfrac{\mathrm{tg}\,29^\circ + \mathrm{tg}\,31^\circ}{1 - \mathrm{tg}\,29^\circ\,\mathrm{tg}\,31^\circ}$; 2) $\dfrac{\mathrm{tg}\frac{7}{16}\pi - \mathrm{tg}\frac{3}{16}\pi}{1 + \mathrm{tg}\frac{7}{16}\pi \cdot \mathrm{tg}\frac{3}{16}\pi}$.

27-§. QOS MU'YESHTIN' SINUSI' HA'M KOSINUSI'

Qosi'w formulalari'nan paydalani'p, *qos mu'yeshtin' sinusi' ha'm kosinusi' formulalari'n* keltirip shi'g'arami'z.

1) $\sin 2\alpha = \sin(\alpha + \alpha) = \sin\alpha\cos\alpha + \sin\alpha\cos\alpha = 2\sin\alpha\cos\alpha$.

Solay yetip,

$$\sin 2\alpha = 2\sin\alpha\cos\alpha. \quad (1)$$

1-ma'sele. Yeger $\sin\alpha = -0{,}6$ ha'm $\pi < \alpha < \frac{3\pi}{2}$ bolsa, $\sin 2\alpha$ ni' yesaplan'.

△ (1) formula boyi'nsha tabami'z:

$$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2 \cdot (-0{,}6) \cdot \cos\alpha = -1{,}2\cos\alpha.$$

$\pi < \alpha < \frac{3\pi}{2}$ bolg'ani' ushi'n $\cos\alpha < 0$ boladi' ha'm sonli'qtan:

$$\cos\alpha = -\sqrt{1 - \sin^2\alpha} = -\sqrt{1 - 0{,}36} = -0{,}8.$$

Demek, $\sin 2\alpha = -1{,}2 \cdot (-0{,}8) = 0{,}96$. ▲

2) $\cos 2\alpha = \cos(\alpha + \alpha) = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha = \cos^2\alpha - \sin^2\alpha$.

Solay yetip,

$$\mathbf{\cos 2\alpha = \cos^2\alpha - \sin^2\alpha.} \qquad \textbf{(2)}$$

2-ma'sele. Yeger $\cos\alpha = 0,3$ bolsa, $\cos 2\alpha$ ni' yesaplan'.

△ (2) formuladan ha'm tiykarg'i' trigonometriyali'q birdeylikten paydalani'p, to'mendegige iye bolami'z:

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \cos^2\alpha - (1 - \cos^2\alpha) =$$

$$= 2\cos^2\alpha - 1 = 2\cdot(0,3)^2 - 1 = -0,82. \ \blacktriangle$$

3-ma'sele. An'latpani' a'piwayi'lasti'ri'n': $\frac{\sin\alpha\cos\alpha}{1-2\sin^2\alpha}$.

$$\triangle \ \frac{\sin\alpha\cos\alpha}{1-2\sin^2\alpha} = \frac{2\sin\alpha\cos\alpha}{2(\sin^2\alpha+\cos^2\alpha-2\sin^2\alpha)} = \frac{\sin 2\alpha}{2(\cos^2\alpha-\sin^2\alpha)} =$$

$$= \frac{\sin 2\alpha}{2\cos 2\alpha} = \frac{1}{2}\,\mathrm{tg}\,2\alpha. \ \blacktriangle$$

4-ma'sele. Yeger $\mathrm{tg}\,\alpha = \frac{1}{2}$ bolsa, $\mathrm{tg}\,2\alpha$ ni' yesaplan'.

$$\triangle \ \mathrm{tg}(\alpha+\beta) = \frac{\mathrm{tg}\,\alpha+\mathrm{tg}\,\beta}{1-\mathrm{tg}\,\alpha\,\mathrm{tg}\,\beta}$$

formulada $\beta = \alpha$ dep boljap (26-§ qa qaran'), to'mendegige iye bolami'z:

$$\mathbf{tg\,2\alpha = \frac{2tg\,\alpha}{1-tg^2\alpha}}. \qquad \textbf{(3)}$$

Yeger $\mathrm{tg}\,\alpha = \frac{1}{2}$ bolsa, wonda (3) formula boyi'nsha to'mendegige iye bolami'z:

$$\mathrm{tg}\,2\alpha = \frac{2\cdot\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} = \frac{4}{3}. \ \blacktriangle$$

Shi'ni'g'i'wlar

Yesaplan' **(338–339):**

338. 1) $2\sin 15^\circ\cos 15^\circ$; 2) $\cos^2 15^\circ - \sin^2 15^\circ$;

3) $(\cos 75^\circ - \sin 75^\circ)^2$; 4) $(\cos 15^\circ + \sin 15^\circ)^2$.

339. 1) $2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$; 2) $\cos^2\frac{\pi}{8}-\sin^2\frac{\pi}{8}$;

3) $\sin\frac{\pi}{8}\cos\frac{\pi}{8}+\frac{1}{4}$; 4) $\frac{\sqrt{2}}{2}-\left(\cos\frac{\pi}{8}+\sin\frac{\pi}{8}\right)^2$.

340. Yeger:

1) $\sin\alpha=\frac{3}{5}$ ha'm $\frac{\pi}{2}<\alpha<\pi$; 2) $\cos\alpha=-\frac{4}{5}$ ha'm $\pi<\alpha<\frac{3\pi}{2}$

bolsa, $\sin 2\alpha$ ni' yesaplan'.

341. Yeger:

1) $\cos\alpha=\frac{4}{5}$; 2) $\sin\alpha=-\frac{3}{5}$ bolsa, $\cos 2\alpha$ ni' yesaplan'.

An'latpani' a'piwayi'lasti'ri'n' **(342–343):**

342. 1) $\sin\alpha\cos\alpha$; 2) $\cos\alpha\cos\left(\frac{\pi}{2}-\alpha\right)$;

3) $\cos 4\alpha+\sin^2 2\alpha$; 4) $\sin 2\alpha+(\sin\alpha-\cos\alpha)^2$.

343. 1) $\frac{\cos 2\alpha+1}{2\cos\alpha}$; 2) $\frac{\sin 2\alpha}{1-\cos^2\alpha}$; 3) $\frac{\sin^2\alpha}{(\sin\alpha+\cos\alpha)^2-1}$; 4) $\frac{1+\cos 2\alpha}{1-\cos 2\alpha}$.

344. Birdeylikti da'lillen':

1) $\sin 2\alpha=(\sin\alpha+\cos\alpha)^2-1$; 2) $(\sin\alpha-\cos\alpha)^2=1-\sin 2\alpha$;

3) $\cos^4\alpha-\sin^4\alpha=\cos 2\alpha$; 4) $2\cos^2\alpha-\cos 2\alpha=1$.

345. Yeger:

1) $\sin\alpha+\cos\alpha=\frac{1}{2}$; 2) $\sin\alpha-\cos\alpha=-\frac{1}{3}$; 3) $\sin\alpha-\cos\alpha=\frac{1}{4}$

bolsa, $\sin 2\alpha$ ni' yesaplan':

346. Birdeylikti da'lillen':

1) $1+\cos 2\alpha=2\cos^2\alpha$; 2) $1-\cos 2\alpha=2\sin^2\alpha$.

347. Yesaplan':

1) $2\cos^2 15°-1$; 2) $1-2\sin^2 22{,}5°$;

3) $2\cos^2\frac{\pi}{8}-1$; 4) $1-2\sin^2\frac{\pi}{12}$.

348. An'latpani' a'piwayi'lasti'ri'n':

1) $1-2\sin^2 5\alpha$; 2) $2\cos^2 3\alpha-1$; 3) $\frac{1-\cos 2\alpha}{\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$;

4) $\frac{2\cos^2\frac{\alpha}{2}-1}{\sin 2\alpha}$; 5) $1+\cos 4\alpha$; 6) $1-2\cos^2 5\alpha$.

349. Birdeylikti da'lillen':

1) $\frac{\cos2\alpha}{\sin\alpha\cos\alpha+\sin^2\alpha}=\text{ctg}\alpha-1$; 2) $\frac{\sin2\alpha-2\cos\alpha}{\sin\alpha-\sin^2\alpha}=-2\text{ctg}\alpha$;

3) $\text{tg}\alpha(1+\cos2\alpha)=\sin2\alpha$; 4) $\frac{1-\cos2\alpha+\sin2\alpha}{1+\cos2\alpha+\sin2\alpha}\cdot\text{ctg}\alpha=1$.

350. Yeger $\text{tg}\alpha=0{,}6$ bolsa, $\text{tg}2\alpha$ ni' yesaplan'.

351. Yesaplan': 1) $\frac{2\text{tg}\frac{\pi}{8}}{1-\text{tg}^2\frac{\pi}{8}}$; 2) $\frac{6\text{tg}15^\circ}{1-\text{tg}^2 15^\circ}$; 3) $\frac{4\text{tg}75^\circ}{1-\text{tg}^2 75^\circ}$.

28- §. KELTIRIW FORMULALARI'

Sinus, kosinus, tangens ha'm kotangenstin' ma'nislerinin' kesteleri 0° dan 90° qa shekem (yaki 0 den $\frac{\pi}{2}$ ge shekem) mu'yeshler ushi'n du'ziledi. Bul jag'day wolardi'n' basqa mu'yeshler ushi'n ma'nisleri su'yir mu'yeshler ushi'n ma'nislerine keltiriw menen tu'sindiriledi.

1-ma'sele. $\sin870^\circ$ ha'm $\cos870^\circ$ ti' yesaplan'.

△ $870^\circ=2\cdot360^\circ+150^\circ$. Sonli'qtan da $P(1;\,0)$ noqati'n koordinatalar basi' a'tirapi'nda 870° qa burg'anda noqat yeki toli'q aynali's jasaydi' ha'm ja'ne 150° mu'yeshke buri'ladi', yag'ni'y 150° qa buri'wdag'i' M noqati'ni'n' da'l wo'zi payda boladi' (70-su'wret). Sonli'qtan $\sin870^\circ=\sin150^\circ$, $\cos870^\circ=\cos150^\circ$.

M noqati'na Oy ko'sherine qarata simmetriyali' bolg'an M_1 noqati'n jasaymi'z (71-su'wret). M ha'm M_1 noqatlari'ni'n' ordinatalari' birdey, al abcissalari' tek belgileri menen g'ana pari'qlanadi'. Sonli'qtan

$$\sin150^\circ=\sin30^\circ=\frac{1}{2};\ \cos150^\circ=-\cos30^\circ=-\frac{\sqrt{3}}{2}.$$

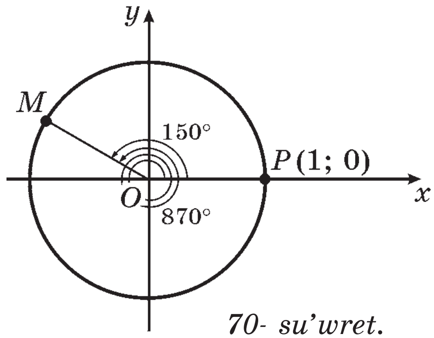
Juwabi': $\sin870^\circ=\frac{1}{2}$, $\cos870^\circ=-\frac{\sqrt{3}}{2}$. ▲

1-ma'seleni sheshiwde

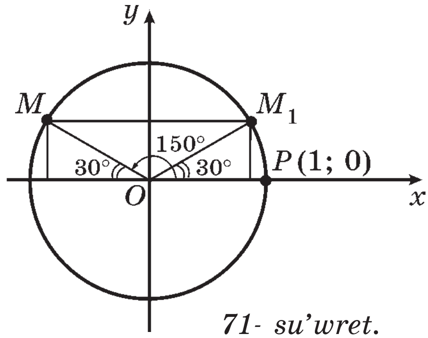
$$\sin(2\cdot360^\circ+150^\circ)=\sin150^\circ,\ \cos(2\cdot360^\circ+150^\circ)=\cos150^\circ \qquad (1)$$

$$\sin(180^\circ-30^\circ)=\sin30^\circ,\ \cos(180^\circ-30^\circ)=-\cos30^\circ \qquad (2)$$

ten'liklerden paydalani'ldi'.



70- su'wret.



71- su'wret.

(1) ten'lik duri's ten'lik, sebebi $P(1;\ 0)$ noqati'n $\alpha + 2\pi k$, $k \in \mathbf{Z}$ mu'yeshke burg'anda woni' α mu'yeshke burg'andag'i' noqatti'n' da'l wo'zi payda boladi'.

Sonli'qtan da to'mendegi formulalar duri's boladi':

$$\mathbf{\sin(\alpha + 2\pi k) = \sin\alpha,\ \cos(\alpha + 2\pi k) = \cos\alpha,\ k \in Z.} \qquad \mathbf{(3)}$$

Atap aytqanda, $k = 1$ bolg'anda:

$$\sin(\alpha + 2\pi) = \sin\alpha,\ \cos(\alpha + 2\pi) = \cos\alpha$$

ten'likleri wori'nli' boladi'.

(2) ten'lik

$$\mathbf{\sin(\pi - \alpha) = \sin\alpha,\ \cos(\pi - \alpha) = -\cos\alpha} \qquad \mathbf{(4)}$$

formulalardi'n' dara jag'dayi' boli'p sanaladi'.

$\sin(\pi - \alpha) = \sin\alpha$ formulasi'n da'lilleymiz.

◯ Sinus ushi'n qosi'w formulasi'n paydalani'p, to'mendegige iye bolami'z:

$$\sin(\pi - \alpha) = \sin\pi\cos\alpha - \cos\pi\sin\alpha =$$

$$= 0 \cdot \cos\alpha - (-1) \cdot \sin\alpha = \sin\alpha. \ ●$$

(4) formulalardi'n' yekinshisi de usi'g'an uqsas da'lillenedi. (4) formulalar *keltiriw formulalari'* dep ataladi'. (3) ha'm (4) formulalar ja'rdeminde qa'legen mu'yeshtin' sinusi' ha'm kosinusi'n yesaplawdi' wolardi'n' su'yir mu'yeshler ushi'n ma'nislerin yesaplawg'a keltiriw mu'mkin.

2-ma'sele. sin930° ti' yesaplan'.

△ (3) formuladan paydalani'p, to'mendegige iye bolami'z:

$$\sin930^\circ = \sin(3\cdot360^\circ - 150^\circ) = \sin(-150^\circ).$$

$\sin(-\alpha) = -\sin\alpha$ formulasi' boyi'nsha $\sin(-150^\circ) = -\sin150^\circ$ ti' payda yetemiz.

(4) formula boyi'nsha to'mendegige iye bolami'z:

$$-\sin150^\circ = -\sin(180^\circ - 30^\circ) = -\sin30^\circ = -\frac{1}{2}.$$

Juwabi': $\sin930^\circ = -\frac{1}{2}$. ▲

3-ma'sele. $\cos\frac{15\pi}{4}$ ti' yesaplan'.

△ $$\cos\frac{15\pi}{4} = \cos(4\pi - \frac{\pi}{4}) = \cos(-\frac{\pi}{4}) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}.$$ ▲

Yendi qa'legen mu'yeshtin' tangensin yesaplawdi' su'yir mu'yeshtin' tangensin yesaplawg'a qalay keltiriw mu'mkinligin ko'rsetemiz.

(3) formuladan ham tangenstin' ani'qlamasi'nan

$$\operatorname{tg}(\alpha + 2\pi k) = \operatorname{tg}\alpha,\ k \in \boldsymbol{Z}$$

ten'ligi kelip shi'g'adi'.

Usi' ten'lik ha'm (4) formuladan paydalani'p, to'mendegige iye bolami'z:

$$\operatorname{tg}(\alpha + \pi) = \operatorname{tg}(\alpha + \pi - 2\pi) = \operatorname{tg}(\alpha - \pi) = -\operatorname{tg}(\pi - \alpha) =$$
$$= -\frac{\sin(\pi-\alpha)}{\cos(\pi-\alpha)} = -\frac{\sin\alpha}{-\cos\alpha} = \operatorname{tg}\alpha.$$

Sonli'qtan to'mendegi formula wori'nli' boladi':

$$\boldsymbol{\operatorname{tg}(\alpha + \pi k) = \operatorname{tg}\alpha,\ k \in Z.} \quad (5)$$

4-ma'sele. Yesaplan': 1) $\operatorname{tg}\frac{11\pi}{3}$; 2) $\operatorname{tg}\frac{13\pi}{4}$.

△ 1) $\operatorname{tg}\frac{11\pi}{3} = \operatorname{tg}(4\pi - \frac{\pi}{3}) = \operatorname{tg}(-\frac{\pi}{3}) = -\operatorname{tg}\frac{\pi}{3} = -\sqrt{3}$.

2) $\operatorname{tg}\frac{13\pi}{4} = \operatorname{tg}(3\pi + \frac{\pi}{4}) = \operatorname{tg}\frac{\pi}{4} = 1$. ▲

26- § da (3-ma'sele)

$$\mathbf{sin}\left(\frac{\pi}{2}-\alpha\right)=\mathbf{cos}\alpha, \mathbf{cos}\left(\frac{\pi}{2}-\alpha\right)=\mathbf{sin}\alpha$$

formulalari' da'lillengen yedi, bul formulalar da *keltiriw formulalari'* dep ataladi'. Usi' formulalardan paydalani'p, ma'selen, $\sin\frac{\pi}{6}=\cos\frac{\pi}{6}$, $\cos\frac{\pi}{3}=\sin\frac{\pi}{6}$ ni' payda yetemiz.

x ti'n' qa'legen ma'nisi ushi'n $\sin(x+2\pi)=\sin x$, $\cos(x+2\pi)=\cos x$ ten'liklerinin' duri'sli'g'i' ma'lim.

Bul ten'liklerden argument 2π ge wo'zgergende sinus ha'm kosinusti'n' ma'nisleri periodli' ta'kirarlanatug'i'nli'g'i' ko'rinedi. Bunday funkciyalar *periodi'* 2π *bolg'an periodli' funkciyalar* dep ataladi'.

Yeger sonday bir $T \neq 0$ sani' bar boli'p, $y = f(x)$ funkciyasi'ni'n' ani'qlani'w oblasti'ndag'i' qa'legen x ushi'n

$$f(x - T) = f(x) = f(x + T)$$

ten'ligi wori'nlansa, wonda $f(x)$ *funkciyasi' periodli' funkciya* dep ataladi'.

T sani' $f(x)$ funkciyasi'ni'n' *periodi'* dep ataladi'.

Bul ani'qlamadan, yeger x san $f(x)$ funkciyasi'ni'n' ani'qlani'w oblasti'na tiyisli bolsa, wonda $x+T$, $x-T$ sanlari' ha'm, uluwma $x+$ $+Tn$, $n\in\boldsymbol{Z}$ sanlari' da usi' periodli' funkciyani'n' ani'qlani'w oblasti'na tiyisli ha'm $f(x+Tn)=f(x)$, $n\in\boldsymbol{Z}$ boladi'.

2π sani' $y=\cos x$ funkciyasi'ni'n' *yen' kishi won' periodi'* yekenligin ko'rsetemiz.

○ $T>0$ kosinusti'n' periodi' bolsi'n, yag'ni'y qa'legen x ushi'n $\cos(x+T)=\cos x$ ten'ligi wori'nlanadi'. $x=0$ dep, $\cos T=1$ di payda yetemiz. Bunda $T=2\pi k$, $k\in\boldsymbol{Z}$. $T>0$ bolg'anli'qtan T to'mendegi 2π, 4π, 6π, ... ma'nislerin qabi'l yete aladi' ha'm sonli'qtan da T ni'n' ma'nisi 2π den kishi boli'wi' mu'mkin yemes. ●

$y=\sin x$ funkciyasi'ni'n' *yen' kishi won' periodi'* 2π ge ten' yekenin da'lillew mu'mkin.

Shi'ni'g'i'wlar

Yesaplan' **(352–355):**

352. 1) $\sin\frac{13}{2}\pi$; 2) $\sin 17\pi$; 3) $\cos 7\pi$; 4) $\cos\frac{11}{2}\pi$;
5) $\sin 720^\circ$; 6) $\cos 540^\circ$; 7) $\sin 12{,}5\pi$; 8) $\cos 2025^\circ$.

353. 1) $\cos 420^\circ$; 2) $\mathrm{tg}\,570^\circ$; 3) $\sin 3630^\circ$; 4) $\mathrm{ctg}\,960^\circ$;
5) $\sin\frac{13\pi}{6}$; 6) $\mathrm{tg}\,\frac{11}{6}\pi$; 7) $\mathrm{tg}\,585^\circ$; 8) $\mathrm{ctg}\,\frac{13\pi}{4}$.

354. 1) $\cos 150^\circ$; 2) $\sin 135^\circ$; 3) $\cos 120^\circ$; 4) $\sin 315^\circ$.

355. 1) $\mathrm{tg}\,\frac{5\pi}{4}$; 2) $\sin\frac{7\pi}{6}$; 3) $\cos\frac{5\pi}{3}$;
4) $\sin\left(-\frac{11\pi}{6}\right)$; 5) $\cos\left(-\frac{7\pi}{3}\right)$; 6) $\mathrm{tg}\left(-\frac{2\pi}{3}\right)$.

356. An'latpani'n' san ma'nisin tabi'n':
1) $\cos 630^\circ - \sin 1470^\circ - \mathrm{ctg}\,1125^\circ$;
2) $\mathrm{tg}\,1800^\circ - \sin 495^\circ + \cos 945^\circ$;
3) $\sin(-7\pi) - 2\cos\frac{13\pi}{3} - \mathrm{tg}\,\frac{7\pi}{4}$;
4) $\cos(-9\pi) + 2\sin\left(-\frac{49\pi}{6}\right) - \mathrm{ctg}\left(-\frac{21\pi}{4}\right)$.

357. An'latpani' a'piwayi'lasti'ri'n':
1) $\cos^2(\pi-\alpha) + \sin^2(\alpha-\pi)$;
2) $\cos(\pi-\alpha)\cos(3\pi-\alpha) - \sin(\alpha-\pi)\sin(\alpha-3\pi)$.

358. Yesaplan':
1) $\cos 7230^\circ + \sin 900^\circ$; 2) $\sin 300^\circ + \mathrm{tg}\,150^\circ$;
3) $2\sin 6{,}5\pi - \sqrt{3}\sin\frac{19\pi}{3}$; 4) $\sqrt{2}\cos 4{,}25\pi - \frac{1}{\sqrt{3}}\cos\frac{61\pi}{6}$;
5) $\frac{\sin(-6{,}5\pi)+\mathrm{tg}(-7\pi)}{\cos(-7\pi)+\mathrm{ctg}(-16{,}25\pi)}$; 6) $\frac{\cos(-540^\circ)+\sin 480^\circ}{\mathrm{tg}\,405^\circ-\mathrm{ctg}\,330^\circ}$.

359. An'latpani' a'piwayi'lasti'ri'n':

1) $\dfrac{\sin\left(\frac{\pi}{2}-\alpha\right)+\sin(\pi-\alpha)}{\cos(\pi-\alpha)+\sin(2\pi-\alpha)}$; 2) $\dfrac{\cos(\pi-\alpha)+\cos\left(\frac{\pi}{2}-\alpha\right)}{\sin(\pi-\alpha)-\sin\left(\frac{\pi}{2}-\alpha\right)}$;

3) $\dfrac{\sin(\alpha-\pi)}{\operatorname{tg}(\alpha+\pi)}\cdot\dfrac{\operatorname{tg}(\pi-\alpha)}{\cos\left(\frac{\pi}{2}-\alpha\right)}$; 4) $\dfrac{\sin^2(\pi-\alpha)+\sin^2\left(\frac{\pi}{2}-\alpha\right)}{\sin(\pi-\alpha)}\cdot\operatorname{tg}(\pi-\alpha)$.

360. U'shmu'yeshliktin' yeki ishki mu'yeshinin' qosi'ndi'si'ni'n' sinusi' u'shinshi mu'yeshinin' sinusi'na ten'ligin da'lillen'.

361. Birdeylikti da'lillen'.

1) $\sin\left(\frac{\pi}{2}+\alpha\right)=\cos\alpha$; 2) $\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha$;

3) $\cos\left(\frac{3}{2}\pi-\alpha\right)=-\sin\alpha$; 4) $\sin\left(\frac{3}{2}\pi-\alpha\right)=-\cos\alpha$.

362. Ten'lemeni sheshin':

1) $\cos\left(\frac{\pi}{2}-x\right)=1$; 2) $\sin(\pi-x)=1$; 3) $\cos(x-\pi)=0$;

4) $\sin\left(x-\frac{\pi}{2}\right)=1$; 5) $\cos(\pi-2x)=1$; 6) $\sin\left(\frac{\pi}{2}+x\right)=0$.

29- §. SINUSLARDI'N' QOSI'NDI'SI' HA'M AYI'RMASI'. KOSINUSLARDI'N' QOSI'NDI'SI' HA'M AYI'RMASI'

1-ma'sele. An'latpani' a'piwayi'lasti'ri'n':

$$\left(\sin\left(\alpha+\frac{\pi}{12}\right)+\sin\left(\alpha-\frac{\pi}{12}\right)\right)\sin\frac{\pi}{12}.$$

△ Qosi'w formulasi' ha'm qos mu'yeshtin' sinusi' formulasi'nan paydalani'p, to'mendegige iye bolami'z:

$$\left(\sin\left(\alpha+\frac{\pi}{12}\right)+\sin\left(\alpha-\frac{\pi}{12}\right)\right)\sin\frac{\pi}{12}=$$

$$=\left(\sin\alpha\cos\frac{\pi}{12}+\cos\alpha\sin\frac{\pi}{12}+\sin\alpha\cos\frac{\pi}{12}-\cos\alpha\sin\frac{\pi}{12}\right)\sin\frac{\pi}{12}=$$

$$=2\sin\alpha\cos\frac{\pi}{12}\cdot\sin\frac{\pi}{12}=\sin\alpha\sin\frac{\pi}{6}=\frac{1}{2}\sin\alpha.\ \blacktriangle$$

Yeger *sinuslardi'n' qosi'ndi'si'ni'n' formulasi'*

$$\boxed{\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}} \qquad (1)$$

den paydalani'lsa, bul ma'seleni a'piwayi'raq sheshiw mu'mkin. Usi' formula ja'rdeminde to'mendegini payda yetemiz:

$$\left(\sin\left(\alpha+\frac{\pi}{12}\right)+\sin\left(\alpha-\frac{\pi}{12}\right)\right)\sin\frac{\pi}{12}=$$

$$=2\sin\alpha\cos\frac{\pi}{12}\cdot\sin\frac{\pi}{12}=\frac{1}{2}\sin\alpha.$$

Yendi (1) formulani'n' wori'nli' yekenin da'lilleymiz.

○ $\frac{\alpha+\beta}{2}=x$, $\frac{\alpha-\beta}{2}=y$ belgilewlerin kiritemiz. Sonda $x+y=\alpha$, $x-y=\beta$ boladi' ha'm sonli'qtan $\sin\alpha+\sin\beta=\sin(x+y)+\sin(x-y)=\sin x\cos y+$ $+\cos x\sin y+\sin x\cos y-\cos x\sin y=2\sin x\cos y=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$. ●

(1) formula menen bir qatarda to'mendegi *sinuslar ayi'rmasi'ni'n' formulasi', kosinuslardi'n' qosi'ndi'si' ha'm ayi'rmasi'ni'n' formulalari'nan* da paydalani'ladi':

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha-\beta}{2}, \qquad (2)$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}, \qquad (3)$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}. \qquad (4)$$

(3) ha'm (4) formulalar da (1) formulani'n' da'lilleniwine uqsas da'lillenedi; (2) formula β ni' $-\beta$ menen almasti'ri'w arqali' (1) formuladan keltirip shi'g'ari'ladi' (*buni' wo'zin'izshe da'lillen'*).

2-ma'sele. $\sin 75^\circ+\cos 75^\circ$ ti' yesaplan'.

$$\triangle\ \sin 75^\circ+\cos 75^\circ=\sin 75^\circ+\sin 15^\circ=$$

$$=2\sin\frac{75^\circ+15^\circ}{2}\cos\frac{75^\circ-15^\circ}{2}=2\sin 45^\circ\cos 30^\circ=2\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}=\frac{\sqrt{6}}{2}.\ \blacktriangle$$

3-ma'sele. $2\sin\alpha+\sqrt{3}$ ti ko'beymege tu'rlendirin'.

$$\triangle\ 2\sin\alpha+\sqrt{3}=2\left(\sin\alpha+\frac{\sqrt{3}}{2}\right)=2\left(\sin\alpha+\sin\frac{\pi}{3}\right)=$$

$$=4\sin\left(\frac{\alpha}{2}+\frac{\pi}{6}\right)\cos\left(\frac{\alpha}{2}-\frac{\pi}{6}\right).\ \blacktriangle$$

4-ma'sele. $\sin\alpha+\cos\alpha$ an'latpasi'ni'n' yen' kishi ma'nisi $-\sqrt{2}$ ge, al yen' u'lken ma'nisi $\sqrt{2}$ ge ten' yekenin da'lillen'.

$\triangle$ Berilgen an'latpani' ko'beymege tu'rlendiremiz:

$$\sin\alpha+\cos\alpha=\sin\alpha+\sin\left(\frac{\pi}{2}-\alpha\right)=2\sin\frac{\pi}{4}\cos\left(\alpha-\frac{\pi}{4}\right)=\sqrt{2}\cos\left(\alpha-\frac{\pi}{4}\right).$$

Kosinusti'n' yen' kishi ma'nisi –1 ge, al yen' u'lken ma'nisi 1 ge ten' bolg'anli'qtan berilgen an'latpani'n' yen' kishi ma'nisi $\sqrt{2}\cdot(-1)=-\sqrt{2}$ ge, al yen' u'lken ma'nisi $\sqrt{2}\cdot 1=\sqrt{2}$ ge ten'. $\blacktriangle$

Shi'ni'g'i'wlar

363. An'latpani' a'piwayi'lasti'ri'n':

1) $\sin(\frac{\pi}{3}+\alpha)+\sin(\frac{\pi}{3}-\alpha)$; 2) $\cos(\frac{\pi}{4}-\beta)-\cos(\frac{\pi}{4}+\beta)$;

3) $\sin^2(\frac{\pi}{4}+\alpha)-\sin^2(\frac{\pi}{4}-\alpha)$; 4) $\cos^2(\alpha-\frac{\pi}{4})-\cos^2(\alpha+\frac{\pi}{4})$.

364. Yesaplan':

1) $\cos 105^\circ+\cos 75^\circ$; 2) $\sin 105^\circ-\sin 75^\circ$;

3) $\cos\frac{11\pi}{12}+\cos\frac{5\pi}{12}$; 4) $\cos\frac{11\pi}{12}-\cos\frac{5\pi}{12}$;

5) $\sin\frac{7\pi}{12}-\cos\frac{\pi}{12}$; 6) $\sin 105^\circ+\sin 165^\circ$.

365. Ko'beymege tu'rlendirin':

1) $1+2\sin\alpha$; 2) $1-2\sin\alpha$; 3) $1+2\cos\alpha$;

4) $1+\sin\alpha$; 5) $1-\cos\alpha$; 6) $1+\cos\alpha$;

366. Birdeylikti da'lillen':

1) $\frac{\sin\alpha+\sin 3\alpha}{\cos\alpha+\cos 3\alpha}=\operatorname{tg}2\alpha$; 2) $\frac{\sin 2\alpha+\sin 4\alpha}{\cos 2\alpha-\cos 4\alpha}=\operatorname{ctg}\alpha$.

367. An'latpani' a'piwayi'lasti'ri'n':

1) $\frac{2(\cos\alpha+\cos3\alpha)}{2\sin2\alpha+\sin4\alpha}$; 2) $\frac{1+\sin\alpha-\cos2\alpha-\sin3\alpha}{2\sin^2\alpha+\sin\alpha-1}$.

Birdeylikti da'lillen' **(368–369)**:

368. 1) $\cos^4\alpha-\sin^4\alpha+\sin2\alpha=\sqrt{2}\cos\left(2\alpha-\frac{\pi}{4}\right)$;

2) $\cos\alpha+\cos\left(\frac{2\pi}{3}+\alpha\right)+\cos\left(\frac{2\pi}{3}-\alpha\right)=0$.

369. 1) $\frac{\sin2\alpha+\sin5\alpha-\sin3\alpha}{\cos\alpha+1-2\sin^2 2\alpha}=2\sin\alpha$;

2) $\frac{\sin\alpha+\sin3\alpha+\sin5\alpha+\sin7\alpha}{\cos\alpha-\cos3\alpha+\cos5\alpha-\cos7\alpha}=\mathrm{ctg}\,\alpha$.

370. Ko'beyme tu'rinde jazi'n':

1) $\cos22^\circ+\cos24^\circ+\cos26^\circ+\cos28^\circ$; | 2) $\cos\frac{\pi}{12}+\cos\frac{\pi}{4}+\cos\frac{5\pi}{6}$.

371. $\mathrm{tg}\,\alpha+\mathrm{tg}\,\beta=\frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$ birdeyligin da'lillen' ha'm yesaplan':

1) $\mathrm{tg}\,267^\circ+\mathrm{tg}\,93^\circ$; | 2) $\mathrm{tg}\,\frac{5\pi}{12}+\mathrm{tg}\,\frac{5\pi}{12}$; | 3) $\mathrm{tg}\,99^\circ+\mathrm{tg}\,81^\circ$.

372. Ko'beytiwshilerge jiklen':

1) $1-\cos\alpha+\sin\alpha$; 2) $1-2\cos\alpha+\cos2\alpha$;
3) $1+\sin\alpha-\cos\alpha-\mathrm{tg}\,\alpha$; 4) $1+\sin\alpha+\cos\alpha+\mathrm{tg}\,\alpha$.

V bapqa tiyisli shi'ni'g'i'wlar

373. $0<\alpha<\frac{\pi}{2}$ bolsi'n. $P(1;\ 0)$ noqati'n:

1) $\frac{\pi}{2}-\alpha$; | 2) $\alpha-\pi$; | 3) $\frac{3\pi}{2}-\alpha$; | 4) $\frac{\pi}{2}+\alpha$; | 5) $\alpha-\frac{\pi}{2}$; | 6) $\pi-\alpha$

mu'yeshke buri'w na'tiyjesinde payda bolg'an noqat qaysi' sherekte jatatug'i'ni'n ani'qlan'.

374. Mu'yeshtin' sinusi' ha'm kosinusi'ni'n' ma'nisin tabi'n':

1) 3π; 2) 4π; 3) $3{,}5\pi$; 4) $\frac{5}{2}\pi$; 5) $\pi k,\ k\in\mathbb{Z}$;
6) $(2k+1)\pi,\ k\in\mathbb{Z}$; 7) $2k\pi,\ k\in\mathbb{Z}$; 8) $6{,}5\pi$.

375. Yesaplan':

1) $\sin 3\pi - \cos\frac{3\pi}{2}$; 2) $\cos 0 - \cos 3\pi + \cos 3,5\pi$;

3) $\sin \pi k + \cos 2\pi k$, bunda k – pu'tin san;

4) $\cos\frac{(2k+1)\pi}{2} - \sin\frac{(4k+1)\pi}{2}$, bunda k – pu'tin san.

376. Tabi'n':

1) yeger $\sin\alpha = \frac{\sqrt{3}}{3}$ ha'm $\frac{\pi}{2} < \alpha < \pi$ bolsa, $\cos\alpha$ ni';

2) yeger $\cos\alpha = -\frac{\sqrt{5}}{3}$ ha'm $\pi < \alpha < \frac{3\pi}{2}$ bolsa, $\mathrm{tg}\alpha$ ni';

3) yeger $\mathrm{tg}\alpha = 2\sqrt{2}$ ha'm $0 < \alpha < \frac{\pi}{2}$ bolsa, $\sin\alpha$ ni';

4) yeger $\mathrm{ctg}\alpha = \sqrt{2}$ ha'm $\pi < \alpha < \frac{3\pi}{2}$ bolsa, $\sin\alpha$ ni'.

377. Birdeylikti da'lillen':

1) $5\sin^2 a + \mathrm{tg}a\cos a + 5\cos^2 a = 5 + \sin a$;

2) $\mathrm{ctg}a\sin a - 2\cos^2 a - 2\sin^2 a = \cos a - 2$;

3) $\frac{3}{1+\mathrm{tg}^2\alpha} = 3\cos^2\alpha$; 4) $\frac{5}{1+\mathrm{ctg}^2\alpha} = 5\sin^2\alpha$.

378. An'latpani' a'piwayi'lasti'ri'n':

1) $2\sin(-\alpha)\cos\left(\frac{\pi}{2} - \alpha\right) - 2\cos(-\alpha)\sin\left(\frac{\pi}{2} - \alpha\right)$;

2) $3\sin(\pi - \alpha)\cos\left(\frac{\pi}{2} - \alpha\right) + 3\sin^2\left(\frac{\pi}{2} - \alpha\right)$;

3) $(1 - \mathrm{tg}(-\alpha))(1 - \mathrm{tg}(\pi + \alpha)\cos^2\alpha$;

4) $(1 + \mathrm{tg}^2(-\alpha))\left(\frac{1}{1+\mathrm{ctg}^2(-\alpha)}\right)$.

379. An'latpani' a'piwayi'lasti'ri'n' ha'm woni'n' san ma'nisin tabi'n':

1) $\sin\left(\frac{3}{2}\pi - \alpha\right) + \sin\left(\frac{3}{2}\pi + \alpha\right)$, bunda $\cos\alpha = \frac{1}{4}$;

2) $\cos\left(\frac{\pi}{2} + \alpha\right) + \cos\left(\frac{3}{2}\pi - \alpha\right)$, bunda $\sin\alpha = \frac{1}{6}$.

380. Yesaplan':

1) $2\sin 75^\circ \cos 75^\circ$; 2) $\sin 15^\circ$; 3) $\cos^2 75^\circ - \sin^2 75^\circ$;

4) $\sin 75^\circ$; 5) $\cos 75^\circ$; 6) $\sin 135^\circ$.

WO'ZIN'IZDI TEKSERIP KO'RIN'!

1. Yeger: 1) $\sin\alpha = \frac{4}{5}$ ha'm $\frac{\pi}{2} < \alpha < \pi$ bolsa, $\cos\alpha$, $tg\alpha$, $\sin 2\alpha$ ni',

2) $\cos\alpha = -0,6$ ha'm $\pi < \alpha < \frac{3\pi}{2}$ bolsa, $\sin\alpha$, $ctg\alpha$, $\cos 2\alpha$ ni' yesaplan'.

2. An'latpani'n' ma'nisin tabi'n':

1) $4\cos\left(-\frac{\pi}{3}\right) - tg\frac{\pi}{4} + 2\sin\left(-\frac{\pi}{6}\right) - \cos\pi$;

2) $\cos 150^\circ$; 3) $\sin\frac{8\pi}{3}$; 4) $tg\frac{5\pi}{3}$; 5) $\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}$.

3. (*G'iyosiddin Jamshid al-Koshiy ma'selesi.*)

$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$ yekenin da'lillen'.

4. Birdeylikti da'lillen'.

1) $3 - \cos^2\alpha - \sin^2\alpha = 2$; 2) $1 - \sin\alpha\cos\alpha\, ctg\alpha = \sin^2\alpha$.

5. An'latpani' a'piwayi'lasti'ri'n':

1) $\sin(\alpha - \beta) - \sin\left(\frac{\pi}{2} - \alpha\right)\sin(-\beta)$; 2) $\sin^2\alpha + \cos 2\alpha$;

3) $tg(\pi - \alpha)\cos(\pi - \alpha) + \sin(4\pi + \alpha)$.

381. An'latpani' a'piwayi'lasti'ri'n':

1) $\cos^2(\pi - \alpha) - \cos^2\left(\frac{\pi}{2} - \alpha\right)$; 2) $2\sin\left(\frac{\pi}{2} - \alpha\right)\cos\left(\frac{\pi}{2} - \alpha\right)$;

3) $\dfrac{\cos^2(2\pi+\alpha) - \sin^2(\alpha+2\pi)}{2\cos(\alpha+2\pi)\cos\left(\frac{\pi}{2}-\alpha\right)}$; 4) $\dfrac{2\sin(\pi-\alpha)\sin\left(\frac{\pi}{2}-\alpha\right)}{\sin^2\left(\alpha-\frac{\pi}{2}\right) - \sin^2(\alpha-\pi)}$.

Yesaplan' **(382–383):**

382. 1) $\sin\frac{47\pi}{6}$; 2) $tg\frac{25\pi}{4}$; 3) $ctg\frac{27\pi}{4}$; 4) $\cos\frac{21\pi}{4}$.

383. 1) $\cos\frac{23\pi}{4} - \sin\frac{15\pi}{4}$; 2) $\sin\frac{25\pi}{3} - tg\frac{10\pi}{3}$;

3) $3\cos 3660^\circ + \sin(-1560^\circ)$; 4) $\cos(-945^\circ) + tg 1035^\circ$.

384. Sanlardi’ sali’sti’ri’n’:

1) sin3 ha’m cos4; 2) cos0 ha’m sin5; 3) sin1 ha’m cos1.

385. Sanni’n’ belgisin ani’qlan’:

1) sin3,5tg3,5; 2) cos5,01sin0,73; 3) $\frac{\text{tg}13}{\cos 15}$;

4) sin1cos2tg3; 5) sin2cos2; 6) tg1cos1.

386. Yesaplan’:

1) $\sin\frac{\pi}{8}\cos\frac{3\pi}{8}+\sin\frac{3\pi}{8}\cos\frac{\pi}{8}$; 2) sin165°; 3) sin105°;

4) $\sin\frac{\pi}{12}$; 5) $1-2\sin^2 195°$; 6) $2\cos^2\frac{3\pi}{8}-1$.

387. An’latpani’ a’piwayi’lasti’ri’n’:

1) $(1+\text{tg}(-\alpha))(1-\text{ctg}(-\alpha))-\frac{\sin(-\alpha)}{\cos(-\alpha)}$; 2) $\frac{\text{ctg}\alpha+\text{tg}(-\alpha)}{\cos\alpha+\sin(-\alpha)}+\frac{\text{tg}(-\alpha)}{\sin\alpha}$.

388. Berilgeni: $\sin\alpha=\frac{\sqrt{5}}{3}$ ha’m $\frac{\pi}{2}<\alpha<\pi$. cosα, tgα, ctgα, sin2α, cos2α lardi’n’ ma’nislerin yesaplan’.

An’latpani’ a’piwayi’lasti’ri’n’ **(389–391):**

389. 1) $\cos^3\alpha\sin\alpha-\sin^3\alpha\cos\alpha$; 2) $\frac{\sin\alpha+\sin2\alpha}{1+\cos\alpha+\cos2\alpha}$.

390. 1) $\frac{\sin2\alpha-\sin2\alpha\cos2\alpha}{4\cos\alpha}$; 2) $\frac{2\cos^2 2\alpha}{\sin4\alpha\cos4\alpha+\sin4\alpha}$;

3) $\frac{\cos2\alpha+\sin2\alpha\cos2\alpha}{2\sin^2\alpha-1}$; 4) $\frac{(\cos\alpha-\sin\alpha)^2}{\sin2\alpha\cos2\alpha-\cos2\alpha}$.

391. 1) $\frac{\cos^2 x}{1-\sin x}-\sin(\pi-x)$; 2) $\frac{\cos^2 x}{1+\sin x}+\cos(1,5\pi+x)$;

3) $\frac{\sin^2 x}{1+\cos x}-\sin(1,5\pi+x)$; 4) $\frac{\sin^2 x}{1-\cos x}+\cos(3\pi-x)$.

392. 1) Yeger $\text{tg}\alpha=-\frac{3}{4}$ ha’m $\text{tg}\beta=2{,}4$ bolsa, $\text{tg}(\alpha+\beta)$ ni’;

2) yeger $\text{ctg}\alpha=\frac{4}{3}$ ha’m $\text{ctg}\beta=-1$ bolsa, $\text{ctg}(\alpha+\beta)$ ni’ yesaplan’.

393. An’latpani’ a’piwayi’lasti’ri’n’:

1) $2\sin\left(\frac{\pi}{4}+2\alpha\right)\sin\left(\frac{\pi}{4}-2\alpha\right)$; 2) $2\cos\left(\frac{\pi}{4}+2\alpha\right)\cos\left(\frac{\pi}{4}-2\alpha\right)$.

V bapqa tiyisli si'naq (test) shi'ni'g'i'wlari'

1. 153° ti'n' radian wo'lshemin tabi'n'.

A) $\frac{17\pi}{20}$; B) $\frac{19\pi}{20}$; C) 17π; D) $\frac{2\pi}{9}$.

2. $0{,}65\pi$ din' gradus wo'lshemin tabi'n'.

A) $11{,}7^\circ$; B) 117°; C) 116°; D) 118°.

3. Ko'beymenin' qaysi' biri teris?

A) $\cos 314^\circ \sin 147^\circ$; B) $\operatorname{tg} 200^\circ \operatorname{ctg} 201^\circ$;
C) $\cos 163^\circ \cos 295^\circ$; D) $\sin 170^\circ \operatorname{ctg} 250^\circ$.

4. Ko'beymenin' qaysi' biri won'?

A) $\sin 2 \cos 2 \sin 1 \sin 1^\circ$; B) $\operatorname{tg} 8^\circ \operatorname{ctg} 8 \operatorname{ctg} 10^\circ \operatorname{ctg} \sqrt{10}$;
C) $\sin 9^\circ \sin 9 \cos 9^\circ \cos 9$; D) $\cos 10^\circ \cos 10 \cos 11^\circ \cos \sqrt{11}$.

5. $\left(\frac{\sqrt{3}}{2}; \frac{1}{2}\right)$ noqati'na tu'siw ushi'n (1; 0) noqati'n buri'w kerek bolg'an barli'q mu'yeshlerdi tabi'n':

A) $\frac{\pi}{6} + 2\pi k$, $k \in \boldsymbol{Z}$; B) $-\frac{\pi}{6} + \pi k$, $k \in \boldsymbol{Z}$;

C) $\frac{\pi}{6} + \pi k$, $k \in \boldsymbol{Z}$; D) $2\pi + \pi k$, $k \in \boldsymbol{Z}$.

6. (1; 0) noqati'n $\frac{5\pi}{2} + 2\pi k$, $k \in \boldsymbol{Z}$ mu'yeshke buri'wdan payda bolatug'i'n noqatti'n' koordinatalari'n tabi'n':

A) (0; 1); B) (0; −1); C) (1; 0); D) (−1; 0).

7. Sanlardi' wo'siw ta'rtibinde jazi'n':

$$a = \sin 1{,}57; \quad b = \cos 1{,}58; \quad c = \sin 3.$$

A) $a < c < b$; B) $b < c < a$; C) $c < a < b$; D) $b < a < c$.

8. Sanlardi' kemeyiw ta'rtibinde jazi'n':

$$a = \cos 2; \quad b = \cos 2^\circ; \quad c = \sin 2; \quad d = \sin 2^\circ.$$

A) $a > c > d > b$; B) $d > c > b > a$;
C) $b > c > d > a$; D) $c > d > b > a$.

9. Yesaplan’: $\dfrac{\sin136^\circ\cdot\cos46^\circ-\sin46^\circ\cdot\cos224^\circ}{\sin110^\circ\cdot\cos40^\circ-\sin20^\circ\cdot\cos50^\circ}$.

A) $\cos40^\circ$; B) 0,5; C) $\sin44^\circ$; D) 2.

10. Yesaplan’: $\dfrac{\sin10^\circ\cdot\sin130^\circ-\sin100^\circ\cdot\sin220^\circ}{\sin27^\circ\cdot\cos23^\circ-\sin157^\circ\cdot\cos153^\circ}$.

A) 1; B) -1; C) $\dfrac{\sqrt{3}}{2}$; D) $-\dfrac{\sqrt{3}}{2}$.

11. Yesaplan’: $\cos(-225^\circ)+\sin675^\circ+\mathrm{tg}(-1035^\circ)$.

A) 1; B) -1; C) $\sqrt{2}$; D) $-\dfrac{\sqrt{2}}{2}$.

12. $\sin\alpha = 0{,}6$ bolsa, $\mathrm{tg}2\alpha$ ni’ tabi’n’ $\left(0<\alpha<\dfrac{\pi}{2}\right)$.

A) 3,42; B) $3\dfrac{3}{7}$; C) $\dfrac{7}{24}$; D) $-\dfrac{7}{24}$.

13. $\mathrm{tg}\alpha = \sqrt{5}$ bolsa, $\sin2\alpha$ ni’ tabi’n’.

A) $\dfrac{3\sqrt{5}}{5}$; B) $-\dfrac{\sqrt{5}}{3}$; C) $\dfrac{\sqrt{5}}{3}$; D) $\sqrt{5}$.

14. $\mathrm{tg}\alpha = \sqrt{7}$ bolsa, $\cos2\alpha$ ni’ tabi’n’ .

A) $\dfrac{4}{3}$; B) $-\dfrac{4}{3}$; C) $\dfrac{3}{4}$; D) $-\dfrac{3}{4}$.

15. A’piwayi’lasti’ri’n’: $\dfrac{\cos\left(\dfrac{\pi}{2}-\alpha\right)}{\sin(\pi+\alpha)}$.

A) -1; B) 1; C) 0,5; D) $-\dfrac{1}{2}$.

16. A’piwayi’lasti’ri’n’: $\dfrac{\sin2\alpha+\sin(\pi-\alpha)\cdot\cos\alpha}{\sin\left(\dfrac{\pi}{2}-\alpha\right)}$.

A) $3\sin\alpha$; B) $\dfrac{1}{3}\sin\alpha$; C) $-\sin\alpha$; D) $\dfrac{1}{3}\cos\alpha$.

17. $\mathrm{tg}\alpha = \sqrt{7}$ bolsa, $\dfrac{4\sin^4\alpha}{5\sin^2\alpha + 15\cos^2\alpha}$ ni' yesaplan'.

A) 0,59; B) 0,49; C) –0,49; D) 0,2.

18. $\cos\alpha + \sin\alpha = \dfrac{1}{3}$ bolsa, $\sin^4\alpha + \cos^4\alpha$ ni' tabi'n'.

A) $\dfrac{81}{49}$; B) $-\left(\dfrac{7}{9}\right)^2$; C) $\dfrac{49}{81}$; D) $-1\dfrac{32}{49}$.

19. Yesaplan': $\sin 100^\circ \cdot \cos 440^\circ + \sin 800^\circ \cdot \cos 460^\circ$.

A) $\dfrac{\sqrt{3}}{2}$; B) 1; C) –1; D) 0.

20. A'piwayi'lasti'ri'n': $\dfrac{\sin 3\alpha}{\sin\alpha} + \dfrac{\cos 3\alpha}{\cos\alpha}$.

A) $4\cos 2\alpha$; B) $-2\sin 4\alpha$; C) $\sin 4\alpha$; D) $2\cos 2\alpha$.

21. $8x^2 - 6x + 1 = 0$ ten'lemesinin' korenleri $\sin\alpha$ ha'm $\sin\beta$ boli'p, α, β lar I sherekte bolsa, $\sin(\alpha+\beta)$ ni' tabi'n':

A) $\dfrac{\sqrt{3}(1+\sqrt{5})}{8}$; B) $\dfrac{\sqrt{2}(1+\sqrt{5})}{8}$;

C) $\dfrac{\sqrt{3}(4-\sqrt{5})}{16}$; D) $-\dfrac{\sqrt{3}(4+\sqrt{5})}{16}$.

22. $6x^2 - 5x + 1 = 0$ ten'lemesinin' korenleri $\cos\alpha$ ha'm $\cos\beta$ boli'p, α, β lar I sherekte bolsa, $\cos(\alpha+\beta)$ ni' tabi'n'.

A) $\dfrac{2\sqrt{6}-1}{6}$; B) $\dfrac{1-2\sqrt{6}}{6}$; C) $\dfrac{2\sqrt{6}-1}{7}$; D) $\dfrac{1-2\sqrt{6}}{5}$.

23. x ti' tabi'n': $2(x+\sqrt{2}) = \cos\left(\dfrac{\pi}{2} - 2\alpha\right) + 2\sin\left(\dfrac{3\pi}{2} + \alpha\right) \cdot \sin(\pi - \alpha)$.

A) $\dfrac{\sqrt{2}}{2}$; B) $\sqrt{2}$; C) $-\sqrt{2}$; D) $2\sqrt{2}$.

24. $x^2 - 7x + 12 = 0$ ten'lemesinin' korenleri $\mathrm{tg}\alpha$ ha'm $\mathrm{tg}\beta$ bolsa, $\mathrm{tg}(\alpha+\beta)$ ni' tabi'n'.

A) 1; B) $\dfrac{7}{11}$; C) $\sqrt{3}$; D) $-\dfrac{7}{11}$.

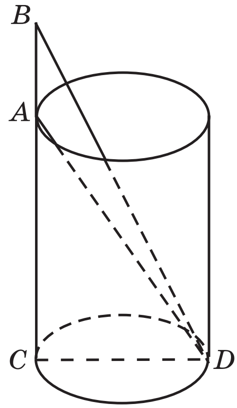
Abu Rayxan Beruniy ma'seleleri

1. Qudi'q cilindr formasi'nda boli'p, woni'n' tu'bi qudi'qti'n' yernegindegi A noqati'nan α mu'yeshi asti'nda, qudi'qti'n' diywali' dawami'ndag'i' B noqati'nan β mu'yeshi asti'nda ko'rinedi (72-su'wret). Yeger $AB = a$ bolsa, qudi'qti'n' teren'ligin tabi'n'.

Berilgeni:

$\angle CAD = \alpha$, $\angle ABD = \beta$, $AB = a$.

Tabi'w kerek: $AC = ?$



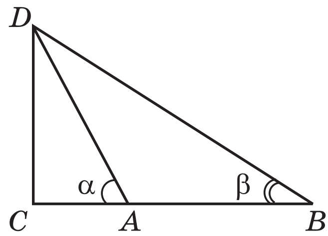
72- su'wret.

2. Minara jerdegi A noqati'nan α mu'yeshi asti'nda, al B noqati'nan β mu'yeshi asti'nda ko'rinedi (73-su'wret). $AB = a$ bolsa, minarani'n' biyikligin tabi'n'.

Berilgeni:

$\angle CAD = \alpha$, $\angle ABD = \beta$, $AB = a$.

Tabi'w kerek: $CD = ?$



73- su'wret.

G'iyosiddin Jamshid al-Koshiy ma'selesi:

3. Qa'legen α mu'yeshi ushi'n

$$\sin\left(45^\circ + \frac{\alpha}{2}\right) = \sqrt{\frac{1+\sin\alpha}{2}}$$

bolatug'i'nli'g'i'n da'lillen'.

Ataqli' matematik Abulvafa Muhammad al-Buzjaniydin' (940—998) ma'selesi:

4. Qa'legen α ha'm β ushi'n,

$$\sin(\alpha - \beta) = \sqrt{\sin^2\alpha - \sin^2\alpha \cdot \sin^2\beta} - \sqrt{\sin^2\beta - \sin^2\alpha \cdot \sin^2\beta}$$

bolatug'i'nli'g'i'n da'lillen'.

Tariyxi'y mag'luwmatlar

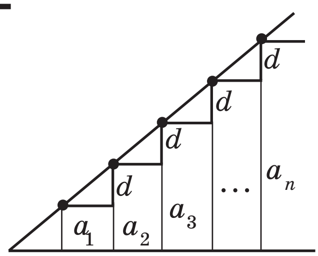
Mi'rza Ulug'bek (1394–1449)

Matematikani'n', atap aytqanda trigonometriyani'n' rawajlani'wi'na ulli' ilimpazlar Muhammed al-Xorezmiy, Ahmad Farg'oniy, Abu Rayxan Beruniy, Mi'rza Ulug'bek, Ali Qusshi', G'iyosiddin Jamshid al-Koshiy u'lken u'les qosqan. Juldi'zlardi'n' aspan sferasi'ndag'i' koordinatalari'n ani'qlaw, planetalardi'n' ha'reketin baqlaw, Ay ha'm Quyashti'n' tuti'li'wi'n aldi'nala ayti'p beriw ha'm basqa ilimiy, a'meliy a'hmiyetke iye ma'seleler ani'q yesaplawlardi', bul yesaplawlarg'a tiykarlang'an kesteler du'ziwdi talap yetiletug'i'n yedi. Mine usi'nday astronomiyali'q (trigonometriyali'q) kesteler Shi'g'i'sta «Zij» ler dep atalg'an.

Muhammed al-Xorezmiy, Abu Rayxan Beruniy, Mi'rza Ulug'bek si'yaqli' ilimpazlardi'n' matematikali'q miynetleri menen birge «Zij» leri de du'nyag'a tani'lg'an, wolar lati'n ha'm basqa tillerde awdarmasi' islengen, Evropada matematikani'n', astronomiyani'n' rawajlani'wi'na salmaqli' ta'sir wo'tkizgen. Beruniydin' «Qonuni Ma'sudiy» miynetinde sinuslar kestesi 15 minut arali'q penen, tangensler kestesi 1° arali'q penen 10^{-8} ge shekemgi da'llikte berilgen. Ju'da' da'l «Zij» lerden biri Mi'rza Ulug'bektin' «Zij»i-«Ziji Ko'ragoniy» den ibarat. Bunda sinuslar kestesi 1 minut arali'q penen, tangensler kestesi 0° tan 45° qa shekem 1 minutli'q arali'q penen, al 46° tan 90° qa shekem 5 minutli'q arali'q penen 10^{-10} g'a shekemgi da'llikte berilgen. G'iyosiddin Jamshid al-Koshiy «Vatar va sinus haqida risola» si'nda $\sin 1^\circ$ ti' u'tirden son' 17 tan'ba da'lliginde yesaplaydi': $\sin 1^\circ = 0{,}01745240643\ 7283512\ldots$

Shen'berdin' uzi'nli'g'i' wog'an ishley ha'm si'rtlay si'zi'lg'an duri's $3 \cdot 2^n$ – ko'pmu'yeshliklerdin' perimetrlerinin' worta arifmetikali'q shamasi'na ten' dep, $n = 28$ bolg'anda Jamshid al-Koshiy «Aylana haqida risola» miynetinde 2π ushi'n, $2\pi = 6{,}28318\ 53071795865\ldots$ na'tiyjeni aldi':

VI BAP. PROGRESSIYALAR



30- §. ARIFMETIKALI’Q PROGRESSIYA

To’mendegi ma’seleni ko’remiz.

M a ’ s e l e . Woqi’wshi’ si’naqtan wo’tiwi ushi’n tayarli’q ko’rip ha’r ku’ni 5 si’naq ma’selelerin sheshiwdi jobalasti’rdi’. Ha’r ku’ni sheshiliwi jobalasti’ri’lg’an si’naq ma’selelerinin’ sani’ qalay wo’zgerip baradi’?

Jobalasti’ri’lg’an ma’seleler sani’ ha’r ku’n sayi’n to’mendegishe wo’zgerip baradi’:

1- ku’n	2- ku’n	3- ku’n	4- ku’n …
5	10	15	20 …

Na’tiyjede to’mendegi izbe-izlikti payda yetemiz:

$$5, 10, 15, 20, 25, \ldots .$$

a_n –arqali’ n-ku’nde sheshiliwi kerek bolg’an barli’q ma’seleler sani’n belgileyik. Mi’sali’:

$$a_1 = 5,\ a_2 = 10,\ a_3 = 15,\ \ldots .$$

Payda bolg’an

$$a_1, a_3, a_3, \ \ldots, a_n, \ldots$$

sanlari’ *sanli’ izbe-izlik* dep ataladi’.

Bul izbe-izlikte yekinshisinen baslap woni’n’ ha’rbir ag’zasi’ aldi’ng’i’ ag’zag’a birdey 5 sani’ni’n’ qosi’lg’ani’na ten’. Bunday izbe-izlik *arifmetikali’q progressiya* dep ataladi’.

A n i ' q l a m a . ***Yeger $a_1, a_2, \dots, a_n, \dots$ sanli' izbe-izliginde barli'q natural n ler ushi'n***

$$a_{n+1} = a_n + d$$

(bunda d — qanday da bir san) ten'ligi wori'nlansa, bunday izbe-izlik arifmetikali'q progressiya dep ataladi'.

Bul formuladan $a_{n+1} - a_n = d$ yekenligi kelip shi'g'adi'. d sani' *arifmetikali'q progressiyani'n' ayi'rmasi'* dep ataladi'.

M i ' s a l l a r .

1) Sanlardi'n' 1, 2, 3, 4 ..., n, ... natural qatari' arifmetikali'q progressiyani' du'zedi. Bul progressiyani'n' ayi'rmasi' $d = 1$.

2) Pu'tin teris sanlardi'n' $-1, -2, -3, \dots, -n, \dots$ izbe-izligi, ayi'rmasi' $d = -1$ bolg'an arifmetikali'q progressiya boladi'.

3) 3, 3, 3, ..., 3, ... izbe-izligi ayi'rmasi' $d = 0$ bolg'an arifmetikali'q progressiyadan ibarat.

1 - m a ' s e l e . $a_n = 1{,}5 + 3n$ formulasi' menen berilgen izbe-izlik arifmetikali'q progressiya bolatug'i'nli'g'i'n da'lillen'.

△ $a_{n+1} - a_n$ ayi'rmasi' barli'q n ushi'n birdey (n ge g'a'rezli yemes) yekenligin ko'rsetiw talap yetiledi.

Berilgen izbe-izliktin' $(n + 1)$-ag'zasi'n jazami'z:

$$a_{n+1} = 1{,}5 + 3(n + 1).$$

Sonli'qtan

$$a_{n+1} - a_n = 1{,}5 + 3(n + 1) - (1{,}5 + 3n) = 3.$$

Demek, $a_{n+1} - a_n$ ayi'rmasi' n ge g'a'rezli yemes. ▲

Arifmetikali'q progressiyani'n' ani'qlamasi'na muwapi'q $a_{n+1} = a_n + d$, $a_{n-1} = a_n - d$, bunnan

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}, \; n > 1.$$

Solay yetip, arifmetikali'q progressiyani'n' yekinshi ag'zasi'nan baslap ha'rbir ag'zasi' wog'an qon'si'las bolg'an yeki ag'zani'n' arifmetikali'q wortasi'na ten'. «Arifmetikali'q» progressiya degen atama da usi'ni'n' menen tu'sindiriledi.

Yeger a_1 ha'm d berilgen bolsa, wonda arifmetikali'q progressiyani'n' qalg'an ag'zalari'n $a_{n+1} = a_n + d$ formulasi' boyi'nsha yesaplaw mu’mkin yekenligin atap wo’temiz. Bunday usi’l menen progressiyani’n’ birneshe da’slepki ag’zasi’n yesaplaw qi’yi’nshi’li’q tuwdi’rmaydi’: biraq, ma’selen, a_{100} ushi’n birqansha yesaplawlar talap yetiledi. A’dette buni’n’ ushi’n n-ag’za formulasi’nan paydalani’ladi’.

Arifmetikali’q progressiyani’n’ ani’qlamasi’na muwapi’q

$$a_2 = a_1 + d,$$
$$a_3 = a_2 + d = a_1 + 2d,$$
$$a_4 = a_3 + d = a_1 + 3d \text{ ha'm t.b.}$$

Uluwma,

$$\boldsymbol{a_n = a_1 + (n-1)d,} \qquad \textbf{(1)}$$

sebebi arifmetikali’q progressiyani’n’ n- ag’zasi’ woni’n’ birinshi ag’zasi’na d sani’n $(n-1)$ ma’rte qosi’w na’tiyjesinde payda yetiledi.

(1) formula *arifmetikali’q progressiyani’n’ n–ag’zasi’ni’n’ formulasi’* dep ataladi’.

2-ma’sele. Yeger $a_1 = -6$ ha’m $d = 4$ bolsa, arifmetikali’q progressiyani’n’ ju’zinshi ag’zasi’n tabi’n’.

△ (1) formula boyi’nsha: $a_{100} = -6 + (100-1) \cdot 4 = 390$. ▲

3-ma’sele. 99 sani’ 3, 5, 7, 9, ... arifmetikali’q progressiyani’n’ ag’zasi’. Usi’ ag’zani’n’ nomerin tabi’n’.

△ Meyli, n − izlengen nomer bolsi’n. $a_1 = 3$ ha’m $d = 2$ bolg’anli’qtan $a_n = a_1 + (n-1)d$ formulasi’na muwapi’q: $99 = 3 + (n-1) \cdot 2$. Bunnan $99 = 3 + 2n - 2$; $98 = 2n$, $n = 49$.

Juwabi’: $n = 49$. ▲

4-ma’sele. Arifmetikali’q progressiyada $a_8 = 130$ ha’m $a_{12} = 166$. n-ag’zasi’ni’n’ formulasi’n tabi’n’.

△ (1) formuladan paydalani’p, tabami’z:

$$a_8 = a_1 + 7d,\ a_{12} = a_1 + 11d.$$

a_8 ha’m a_{12} lerdin’ berilgen ma’nislerin worni’na qoyi’p, a_1 ha’m d g’a qarata ten’lemeler sistemasi’n payda yetemiz:

$$\begin{cases} a_1 + 7d = 130, \\ a_1 + 11d = 166. \end{cases}$$

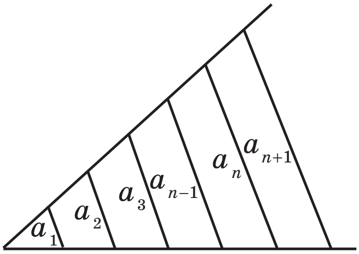
Yekinshi ten’lemeden birinshi ten’lemeni ali’p, mi’nag’an iye bolami’z: $4d = 36$, $d = 9$.

Demek, $a_1 = 130 - 7d = 130 - 63 = 67$. Progressiyani'n' n-ag'zasi'ni'n' formulasi'n jazami'z:

$a_n = 67 + 9(n - 1) = 67 + 9n - 9 = 58 + 9n$.

J u w a b i ' : $a_n = 9n + 58$. ▲

5 - m a ' s e l e . Mu'yeshtin' bir ta'repine woni'n' to'besinen baslap ten' kesindiler aji'rati'ldi'. Bul kesindilerdin' aqi'rg'i' ushlari'nan parallel tuwri'-lar ju'rgizildi (74-su'wret). Usi' tuwri'-lardi'n' mu'yeshtin' ta'repleri arasi'ndag'i' $a_1, a_2, a_3, \ldots$ kesindilerinin' uzi'nli'qlari' arifmetikali'q progressiya du'zetug'i'nli'g'i'n da'lillen'.



74- su'wret.

△ Ultanlari' a_{n-1} ha'm a_{n+1} bolg'an tra-peciyada woni'n' worta si'zi'g'i' a_n ge ten'. Sonli'qtan, $a_n = \frac{a_{n-1}+a_{n+1}}{2}$.

Bunnan $2a_n = a_{n-1} + a_{n+1}$ yamasa $a_{n+1} - a_n = a_n - a_{n-1}$.

Izbe-izliktin' ha'rbir ag'zasi' menen wonnan aldi'ng'i' ag'zasi'ni'n' ayi'rmasi' birdey san bolg'anli'qtan bul izbe-izlik arifmetikali'q progressiya boladi'. ▲

S h i ' n i ' g ' i ' w l a r

394. (Awi'zeki.) Arifmetikali'q progressiyani'n' birinshi ag'zasi'n ha'm ayi'rmasi'n ayti'n':

1) 6, 8, 10, ...; 2) 7, 9, 11, ...;
3) 25, 21, 17, ...; 4) –12, –9, –6, ...

395. Yeger: 1) $a_1 = 2$ ha'm $d = 5$; | 2) $a_1 = -3$ ha'm $d = 2$; | 3) $a_1 = 4$ ha'm $d = -1$ bolsa, arifmetikali'q progressiyani'n' da'slepki bes ag'zasi'n jazi'n'.

396. n- ag'zasi'ni'n' formulasi' menen berilgen to'mendegi izbe-izlik arifmetikali'q progressiya bolatug'i'ni'n da'lillen'.

1) $a_n = 3 - 4n$; 2) $a_n = -5 + 2n$; 3) $a_n = 3(n + 1)$;
4) $a_n = 2(3 - n)$; 5) $a_n = 3 - 5n$; 6) $a_n = -7 + 3n$.

397. Arifmetikali'q progressiyada:

1) yeger $a_1 = 2$, $d = 3$ bolsa, a_{15} ni' tabi'n';

2) yeger $a_1 = 3$, $d = 4$ bolsa, a_{20} ni' tabi'n';
3) yeger $a_1 = -3$, $d = -2$ bolsa, a_{18} ni' tabi'n';
4) yeger $a_1 = -2$, $d = -4$ bolsa, a_{11} ni' tabi'n'.

398. Arifmetikali'q progressiyani'n' n- ag'zasi'ni'n' formulasi'n jazi'n':

1) 1, 6, 11, 16, ...; 2) 25, 21, 17, 13, ...;
3) –4, –6, –8, –10, ...; 4) 1, –4, –9, –14, ...

399. –22 sani' 44, 38, 32, ... arifmetikali'q progressiyasi'ni'n' ag'zasi'. Usi' sanni'n' nomerin tabi'n'.

400. 12 sani' –18, –15, –12, ...arifmetikali'q progressiyasi'ni'n' ag'zasi' bola ma?

401. –59 sani' 1, –5 ... arifmetikali'q progressiyasi'ni'n' ag'zasi'. Usi' sanni'n' nomerin tabi'n'. –46 sani' usi' progressiyani'n' ag'zasi' bola ma?

402. Yeger arifmetikali'q progressiyada:

1) $a_1 = 7$, $a_{16} = 67$; 2) $a_1 = -4$, $a_9 = 0$, 3) $a_2 = 8$, $a_{10} = 64$ bolsa, woni'n' ayi'rmasi'n tabi'n'.

403. Arifmetikali'q progressiyani'n' ayi'rmasi' 1,5 ke ten'. Yeger:
1) $a_9 = 12$; 2) $a_7 = -4$; 3) $a_{16} = 32{,}5$ bolsa, a_1 ni' tabi'n'.

404. Yeger arifmetikali'q progressiyada:
1) $d = -3$, $a_{11} = 20$; 2) $a_{21} = -10$, $a_{22} = -5{,}5$;
3) $a_3 = -1$, $a_9 = 17$ bolsa, woni'n' birinshi ag'zasi'n tabi'n'.

405. Yeger arifmetikali'q progressiyada:
1) $a_3 = 13$, $a_6 = 22$; 2) $a_2 = -7$, $a_7 = 18$; 3) $a_7 = 11$, $a_{13} = 29$ bolsa, woni'n' n- ag'zasi'ni'n' formulasi'n tabi'n'.

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406. n nin' qanday ma'nislerinde 15, 13, 11, ... arifmetikali'q progressiyani'n' ag'zalari' teris boladi'?

407. Arifmetikali'q progressiyada $a_1 = -10$, $d = 0{,}5$ bolsa, n nin' qanday ma'nislerinde $a_n < 2$ ten'sizligi wori'nlanadi'?

408. Yeger arifmetikali'q progressiyada:

1) $a_8 = 126$, $a_{10} = 146$; 2) $a_8 = -64$, $a_{10} = -50$;
3) $a_8 = -7$, $a_{10} = 3$; 4) $a_8 = 0{,}5$, $a_{10} = -2{,}5$;
5) $a_7 = 13$, $a_{11} = 21$; 6) $a_6 = 3$, $a_{10} = 11$

bolsa, woni'n' tog'i'zi'nshi' ag'zasi'n ha'm ayi'rmasi'n tabi'n'.

409. Yerkin tu'siwshi dene birinshi sekundta 4,9 m arali'qti' wo'tedi, al keyingi ha'rbir sekundta aldi'ng'i'si'nan 9,8 m arti'q arali'qti' wo'tedi. Yerkin tu'siwshi dene besinshi sekundta qanday arali'qti' wo'tedi?

410. Hawa vannasi'n ali'w joli' menen yemleniwde birinshi ku'ni yemleniw 15 minut dawam yetedi, keyninen ha'r ku'n sayi'n yemleniw 10 minutqa artti'ri'li'p bari'ladi'. Vanna ali'w arti'g'i' menen 1 saat 45 min dawam yetiwi ushi'n ko'rsetilgen ta'rtipte hawa vannasi'nda yemleniw neshe ku'n dawam yetedi?

411. Arifmetikali'q progressiya ushi'n $a_n + a_k = a_{n-l} + a_{k+l}$ ten'ligi wori'nli' yekenin da'lillen'. Yeger $a_7 + a_8 = 30$ bolsa, $a_{10} + a_5$ ti tabi'n'.

412. Arifmetikali'q progressiya ushi'n

$$a_n = \frac{a_{n+k} + a_{n-k}}{2}$$

ten'ligi wori'nli' yekenin da'lillen'. Yeger $a_{10} + a_{30} = 120$ bolsa, a_{20} ni' tabi'n'.

31- §. ARIFMETIKALI'Q PROGRESSIYANI'N' DA'SLEPKI *n* AG'ZASI'NI'N' QOSI'NDI'SI'

1-ma'sele. 1 den 100 ge shekemgi barli'q natural sanlardi'n' qosi'ndi'si'n tabi'n'.

△ Bul qosi'ndi'ni' yeki usi'l menen jazami'z:

$$S = 1 + 2 + 3 + \ldots + 99 + 100,$$

$$S = 100 + 99 + 98 + \ldots + 2 + 1.$$

Bul ten'liklerdi ag'zama-ag'za qosami'z:

$$2S = \underbrace{101 + 101 + 101 + \ldots + 101 + 101}_{100 \text{ qosi'li'wshi'}}$$

Sonli'qtan da $2S = 101 \cdot 100$, bunnan $S = 101 \cdot 50 = 5050$. ▲

Yendi qa'legen, $a_1, a_2, \ldots, a_n, \ldots$ arifmetikali'q progressiyani' qaraymi'z. S_n – usi' progressiyani'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si' bolsi'n:

$$S_n = a_1 + a_2 + \ldots + a_{n-1} + a_n.$$

Teorema. **Arifmetikali'q progressiyani'n' da'slepki *n* ag'zasi'ni'n' qosi'ndi'si' to'mendegige ten':**

$$S_n = \frac{a_1+a_n}{2}n. \qquad \textbf{(1)}$$

○ S_n di yeki usi'l menen jazi'p alami'z:

$$S_n = a_1 + a_2 + \ldots + a_{n-1} + a_n,$$
$$S_n = a_n + a_{n-1} + \ldots + a_2 + a_1.$$

Arifmetikali'q progressiyani'n' ani'qlamasi'na muwapi'q, bul ten'liklerdi to'mendegishe jazi'w mu'mkin:

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + \ldots + (a_1 + (n-1)d), \qquad (2)$$
$$S_n = a_n + (a_n - d) + (a_n - 2d) + \ldots + (a_n - (n-1)d). \qquad (3)$$

(2) ha'm (3) ten'liklerdi ag'zama-ag'za qosami'z:

$$2S_n = \underbrace{(a_1 + a_n) + (a_1 + a_n) + \ldots + (a_1 + a_n)}_{n \text{ qosi'li'wshi'}}$$

Demek, $2S_n = (a_1 + a_n)n$, bunnan $S_n = \frac{a_1+a_n}{2}n$. ●

2-ma'sele. Da'slepki n natural sanni'n' qosi'ndi'si'n tabi'n'.

△ Natural sanlardi'n'

$$1, 2, 3, 4, 5, 6, \ldots, n, \ldots$$

izbe-izligi ayi'rmasi' $d = 1$ bolg'an arifmetikali'q progressiyadan ibarat. $a_1 = 1$ ha'm $a_n = n$ bolg'anli'qtan (1) formula boyi'nsha to'mendegige iye bolami'z:

$$S_n = 1 + 2 + 3 + \ldots + n = \frac{1+n}{2} \cdot n.$$

Solay yetip, $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$. ▲

3-ma'sele. Yeger $38 + 35 + 32 + \ldots + (-7)$ qosi'ndi'si'ni'n' qosi'li'wshi'lari' arifmetikali'q progressiyani'n' izbe-iz ag'zalari' bolsa, usi' qosi'ndi'ni' tabi'n'.

△ Sha'rt boyi'nsha, $a_1 = 38$, $d = -3$, $a_n = -7$. Yendi $a_n = a_1 + (n-1)d$ formulasi'n qollani'p, $-7 = 38 + (n-1)(-3)$ payda yetemiz, bunnan $n = 16$.

$S_n = \frac{a_1+a_n}{2}n$ formulasi' boyi'nsha to'mendegige iye bolami'z:

$$S_{16} = \frac{38-7}{2} \cdot 16 = 248.$$ ▲

4*-ma'sele. Qosi'ndi'si' 153 ke ten' boli'wi' ushi'n 1 den baslap neshe natural sanlardi' izbe-iz qosi'w kerek?

△ Natural sanlar qatari' — ayi'rmasi' $d = 1$ bolg'an arifmetikali'q progressiya boladi'. Sha'rtke muwapi'q $a_1 = 1$, $S_n = 153$. Da'slepki n ag'zani'n' qosi'ndi'si'ni'n' formulasi'n to'mendegishe wo'zgertemiz:

$$S_n = \frac{a_1 + a_n}{2} \cdot n = \frac{a_1 + a_1 + (n-1)d}{2} \cdot n = \frac{2a_1 + (n-1)d}{2} \cdot n .$$

Berilgenlerden paydalani'p, n belgisizine qarata ten'leme payda yetemiz:

$$153 = \frac{2 \cdot 1 + (n-1) \cdot 1}{2} \cdot n ,$$

bunnan, $306 = 2n + (n-1)n$, $n^2 + n - 306 = 0$.
Bul ten'lemeni sheship, to'mendegige iye bolami'z:

$$n_{1,2} = \frac{-1 \pm \sqrt{1+1224}}{2} = \frac{-1 \pm 35}{2},$$

$$n_1 = -18, \ n_2 = 17.$$

Qosi'li'wshi'lardi'n' sani' teris boli'wi' mu'mkin yemes, sonli'qtan $n = 17$. ▲

Shi'ni'g'i'wlar

413. Yeger arifmetikali'q progressiyada:

1) $a_1 = 1$, $a_n = 20$, $n = 50$;
3) $a_1 = -1$, $a_n = -40$, $n = 20$;
2) $a_1 = 1$, $a_n = 200$, $n = 100$;
4) $a_1 = 2$, $a_n = 100$, $n = 50$

bolsa, woni'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

414. 2 den 98 ge shekemgi barli'q natural sanlardi'n' qosi'ndi'si'n tabi'n' (98 de qosi'ndi'g'a kiredi).

415. 1 den 133 ke shekemgi barli'q taq sanlardi'n' qosi'ndi'si'n tabi'n' (133 te qosi'ndi'g'a kiredi).

416. Yeger arifmetikali'q progressiyada:

1) $a_1 = -5$, $d = 0{,}5$;
2) $a_1 = \frac{1}{2}$, $d = -3$;
3) $a_1 = 36$, $d = -2{,}5$

bolsa, woni'n' da'slepki won yeki ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

417. 1) yeger $n = 11$ bolsa, 9; 13; 17; ...;
2) yeger $n = 12$ bolsa, –16; –10; –4; ...

arifmetikali'q progressiyani'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

418. Yeger:

1) $3 + 6 + 9 + \ldots + 273$; 2) $90 + 80 + 70 + \ldots + (-60)$

qosi'ndi'si'ni'n' qosi'li'wshi'lari' arifmetikali'q progressiyani'n' izbe-iz ag'zalari' bolsa, usi' qosi'ndi'ni' tabi'n'.

419. Barli'q yeki ha'm u'sh tan'bali' sanlardi'n' qosi'ndi'si'n tabi'n'.

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420. Arifmetikali'q progressiya n - ag'zasi'ni'n' formulasi' menen berilgen. Yeger: 1) $a_n = 3n + 5$; 2) $a_n = 7 + 2n$ bolsa, S_{50} di tabi'n'.

421. Qosi'ndi' 75 ke ten' boli'wi' ushi'n 3 ten baslap neshe natural sandi' izbe-iz qosi'w kerek?

422. Yeger arifmetikali'q progressiyada:

1) $a_1 = 10$, $n = 14$, $S_{14} = 1050$; 2) $a_1 = 2\frac{1}{3}$, $n = 10$, $S_{10} = 90\frac{5}{6}$

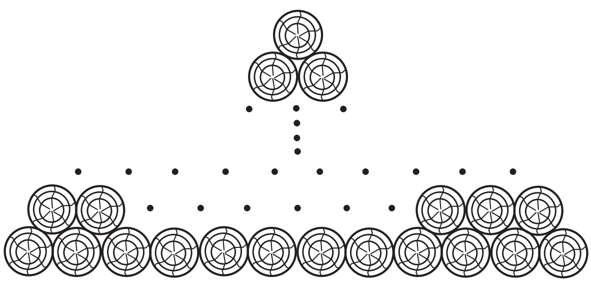
bolsa, a_1 ha'm d ni' tabi'n'.

423. Yeger arifmetikali'q progressiyada:

1) $a_7 = 21$, $S_7 = 205$; 2) $a_{11} = 92$, $S_{11} = 22$; 3) $a_{20} = 65$, $S_{20} = 350$

bolsa, a_1 ha'm d ni' tabi'n'.

424. Imaratlar quri'wg'a arnalg'an bo'renelerdi saqlawda wolardi' 75-su'wrette ko'rsetilgenindey yetip taqlaydi'. Yeger taqlamni'n' ultani'nda 12 bo'rene turg'an bolsa, ha'rbir taqlamda neshe bo'rene boladi'?



75- su'wret.

425. Arifmetikali'q progressiyada $a_3 + a_9 = 8$. S_{11} di tabi'n'.

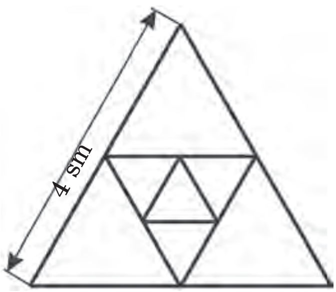
426. Yeger arifmetikali'q progressiyada $S_5 = 65$ ha'm $S_{10} = 230$ bolsa, woni'n' birinshi ag'zasi'n ha'm ayi'rmasi'n tabi'n'.

427. Arifmetikali'q progressiya ushi'n

$$S_{12} = 3(S_8 - S_4)$$

ten'ligi wori'nlanatug'i'ni'n da'lillen'.

32- §. GEOMETRIYALI'Q PROGRESSIYA



76- su'wret.

Ta'repi 4 sm bolg'an ten' ta'repli duri's u'shmu'yeshlikti qaraymi'z. To'beleri berilgen u'shmu'yeshliktin' ta'replerinin' wortalari'nan ibarat bolg'an u'shmu'yeshlik jasaymi'z (76-su'wret). U'shmu'yeshliktin' worta si'zi'g'i'ni'n' qa'siyetine muwapi'q yekinshi u'shmu'yeshliktin' ta'repi 2 sm ge ten'. Sog'an uqsas jasawlardi' dawam yettirip, ta'repleri 1, $\frac{1}{2}$, $\frac{1}{4}$ sm ha'm t.b. bolg'an u'shmu'yeshliklerdi payda yetemiz. Usi' u'shmu'yeshliklerdin' ta'replerinin' uzi'nli'qlari'ni'n' izbe-izligin jazami'z:

$$4,\ 2,\ 1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \ldots .$$

Bul izbe-izlikte, yekinshisinen baslap, woni'n' ha'rbir ag'zasi' bunnan aldi'ng'i' ag'zani' da'l bir qi'yli' $\frac{1}{2}$ sani'na ko'beytkenge ten'. Bunday izbe-izlikler *geometriyali'q progressiyalar* dep ataladi':

A n i ' q l a m a . ***Yeger***

$$b_1,\ b_2,\ b_3,\ \ldots,\ b_n,\ \ldots$$

sanli' izbe-izliginde barli'q natural n ushi'n

$$b_{n+1} = b_n q$$

ten'ligi wori'nlansa, bunday izbe-izlik geometriyali'q progressiya dep ataladi', bunda $b_n \neq 0$, q – nolge ten' bolmag'an bazi' bir san.

Bul formuladan $\frac{b_{n+1}}{b_n}=q$ yekenligi kelip shi'g'adi'. q sani' *geometriyali'q progressiyani'n' bo'limi* dep ataladi'.

Mi'sallar.

1) 2, 8, 32, 128, ... – bo'limi $q=4$ bolg'an geometriyali'q progressiya;

2) $1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \ldots$ – bo'limi $q=\frac{2}{3}$ bolg'an geometriyali'q progressiya;

3) $-\frac{1}{12}$, 1, –12, 144, ... – bo'limi $q=-12$ bolg'an geometriyali'q progressiya;

4) 7, 7, 7, 7, ... – bo'limi $q=1$ bolg'an geometriyali'q progressiya;

1-ma'sele. $b_n=7^{2n}$ formulasi' menen berilgen izbe-izlik geometriyali'q progressiya bolatug'i'ni'n da'lillen'.

△ Barli'q n lerde $b_n=7^{2n}\neq 0$ yekenligin atap wo'temiz. $\frac{b_{n+1}}{b_n}$ tiyindisi barli'q n ler ushi'n n ge g'a'rezli bolmag'an birdey sang'a ten'ligin da'lillew talap yetiledi. Haqi'yqati'nda da,

$$\frac{b_{n+1}}{b_n}=\frac{7^{2(n+1)}}{7^{2n}}=\frac{7^{2n+2}}{7^{2n}}=49,$$

yag'ni'y $\frac{b_{n+1}}{b_n}$ tiyindisi n ge g'a'rezli yemes. ▲

Geometriyali'q progressiyani'n' ani'qlamasi'na muwapi'q

$$b_{n+1}=b_nq,\ b_{n-1}=\frac{b_n}{q},$$

bunnan $b_n^2=b_{n-1}b_{n+1}$, $n>1$.

Yeger progressiyani'n' barli'q ag'zalari' won' bolsa, wonda $b_n=\sqrt{b_{n-1}b_{n+1}}$ boladi', yag'ni'y geometriyali'q progressiyani'n' yekinshi ag'zasi'nan baslap ha'rbir ag'zasi' wog'an qon'si'las bolg'an yeki ag'zani'n' geometriyali'q wortasi'na ten'. «Geometriyali'q» progressiya degen atama usi'ni'n' menen tu'sindiriledi.

Yeger b_1 ha'm q berilgen bolsa, wonda geometriyali'q progressiyani'n' qalg'an ag'zalari'n $b_{n+1}=b_nq$ rekkurent formulasi' boyi'nsha yesaplaw mu'mkinligin atap wo'temiz. Biraq n u'lken bolg'anda bunday yesaplaw ko'p miynet talap yetedi. A'dette n-ag'zani'n' formulasi'nan paydalani'ladi'.

Geometriyali'q progressiyani'n' ani'qlamasi'na muwapi'q

$$b_2 = b_1q,$$
$$b_3 = b_2q = b_1q^2,$$
$$b_4 = b_3q = b_1q^3 \text{ ha'm h.k.}$$

Uluwma,

$$\boldsymbol{b_n = b_1q^{n-1}}, \qquad \textbf{(1)}$$

sebebi geometriyali'q progressiyani'n' n- ag'zasi' woni'n' birinshi ag'zasi'n q sani'na **(**n**–1)** ma'rte ko'beytiw menen payda yetiledi.

(1) formula geometriyali'q progressiyani'n' *n-ag'zasi'ni'n' formulasi'* dep ataladi'.

2-ma'sele. Yeger $b_1 = 81$ ha'm $q = \frac{1}{3}$ bolsa, geometriyali'q progressiyani'n' jetinshi ag'zasi'n tabi'n'.

△ (1) formulag'a muwapi'q:

$$b_7 = 81 \cdot \left(\frac{1}{3}\right)^{7-1} = \frac{81}{3^6} = \frac{1}{9}. \ ▲$$

3-ma'sele. 486 sani' 2, 6, 18, ... geometriyali'q progressiyasi'ni'n' ag'zasi'. Usi' ag'zani'n' nomerin tabi'n'.

△ Meyli, n – izlengen nomer bolsi'n. $b_1 = 2$, $q = 3$ bolg'anli'qtan $b_n = b_1q^{n-1}$ formulasi'na muwapi'q:

$$486 = 2 \cdot 3^{n-1},\ 243 = 3^{n-1},\ 3^5 = 3^{n-1},$$

bunnan $n - 1 = 5$, $n = 6$. ▲

4-ma'sele. Geometriyali'q progressiyada $b_6 = 96$ ha'm $b_8 = 384$. n-ag'zasi'ni'n' formulasi'n tabi'n'.

△ $b_n = b_1q^{n-1}$ formulasi'na muwapi'q: $b_6 = b_1q^5$, $b_8 = b_1q^7$. b_6 ha'm b_8 din' berilgen ma'nislerin wori'nlari'na qoyi'p, to'mendegini payda yetemiz: $96 = b_1q^5$, $384 = b_1q^7$. Bul ten'liklerdin' yekinshisin birinshisine bo'lemiz:

$$\frac{384}{96} = \frac{b_1q^7}{b_1q^5},$$

bunnan $4 = q^2$ yamasa $q^2 = 4$. Aqi'rg'i' ten'likten $q = 2$ yamasa $q = -2$ yekenin tabami'z. Progressiyani'n' birinshi ag'zasi'n tabi'w ushi'n $96 = b_1q^5$ ten'liginen paydalanami'z:

1) $q = 2$ bolsi'n. Sonda $96 = b_1 \cdot 2^5$, $96 = b_1 \cdot 32$, $b_1 = 3$.

Demek, $b_1 = 3$ ha'm $q = 2$ bolg'anda n-ag'zani'n' formulasi'

$$b_n = 3 \cdot 2^{n-1}$$

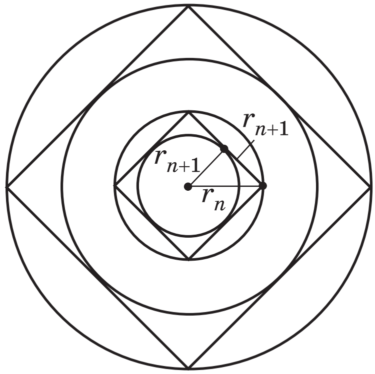
boladi'.

2) $q = -2$ bolsi'n. Bul jag'dayda $96 = b_1(-2)^5$, $96 = b_1(-32)$, $b_1 = -3$.

Demek, $b_1 = -3$ ha'm $q = -2$ bolg'anda n-ag'zani'n' formulasi'

$$b_n = -3 \cdot (-2)^{n-1}$$

boladi'.



77- su'wret.

J u w a b i ': $b_n = 3 \cdot 2^{n-1}$ yamasa $b_n = -3 \cdot (-2)^{n-1}$. ▲

5 - m a ' s e l e . Shen'berge kvadrat ishley si'zi'lg'an, al kvadratqa yekinshi shen'ber ishley si'zi'lg'an. Yekinshi shen'berge yekinshi kvadrat ishley si'zi'lg'an, al wog'an u'shinshi shen'ber ishley si'zi'lg'an ha'm t.b. (77-su'wret). Shen'berlerdin' radiuslari' geometriyali'q progressiya du'zetug'i'ni'n da'lillen'.

△ n-shen'berdin' radiusi' r_n bolsi'n. Sonda Pifagor teoremasi'na muwapi'q

$$r_{n+1}^2 + r_{n+1}^2 = r_n^2,$$

bunnan

$$r_{n+1}^2 = \frac{1}{2} r_n^2, \text{ yag'ni'y } r_{n+1} = \frac{1}{\sqrt{2}} r_n.$$

Demek, shen'berlerdin' radiuslari'ni'n' izbe-izligi bo'limi $\frac{1}{\sqrt{2}}$ bolg'an geometriyali'q progressiyani' du'zedi. ▲

S h i ' n i ' g ' i ' w l a r

428. (Awi'zeki.) To'mendegi geometriyali'q progressiyani'n' birinshi ag'zasi' ha'm bo'limi nege ten':

1) 8, 16, 32, ... ; 2) –10, 20, –40, ... ;
3) 4, 2, 1, ... ; 4) –50, 10, –2, ... ?

429. Yeger geometriyali'q progressiyada:

1) $b_1 = 12$, $q = 2$; 2) $b_1 = -3$, $q = -4$; 3) $b_1 = 16$, $q = -2$ bolsa, bolsa, woni'n' da'slepki bes ag'zasi'n jazi'n'.

430. n-ag'zasi'ni'n' formulasi' menen berilgen to'mendegi izbe-izlik geometriyali'q progressiya bolatug'i'ni'n da'lillen':

1) $b_n = 3 \cdot 2^n$; 2) $b_n = 5^{n+3}$; 3) $b_n = \left(\frac{1}{3}\right)^{n-2}$; 4) $b_n = \frac{1}{5^{n-1}}$.

431. Geometriyali'q progressiyada:

1) $b_1 = 3$ ha'm $q = 10$ bolsa, b_4 ti;

2) $b_1 = 4$ ha'm $q = \frac{1}{2}$ bolsa, b_7 ni;

3) $b_1 = 1$ ha'm $q = -2$ bolsa, b_5 ti;

4) $b_1 = -3$ ha'm $q = -\frac{1}{3}$ bolsa, b_6 ni' yesaplan'.

432. Geometriyali'q progressiyani'n' n-ag'zasi'ni'n' formulasi'n jazi'n':

1) 4, 12, 36, ...; 2) 3, 1, $\frac{1}{3}$, ...; 3) 4, –1, $\frac{1}{4}$, ...;

4) 3, –4, $\frac{16}{3}$, ... ; 5) 16, 8, 4, 2, ... ; 6) –9, 3, –1, $\frac{1}{3}$, ...

433. Geometriyali'q progressiyada asti'nan si'zi'lg'an ag'zani'n' nomerin tabi'n':

1) 6, 12, 24, ... , <u>192</u>, ...; 3) 625, 125, 25, ... , $\underline{\frac{1}{25}}$;

2) 4, 12, 36, ... , <u>324</u>, ...; 4) –1, 2, –4, ... , <u>128</u>, ...

434. Yeger geometriyali'q progressiyada:

1) $b_1 = 2$, $b_5 = 162$; 3) $b_1 = -128$, $b_7 = -2$;

2) $b_1 = 3$, $b_4 = 81$; 4) $b_1 = 250$, $b_4 = -2$

bolsa, woni'n' bo'limin tabi'n'.

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435. 2, 6, 18, ... geometriyali'q progressiyasi' berilgen.

1) usi' progressiyani'n' segizinshi ag'zasi'n yesaplan';

2) izbe-izliktin' 162 ge ten' ag'zasi'ni'n' nomerin tabi'n'.

436. Yeger won' ag'zali' geometriyali'q progressiyada:

1) $b_8 = \frac{1}{9}$, $b_6 = 81$; 2) $b_6 = 9$, $b_8 = 3$; 3) $b_6 = 3$, $b_8 = \frac{1}{3}$ bolsa, woni'n' jetinshi ag'zasi'n ha'm bo'limin tabi'n'.

437. Yeger geometriyali'q progressiyada:

1) $b_4 = 9$, $b_6 = 20$; 2) $b_4 = 9$, $b_6 = 4$; 3) $b_4 = 320$, $b_6 = 204{,}8$ bolsa, woni'n' besinshi ha'm birinshi ag'zalari'n tabi'n'.

438. Amanatshi' amanat bankine 2009-ji'ldi'n' 4-yanvari' ku'ni 300000 swm pul qoydi'. Yeger amanat banki ji'li'na toplang'an qarji'ni'n' 30% i mug'dari'nda payda berse, amanatshi'ni'n' puli' 2012-ji'ldi'n' 4-yanvari'na bari'p qansha boladi'?

439. Ta'repi 4 sm bolg'an kvadrat berilgen. Woni'n' ta'replerinin' wortalari' yekinshi kvadratlardi'n' to'beleri boladi'. Yekinshi kvadratti'n' ta'replerinin' wortalari' u'shinshi kvadratti'n' to'beleri boladi' ha'm t.b. Usi' kvadratti'n' maydanlari'ni'n' izbe-izligi geometriyali'q progressiyani' du'zetug'i'ni'n da'lillen'. Jetinshi kvadratti'n' maydani'n tabi'n'.

33- §. GEOMETRIYALI'Q PROGRESSIYANI'N' DA'SLEPKI *n* AG'ZASI'NI'N' QOSI'NDI'SI'

1-ma'sele. Mi'na qosi'ndi'ni' tabi'n':

$$S = 1 + 3 + 3^2 + 3^3 + 3^4 + 3^5. \qquad (1)$$

△ Ten'liktin' yeki ta'repin de 3 ke ko'beytemiz:

$$3S = 3 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6. \qquad (2)$$

(1) ha'm (2) ten'liklerdi to'mendegishe jazi'p shi'g'ami'z:

$$S = 1 + (3 + 3^2 + 3^3 + 3^4 + 3^5);$$

$$3S = (3 + 3^2 + 3^3 + 3^4 + 3^5) + 3^6.$$

Qawsi'rmalar ishinde turg'an an'latpalar birdey. Sonli'qtan bul ten'liklerdin' yekinshisinen birinshisin ali'p, to'mendegige iye bolami'z:

$$3S - S = 3^6 - 1, \quad 2S = 3^6 - 1,$$

$$S = \frac{3^6-1}{2} = \frac{729-1}{2} = 364. \ \blacktriangle$$

Yendi bo'limi $q \neq 1$ bolg'an qa'legen $b_1, b_1q, \ldots, b_1q^n, \ldots$ geometriyali'q progressiyasi'n qaraymi'z: S_n – usi' geometriyali'q progressiyani'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si' bolsi'n:

$$S_n = b_1 + b_1q + b_1q^2 + \ldots + b_1q^{n-1}. \qquad (3)$$

Teorema: ***Bo'limi $q \neq 1$ bolg'an geometriyali'q progressiyani'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si' tomendegige ten':***

$$S_n = \frac{b_1(1-q^n)}{1-q}. \qquad (4)$$

○ (3) ten'liktin' yeki ta'repin de q g'a ko'beytemiz:

$$qS_n = b_1q + b_1q^2 + b_1q^3 + ... + b_1q^n. \qquad (5)$$

(3) ha'm (5) ten'liklerdi, wolardag'i' bir qi'yli' qosi'li'wshi'lardi' aji'rati'p, jazi'p shi'g'ami'z:

$$S_n = b_1 + (b_1q + b_1q^2 + ... + b_1q^{n-1}),$$

$$qS_n = (b_1q + b_1q^2 + b_1q^3 + ... + b_1q^{n-1}) + b_1q^n.$$

Qawsi'rmalar ishinde turg'an an'latpalar ten'. Sonli'qtan bul ten'liklerdin' birinshisinen yekinshisin ali'p, to'mendegige iye bolami'z:

$$S_n - qS_n = b_1 - b_1q^n.$$

Bunnan

$$S_n(1-q) = b_1(1-q^n), \; S_n = \frac{b_1(1-q^n)}{1-q}. \; ●$$

Yeger $q = 1$ bolsa, wonda

$$S_n = \underbrace{b_1 + b_1 + ... + b_1}_{n \text{ qosi'li'wshi'}} = b_1n, \text{ yag'ni'y } S_n = b_1n.$$

2-ma'sele. 6, 2, $\frac{2}{3}$, ... geometriyali'q progressiyasi'ni'n' da'slepki bes ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

△ Bul progressiyada $b_1 = 6$, $q = \frac{1}{3}$. (4) formula boyi'nsha tabami'z:

$$S_5 = \frac{6\cdot\left(1-\left(\frac{1}{3}\right)^5\right)}{1-\frac{1}{3}} = \frac{6\cdot\left(1-\frac{1}{243}\right)}{\frac{2}{3}} = \frac{6\cdot242\cdot3}{2\cdot243} = \frac{242}{27}. \; ▲$$

3-ma'sele. Bo'limi $q = \frac{1}{2}$ bolg'an geometriyali'q progressiyada da'slepki alti' ag'zasi'ni'n' qosi'ndi'si' 252 ge ten'. Usi' progressiyani'n' birinshi ag'zasi'n tabi'n'.

△ (4) formuladan paydalani'p, to'mendegige iye bolami'z:

$$252 = \frac{b_1\left(1-\frac{1}{2^6}\right)}{1-\frac{1}{2}}.$$

Bunnan $252 = 2b_1\left(1-\frac{1}{64}\right)$, $252 = \frac{b_1\cdot63}{32}$, $b_1 = 128$. ▲

4-ma'sele. Geometriyali'q progressiyani'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si' −93 ke ten'. Usi' progressiyani'n' birinshi ag'zasi' −3 ke, al bo'limi 2 ge ten'. n di tabi'n'.

△ (4) formuladan paydalani'p, to'mendegige iye bolami'z:

$$-93=\frac{-3(1-2^n)}{1-2}.$$

Bunnan $-31=1-2^n$, $2^n=32$, $2^5=2^n$, $n=5$. ▲

5-ma'sele. 5, 15, 45, ..., 1215, ... − geometriyali'q progressiya. 5 + 15 + + 45 + ... + 1215 qosi'ndi'si'n tabi'n'.

△ Bul progressiyada $b_1=5$, $q=3$, $b_n=1215$. Da'slepki n ag'zasi'ni'n' qosi'ndi'si'ni'n' formulasi'n to'mendegishe tu'rlendiremiz:

$$S_n=\frac{b_1(1-q^n)}{1-q}=\frac{b_1-b_1q^{n-1}q}{1-q}=\frac{b_1-b_nq}{1-q}=\frac{b_nq-b_1}{q-1}.$$

Ma'selenin' sha'rtinen paydalani'p, to'mendegige iye bolami'z:

$$S_n=\frac{1215\cdot 3-5}{3-1}=\frac{3645-5}{2}=1820.$$ ▲

Shi'ni'g'i'wlar

440. Yeger geometriyali'q progressiyada:

1) $b_1=\frac{1}{2}$, $q=2$, $n=6$; 2) $b_1=-2$, $q=\frac{1}{2}$, $n=5$;

3) $b_1=1$, $q=-\frac{1}{3}$, $n=4$; 4) $b_1=-5$, $q=-\frac{2}{3}$, $n=5$;

5) $b_1=6$, $q=1$, $n=200$; 6) $b_1=-4$, $q=1$, $n=100$

bolsa, woni'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

441. Geometriyali'q progressiyani'n' da'slepki jeti ag'zasi'ni'n' qosi'ndi'si'n tabi'n':

1) 5, 10, 20, ... ; 2) 2, 6, 18, ... ; 3) $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, ...

442. Yeger geometriyali'q progressiyada:

1) $q=2$, $S_7=635$ bolsa, b_1 ha'm b_7 ni tabi'n';

2) $q=-2$, $S_8=85$ bolsa, b_1 ha'm b_8 di tabi'n'.

443. Yeger geometriyali'q progressiyada:

1) $S_n = 189$, $b_1 = 3$, $q = 2$;

2) $S_n = 635$, $b_1 = 5$, $q = 2$;

3) $S_n = 170$, $b_1 = 256$, $q = -\frac{1}{2}$;

4) $S_n = -99$, $b_1 = -9$, $q = -2$

bolsa, woni'n' ag'zalari'ni'n' sani' n di tabi'n'.

444. Yeger geometriyali'q progressiyada:

1) $b_1 = 7$, $q = 3$, $S_n = 847$ bolsa, n ha'm b_n ni;

2) $b_1 = 8$, $q = 2$, $S_n = 4088$ bolsa, n ha'm b_n ni;

3) $b_1 = 2$, $b_n = 1458$, $S_n = 2186$ bolsa, n ha'm q ni';

4) $b_1 = 1$, $b_n = 2401$, $S_n = 2801$ bolsa, n ha'm q ni' tabi'n'.

445. Sanlardi'n' qosi'ndi'si'ni'n' qosi'li'wshi'lari' geometriyali'q progressiyani'n' izbe-iz ag'zalari' bolsa, usi' qosi'ndi'ni' tabi'n':

1) $1 + 2 + 4 + \ldots + 128$; 2) $1 + 3 + 9 + \ldots + 243$;

3) $-1 + 2 - 4 + \ldots + 128$; 4) $5 - 15 + 45 - \ldots + 405$.

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446. Yeger geometriyali'q progressiyada:

1) $b_2 = 15$, $b_3 = 25$; 2) $b_2 = 14$, $b_4 = 686$, 3) $b_2 = 15$, $b_4 = 375$, $q > 0$ bolsa, b_5 ha'm S_4 ti tabi'n'.

447. Geometriyali'q progressiya n- ag'zasi'ni'n' formulasi' menen berilgen:

1) $b_n = 3 \cdot 2^{n-1}$ bolsa, S_5 ti tabi'n';

2) $b_n = -2\left(\frac{1}{2}\right)^n$ bolsa, S_6 ni' tabi'n'.

448. Birdeylikti da'lillen':

$(x-1)(x^{n-1} + x^{n-2} + \ldots + 1) = x^n - 1$, bunda n da'reje ko'rsetkishi ha'm wol 1 den u'lken natural san.

449. Geometriyali'q progressiyada:

1) $b_3 = 135$, $S_3 = 195$ bolsa, b_1 ha'm q di' tabi'n';

2) $b_1 = 12$, $S_3 = 372$ bolsa, q ha'm b_3 ti tabi'n'.

450. Geometriyali'q progressiyada:

1) $b_1 = 1$ ha'm $b_3 + b_5 = 90$ bolsa, q di';
2) $b_2 = 3$ ha'm $b_4 + b_6 = 60$ bolsa, q di';
3) $b_1 - b_3 = 15$ ha'm $b_2 - b_4 = 30$ bolsa, S_{10} di';
4) $b_3 - b_1 = 24$ ha'm $b_5 - b_1 = 624$ bolsa, S_5 di' tabi'n'.

34- §. SHEKSIZ KEMEYIWSHI GEOMETRIYALI'Q PROGRESSIYA

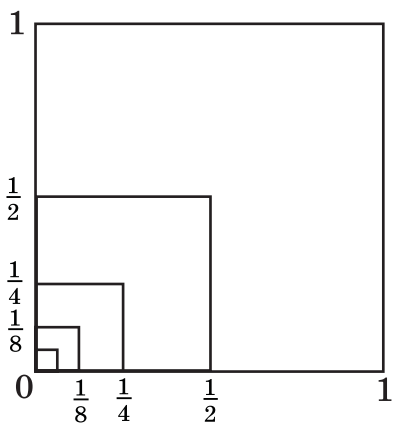
78-su'wrette ko'rsetilgen kvadratlardi' qaraymi'z. Birinshi kvadratti'n' ta'repi 1 ge ten', yekinshisiniki $\frac{1}{2}$ ge, al u'shinshisiniki $\frac{1}{2^2}$ ge ten' ha'm t.b. Solay yetip, kvadratti'n' ta'repinin' bo'limi $\frac{1}{2}$ bolg'an to'mendegi geometriyali'q progressiyani' du'zedi:

$$1,\ \frac{1}{2},\ \frac{1}{2^2},\ \frac{1}{2^3},\ \ldots,\ \frac{1}{2^{n-1}},\ \ldots\ . \qquad (1)$$

Bul kvadratlardi'n' maydanlari' bolsa bo'limi $\frac{1}{4}$ bolg'an to'mendegi geometriyali'q progressiyani' du'zedi:

$$1,\ \frac{1}{4},\ \frac{1}{4^2},\ \frac{1}{4^3},\ \ldots,\ \frac{1}{4^{n-1}},\ \ldots\ . \qquad (2)$$

78-su'wretten ko'rinip turg'ani'nday kvadratlardi'n' ta'repleri ha'm wolardi'n' maydanlari' n nomerinin' arti'wi' menen barg'an sayi'n kemeyip, nolge jaqi'nlasi'p baradi'. Sonli'qtan (1) ha'm (2) progressiyalar sheksiz kemeyiwshi progressiyalar dep ataladi'. Bul progressiyalardi'n' bo'limleri birden kishi yekenligin atap wo'temiz.



78- su'wret.

Yendi to'mendegi geometriyali'q progressiyani' qaraymi'z:

$$1,\ -\frac{1}{3},\ \frac{1}{3^2},\ -\frac{1}{3^3},\ \ldots,\ \frac{(-1)^{n-1}}{3^{n-1}},\ \ldots\ . \qquad (3)$$

Bul progressiyani'n' bo'limi $q = -\frac{1}{3}$, al ag'zalari' bolsa $b_1 = 1$, $b_2 = -\frac{1}{3}$, $b_3 = \frac{1}{9}$, $b_4 = -\frac{1}{27}$ ha'm t.b. n nomerinin' arti'wi' menen bul progressiyani'n' ag'zalari' nolge jaqi'nlasadi'. (3) progressiya da *sheksiz kemeyiwshi progressiya* dep ataladi'. Woni'n' bo'liminin' moduli birden kishi yekenligin atap wo'temiz: $|q|<1$.

Bo'liminin' moduli birden kishi bolg'an geometriyali'q progressiya sheksiz *kemeyiwshi* geometriyali'q progressiya dep ataladi'.

1-ma'sele n- ag'zasi'ni'n' $b_n=\frac{3}{5^n}$ formulasi' menen berilgen geometriyali'q progressiya sheksiz kemeyiwshi progressiya bolatug'i'ni'n da'lillen'.

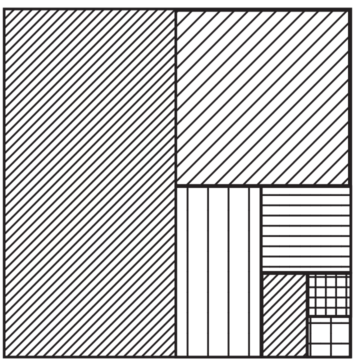
△ Sha'rt boyi'nsha $b_1=\frac{3}{5}$, $b_2=\frac{3}{5^2}=\frac{3}{25}$, bunnan $q=\frac{b_2}{b_1}=\frac{1}{5}$. $|q|<1$ bolg'anli'qtan berilgen geometriyali'q progressiya sheksiz kemeyiwshi boladi'. ▲

79-su'wrette ta'repi 1 bolg'an kvadrat ko'rsetilgen. Woni'n' yari'mi'n shtrixlaymi'z. Son'i'nan qalg'an bo'leginin' yari'mi'n shtrixlaymi'z ha'm t.b. Shtrixlang'an tuwri'mu'yeshliklerdin' maydanlari' to'mendegi sheksiz kemeyiwshi geometriyali'q progressiyani' du'zedi:

$$\frac{1}{2},\ \frac{1}{4},\ \frac{1}{8},\ \frac{1}{16},\ \frac{1}{32},\ \dots\ .$$

Yeger usi'nday jol menen payda yetilgen barli'q tuwri'mu'yeshliklerdi shtrixlap shi'qsaq, wonda kvadrat pu'tkilley shtrix penen qaplanadi'. Barli'q shtrixlang'an tuwri'mu'yeshliklerdin' maydanlari'ni'n' qosi'ndi'si' 1 ge ten' dep yesaplaw ta'biyiy, yag'ni'y:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\dots=1. \qquad (2)$$



79- su'wret.

Bul ten'liktin' shep jag'i'nda sheksiz sandag'i' qosi'li'wshi'ni'n' qosi'ndi'si' bar. Da'slepki n qosi'li'wshi'ni'n' qosi'ndi'si'n qaraymi'z:

$$S_n=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots+\frac{1}{2^n}.$$

Geometriyali'q progressiyani'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si'ni'n' formulasi'na muwapi'q:

$$S_n=\frac{1}{2}\cdot\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}=1-\frac{1}{2^n}.$$

Yeger n sheksiz wo'sip barsa, wonda $\frac{1}{2^n}$ nolge barg'an sayi'n jaqi'nlasi'p baradi' (nolge umti'ladi'). Bul to'mendegishe jazi'ladi':

$$n \to \infty \text{ da } \frac{1}{2^n} \to 0$$

(woqi'li'wi': n sheksizlikke umti'lg'anda $\frac{1}{2^n}$ nolge umti'ladi') yaki

$$\lim_{n\to\infty} \frac{1}{2^n} = 0$$

(woqi'li'wi': n sheksizlikke umti'lg'anda $\frac{1}{2^n}$ izbe-izliginin' limiti nolge ten').

Uluwma, qa'legen bir a_n izbe-izligi ushi'n $n\to\infty$ da $a_n - a \to 0$ bolsa, wonda a_n izbe-izligi a sani'na umti'ladi' (a_n izbe-izliginin' $n\to\infty$ dag'i' limiti a g'a ten') dep ayti'ladi' ha'm $\lim\limits_{n\to\infty} a_n = a$ tu'rinde jazi'ladi'.

$n\to\infty$ da $\frac{1}{2^n} \to 0$ bolg'anli'qtan $n\to\infty$ da $\left(1 - \frac{1}{2^n}\right) \to 1$, yag'ni'y $n\to\infty$ da $S_n \to 1$. Sonli'qtan da $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$ sheksiz qosi'ndi'si' 1 ge ten' dep yesaplanadi'.

Yendi qa'legen sheksiz kemeyiwshi geometriyali'q progressiyani' qaraymi'z: $b_1,\ b_1q,\ b_1q^2,\ \ldots,\ b_1q^{n-1},\ \ldots,$ bunda $|q| < 1$.

Sheksiz kemeyiwshi geometriyali'q progressiyani'n' qosi'ndi'si' dep n → ∞ da woni'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si' umti'latug'i'n sang'a ayti'ladi'.

$S_n = \frac{b_1(1-q^n)}{1-q}$ formulasi'nan paydalanami'z. Woni' to'mendegishe jazami'z:

$$\boxed{S_n = \frac{b_1}{1-q} - \frac{b_1}{1-q} q^n.} \qquad (4)$$

Yeger n sheksiz wo'sse, $|q| < 1$ bolg'anli'qtan $q^n \to 0$. Sonli'qtan $\frac{b_1}{1-q} \cdot q^n$ de $n\to\infty$ da nolge umti'ladi'. (4) formulada birinshi qosi'li'wshi' n ge baylani'sli' yemes. Demek, $n\to\infty$ da S_n qosi'ni'di'si' $\frac{b_1}{1-q}$ sani'na umti'ladi'.

Solay yetip, sheksiz kemeyiwshi geometriyali'q progressiyani'n' S qosi'ndi'si' to'mendegige ten':

$$\boxed{S = \frac{b_1}{1-q}.} \quad (5)$$

Ayi'ri'm jag'dayda, $b_1 = 1$ bolg'anda, $S = \frac{1}{1-q}$ boladi'. Bul ten'lik a'dette mi'na ko'riniste jazi'ladi':

$$1 + q + q^2 + \ldots + q^{n-1} + \ldots = \frac{1}{1-q}.$$

Bul ten'lik ha'm (5) ten'lik tek $|q| < 1$ bolg'anda g'ana wori'nli' bolatug'i'ni'n atap wo'temiz.

2-ma'sele. $\frac{1}{2}, -\frac{1}{6}, \frac{1}{18}, -\frac{1}{54}, \ldots$ sheksiz kemeyiwshi geometriyali'q progressiyani'n' qosi'ndi'si'n tabi'n'.

△ $b_1 = \frac{1}{2}$, $b_2 = -\frac{1}{6}$ bolg'anli'qtan $q = \frac{b_2}{b_1} = -\frac{1}{3}$, $S = \frac{b_1}{1-q}$ formulasi' boyi'nsha:

$$S = \frac{\frac{1}{2}}{1-\left(-\frac{1}{3}\right)} = \frac{3}{8}. \ ▲$$

3-ma'sele. Yeger $b_3 = -1$, $q = \frac{1}{7}$ bolsa, sheksiz kemeyiwshi geometriyali'q progressiyani'n' qosi'ndi'si'n tabi'n'.

△ $n = 3$ bolg'anda $b_n = b_1 q^{n-1}$ formulasi'n qollansaq, $-1 = b_1 \cdot \left(\frac{1}{7}\right)^{3-1}$, $-1 = b_1 \cdot \frac{1}{49}$ payda boladi', bunnan $b_1 = -49$.

(5) formula boyi'nsha S qosi'ndi'si'n tabami'z:

$$S = \frac{-49}{1-\frac{1}{7}} = -57\frac{1}{6}. \ ▲$$

4-ma'sele. (5) formuladan paydalani'p, $a = 0{,}(15) = 0{,}151515\ldots$ sheksiz periodli' wonli'q bo'lshekti a'piwayi' bo'lshek tu'rinde jazi'n'.

△ Berilgen sheksiz bo'lshektin' juwi'q ma'nislerinin' to'mendegi izbe-izligin jazami'z:

$$a_1 = 0{,}15 = \frac{15}{100},$$

$$a_2 = 0{,}1515 = \frac{15}{100} + \frac{15}{100^2},$$

$$a_3 = 0{,}151515 = \frac{15}{100} + \frac{15}{100^2} + \frac{15}{100^3}.$$

Juwi'q ma'nislerdi usi'layi'nsha jazi'w berilgen periodli' bo'lshekti sheksiz kemeyiwshi geometriyali'q progressiyani'n' qosi'ndi'si' tu'rinde su'wretlew mu'mkinligin ko'rsetedi:

$$a = \frac{15}{100} + \frac{15}{100^2} + \frac{15}{100^3} + \dots .$$

(5) formulag'a muwapi'q:

$$a = \frac{\frac{15}{100}}{1 - \frac{1}{100}} = \frac{15}{99} = \frac{5}{33}. \blacktriangle$$

Shi'ni'g'i'wlar

451. Geometriyali'q progressiya sheksiz kemeyiwshi bolatug'i'ni'n da'lillen':

1) $1, \frac{1}{2}, \frac{1}{4}, \dots;$ 2) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots;$ 3) $-81, -27, -9, \dots;$
4) $-16, -8, -4, \dots;$ 5) $3, 2, \frac{4}{3}, \frac{8}{9}, \dots;$ 6) $8, 6, \frac{9}{2}, \frac{27}{8}, \dots .$

452. Yeger geometriyali'q progressiyada:

1) $b_1 = 40, b_2 = -20;$ 2) $b_7 = 12, b_{11} = \frac{3}{4};$
3) $b_7 = -30, b_6 = 15;$ 4) $b_5 = -9, b_9 = -\frac{1}{27}$

bolsa, wol sheksiz kemeyiwshi bola ma? Usi'ni' ani'qlan'.

453. Sheksiz kemeyiwshi geometriyali'q progressiyani'n' qosi'n-di'si'n tabi'n':

1) $1, \frac{1}{3}, \frac{1}{9}, \dots;$ 2) $6, 1, \frac{1}{6}, \dots;$ 3) $-25, -5, -1, \dots;$
4) $-7, -1, -\frac{1}{7}, \dots;$ 5) $128, 64, 2, \dots;$ 6) $-81, -27, -9, \dots .$

454. Yeger sheksiz kemeyiwshi geometriyali'q progressiyada:

1) $q = \frac{1}{2}, b_1 = \frac{1}{8};$ 2) $q = -\frac{1}{3}, b_1 = 9;$
3) $q = \frac{1}{3}, b_5 = \frac{1}{81};$ 4) $q = -\frac{1}{2}, b_4 = -\frac{1}{8}$

bolsa, woni'n' qosi'ndi'si'n tabi'n'.

455. n- ag'zasi'ni'n' formulasi' menen berilgen to'mendegi izbe-izlik sheksiz kemeyiwshi geometriyali'q progressiya bola ma?

1) $b_n = 3 \cdot (-2)^n$; 2) $b_n = -3 \cdot 4^n$; 3) $b_n = -2 \cdot \left(-\frac{1}{3}\right)^{n-1}$;

4) $b_n = 5 \cdot \left(-\frac{1}{2}\right)^{n-1}$; 5) $b_n = -2 \cdot (-3)^n$; 6) $b_n = 8 \cdot \left(-\frac{1}{4}\right)^{n-1}$.

456. Sheksiz kemeyiwshi geometriyali'q progressiyani'n' qosi'n-di'si'n tabi'n':

1) $12,\ 4,\ \frac{4}{3},\ \ldots$; 2) $100,\ -10,\ 1\ \ldots$; 3) $98,\ 28,\ 8,\ \ldots$.

457. Yeger sheksiz kemeyiwshi geometriyali'q progressiyada:

1) $q = \frac{1}{2}$, $b_5 = \frac{\sqrt{2}}{16}$; 2) $q = \frac{\sqrt{3}}{2}$, $b_4 = \frac{9}{8}$; 3) $q = \frac{\sqrt{2}}{2}$, $b_9 = 4$

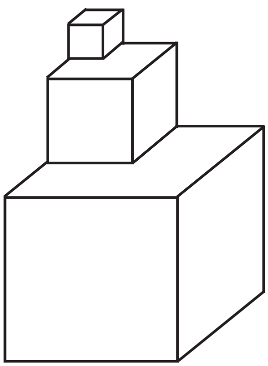
bolsa, woni'n' qosi'ndi'si'n tabi'n'.

458. Sheksiz kemeyiwshi geometriyali'q progressiyani'n' qosi'ndi'si' 150 ge ten'. Yeger:

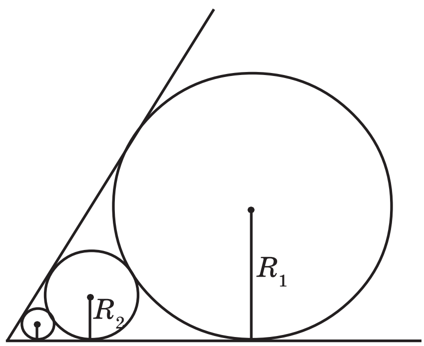
1) $q = \frac{1}{3}$ bolsa, b_1 di; 2) $b_1 = 75$; 3) $b_1 = 15$

bolsa, q di' tabi'n'.

459. Qi'ri' a bolg'an kubti'n' u'stine qi'ri' $\frac{a}{2}$ bolg'an kub qoyi'ldi', woni'n' u'stine qi'ri' $\frac{a}{4}$ bolg'an kub qoyi'ldi', son'i'nan woni'n' u'stine qi'ri' $\frac{a}{8}$ bolg'an kub qoyi'ldi' ha'm t.b. (80-su'wret). Payda bolg'an figurani'n' biyikligin tabi'n'.



80- su'wret.



81- su'wret.

460. $60°$ li' mu'yeshke bir-birine uri'ni'wshi' shen'berler ishley si'zi'lg'an (81-su'wret). Birinshi shen'berdin' radiusi' R_1 ge ten'. Qalg'an shen'berlerdin' $R_2, R_3, \ldots, R_n, \ldots$ radiuslari'n tabi'n' ha'm wolardi'n' sheksiz kemeyiwshi geometriyali'q progressiya du'zetug'i'nli'g'i'n ko'rsetin'. $R_1 + 2(R_2 + R_3 + \ldots + R_n + \ldots)$ qosi'ndi'si' birinshi shen'berdin' worayi'nan mu'yeshtin' to'besine shekemgi arali'qqa ten' yekenin da'lillen'.

461. Sheksiz periodli' wonli'q bo'lshekti a'piwayi' bo'lshek tu'rinde jazi'n':

1) 0,(5); 2) 0,(9); 3) 0,(12); 4) 0,2(3); 5) 0,25(18).

VI bapqa tiyisli shi'ni'g'i'wlar

462. Arifmetikali'q progressiyani'n' ayi'rmasi'n tabi'n', woni'n' to'rtinshi ha'm besinshi ag'zalari'n jazi'n':

1) $4,\ 4\frac{1}{3},\ 4\frac{2}{3},\ \ldots;$ 2) $3\frac{1}{2},\ 3,\ 2\frac{1}{2},\ \ldots;$

3) $1,\ 1+\sqrt{3},\ 1+2\sqrt{3},\ \ldots;$ 4) $\sqrt{2},\ \sqrt{2}-3,\ \sqrt{2}-6,\ \ldots.$

463. n- ag'zasi'ni'n' $a_n = -2(1 - n)$ formulasi' menen berilgen izbe-izlik arifmetikali'q progressiya bolatug'i'ni'n da'lillen'.

464. Yeger arifmetikali'q progressiyada:

1) $a_1 = 6,\ d = \frac{1}{2}$ bolsa, a_5 ti; 2) $a_1 = -3\frac{1}{3},\ d = -\frac{1}{3}$ bolsa, a_7 di;

3) $a_1 = 4{,}8,\ d = 1{,}2$ bolsa, a_{11} di yesaplan'.

465. Yeger arifmetikali'q progressiyada:

1) $a_1 = -1,\ a_2 = 1;$ 2) $a_1 = 3,\ a_2 = -3;$ 3) $a_3 = -2,\ a_5 = 6$ bolsa, woni'n' da'slepki jigirma ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

466. Yeger arifmetikali'q progressiyada:

1) $a_1 = -2,\ a_n = -60,\ n = 10;$ 2) $a_1 = \frac{1}{2},\ a_n = 25\frac{1}{2},\ n = 11$ bolsa, woni'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

467. Yeger:

1) $-38 + (-33) + (-28) + \ldots + 12;$ 2) $-17 + (-14) + (-11) + \ldots + 13$ qosi'ndi'ni'n' qosi'li'wshi'lari' arifmetikali'q progressiyani'n' izbe-iz ag'zalari' bolsa, usi' qosi'ndi'ni' tabi'n'.

468. Geometriyali'q progressiyani'n' bo'limin tabi'n' ha'm de woni'n' to'rtinshi ha'm besinshi ag'zalari'n jazi'n':

1) $3, 1, \frac{1}{3}, ...;$ 2) $\frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, ...;$ 3) $3, \sqrt{3}, 1, ...;$

4) $5, -5\sqrt{2}, 10, ...;$ 5) $16, 4, 1, ...;$ 6) $8, -4, 2, ...$

469. Geometriyali'q progressiyani'n' n-ag'zasi' formulasi'n jazi'n':

1) $-2, 4, -8, ...;$ 2) $-\frac{1}{2}, 1, -2, ...;$ 3) $-27, -9, -3....$

470. Yeger geometriyali'q progressiyada:

1) $b_1 = 2, q = 2, n = 6;$ 2) $b_1 = \frac{1}{8}, q = 5, n = 4;$

3) $b_1 = -8, q = \frac{1}{2}, n = 5$ bolsa, b_n di tabi'n'.

471. Yeger geometriyali'q progressiyada:

1) $b_1 = \frac{1}{2}, q = -4, n = 5;$ 2) $b_1 = 2, q = -\frac{1}{2}, n = 10;$

3) $b_1 = 10, q = 1, n = 6;$ 4) $b_1 = 5, q = -1, n = 9$

bolsa, woni'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

472. Geometriyali'q progressiyani'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si'n tabi'n':

1) $128, 64, 31, ..., n = 6;$ 2) $162, 54, 18, ..., n = 5;$

3) $\frac{2}{3}, \frac{1}{2}, \frac{3}{8}, ..., n = 5;$ 4) $\frac{3}{4}, \frac{1}{2}, \frac{1}{3}, ..., n = 4.$

473. Berilgen geometriyali'q progressiya sheksiz kemeyiwshi yekenligin da'lillen' ha'm woni'n' qosi'ndi'si'n tabi'n':

1) $-\frac{1}{2}, -\frac{1}{4}, -\frac{1}{8}, ...;$ 2) $-1, \frac{1}{4}, -\frac{1}{16}, ...;$ 3) $7, 1, \frac{1}{7}, ...$

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474. Yeger arifmetikali'q progressiyada $a_1 = 2\frac{1}{2}$ ha'm $a_8 = 23\frac{1}{2}$ bolsa, woni'n' ayi'rmasi'n tabi'n'.

475. Yeger arifmetikali'q progressiyada:

1) $a_1 = 5, a_3 = 15;$ 2) $a_3 = 8, a_5 = 2;$ 3) $a_2 = 18, a_4 = 14$

bolsa, woni'n' da'slepki bes ag'zasi'n jazi'n'.

476. -10 ha'm 5 sanlari' arasi'na bir san jazi'n', na'tiyjede arifmetikali'q progressiyani'n' izbe-iz u'sh ag'zasi' payda bolsi'n.

477. Yeger arifmetikali'q progressiyada: 1) $a_{13} = 28, a_{20} = 38;$ 2) $a_{18} = -6, a_{20} = 6;$ 3) $a_6 = 10, a_{11} = 0$ bolsa, woni'n' won tog'i'zi'nshi' ha'm birinshi ag'zalari'n tabi'n'.

WO'ZIN'IZDI TEKSERIP KO'RIN'!

1. Arifmetikali'q progressiyada: 1) $a_1 = 2$, $d = -3$; 2) $a_1 = -7$, $d = 2$ bolsa, a_{10} ha'm da'slepki won ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

2. Geometriyali'q progressiyada:1) $b_1 = 4, q = \frac{1}{2}$; 2) $b_1 = \frac{1}{9}$, $q = 3$ bolsa, b_6 ha'm da'slepki alti' ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

3. 1) $1, \frac{1}{3}, \frac{1}{9}, \ldots$; 2) $128, 32, 8 \ldots$, izbe-izligi sheksiz kemeyiwshi geometriyali'q progressiya yekenligin da'lillen' ha'm woni'n' ag'zalari'ni'n' qosi'ndi'si'n tabi'n'.

478. x ti'n' qanday ma'nislerinde:

1) $3x$, $\frac{x+2}{2}$, $2x-1$; 2) $3x^2$, 2, $11x$; 3) x^2, $10x$, 25

sanlari' arifmetikali'q progressiyani'n' izbe-iz ag'zalari' boladi'?

479. To'mendegi sanlar arifmetikali'q progressiyani'n' izbe-iz u'sh ag'zasi' boli'wi'n ko'rsetin':

1) $\sin(\alpha+\beta)$, $\sin\alpha\cos\beta$, $\sin(\alpha-\beta)$;
2) $\cos(\alpha+\beta)$, $\cos\alpha\cos\beta$, $\cos(\alpha-\beta)$;
3) $\cos2\alpha$, $\cos^2\alpha$, 1; 4) $\sin5\alpha$, $\sin3\alpha\cos2\alpha$, $\sin\alpha$.

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480. Qosi'ndi' 252 ge ten' boli'wi' ushi'n 5 ten baslap neshe izbe-iz taq sandi' qosi'w kerek?

481. Yeger arifmetikali'q progressiyada:

1) $a_1 = 40$, $n = 20$, $S_{20} = -40$; 2) $a_1 = \frac{1}{3}$, $n = 16$, $S_{16} = -10\frac{2}{3}$
3) $a_1 = -4$, $n = 11$, $S_{11} = 231$ bolsa, a_n ha'm d ni' tabi'n'.

482. Geometriyali'q progressiyada:

1) yeger $b_1 = 4$ ha'm $q = -1$ bolsa, b_9 di' yesaplan';
2) yeger $b_1 = 1$ ha'm $q = \sqrt{3}$ bolsa, b_7 ni yesaplan'.

483. Yeger geometriyali'q progressiyada: 1) $b_2 = \frac{1}{2}$, $b_7 = 16$; 2) $b_3 = -3$, $b_6 = -81$; 3) $b_2 = 4$, $b_4 = 1$; 4) $b_4 = -\frac{1}{5}$, $b_6 = -\frac{1}{125}$ bolsa, woni'n' birinshi ag'zasi'n tabi'n'.

484. 4 ha'm 9 sanlari' arasi'na sonday bir won' sandi' qoyi'n', na'tiyjede geometriyali'q progressiyani'n' izbe-iz u'sh ag'zasi' payda bolsi'n.

485. Yeger izbe-izliktin' n- ag'zasi': 1) $b_n = 5^{n+1}$; 2) $b_n = (-4)^{n+2}$; 3) $b_n = \frac{10}{7^n}$; 4) $b_n = -\frac{50}{3^{n+3}}$ formulasi' menen berilgen, wol sheksiz kemeyiwshi geometriyali'q progressiya bola ma?

486. Geometriyali'q progressiyada: 1) $b_2 = -81$, $S_2 = 162$; 2) $b_2 = 33$, $S_2 = 67$; 3) $b_1 + b_3 = 130$, $b_1 - b_3 = 120$; 4) $b_2 + b_4 = 68$, $b_2 - b_4 = 60$ bolsa, wol sheksiz kemeyiwshi yekenligin ko'rsetin'.

487. Dem ali'wshi' shi'pakerdin' usi'ni'si' boyi'nsha birinshi ku'ni Quyash nuri'na 5 minut taplandi', al keyingi ha'r ku'n sayi'n taplani'wdi' 5 minutqa artti'ri'p bardi'. Yeger wol taplani'wdi' sa'rsembi ku'ninen baslag'an bolsa, ha'ptenin' qaysi' ku'ni woni'n' Quyashta taplani'wi' 40 minutqa ten' boladi'?

488. Yeger arifmetikali'q progressiyada $a_1 + a_2 + a_3 = 15$ ha'm $a_1 \cdot a_2 \cdot a_3 = 80$ bolsa, woni'n' birinshi ag'zasi'n ha'm ayi'rmasi'n tabi'n'.

489. Yeger arifmetikali'q progressiyada $a_1 + a_2 + a_3 = 0$ ha'm $a_1^2 + a_2^2 + a_3^2 = 50$, woni'n' ag'zasi'n ha'm ayi'rmasi'n tabi'n'.

490. Saat 1 de 1 ma'rte, 2 de 2 ma'rte, ... 12 de 12 ma'rte ses shi'g'aradi'. Al saat tili na'wbettegi ha'rbir saatti'n' yari'mi'n ko'rsetkende bir ma'rte ses shi'g'aradi'. Bul saat bir sutka dawami'nda neshe ma'rte ses shi'g'aradi'?

VI bapqa tiyisli si'naq (test) shi'ni'g'i'wlari'

1. Arifmetikali'q progressiyada $a_1 = 3$, $d = -2$. S_{101} di tabi'n'.
A) −9797; B) −9798; C) −7979; D) −2009.

2. Arifmetikali'q progressiyada $d = 4$, $S_{50} = 5000$ bolsa, a_1 di tabi'n'.
A) −2; B) 2; C) 100; D) 1250.

3. Arifmetikali'q progressiyada $a_1 = 1$, $a_{101} = 301$, d ni' tabi'n'.
A) 4; B) 2; C) 3; D) 3,5.

4. Arifmetikali'q progressiyada $a_2 + a_9 = 20$ bolsa, S_{10} di' tabi'n'.
A) 90; B) 110; C) 200; D) 100.

5. 8 ge bo'lgende 7 qaldi'q shi'g'atug'i'n izbe-izliktin' 5-ag'zasi'n belgilen'.

A) 47; B) 55; C) 39; D) 63.

6. 701 sani' 1, 8, 15, 22, ... progressiyani'n' neshinshi nomerli ag'zasi'?

A) 101; B) 100; C) 102; D) 99.

7. 1002, 999, 996, ... progressiyani'n' neshinshi nomerli ag'zasi'nan baslap, woni'n' ag'zalari' teris sanlar boladi'?

A) 335; B) 336; C) 337; D) 334.

8. Arifmetikali'q progressiyada $a_2 + a_6 = 44$, $a_5 - a_1 = 20$, a_{100} di tabi'n'.

A) 507; B) 495; C) 502; D) 595.

9. Arifmetikali'q progressiyada $a_1 = 7$, $d = 5$, $S_n = 25450$ bolsa, n di tabi'n'.

A) 99; B) 101; C) 10; D) 100.

10. Arifmetikali'q progressiyada $a_{12} + a_{15} = 20$, S_{26} ni' tabi'n'.

A) 260; B) 270; C) 520; D) 130.

11. 1 ha'm 11 sanlari' arasi'nda sonday 99 sandi' jaylasti'ri'n', na'tiyjede wolar bul sanlar menen birgelikte arifmetikali'q progressiya payda yetsin. Bul progressiya ushi'n S_{50} di tabi'n'.

A) $172\frac{1}{2}$; B) 495; C) 300; D) 178.

12. Arifmetikali'q progressiyada $a_1 = -20{,}7$, $d = 1{,}8$, qaysi' nomerli ag'zadan baslap progressiyani'n' barli'q ag'zalari' won' boladi'?

A) 18; B) 13; C) 12; D) 15.

13. 7 ge yeseli da'slepki neshe natural sandi' qosqanda 385 kelip shi'g'adi'?

A) 12; B) 11; C) 10; D) 55.

14. Geometriyali'q progressiyada $b_1 = 2$, $q = 3$ bolsa, S_6 ni' tabi'n'.

A) 1458; B) 729; C) 364; D) 728.

15. Geometriyali'q progressiyada $q = \frac{1}{3}$, $S = 364$ bolsa, b_1 di tabi'n'.

A) $242\frac{2}{3}$; B) 81; C) $121\frac{1}{3}$; D) 240.

16. Geometriyali'q progressiyada $S_4 = 10\frac{5}{8}$, $S_5 = 42\frac{5}{8}$, $b_1 = \frac{1}{8}$ bolsa, q di' tabi'n'. A) 4; B) 2; C) 8; D) $\frac{1}{2}$.

17. Geometriyali'q progressiyada 6 ag'za bar. Da'slepki 3 ag'zasi'ni'n' qosi'ndi'si' 26 g'a, keyingi 3 ag'zasi'ni'n' qosi'ndi'si' bolsa 702 ge ten'. Progressiya bo'limin tabi'n'.

A) 4; B) 3; C) $\frac{1}{3}$; D) $2\sqrt{3}$.

18. Sheksiz kemeyiwshi geometriyali'q progressiyada $b_1 = \frac{1}{4}$, $S = 16$ bolsa, q di' tabi'n'.

A) $\frac{1}{2}$; B) $\frac{64}{65}$; C) $\frac{63}{64}$; D) $\frac{1}{4}$.

19. Geometriyali'q progressiyada $q = \frac{\sqrt{3}}{2}$, $b_1 = 2 - \sqrt{3}$, S ti tabi'n'.

A) $2+\sqrt{3}$; B) 3; C) $\frac{2\sqrt{3}}{3}$; D) 2.

Tariyxi'y ma'seleler

1. *Beruniy ma'selesi.* Yeger ag'zalari' won' bolg'an geometriyali'q progressiyani'n': ag'zalar sani' taq bolsa, wonda $b_{k+1}^2 = b_1 \cdot b_{2k+1}$; ag'zalar sani' jup bolsa $b_k \cdot b_{k+1} = b_1 \cdot b_{2k}$ bolatug'i'ni'n da'lillen'.
2. *Axmes papirusi'nan ali'ng'an ma'sele (bizin' erami'zg'a shekemgi 2000-ji'llar)*. 10 wo'lshem g'a'lleni 10 adamg'a sonday yetip u'lestirin', na'tiyjede bul adamlardan biri menen wonnan keyingisi (yaki aldi'ng'i'si') alg'an g'a'llenin' parqi' $\frac{1}{8}$ wo'lshemge ten' bolsi'n.

Tariyxi'y mag'luwmatlar

«A'yyemgi xali'qlardan qalg'an yestelikler» miynetinde Abu Rayxan Beruniy shaxmat woyi'ni'ni'n' da'slepki woylap tabi'li'wi' haqqi'ndag'i' rawiyat penen baylani'sli' birinshi ag'zasi' $b_1 = 1$ ha'm bo'limi $q = 2$ bolg'an geometriyali'q progressiyani'n' da'slepki 64 ag'zasi'ni'n' qosi'ndi'si'n yesaplaydi'; shaxmat taxtasi'ndag'i' k-ko'zgenekke sa'ykes sannan 1 sani' ali'nsa, ayi'rma k-ko'zgenekten aldi'ng'i' barli'q ko'zgeneklerge sa'ykes sanlardi'n' qosi'ndi'si'na ten' bolatug'i'ni'n ko'rsetedi, yag'ni'y $q^k - 1 = 1 + q + q^2 + \ldots + q^{k-1}$ yekenin da'lilleydi.

IX KLASS «ALGEBRA» KURSI'N TA'KIRARLAW USHI'N SHI'NI'G'I'WLAR

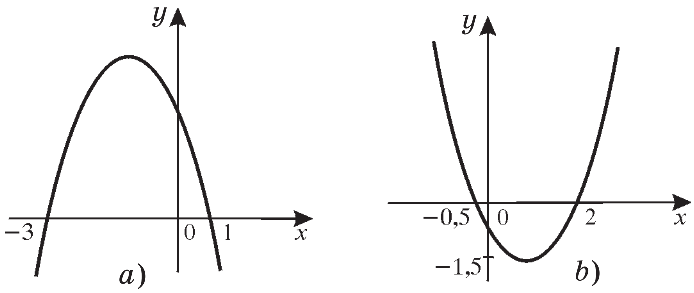
491. Funkciyani'n' grafigin jasan':

1) $y = x^2 + 6x + 9;$ 2) $y = x^2 - \frac{7}{2};$ 3) $y = x^2 - 12x + 4;$

4) $y = x^2 + 3x - 1;$ 5) $y = x^2 + x;$ 6) $y = x^2 - x;$

7) $y = (x-2)(x+5);$ 8) $y = \left(x + \frac{1}{8}\right)(x+4).$

492. (Awi'zeki.) $y = ax^2 + bx + c$ funkciyasi' grafiginen paydalani'p (82-su'wret), woni'n' qa'siyetlerin ani'qlan'.



82- su'wret.

493. Funkciyani'n' grafigin jasan' ha'm qa'siyetlerin ani'qlan':

1) $y = -2x^2 - 8x - 8;$ 2) $y = 3x^2 + 12x + 16;$

3) $y = 2x^2 - 12x + 19;$ 4) $y = 3 + 2x - x^2;$

5) $y = -4x^2 - 4x;$ 6) $y = 12x - 4x^2 - 9.$

494. Funkciyani'n' grafigin bir koordinata tegisliginde jasan':

1) $y = \frac{1}{3}x^2$ ha'm $y = -\frac{1}{3}x^2;$ 2) $y = 3x^2$ ha'm $y = 3x^2 - 2;$

3) $y = -\frac{1}{2}x^2$ ha'm $y = -\frac{1}{2}(x+3)^2;$ 4) $y = 2x^2$ ha'm $y = 2(x-5)^2 + 3.$

Ten'sizlikti sheshin' **(495–499)**:

495. 1) $(x-5)(x+3) > 0;$ 2) $(x+15)(x+4) < 0;$

3) $(x-7)(x+11) \le 0;$ 4) $(x-12)(x-13) \ge 0.$

496. 1) $x^2 + 3x > 0$; 2) $x^2 - x\sqrt{5} < 0$; 3) $x^2 - 16 \le 0$;
4) $x^2 - 3 > 0$; 5) $x^2 - 4x \le 0$; 6) $x^2 - 7 \ge 0$.

497. 1) $x^2 - 8x + 7 > 0$; 2) $x^2 + 3x - 54 < 0$;
3) $\frac{1}{2}x^2 + 0,5x - 1 > 0$; 4) $5x^2 + 9,5x - 1 < 0$;
5) $-x^2 - 3x + 4 > 0$; 6) $-8x^2 + 17x - 2 \le 0$.

498. 1) $x^2 - 6x + 9 > 0$; 2) $x^2 - 24x + 144 \le 0$;
3) $\frac{1}{2}x^2 - 4x + 8 < 0$; 4) $\frac{1}{3}x^2 + 4x + 12 \ge 0$;
5) $4x^2 - 4x + 1 > 0$; 6) $5x^2 + 2x + \frac{1}{5} < 0$.

499. 1) $x^2 - 10x + 30 > 0$; 2) $-x^2 + x - 1 < 0$;
3) $x^2 + 4x + 5 < 0$; 4) $2x^2 - 4x + 13 > 0$;
5) $4x^2 - 9x + 7 < 0$; 6) $-11 + 8x - 2x^2 < 0$.

Ten'sizlikti intervallar usi'li' menen sheshin' **(500–502):**

500. 1) $(x+3)(x-4) > 0$; 2) $\left(x - \frac{1}{2}\right)(x + 0,7) < 0$;
3) $(x - 2,3)(x + 3,7) < 0$; 4) $(x+2)(x-1) \le 0$.

501. 1) $(x+2)(x-1) \ge 0$; 2) $(x+2)(x-1)^2 \le 0$;
3) $(x+2)(x-1)^2 > 0$; 4) $(2-x)(x+3x)^2 \ge 0$.

502. 1) $\frac{3-x}{2+x} \ge 0$; 2) $\frac{0,5+x}{x-2} \le 0$; 3) $\frac{(x-1)(x+2)}{x} < 0$;
4) $\frac{2x}{(3+x)(1-x)} < 0$; 5) $\frac{x+3}{4-x} \ge 0$; 6) $\frac{x(x+1)}{1-x} > 0$.

503. Trapeciyani'n' maydani' 19,22 sm^2 tan arti'q. Woni'n' worta si'zi'g'i' biyikliginen yeki ma'rte u'lken. Trapeciyani'n' worta si'zi'g'i'n ha'm biyikligin tabi'n'.

504. 320 m den arti'q biyiklikte ushi'p barati'rg'an samolyottan geologlarg'a ju'k taslap jiberildi. Ju'k qansha waqi'tta jerge kelip tu'sedi? Yerkin tu'siw tezleniwi 10 m/s^2 qa ten' dep qabi'l yetin'.

505. Parallelogrammni'n' ta'repi usi' ta'repke tu'sirilgen biyiklikten 2 sm arti'q. Yeger parallelogrammni'n' maydani' 15 sm^2 tan arti'q bolsa, usi' ta'reptin' uzi'nli'g'i'n tabi'n'.

506. Ten'sizlikti intervallar usi'li' menen sheshin':

1) $(x+2)(x+5)(x-1)(x+4)>0$;

2) $(x+1)(3x^2+2)(x-2)(x+7)<0$;

3) $\frac{3x-1}{3x+1}+\frac{x-3}{x+3}\geq 2$; 4) $\frac{1-3x}{1+3x}+\frac{1+3x}{3x-1}\geq\frac{12}{1-9x^2}$.

507. x^2+px+q kvadrat u'sh ag'zali' $x=0$ bolg'anda -14 ke ten' ma'nisti, $x=-2$ bolg'anda -20 g'a ten' ma'nisti qabi'l yetken, usi' kvadrat u'sh ag'zali'ni'n' p ha'm q koefficientin tabi'n'.

508. Yeger $y=x^2+px+q$ parabola:

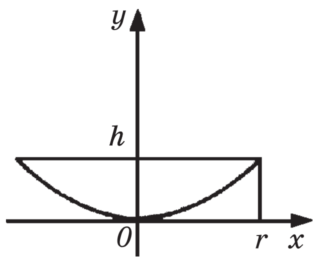
1) abcissa ko'sherin $x=-\frac{1}{2}$ ha'm $x=\frac{2}{3}$ noqatlari'nda kesip wo'tse;

2) abcissa ko'sheri menen $x=-7$ noqati'nda uri'nsa;

3) abcissa ko'sherin $x=2$ ha'm ordinata ko'sherin $y=-1$ noqati'nda kesip wo'tse, $p-q$ di tabi'n'.

509. Yeger parabola abcissalar ko'sherin 5 noqatta kesip wo'tse ha'm woni'n' to'besi $\left(2\frac{3}{4};10\frac{1}{8}\right)$ noqati'nda bolsa, usi' parabolani'n' ten'lemesin jazi'n'.

510. Teleskopti'n' (reflektor) shag'i'li'sti'ri'wshi' aynasi' ko'sher kesimi boyi'nsha parabola formasi'na iye (83-su'wret). Usi' parabolani'n' ten'lemesin jazi'n'.



83- su'wret.

511. Yeger $y=ax^2+bx+c$ kvadrat funkciyasi'ni'n' grafigi:

1) $A(-1;\ 0)$, $B(3;\ 0)$ ha'm $C(0;\ -6)$ noqatlari'nan wo'tse; 2) $K(-2;0)$, $L(1;0)$, $M(0;\ 2)$ noqatlari'na wo'tse, woni'n' koefficientlerin tabi'n'.

512. Qa'legen teris yemes a ha'm b sanlari' ushi'n:

1) $a^2+b^2\leq(a+b)^2$; 2) $a^3+b^3\leq(a+b)^3$;

3) $a^3+b^3\geq a^2b+ab^2$; 4) $(a+b)^3\leq 4(a^3+b^3)$

ten'sizliginin' duri's bolatug'i'ni'n da'lillen'.

513. Qa'legen won' a, b, c sanlari' ushi'n

1) $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq 3$; 2) $\frac{bc}{a}+\frac{ac}{b}+\frac{ab}{c}\geq a+b+c$;

3) $\frac{a^3+b^3+c^3}{a^2+b^2+c^2}\geq\frac{a+b+c}{3}$; 4) $\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\geq\frac{3}{2}$

ten'sizliginin' duri's bolatug'i'ni'n da'lillen'.

514. Funkciyani'n' grafigin jasan':

1) $y=\sqrt{x^2}$; 2) $y=|x-1|$;

3) $y=\sqrt{x^2-6x+9}$; 4) $y=\sqrt{x^2+4x+4}$;

5) $y=\sqrt{(x-1)^2}+\sqrt{x+1)^2}$; 6) $y=\sqrt{x^2-4x+4}+|x+2|$.

515. Ten'lemenin' haqi'yqi'y korenlerin tabi'n':

1) $x^2-|x|-2=0$; 2) $x^2-4|x|+3=0$; 3) $|x^2-x|=2$;
4) $|x^2+x|=1$; 5) $|x^2-2|=2$; 6) $|x^2-26|=10$.

516. Sanlardi' koren belgisi asti'nan shi'g'ari'n':

1) $\sqrt[5]{7\frac{19}{32}}$; 2) $\sqrt{5\frac{4}{9}}$; 3) $\sqrt[3]{\frac{8b^6}{343a^9}}$, $a\neq 0$; 4) $\sqrt[4]{\frac{16x^8}{81y^4}}$, $y>0$.

517. A'piwayi'lasti'ri'n':

1) $\left(3\sqrt{20}+7\sqrt{15}-\sqrt{5}\right):\sqrt{5}$; 2) $\left(\sqrt[3]{7}-\sqrt[3]{14}+\sqrt[3]{56}\right):\sqrt[3]{7}$;

3) $2\sqrt{\frac{3}{2}}+\sqrt{6}-3\sqrt{\frac{2}{3}}$; 4) $7\sqrt{1\frac{3}{4}}-\sqrt{7}+0{,}5\sqrt{343}$.

518. An'latpalardi'n' ma'nislerin sali'sti'ri'n':

1) $\left(\frac{\sqrt{5}}{3}\right)^{-1/3}$ ha'm $\left(\frac{\sqrt{5}}{3}\right)^{-1/2}$; 2) $(2\sqrt{0{,}5})^{0{,}3}$ ha'm $(2\sqrt{0{,}5})^{0{,}37}$.

519. An'latpani' a'piwayi'lasti'ri'n':

1) $\frac{\sqrt[6]{a\sqrt[3]{a^{-1}}}}{a^{-\frac{2}{9}}}$; 2) $\frac{\sqrt[4]{x^3\sqrt[3]{x}}}{x^{\frac{1}{3}}}$; 3) $(16a^{-4})^{-\frac{3}{4}}$; 4) $(27b^{-6})^{\frac{2}{3}}$.

520. Koren belgisi asti'nan ko'beytiwshini shi'g'ari'n':

1) $\sqrt{9a^2b}$, bunda $a<0$, $b>0$; 2) $\sqrt{25a^2b^3}$, bunda $a>0$, $b>0$;
3) $\sqrt{8a^3b^5}$, bunda $a<0$, $b<0$; 4) $\sqrt{12a^3b^3}$, bunda $a<0$, $b<0$.

521. Ko'beytiwshini koren belgisi asti'na kirgizin':

1) $x\sqrt{5}$, bunda $x \geq 0$; 2) $x\sqrt{3}$, bunda $x < 0$;

3) $-a\sqrt{3}$, bunda $a \geq 0$; 4) $-a\sqrt{5}$, bunda $a < 0$.

522. Yesaplan':

1) $\sqrt[3]{1000}\cdot(0{,}0001)^{0{,}25}+(0{,}027)^{\frac{1}{3}}\cdot 7{,}1^{0}-\left(\frac{10}{13}\right)^{-1}$;

2) $\left(2\frac{10}{27}\right)^{-\frac{2}{3}}:\frac{1}{\sqrt{11\frac{1}{9}}}+(6{,}25)^{\frac{1}{2}}:(-4)^{-1}$; 3) $\left(1\frac{61}{64}\right)^{-\frac{2}{3}}\cdot(0{,}008)^{\frac{1}{3}}:(-2)^{-2}$.

523. An'latpani'n' ma'nisin tabi'n':

1) $\left(\frac{a^{\frac{1}{2}}}{a^{\frac{1}{2}}-b^{\frac{1}{2}}}-\frac{a^{\frac{1}{2}}b^{\frac{1}{2}}}{a-b}\right)\cdot\frac{a-2a^{\frac{1}{2}}b^{\frac{1}{2}}+b}{a}$, bunda $a=3$, $b=12$.

2) $\frac{m+2\sqrt{mn}+n}{n}\cdot\frac{\sqrt{mn}+n}{m-n}-\frac{\sqrt{m}}{\sqrt{m}+\sqrt{n}}$, bunda $m=5$, $n=20$.

524. Ten'lemeni sheshin':

1) $x^{\frac{1}{2}}=2$; | 2) $x^{-\frac{1}{2}}=3$; | 3) $x^{-3}=8$; | 4) $x^{\frac{5}{2}}=0$; | 5) $x^{-\frac{1}{3}}=27$.

525. $y=-\frac{25}{x}$ funkciyasi'ni'n' grafigine:

1) $A(\sqrt{5};-5\sqrt{5})$; 2) $B(-5\sqrt{2};5\sqrt{2})$; 3) $C(0{,}1;\ 250)$

noqati' tiyisli boli'wi' yaki bolmaytug'i'ni'n ani'qlan'.

526. $y=\sqrt{1-2x}$ funkciyasi'ni'n' grafigine 1) $C\left(\frac{1}{4};\frac{\sqrt{2}}{2}\right)$; 2) $D\left(-\frac{1}{2};1\right)$;

3) $E\,(-4;3)$ noqati' tiyisli boli'wi' yaki bolmaytug'i'ni'n ani'qlan'.

527. Funkciyani'n' ani'qlani'w oblasti'n tabi'n':

1) $y=\sqrt{-x^{2}-3x+10}$; 2) $y=\sqrt[4]{\frac{x-7}{3-2x}}$; 3) $y=\sqrt[3]{\frac{x+4}{6-x}}$;

4) $y=\sqrt[6]{\frac{2x+15}{6}}$; 5) $y=\sqrt[5]{\frac{x}{0,5x+1}}$; 6) $y=\frac{\sqrt{x}}{x^{2}-4}$.

528. Funkciyani'n' grafigin jasan':

1) $y = x^2 + 6x + 10$; 2) $y = -x^2 - 7x - 6$; 3) $y = \frac{4}{x}$;

4) $y = -\frac{6}{x}$; 5) $y = \frac{x^2}{2}$; 6) $y = \frac{1}{4}x^4$.

Qaysi' arali'qlarda funkciyani'n' wo'siwin, kemeyiwin grafik boyi'nsha ani'qlan'; funkciyani'n' jup yaki taq yekenligin ani'qlan':

529. $P(1;\ 0)$ noqati'n: 1) $A(0;\ 1)$; 2) $B)$ $(0;\ -1)$; 3) $C(-1;\ 0)$; 4) $D(1;0)$ noqati'na wo'tkeretug'i'n birneshe buri'w mu'yeshlerin ko'rsetin'.

530. Yesaplan': 1) $\dfrac{\sin\frac{\pi}{4} + \cos\frac{\pi}{3} - \mathrm{tg}\frac{\pi}{3}}{\mathrm{ctg}\frac{\pi}{6} - \sin\frac{\pi}{6} - \cos\frac{\pi}{4}}$; 2) $\dfrac{\cos\frac{\pi}{4} + \sin\frac{\pi}{6} - \mathrm{tg}\frac{\pi}{4}}{\mathrm{ctg}\frac{\pi}{4} - \cos\frac{\pi}{3} - \sin\frac{\pi}{4}}$.

531. Sanni'n' won' yaki teris yekenligin ani'qlan':

1) $\sin\frac{\pi}{5}\sin\frac{4\pi}{5}\cos\frac{\pi}{6}$; 2) $\sin\alpha\cos(\pi + \alpha)\mathrm{tg}\alpha,\ 0 < \alpha < \frac{\pi}{2}$.

532. Berilgeni: $\sin\alpha = 0{,}6,\ \sin\beta = -0{,}28,\ 0 < \alpha < \frac{\pi}{2}, \pi < \beta < \frac{3\pi}{2}$.

Yesaplan': 1) $\cos(\alpha - \beta)$; 2) $\sin(\alpha + \beta)$; 3) $\cos(\alpha + \beta)$.

533. Ko'beytiwshilerge jiklen':

1) $\sin 2\alpha - 2\sin\alpha$; 2) $\sin\alpha + \sin\frac{\alpha}{2}$;

3) $\cos\alpha - \sin 2\alpha$; 4) $1 - \sin 2\alpha - \cos^2\alpha$.

534. Yeger: 1) $\cos\frac{\alpha}{2} = -\frac{8}{17}$ ha'm $\sin\frac{\alpha}{2} < 0$; 2) $\sin\frac{\alpha}{2} = -\frac{5}{13}$ ha'm $\cos\frac{\alpha}{2} < 0$ bolsa $\sin\alpha$, $\cos\alpha$, $\mathrm{tg}\alpha$ ni' yesaplan'.

535. Yeger: 1) $a_1 = 10,\ d = 6,\ n = 23$; 2) $a_1 = 42,\ d = \frac{1}{2},\ n = 12$;

3) $a_1 = 0,\ d = -2,\ n = 7$; 4) $a_1 = \frac{1}{3},\ d = \frac{2}{3},\ n = 18$ bolsa, arifmetikali'q progressiyani'n' n- ag'zasi'n ha'm da'slepki n ag'zasi'ni'n' qosi'ndi'si'n yesaplan'.

536. Yeger $a_1 = 2,\ a_n = 120,\ n = 20$ bolsa, arifmetikali'q progressiyani'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

537. n- ag'zasi' $a_n = \frac{1-2n}{3}$ formulasi' menen berilgen izbe-izlik arifmetikali'q progressiya bolatug'i'ni'n da'lillen'.

538. Yeger geometriyali'q progressiya ushi'n:
1) $b_1 = 5$ ha'm $q = -10$ bolsa, b_4 ti tabi'n';
2) $b_4 = -5000$ ha'm $q = -10$ bolsa, b_1 di tabi'n';

539. Yeger: 1) $b_1 = 3,\ q = 2,\ n = 5$; 2) $b_1 = 1,\ q = 5,\ n = 4$;
3) $b_1 = 8,\ q = \frac{1}{4},\ n = 4$; 4) $b_1 = 1,\ q = -3,\ n = 5$
bolsa, geometriyali'q progressiyani'n' n- ag'zasi'n ha'm da's-lepki n ag'zasi'ni'n' qosi'ndi'si'n yesaplan'.

540. Yeger: 1) $b_1 = \frac{1}{4},\ q = 2,\ n = 6$; 2) $b_1 = \frac{1}{5},\ q = -5,\ n = 5$
bolsa, geometriyali'q progressiyani'n' da'slepki n ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

541. Sheksiz kemeyiwshi geometriyali'q progressiyani'n' qosi'ndi'si'n tabi'n'.
1) $6, 4, \frac{8}{3}, \ldots$; 2) $5, -1, \frac{1}{5}, \ldots$; 3) $1, -\frac{1}{4}, \frac{1}{16}, \ldots$,
4) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ 5) $\sqrt{2}, 1, \frac{\sqrt{2}}{2}, \ldots$; 6) $-\sqrt{5}, -1, -\frac{\sqrt{5}}{5}, \ldots$

542. Koren belgisi asti'nan ko'beytiwshini shi'g'ari'n':
1) $\sqrt{20a^4b}$, bunda $a < 0,\ b > 0$;
2) $\sqrt[3]{8a^3b^4}$, bunda $a < 0,\ b > 0$;
3) $\sqrt{(a-1)^2}$, bunda $a < 1$; 4) $\sqrt{(3+a)^2}$, bunda $a > -3$;
5) $\sqrt[4]{32a^8 8b^6}$, $a < 0$; $b > 0$; 6) $\sqrt[3]{27a^9b^5}$, $a > 0$; $b < 0$.

543. An'latpani' a'piwayi'lasti'ri'n':
1) $\frac{\sqrt{(a-b)^2}}{a-b}$, bunda $a > b$; 2)
3) $\frac{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{\sqrt{x^2+x+1}}$, bunda $x > 0$; 4) $\frac{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{\sqrt{x^2+x+1}}$, bunda $x < 0$.

544. Ten'liklerden qaysi'si' duri's:

$\sqrt{7-4\sqrt{3}}=2-\sqrt{3}$ ma yaki $\sqrt{7-4\sqrt{3}}=\sqrt{3}-2$ ma?

545. Bo'limindegi irracionalli'qti' joq yetin':

1) $\frac{1}{2+\sqrt[3]{3}}$; 2) $\frac{1}{\sqrt{a}-\sqrt[4]{b}}$; 3) $\frac{1}{\sqrt[3]{3}-\sqrt[3]{2}}$; 4) $\frac{2}{\sqrt{5}+\sqrt{5}}$.

546. An'latpani' a'piwayi'lasti'ri'n':

1) $\frac{\sqrt{ab}\,\sqrt[4]{a}}{(a+2)\sqrt[4]{a^{-1}b^2}}-\frac{a^2+4}{a^2-4}$; 2) $\left(\frac{\sqrt{a}}{b+\sqrt{ab}}-\frac{\sqrt{a}}{b-\sqrt{ab}}\right)\cdot\frac{b-a}{2\sqrt{ab}}$;

3) $\left(\frac{a-b}{a^{\frac{3}{4}}+a^{\frac{1}{2}}b^{\frac{1}{4}}}-\frac{a^{\frac{1}{2}}-b^{\frac{1}{2}}}{a^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)\cdot\frac{a^{\frac{1}{4}}+b^{\frac{1}{4}}}{(a^{-1}b)^{\frac{1}{2}}}$; 4) $\left(\frac{a^{\frac{3}{2}}+b^{\frac{3}{2}}}{a-b}-\frac{a-b}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}\right)\cdot\frac{a-b}{\sqrt{ab}}$.

547. $y=\frac{4}{x^2}$ funkciyani'n' $x>0$ arali'qta wo'siwin yaki kemeyiwin ani'qlan'.

548. Funkciyani'n' ani'qlani'w oblasti'n tabi'n':

1) $y=\sqrt{(x-2)(x-3)}$; 2) $y=\sqrt{(x^2-6x}$; 3) $y=\frac{1}{x^2-2\sqrt{2}x+2}$;

4) $y=\frac{3}{2\sqrt{3}x-x^2+3}$; 5) $y=\sqrt{\frac{(x-1)x}{x+5}}$; 6) $y=\sqrt{\frac{x^2-9}{x^2-2x}}$.

549. Funkciyani'n' grafigin jasan' ha'm grafik boyi'nsha woni'n' tiykarg'i' qa'siyetlerin ani'qlan':

1) $y=\frac{3}{x+1}$; 2) $y=\frac{1}{2-x}$; 3) $y=\frac{x+2}{x}$;

4) $y=\frac{3-x}{x}$; 5) $y=\sqrt{x-3}$; 6) $y=\sqrt[3]{2-x}$.

550. Ten'lemeni sheshin':

1) $\sqrt{x-2}=4$; 2) $\sqrt{x+3}=8$; 3) $\sqrt{2x+1}=\sqrt{x-1}$;

4) $\sqrt{3-x}=\sqrt{1+3x}$; 5) $\sqrt[4]{x^2+12}=x$; 6) $\sqrt[3]{6x-x^2}=x$.

551. An'latpalardi' a'piwayi'lasti'ri'n':

1) $\frac{\mathrm{tg}^2\alpha}{1+\mathrm{ctg}^2\alpha}$; 2) $\frac{1+\mathrm{ctg}^2\alpha}{\mathrm{ctg}^2\alpha}$; 3) $\frac{\mathrm{tg}\alpha-\mathrm{tg}\beta}{\mathrm{ctg}\alpha+\mathrm{ctg}\beta}$;

4) $(\mathrm{tg}\alpha+\mathrm{ctg}\alpha)^2-(\mathrm{tg}\alpha-\mathrm{ctg}\alpha)^2$; 5) $(\sin\alpha-\cos\alpha)^2+2\sin\alpha\cos\alpha$.

552. An'latpalardi' a'piwayi'lasti'ri'n':

1) $\dfrac{\operatorname{ctg}\left(\alpha-\dfrac{\pi}{4}\right):\left(\sin\left(\alpha-\dfrac{3}{2}\pi\right)-\sin(\pi+\alpha\right)}{\operatorname{tg}(\pi+\alpha)(\cos(\alpha+2\pi)+\sin(\alpha-2\pi))}$;

2) $\sin(x-2\pi)\cos\left(\dfrac{3\pi}{2}-x\right)+\operatorname{tg}(\pi-x)\operatorname{tg}\left(\dfrac{3}{2}\pi+x\right)$.

553. Ten'lemeni sheshin':

1) $1-\cos x-2\sin\dfrac{x}{2}=0$; 2) $1+\cos 2x+2\cos x=0$.

554. Birdeylikti da'lillen':

1) $\dfrac{\operatorname{tg}(\alpha-\beta)+\operatorname{tg}\beta}{\operatorname{tg}(\alpha+\beta)-\operatorname{tg}\beta}=\dfrac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$; 2) $\dfrac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}=\operatorname{tg}\alpha$.

555. Birdeylikti da'lillen':

1) $1+\sin\alpha=2\cos^2\left(\dfrac{\pi}{4}-\dfrac{\alpha}{2}\right)$; 2) $1-\sin\alpha=2\sin^2\left(\dfrac{\pi}{4}-\dfrac{\alpha}{2}\right)$.

556. U'shmu'yeshliktin' ishki mu'yeshleri ayi'rmasi' $\dfrac{\pi}{6}$ g'a ten' bolg'an arifmetikali'q progressiyani'n' izbe-iz u'sh ag'zasi' boladi'. Usi' mu'yeshlerdi tabi'n'.

557. Arifmetikali'q progressiyada $a_1+a_5=\dfrac{5}{3}$; $a_3a_4=\dfrac{65}{72}$. Progressiyani'n' da'slepki won jeti ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

558. Yekinshi ag'zasi' birinshisinen 35 ke kem, u'shinshi ag'zasi' bolsa, to'rtinshiden 560 qa arti'q bolg'an geometriyali'q progressiyani'n' da'slepki to'rt ag'zasi'n tabi'n'.

559. Geometriyali'q progressiyada $q=3$, $S_6=1820$ bolsa, b_1 ha'm b_5 ti tabi'n'.

560. Sheksiz kemeyiwshi geometriyali'q progressiyani'n' qosi'ndi'si' $\dfrac{8}{5}$ ke ten', yekinshi ag'zasi' $-\dfrac{1}{2}$ ge ten'. U'shinshi ag'zasi'n tabi'n'.

561. Arifmetikali'q progressiyani'n' izbe-iz ag'zasi' bolg'an u'sh sanni'n' qosi'ndi'si' 39 g'a ten'. Yeger birinshi sannan 4 ti, yekinshisinen 5 ti, u'shinshisinen bolsa 2 ni alsaq, payda bolg'an sanlar geometriyali'q progressiyani'n' izbe-iz u'sh ag'zasi' boladi'. Bul sanlardi' tabi'n'.

An’latpani’ a’piwayi’lasti’ri’n’ **(562—563):**

562. 1) $\sqrt{5+\sqrt{21}}$; 2) $\sqrt{4+\sqrt{7}}$; 3) $\sqrt{5+2\sqrt{6}}$; 4) $\sqrt{8-2\sqrt{15}}$.

563. 1) $\frac{1}{\sqrt{5}}\left[4(a+1)+(\sqrt[3]{a\sqrt{a}}-1)^2-\left(\frac{\sqrt[6]{ab^2}+\sqrt{a}}{\sqrt[3]{a}+\sqrt[3]{b}}+\sqrt[3]{a}\right)^3\right]^{\frac{1}{2}}$, bunda $0<a\le 1$; 2) $\frac{a^{-1}b^{-2}-a^{-2}b^{-1}}{a^{-\frac{5}{3}}b^{-2}-b^{-\frac{5}{3}}a^{-2}}-a^{\frac{1}{3}}b^{\frac{1}{3}}$; 3) $\frac{a^{-2}\cdot b^{-3}-a^{-1}\cdot b^{-2}}{a^{-\frac{9}{2}}\cdot b^{-\frac{11}{2}}-a^{-\frac{11}{2}}\cdot b^{-\frac{9}{2}}}$.

564. Funkciyani’n’ grafigin jasan’:

1) $y=\frac{1}{|x-1|}$; 2) $y=\frac{3}{|x|}-1$; 3) $y=\sqrt[3]{|x|}$; 4) $y=x^2-3|x|-4$.

565. Yeger: 1) $\mathrm{tg}\frac{\alpha}{2}=-2{,}4$; 2) $\sin\frac{\alpha}{2}=\frac{12}{13}$ bolsa, $\sin\alpha$ ha’m $\cos\alpha$ ni’ yesaplan’.

566. Birdeylikti da’lillen’:

1) $\cos\left(\alpha-\frac{2\pi}{3}\right)=-\cos\left(\frac{\pi}{3}+\alpha\right)$; 2) $\cos\left(\alpha-\frac{2\pi}{3}\right)=-\cos\left(\alpha+\frac{4\pi}{3}\right)$.

567. To’mendegi u’sh qa’siyetke iye bolg’an to’rt san tabi’n’.
a) birinshi ha’m to’rtinshi sanlardi’n’ qosi’ndi’si’ 11 ge ten’, yekinshi ha’m u’shinshi sanlardi’n’ qosi’ndi’si’ bolsa 2 ge ten’;
b) birinshi, yekinshi ha’m u’shinshi sanlar arifmetikali’q progressiyani’n’ izbe-iz ag’zalari’ boladi’;
d) yekinshi, u’shinshi ha’m to’rtinshi sanlar geometriyali’q progressiyani’n’ izbe-iz ag’zalari’ boladi’.

568. S_n arifmetikali’q progressiyani’n’ da’slepki n ag’zasi’ni’n’ qosi’ndi’si’ bolsi’n. Da’lillen’:

1) $S_{n+3}=3S_{n+2}-3S_{n+1}+S_n$; 2) $S_{3n}=3(S_{2n}-S_n)$;

3) b_1, b_2, ... geometriyali’q progressiyada $b_1+b_2+\ldots+b_n=S$ ha’m $\frac{1}{b_1}+\frac{1}{b_2}+\ldots+\frac{1}{b_n}=S_1$ bolsa, $b_1\cdot b_2\cdot\ldots\cdot b_n$ ko’beymesin tabi’n’.

VII—IX KLASLARDA «ALGEBRA» KURSI'N TA'KIRARLAW USHI'N SHI'NI'G'I'WLAR

1. Sanlar ha'm algebrali'q tu'rlendiriwler

Yesaplan' **(569—570):**

569. 1) $(5{,}4\cdot1{,}2-3{,}7:0{,}8)\ (3{,}14+0{,}86):0{,}25$;

2) $(20{,}88:18+45:0{,}36):(19{,}59+11{,}95)$;

3) $\left(5\frac{8}{9}-3\frac{11}{12}\right)\cdot\frac{18}{71}-7\frac{5}{6}:15\frac{2}{3}$; 4) $\frac{7}{36}\cdot9+8\cdot\frac{11}{32}+\frac{9}{10}\cdot\frac{5}{18}$.

570. 1) $\left(3\frac{4}{25}+20{,}24\right)\cdot2{,}15+\left(5{,}1625-2\frac{3}{16}\right)\cdot\frac{2}{5}$;

2) $0{,}364:\frac{7}{25}+\frac{5}{16}:0{,}125+2{,}5\cdot0{,}8$;

3) $\dfrac{\left(3{,}25-\frac{3}{4}\right)\cdot6{,}25}{(2-0{,}75):\frac{4}{5}}+\dfrac{\left(5{,}5-3\frac{3}{4}\right):5}{(-2-0{,}8)\cdot1\frac{3}{4}}$; 4) $\dfrac{\left(2\frac{3}{20}+1\frac{5}{16}\right):27{,}7}{\left(1{,}75\cdot\frac{2}{3}-1{,}75\cdot1\frac{1}{8}\right):\frac{7}{12}}$.

571. Proporciyani'n' belgisiz ag'zasi'n tabi'n':

1) $x:7=9:3$; 2) $125:25=35:x$; 3) $144:x=36:3$;

4) $9\frac{1}{2}:14\frac{1}{4}=x:0{,}75$; 5) $\dfrac{x}{6\frac{5}{6}}=\dfrac{3{,}9}{4{,}1}$; 6) $0{,}3:x=\frac{4}{9}:3\frac{1}{3}$.

572. Yeger:

1) $a=400,\ p=27$; 2) $a=2{,}5,\ p=120$;

3) $a=2500,\ p=0{,}2$; 4) $a=4{,}5,\ p=2{,}5$

bolsa, a sani'ni'n' p procentin tabi'n'.

573. Yeger sanni'n' p procenti b g'a ten' bolsa, usi' sanni'n' wo'zin tabi'n':

1) $p=23,\ b=690$; 2) $p=3{,}2,\ b=9{,}6$;

3) $p=125,\ b=3{,}75$; 4) $p=0{,}6,\ b=21{,}6$.

574. a sani' b sani'ni'n' qanday procentin quraydi':

1) $a=24,\ b=120$; 2) $a=4{,}5,\ b=90$;

3) $a=650,\ b=13$; 4) $a=0{,}08,\ b=0{,}48$?

575. A'mellerdi wori'nlan':

1) $(-3a^3b)(-2ab^2)(-5a^3b^7)$; 2) $35a^5b^4c : (7ab^3c)$;

3) $(-5ab^4c)^3 \cdot \left(-\frac{1}{5}a^5bc^2\right)^2$; 4) $\left(-\frac{2}{3}a^4b^3c^2\right)^3 : \left(-\frac{1}{3}a^2bc^3\right)^2$.

576. An'latpani' standart tu'rindegi ko'pag'zali' ko'rinisinde jazi'n':

1) $(x - 6)(5 + x) - x^2(x^2 - 5x + 1)$;

2) $(x + 7)(5 - x) - x^2(x^2 + 2x - 1)$;

3) $(b-3a)^2 + 8\left(a - \frac{1}{2}b\right)\left(a + \frac{1}{2}b\right)$;

4) $(3a+6)^2 + 4\left(b - \frac{1}{2}a\right)\left(b + \frac{1}{2}a\right)$.

577. An'latpani'n' san ma'nisin tabi'n':

1) $a^3 - ba^2$, bunda $a = -0{,}6$, $b = 9{,}4$;

2) $ab^2 + b^3$, bunda $a = 10{,}7$, $b = -0{,}7$;

3) $(m - 5)(2m - 3) - 2m(m - 4)$, bunda $m = \frac{3}{5}$;

4) $(3a - 2)(a - 4) - 3a(a - 2)$, bunda $a = \frac{3}{4}$.

578. A'mellerdi wori'nlan':

1) $(-15x^5 + 10x^4 - 25x^3) : (-5x^5) - 3(x - 3)(x^2 + 3x + 9)$;

2) $(9a^2b^3 - 12a^4b^4) : 3a^2b - b^2 \cdot (2 + 3a^2b)$.

Ko'beytiwshilerge jiklen' **(579–583)**:

579. 1) $1 - \frac{a^2}{4}$; | 2) $\frac{b^2}{9} - 1$; | 3) $a^2 - b^4$; | 4) $b^4 - 9$; | 5) $\frac{a^2}{16} - \frac{b^4}{4}$.

580. 1) $1 - a + \frac{a^2}{4}$; 2) $0{,}25b^2 + b + 1$;

3) $49a^2 - 14a + 1$; 4) $1 + 18b + 81b^2$.

581. 1) $y^2 - xy - y + x$; 2) $a^2 - ax - x + a$;

3) $3a^2 + 3ab + a + b$; 4) $5a^2 - 5ax - 7a + 7x$.

582. 1) $6m^4n + 12m^3n + 3m^2n$; 2) $2a^5b - 4a^4b + 2a^3b$;

3) $a^2 - 2ab + b^2 - y^2$; 4) $a^4 + 2a^2b^2 + b^4 - 4a^2b^2$.

583. 1) $x^2 + 3x - 28$; 2) $2x^2 - 12x + 18$;
3) $2x^2 - 5x + 3$; 4) $x^2 + x - 2$.

584. Bo'lshekti qi'sqarti'n':

1) $\dfrac{4-b^2}{4b+2b^2}$; 2) $\dfrac{b^2-9}{3b^2-9b}$; 3) $\dfrac{5a^2-10ab}{ab-2b^2}$; 4) $\dfrac{3xy-21y^2}{4x^2-28xy}$;

5) $\dfrac{x^2-x-12}{x^2-16}$; 6) $\dfrac{x^2-x-20}{x^2-25}$; 7) $\dfrac{3x^2-2x-8}{2x^2-3x-2}$; 8) $\dfrac{2x^2+x-3}{2x^2+7x+6}$.

An'latpani' a'piwayi'lasti'ri'n' **(585–589):**

585. 1) $\dfrac{a^5}{6c^3} : \dfrac{a^2}{4c^3}$; 2) $\dfrac{9a^2}{m^3} : \dfrac{6a^2}{m^5}$; 3) $\left(\dfrac{4a}{b^3}\right) \cdot \dfrac{b^4}{8a}$;

4) $\left(\dfrac{3c}{k^2}\right) : \dfrac{9c}{k^3}$; 5) $\dfrac{5a}{28b^2} \cdot 8ab \cdot \dfrac{7b}{5a^3}$; 6) $\left(-\dfrac{25a^4b^3}{14c^2}\right) \cdot \dfrac{-21c}{10a^3b^3}$;

7) $\dfrac{4x(x-1)+1}{4-x^2} : \dfrac{1-2x}{x-2}$; 8) $\dfrac{x^2-4(x-1)}{x-1} : \dfrac{2-x}{1-x^2}$.

586. 1) $\dfrac{a-3}{a+3} - \dfrac{a^2+27}{a^2-9}$; 2) $\dfrac{a^2+12}{a^2-4} - \dfrac{a+3}{a-2}$; 3) $\dfrac{a+1}{a^2-ax} - \dfrac{x+1}{a^2-x^2}$;

4) $\dfrac{3-a}{ab-a^2} - \dfrac{3-b}{b^2-a^2}$; 5) $\dfrac{a^3+8}{a+2} - (a+2)^2$; 6) $\dfrac{4(a+4)}{a^2-16} + \dfrac{a^2-4a}{a-4}$.

587. 1) $\dfrac{4}{a-b} + \dfrac{9}{a+b} - \dfrac{8a}{a^2-b^2}$; 2) $\dfrac{42}{4a^2-9} + \dfrac{8}{2a+3} + \dfrac{7}{3-2a}$;

3) $\left(\dfrac{a}{b} + \dfrac{b}{a} - 2\right)ab$; 4) $\left(\dfrac{1}{a} + \dfrac{1}{b} - \dfrac{1}{ab}\right)ab$.

588. 1) $\dfrac{1}{(x+3)^2} : \dfrac{x}{x^2-9} - \dfrac{x-9}{x^2-9}$; 2) $\dfrac{a+6}{a^2-4} - \dfrac{1}{a^2-4} \cdot \dfrac{(a+2)^2}{a}$;

3) $a + b - \dfrac{a^2}{a-1}$; 4) $\dfrac{a^2}{a+1} - a + 1$; 5) $\dfrac{a^2+b^2}{a+b} - (a+b).$;

589. 1) $\dfrac{b^2}{a^2-2ab} : \left(\dfrac{2ab}{a^2-4b^2} - \dfrac{b}{a+2b}\right)$; 2) $\left(\dfrac{2xy}{x^2-9y^2} - \dfrac{y}{x-3y}\right) : \dfrac{y^2}{x^2+3xy}$;

3) $\left(\dfrac{xy}{x^2-y^2} - \dfrac{y}{2x-2y}\right) : \dfrac{3y}{x^2-y^2}$; 4) $\left(\dfrac{2a+1}{2a-1} - \dfrac{2a-1}{2a+1}\right) \cdot \dfrac{10a-5}{4a}$.

590. An’latpani’ a’piwayi’lasti’ri’n’ ha’m woni’n’ san ma’nisin tabi’n’:

1) $\frac{a+1}{a-1}+\frac{6}{a^2-1}-\frac{a+3}{a+1}$, bunda $a=-9$;

2) $\frac{b+5}{b+2}-\frac{3}{b^2-4}-\frac{b+1}{b-2}$, bunda $b=-8$;

3) $\frac{a-2}{a-3}:\left(\frac{a^2-6a+10}{a^2-9}+\frac{2}{a+3}\right)$, bunda $a=-1\frac{1}{2}$;

4) $\frac{b+1}{b-4}:\left(\frac{b^2+9}{b^2-16}+\frac{2}{b+4}\right)$, bunda $b=4\frac{1}{3}$.

591. Yesaplan’:

1) $\left(\frac{1}{2}\right)^{-1}-3^{-2}:3^{-5}$; | 2) $(-6)^0\cdot 81^{-2}\cdot 27^3$; | 3) $\left(\frac{1}{4}\right)^{-2}-3^{-3}:3^{-4}$.

592. Bo’lshekti qi’sqarti’n’:

1) $\frac{a+\sqrt{3}}{a^2-3}$; 2) $\frac{x-\sqrt{2}}{x^2-2}$; 3) $\frac{y-9y^{\frac{1}{2}}}{y^{\frac{1}{4}}+3}$; 4) $\frac{x+x^{\frac{1}{2}}}{x-1}$; 5) $\frac{x+\sqrt{5}}{x^2-5}$.

593. Yesaplan’:

1) $(6-3\sqrt{5})(6+3\sqrt{5})$; 2) $(\sqrt{5}-1)(\sqrt{5}+1)$;

3) $(3\sqrt{5}-2\sqrt{20})\sqrt{5}$; 4) $(1-\sqrt{3})^2+(1+\sqrt{3})^2$.

594. Yesaplan’:

1) $4\sqrt{3}-\sqrt{3}(\sqrt{16}-\sqrt{3})$; 2) $6\sqrt{2}-\sqrt{2}(\sqrt{2}+\sqrt{36})$;

3) $\sqrt{48}-\sqrt{27}-\frac{1}{2}\sqrt{12}$; 4) $\sqrt{50}-\sqrt{32}-\frac{1}{3}\sqrt{18}$;

5) $(\sqrt{2}+3)^2-3\sqrt{8}$; 6) $(2-\sqrt{3})^2+2\sqrt{12}$.

595. Yesaplan’:

1) $\left(\sqrt{4+\sqrt{7}}+\sqrt{4-\sqrt{7}}\right)^2$; 2) $\left(\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}\right)^2$;

3) $\frac{1}{5-\sqrt{5}}-\frac{1}{5+\sqrt{5}}$; 4) $\frac{1}{7+4\sqrt{3}}+\frac{1}{7-4\sqrt{3}}$.

596. A'piwayi'lasti'ri'n':

1) $\frac{1}{3-\sqrt{2}}+\frac{1}{3+\sqrt{2}}$; 2) $\frac{1}{5-\sqrt{3}}-\frac{1}{5+\sqrt{3}}$; 3) $\frac{3-\sqrt{2}}{3+\sqrt{2}}+\frac{3+\sqrt{2}}{3-\sqrt{2}}$;

4) $\frac{3}{\sqrt{3}-\sqrt{2}}-\frac{3}{\sqrt{3}+\sqrt{2}}$; 5) $\frac{11}{4-\sqrt{5}}+\frac{4}{3+\sqrt{5}}$; 6) $\frac{2}{\sqrt{5}-\sqrt{3}}-\frac{2}{\sqrt{5}+\sqrt{3}}$.

597. Sandi' standart tu'rinde jazi'n':

1) 0,00051; | 2) $\frac{1}{500}$; | 3) 250000; | 4) $\frac{3}{2500}$; | 5) 0,0000032.

598. Yesaplan': 1) $\frac{(0{,}25)^5\cdot 8^6}{2^8\cdot\left(\frac{1}{2}\right)^3}$; 2) $\frac{16\cdot 4^{-2}+4\left(\frac{2}{3}\right)^{-2}}{4+\cdot\left(\frac{1}{16}\right)^{-\frac{1}{2}}}$; 3) $\frac{(0{,}3)^6\cdot 5^6}{2^{-5}\cdot 9^3}$.

599. Yesaplan': 1) $\sqrt{8{,}75^3+8{,}75^2\cdot 7{,}25}$; 2) $\frac{0{,}625\cdot 6{,}75^2-3{,}25^2\cdot 0{,}625}{\sqrt{3{,}5^2+7\cdot 2{,}75+2{,}75^2}}$.

600. $x > 0$, $y > 0$ bolg'anda a'piwayi'lasti'ri'n':

1) $\sqrt{\frac{4}{81}x^6y^{20}}$; 2) $\sqrt{x^4y^{18}}$; 3) $\sqrt[3]{27x^3y^6}$; 4) $\sqrt[5]{x^5y^{10}}$.

601. An'latpalardi' a'piwayi'lasti'ri'n':

1) $\left(\frac{a^{\frac{1}{2}}-b^{\frac{1}{2}}}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}+\frac{2a^{\frac{1}{2}}b^{\frac{1}{2}}}{a-b}\right)\cdot\frac{a-2a^{\frac{1}{2}}b^{\frac{1}{2}}+b}{a+b}$; 2) $\left(\frac{1}{a^{\frac{1}{2}}+a}-\frac{a^{\frac{1}{2}}}{a^{\frac{1}{2}}+1}\right)\cdot\frac{a^{\frac{1}{2}}}{a^{\frac{1}{2}}-1}$;

3) $\frac{x^{\frac{1}{2}}}{1+x^{\frac{1}{2}}}\cdot\left(\frac{x^{\frac{1}{2}}}{1-x^{\frac{1}{2}}}-\frac{1}{x^{\frac{1}{2}}-x}\right)$; 4) $\frac{m+2m^{\frac{1}{2}}+1}{2m^{\frac{1}{2}}}\cdot\left(\frac{2m^{\frac{1}{2}}}{m^{\frac{1}{2}}-1}-\frac{4m^{\frac{1}{2}}}{m-1}\right)$.

2. Ten'lemeler

Ten'lemeni sheshin' **(602–605)**:

602. 1) $8(3x-7)-3(8-x)=5(2x+1)$;
2) $10(2x-1)-9(x-2)+4(5x+8)=71$;
3) $3+x(5-x)=(2-x)(x+3)$;
4) $7-x(3+x)=(x+2)(5-x)$.

603. 1) $\frac{5x-7}{6}-\frac{x+2}{7}=2$; 2) $\frac{4x-8}{3}-\frac{3+2x}{5}=8$;

3) $\frac{14-x}{4}+\frac{3x+1}{5}=3$; 4) $\frac{2x-5}{4}-\frac{6x+1}{8}=2$.

604. 1) $\frac{4}{3(x+2)}=\frac{9}{8x+11}$; 2) $\frac{1}{3(x-1)}=\frac{3}{2(x+6)}$;

3) $\frac{x}{5-x}+\frac{5-x}{5+x}=-2$; 4) $\frac{x+3}{x-3}+\frac{x}{x+3}=2$.

605. 1) $x(x-1)=0$; 2) $(x+2)(x-3)=0$;

3) $x\left(2x-\frac{1}{2}\right)(4+3x)=0$; 4) $\frac{(x-5)(x+1)}{x^2+1}=0$.

Ten'lemeni sheshin' **(606–608)**:

606. 1) $x^2+3x=0$; 2) $5x-x^2=0$; 3) $4x+5x^2=0$;

4) $-6x^2-x=0$; 5) $2x^2-32=0$; 6) $2-\frac{x^2}{2}=0$;

7) $\left(\frac{x}{2}\right)^2-1=0$; 8) $x^2-8=0$; 9) $\left(\frac{x}{3}\right)^2-4=0$.

607. 1) $2x^2+x-10=0$; 2) $2x^2-x-3=0$.

608. 1) $7x^2-13x-2=0$; 2) $4x^2-17x-15=0$.

Ten'lemeni sheshin' **(609–614)**:

609. 1) $(3x+4)^2+3(x-2)=46$; 2) $2(1-1{,}5x)+2(x-2)^2=1$;

3) $(5x-3)(x+2)-(x+4)^2=0$;

4) $x(11-6x)-20+(2x-5)^2=0$.

610. 1) $|x|=\frac{1}{2}$; 2) $|x-1|=4$; 3) $|3-x|=2$;

4) $|3x|-3x=6$; 5) $|2{,}5-x|+3=5$; 6) $|3{,}7+x|-2=6$.

611. 1) $\frac{7}{2x+9}-6=5x$; 2) $\frac{x^2}{x-2}-\frac{x+2}{x-2}=4$;

3) $\frac{x}{x^2-16}+\frac{x-1}{x+4}=1$; 4) $\frac{12}{(x+6)^2}+\frac{x}{x+6}=1$.

612. 1) $x^4-17x^2+16=0$; 2) $x^4-37x^2+36=0$;

3) $2x^4-5x^2-12=0$; 4) $x^4-3x^2-4=0$.

613. 1) $\sqrt{x+1}-5=0$; 2) $6-\sqrt{x+3}=0$; 3) $\sqrt{5-x}-1=x$;

4) $3+\sqrt{x-5}=x-4$; 5) $7x-\sqrt{2x+2}=5x$;

6) $12x-\sqrt{5x-4}=11x$.

614. 1) $2^{x-1} = 64$; 2) $3^{1-x} = 27$; 3) $3^{x-8} = 27$; 4) $7^{2x-1} = 49$.

615. Ten'lemeni grafikali'q usi'lda sheshin':

1) $x^3 = 3x + 2$; 2) $x^3 = -x - 2$; 3) $\frac{5}{x}=6-x$;

4) $x^{-1} = 2x - 1$; 5) $\sqrt{x}=\frac{x+3}{4}$; 6) $\sqrt{x}=6-x$.

Ten'lemeler sistemasi'n sheshin' (**616–618**):

616. 1) $\begin{cases}x+y=12,\\ x-y=2;\end{cases}$ 2) $\begin{cases}x+y=10,\\ y-x=4;\end{cases}$ 3) $\begin{cases}2x+3y=11,\\ 2x-y=7;\end{cases}$

4) $\begin{cases}3x+5y=21,\\ 6x+5y=27;\end{cases}$ 5) $\begin{cases}3x+5y=4,\\ 2x-y=7;\end{cases}$ 6) $\begin{cases}4x-3y=1,\\ 3x+y=-9.\end{cases}$

617. 1) $\begin{cases}\frac{2x}{3}=\frac{3y}{4}-2,\\ \frac{1}{2}x+\frac{1}{4}y=5;\end{cases}$ 2) $\begin{cases}\frac{3}{7}x-\frac{2}{5}y=2,\\ \frac{3}{4}x+\frac{1}{6}y=12\frac{1}{6};\end{cases}$

3) $\begin{cases}\frac{1}{2}(x+11)=\frac{1}{3}(y+13)+2,\\ 5x=3y+8;\end{cases}$ 4) $\begin{cases}\frac{1}{4}(x+3y)=\frac{1}{3}(x+2y),\\ x+5y=12.\end{cases}$

618. 1) $\begin{cases}x-y=7,\\ xy=18;\end{cases}$ 2) $\begin{cases}x-y=2,\\ xy=15;\end{cases}$ 3) $\begin{cases}x+y=2,\\ xy=-15;\end{cases}$

4) $\begin{cases}x+y=-5,\\ xy=-36;\end{cases}$ 5) $\begin{cases}x^2+y^2=13,\\ xy=6;\end{cases}$ 6) $\begin{cases}x^2+y^2=41,\\ xy=20.\end{cases}$

3. Ten'sizlikler

Ten'sizlikti sheshin' (**619–620**):

619. 1) $3x-7<4(x+2)$; 2) $7-6x\ge\frac{1}{3}(9x-1)$;

3) $1{,}5(x-4)+2{,}5x<x+6$; 4) $1{,}4(x+5)+1{,}6x>9+x$.

620. 1) $\frac{x-1}{3}-\frac{x-4}{2}\le 1$; 2) $\frac{x+4}{5}-\frac{x-1}{4}\ge 1$; 3) $\frac{x-1}{2}+\frac{x+1}{3}\ge 7$;

4) $\frac{2x-5}{4}-\frac{3-2x}{5}<1$; 5) $x+\frac{x-3}{6}>3$; 6) $x+\frac{x+2}{4}<3$.

621. Ten'sizlikler sistemasi'n sheshin':

1) $\begin{cases}x+5\ge 5x-3,\\ 2x-5<0;\end{cases}$ 2) $\begin{cases}2x+3\ge 0,\\ x-7<4x-1;\end{cases}$ 3) $\begin{cases}5x-1\le 7+x,\\ -0,2x>1;\end{cases}$

4) $\begin{cases}3x-2\ge 10-x,\\ -0,5x<1;\end{cases}$ 5) $\begin{cases}2,5x-4\le 0,\\ 9x-5\ge 0;\end{cases}$ 6) $\begin{cases}4x-3\ge 2-x,\\ -0,3x\ge 6.\end{cases}$

622. Ten'sizliktin' natural sanlardan ibarat bolg'an barli'q sheshimin tabi'n':

1) $\frac{x-2}{6}-x\ge\frac{x-8}{3}$; 2) $\frac{x+5}{2}>\frac{x-5}{4}+x$; 3) $\frac{x-3}{2}>\frac{2-x}{5}-1$.

623. Ten'sizlikler sistemasi'ni'n' pu'tin sannan ibarat bolg'an barli'q sheshimin tabi'n':

1) $\begin{cases}2(x+1)<8-x,\\ -5x-9<6;\end{cases}$ 2) $\begin{cases}3(x-1)>x-7,\\ -4x+7>-5;\end{cases}$

3) $\begin{cases}3y+\frac{2y-13}{11}>2,\\ \frac{y}{6}-\frac{3y-20}{9}<-\frac{2}{3}(y-7);\end{cases}$ 4) $\begin{cases}\frac{y-1}{2}-\frac{y-3}{4}\ge\frac{y-2}{3}-y,\\ 1-y\ge\frac{1}{2}y-4.\end{cases}$

624. Ten'sizliktin' pu'tin teris sannan ibarat bolg'an barli'q sheshimin tabi'n':

$$\begin{cases}\frac{3x-2}{4}+2\frac{1}{2}>\frac{2x-1}{3}-\frac{3x+2}{6},\\ \frac{2x-5}{3}-\frac{3x-1}{2}<\frac{3-x}{5}-\frac{2x-1}{4}.\end{cases}$$

625. Kvadrat ten'sizlikti sheshin':

1) $x^2-3x+2>0$; 2) $x^2-2x-3\le 0$;
3) $x^2-7x+12>0$; 4) $-x^2+3x-1\ge 0$;
5) $3+4x+8x^2<0$; 6) $x-x^2-1\ge 0$;
7) $2x^2-x-1<0$; 8) $3x^2+x-4>0$.

626. Ten'sizlikti sheshin':

1) $|x|>\frac{1}{5}$; 2) $|x-1|<2\frac{1}{3}$; 3) $|x-1|>3$; 4) $|x-1|\le 2$.

627. Ten'sizlikti arali'qlar usi'li' menen sheshin':

1) $(x-1)(x+3)>0$; 2) $(x+4)(x-2)<0$;
3) $(x+1{,}5)(x-2)x>0$; 4) $x(x-8)(x-7)>0$;
5) $(x-1)\left(x^2-\frac{1}{9}\right)\geq 0$; 6) $(x+3)\left(x^2-\frac{1}{4}\right)\leq 0$.

628. Sanlardi' sali'sti'ri'n':

1) $5\sqrt{2}$ ha'm 7; 2) 9 ha'm $4\sqrt{5}$; 3) $10\sqrt{11}$ ha'm $11\sqrt{10}$;
4) $5\sqrt{6}$ ha'm $6\sqrt{5}$; 5) $3\sqrt[3]{3}$ ha'm $2\sqrt[3]{10}$; 6) $2\sqrt[6]{3}$ ha'm $\sqrt{2}\cdot\sqrt[3]{5}$.

4. Ten'lemeler du'ziwge baylani'sli' ma'seleler

629. Yeki sanni'n' qosi'ndi'si' 120 g'a ten', wolardi'n' ayi'rmasi' bolsa 5 ke ten'. Usi' sanlardi' tabi'n'.

630. Kater da'rya ag'i'si' boyi'nsha 3 saat, qayti'wdag'i' jolg'a bolsa 4,5 saat sari'pladi'. Yeger katerdin' suwg'a sali'sti'rg'andag'i' tezligi 25 km/saat bolsa, da'rya ag'i'si'ni'n' tezligi qansha?

631. Motorli' qayi'q *A* dan *B* g'a shekemgi joldi' da'rya ag'i'si' boyi'nsha 2,4 saatta, qaytardag'i' joldi' bolsa 4 saatta basi'p wo'tti. Yeger qayi'qti'n' suwg'a sali'sti'rg'andag'i' tezligi 16 km/saat yekenligi belgili bolsa, da'rya ag'i'si' tezligin tabi'n'.

632. Kater da'rya ag'i'si' boyi'nsha 1 saatta 15 km ju'zdi ha'm qaytardag'i' jolg'a 1,5 saat sari'plap, da'slepki worni'na qayti'p keldi. Katerdin' suwg'a sali'sti'rg'andag'i' tezligin ha'm da'rya ag'i'si'ni'n' tezligin tabi'n'.

633. Ten' qaptalli' u'shmu'yeshliktin' perimetri 5,4 dm ge ten'. Qaptal ta'repi ultani'nan 13 ma'rte uzi'n. U'shmu'yeshliktin' ta'replerinin' uzi'nli'qlari'n tabi'n'.

634. Belgili bir jo'nelis boyi'nsha qatnaytug'i'n jan'a tu'rdegi tramvaydi'n' tezligi yeski tu'rdegiden 5 km/saat arti'q. Soni'n' ushi'n da wol 20 km joldi' yeski tu'rdegi tramvayg'a qarag'anda 12 min tezirek basi'p wo'tedi. Jan'a tramvay usi' joldi' qansha waqi'tta basi'p wo'tedi?

635. Avtobus ku'nnin' belgili bo'liminde «tez ju'rer» (ekspress) ta'rtipte isleydi. Soni'n' ushi'n da woni'n' tezligi bul waqi'tta 8 km/ saat artadi', 16 km ge sari'planatug'i'n waqi'ti' bolsa 4 min qa qi'sqaradi'. Avtobus tez ju'rer ta'rtibinde usi' jo'nelisti qanday waqi'tta basi'p wo'tedi?

636. Bir diyqan-fermer xojali'g'i' wo'z jer maydani'nan 875 c biyday, yekinshisi bolsa wonnan 2 ga kem maydannan 920 c biyday ji'ynap aldi'. Yeger bir gektar maydannan yekinshi xojali'q birinshi xojali'qqa qarag'anda 5 c arti'q biyday ji'ynap alg'anli'g'i' belgili bolsa, ha'rbir xojali'q bir gektar maydannan qanshadan biyday ji'ynap alg'an?

637. Yeki nasos bir waqi'tta islegende ha'wiz 2 saat 55 min ta tazalanadi'. Yeger wolardan biri bul jumi'sti' yekinshisine qarag'anda 2 saat tezirek wori'nlasa, ha'rbir nasos wo'z aldi'na islegende ha'wizdi qansha waqi'tta tazalawi' mu'mkin?

5. Funkciyalar ha'm grafikler

638. A noqati' to'mendegishe berilgen funkciyalardi'n' grafigine tiyisli yaki tiyisli yemes yekenligin ani'qlan'; bul funkciyalardi'n' koordinata ko'sherleri menen kesilisiw noqatlari'ni'n' koordinatalari'n ha'm $x = -2$ bolg'anda funkciyalardi'n' ma'nisin tabi'n':

1) $y = 3 - 0{,}5x$, $A(4;\ 1)$; 2) $y = \frac{1}{2}x - 4$, $A(6;\ -1)$;
3) $y = 2{,}5x - 5$, $A(1{,}5;\ -1{,}25)$;
4) $y = -1{,}5x + 6$, $A(4{,}6;\ -0{,}5)$.

639. Funkciyalardi'n' grafiklerin jasan' (bir koordinata tegisliginde):

1) $y = 3x$, $y = -3x$; 2) $y = \frac{1}{3}x$, $y = -\frac{1}{3}x$;
3) $y = x - 2$, $y = x + 2$; 4) $y = -x - 2$, $y = 2 - x$.

640. Funkciyani'n' grafigin jasan':

1) $y = x^2 + 2\frac{1}{4}$;	2) $y = \left(x - \frac{1}{3}\right)^2$;	3) $y = (x + 2{,}5)^2 - \frac{1}{4}$;
4) $y = x^2 - 4x + 5$;	5) $y = x^2 + 2x - 3$;	6) $y = -x^2 - 3x + 4$.

641. Parabola to'besinin' koordinatalari'n tabi'n':

1) $y = x^2 - 8x + 16$; 2) $y = x^2 - 10x + 15$;
3) $y = x^2 + 4x - 3$; 4) $y = 2x^2 - 5x + 3$.

642. Funkciyani'n' yen' u'lken ha'm yen' kishi ma'nislerin tabi'n':

1) $y = x^2 - 7x - 10$; 2) $y = -x^2 + 8x + 7$;
3) $y = x^2 - x - 6$; 4) $y = 4 - 3x - x^2$.

643. Berilgen yeki funkciyani'n' bir koordinata tegisliginde grafiklerin jasan' ha'm x ti'n' qanday ma'nislerinde bul funkciyalardi'n' ma'nislerinin' ten'ligin ani'qlan':

1) $y = x^2 - 4$ ha'm $y = 3x$; 2) $y = (x + 3)^2 + 1$ ha'm $y = -x$.

644. Grafiktin' da'slepki ko'rinisin jasan' ha'm funkciyani'n' qa'siyetlerin ayti'n':

1) $y = x^4$; 2) $y = x^5$; 3) $y = \frac{1}{x^3}$; 4) $y = \frac{1}{x^4}$.

645. An'latpalardi'n' ma'nislerin sali'sti'ri'n':

1) $\sqrt[4]{5,3}$ ha'm $\sqrt[4]{5\frac{1}{3}}$; 2) $\sqrt[5]{-\frac{2}{9}}$ ha'm $\sqrt[5]{-\frac{1}{7}}$; 3) $-\sqrt[3]{4\frac{7}{8}}$ ha'm $\sqrt[3]{-5\frac{5}{8}}$.

646. Funkciyani'n' grafigin jasan' ha'm x ti'n' $y = 0$, $y > 0$, $y < 0$ bolatug'i'n ma'nislerin tabi'n':

1) $y = 2x^2 - 3$;	2) $y = -2x^2 + 1$;	3) $y = 2(x - 1)^2$;
4) $y = 2(x + 2)^2$;	5) $y = 2(x - 3)^2 + 1$;	6) $y = -3(x - 1)^2 + 5$.

6. Trigonometriya elementleri

647. 1) $\left(\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2}\right)$; 2) $\left(-\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2}\right)$; 3) $\left(-\frac{1}{2}; -\frac{\sqrt{3}}{2}\right)$; 4) $\left(-\frac{\sqrt{3}}{2}; -\frac{1}{2}\right)$

koordinatali' noqat payda yetiw ushi'n $P\,(1; 0)$ noqati'n buri'w kerek bolg'an barli'q mu'yeshlerin tabi'n'.

648. An'latpalardi' a'piwayi'lasti'ri'n': $(1+\operatorname{tg}\alpha)(1+\operatorname{ctg}\alpha)-\frac{1}{\sin\alpha\cos\alpha}$.

649. Birdeylikti da'lillen':

1) $\frac{1-(\sin\alpha+\cos\alpha)^2}{\sin\alpha\cos\alpha-\operatorname{ctg}\alpha} = 2\operatorname{tg}^2\alpha$; 2) $\frac{\operatorname{tg}\alpha-\sin\alpha\cos\alpha}{(\sin\alpha-\cos\alpha)^2-1} = -\frac{1}{2}\operatorname{tg}^2\alpha$.

650. An'latpalardi' a'piwayi'lasti'ri'n':

1) $\sin^2(\alpha + 8\pi) + \cos(\alpha + 10\pi)$;

2) $\cos^2(\alpha + 6\pi) + \cos^2(\alpha - 4\pi)$.

651. An'latpalardi' a'piwayi'lasti'ri'n': $\frac{\sin 2\alpha}{2(1-2\cos^2\alpha)} + \frac{\sin\alpha\cos(\pi-\alpha)}{1-2\sin^2\alpha}$.

652. Birdeylikti da'lillen': $\frac{\cos^2 x}{1-\sin x} - \frac{\sin^2 x}{1+\cos x} = \sin x + \cos x$.

653. 1) yeger $\cos\alpha = -\frac{\sqrt{3}}{3}$ ha'm $\frac{\pi}{2} < \alpha < \pi$ bolsa, $\sin 2\alpha$ ni' yesaplan';

2) yeger $\sin\alpha = \frac{1}{3}$ bolsa, cos2α ni' yesaplan'.

654. An'latpani'n' ma'nisin tabi'n':

1) $\cos 765° - \sin 750° - \cos 1035°$; 2) $\sin\frac{11\pi}{3} + \cos 690° - \cos\frac{19\pi}{3}$.

655. Yeger tgα = 2 bolsa, an'latpani'n' ma'nisin tabi'n':

1) $\frac{\sin^2\alpha + \sin\alpha\cos\alpha}{\cos^2\alpha + 3\cos\alpha\sin\alpha}$; 2) $\frac{2 - \sin^2\alpha}{3 + \cos^2\alpha}$; 3) $\frac{2\sin^2\alpha - 3\cos^2\alpha}{5\cos^2\alpha + \sin^2\alpha}$.

656. $\text{tg}\alpha + \text{ctg}\alpha = 3$ yekenligi belgili. $\text{tg}^2\alpha + \text{ctg}^2\alpha$ ni' tabi'n'.

657. An'latpalardi' a'piwayi'lasti'ri'n':

1) $\frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} - \text{tg}\left(\frac{\pi}{4} + \alpha\right)$; 2) $\text{tg}^2\left(\frac{\pi}{4} - \alpha\right) - \frac{1 - \sin 2\alpha}{1 + \sin 2\alpha}$.

658. An'latpani' a'piwayi'lasti'ri'n': $\frac{\cos 2\alpha - \sin 2\alpha - 2\cos^2\alpha}{\cos(-\alpha) - \cos(2{,}5\pi + \alpha)}$.

7. Progressiyalar

659. Yeger: 1) $a_1 = 7$, $a_7 = -5$; 2) $a_1 = -3$, $a_8 = 11$ bolsa, arifmetikali'q progressiyani'n' ayi'rmasi'n tabi'n'.

660. Yeger: 1) $a_{10} = 4$, $d = 0{,}5$; 2) $a_{20} = 35$, $d = 2$ bolsa, arifmetikali'q progressiyani'n' birinshi ag'zasi'n tabi'n'.

661. Yeger: 1) $a_n = 459$, $d = 10$, $n = 45$; 2) $a_n = 121$, $d = -5$, $n = 17$ bolsa, arifmetikali'q progressiyani'n' birinshi ag'zasi'n ha'm da'slepki n ag'zasi'ni'n' qosi'ndi'si'n yesaplan'.

662. Yeger arifmetikali'q progressiyada: 1) $a_1 = -2$, $a_5 = -6$, $a_n = -40$; 2) $a_1 = -3$, $a_7 = 9$, $a_n = 33$ bolsa, n nomerin tabi'n'.

663. $b_{n+1} = -\frac{b_n}{2}$ formula ha'm $b_1 = 1024$ sha'rti menen berilgen izbe-izliktin' da'slepki won ag'zasi'ni'n' qosi'ndi'si'n tabi'n'.

664. Yeger geometriyali'q progressiyada:

1) $b_1 = 5$, $q = -10$ ha'm $b_n = -5000$ bolsa, n di;

2) $b_3 = 16$ ha'm $b_6 = 2$ bolsa, q ni';

3) $b_3 = 16$ ha'm $b_6 = 2$ bolsa, b_1 di;

4) $b_3 = 16$ ha'm $b_6 = 1$ bolsa b_7 ni tabi'n'.

665. Yeger 3 + 6 + 12 + ... + 96 qosi'ndi'si'ni'n' qosi'li'wshi'lari' geometriyali'q progressiyani'n' izbe-iz ag'zalari' bolsa, bul sanlardi'n' qosi'ndi'si'n tabi'n'.

666. Yeger: 1) $a_3 = 25,\ a_{10} = -3;$ 2) $a_1 = 10,\ a_7 = 19;$
3) $a_3 + a_7 = 4,\ a_2 + a_{14} = -8;$ 4) $a_2 + a_4 = 16,\ a_1 \cdot a_5 = 28$
bolsa, arifmetikali'q progressiyani'n' birinshi ag'zasi'n ha'm ayi'rmasi'n tabi'n'.

667. Yeger: 1) $a_9 = -5$ ha'm $a_{11} = 7;$ 2) $a_9 + a_{11} = -10;$
3) $a_9 + a_{10} + a_{11} = 12$ bolsa, arifmetikali'q progressiyani'n' woni'nshi' ag'zasi'n tabi'n'.

668. $S_7 = -35$ ha'm $S_{42} = -1680$ bolsa, arifmetikali'q progressiyani'n' birinshi ag'zasi'n ha'm ayi'rmasi'n tabi'n'.

669. n- ag'zasi'ni'n' formulasi' menen berilgen izbe-izlik geometriyali'q progressiya bola ma:
1) $b_n = -3^{2n};$ 2) $b_n = 2^{3n};$ 3) $b_n = \frac{3}{2n};$ 4) $b_n = \frac{(-1)^n}{2^n}$?

670. Yeger: 1) $b_1 = 12,\ S_3 = 372;$ 2) $b_1 = 1,\ S_3 = 157$
bolsa, geometriyali'q progressiyani'n' bo'limin yesaplan'.

671. Yeger $b_2 = -\frac{1}{2}$ ha'm $b_4 = -\frac{1}{72}$ bolsa, geometriyali'q progressiyani'n' birinshi ag'zasi'n, bo'limin ha'm n- ag'zasi'ni'n' formulasi'n tabi'n'.

672. Yeger $b_3 = -6$ ha'm $b_5 = -24$ bolsa, geometriyali'q progressiyani'n' to'rtinshi ag'zasi'n ha'm bo'limin tabi'n'.

673. $\frac{1}{3}$ ha'm 27 sanlari'ni'n' arasi'na u'sh sandi' sonday yetip jaylasti'ri'n', na'tiyjede geometriyali'q progressiyani'n' izbe-iz bes ag'zasi' payda bolsi'n.

674. Yeger, geometriyali'q progressiyada:
1) $q = 3,\ S_3 = 484$ bolsa, b_1 ha'm b_5 ti tabi'n';
2) $b_3 = 0{,}024,\ S_3 = 0{,}504$ bolsa, b_1 ha'm q di' tabi'n'.

675. Yeger: 1) $b_1 + b_2 = 20,\ b_2 + b_3 = 60;$ 2) $b_1 + b_2 = 60,\ b_1 + b_3 = 51$
bolsa, geometriyali'q progressiyani'n' birinshi ag'zasi'n ha'm bo'limin yesaplan'.

676. Yeger geometriyali'q progressiyada:
1) $b_4 = 88,\ q = 2$ bolsa, S_5 ti; 2) $S_5 = 341,\ q = 2$ bolsa, b_1 di;
3) $b_1 = 11,\ b_4 = 88$ bolsa, S_5 ti; 4) $b_3 = 44,\ b_5 = 176$ bolsa, S_5 ti
tabi'n'.

VII—VIII KLASLARDA «ALGEBRA» KURSI' BOYI'NSHA QI'SQASHA TEORIYALI'Q MAG'LUWMATLAR

Sanlar ha'm an'latpalar

1. San.

Natural sanlar ko'pligi: 1, 2, 3 ... .

Pu'tin sanlar ko'pligi: 0; ±1; ±2; ±3; ... .

Racional sanlar ko'pligi – $\frac{m}{n}$ ko'rinisindegi sanlar, bunda m – pu'tin san, n – natural san. Mi'sali', $\frac{3}{5}$; 2; $\frac{2}{7}$ sanlari' racional sanlar boladi'.

Racional sandi' shekli wonli'q bo'lshek yaki sheksiz periodli' wonli'q bo'lshek tu'rinde ko'rsetiw mu'mkin. Mi'sali',

$$\frac{2}{5} = 0,4; -\frac{1}{3} = -0,333 = -0,(3).$$

Irracional sanlar ko'pligi sheksiz periodli' yemes wonli'q bo'lshekler ko'pligi. Mi'sali', 0,1001000100001... — irracional san.

Sonday-aq, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ sanlari' da irracional sanlar boladi'.

Haqi'yqi'y sanlar ko'pligi — racional ha'm irracional sanlar ko'pligi boladi'.

2. Sanli' arali'qlar — kesindi, interval, yari'm interval, nurlar.

[a; b] *kesindisi* $a \le x \le b$ ten'sizliklerin qanaatlandi'ri'wshi' x sanlari'ni'n' ko'pligi, bunda $a < b$. Mi'sali', [2; 5] kesindisi — bul $2 \le x \le 5$ ten'sizlikti qanaatlandi'ri'wshi' x sanlari'ni'n' ko'pligi.

($a; b$) intervali' (arali'q) $a < x < b$ ten'sizligin qanaatlandi'ri'wshi' x sanlari'ni'n' ko'pligi, bunda $a < b$. Mi'sali', (−2; 3) intervali' – bul $-2 < x < 3$ ten'sizligin qanaatlandi'ri'wshi' x sanlari'ni'n' ko'pligi.

[$a; b$) *yari'm intervali'* $a \le x < b$ ten'sizligin qanaatlandi'ri'wshi' x sanlari'ni'n' ko'pligi, (a; b] yari'm intervali' bolsa, $a < x \le b$ ten'sizligin qanaatlandi'ri'wshi' x sanlari'ni'n' ko'pligi, $a < b$. Mi'sali', [3; 8) yari'm intervali' $3 \le x < 8$ ten'sizligin qanaatlandi'ri'wshi' x sanlari'ni'n' ko'pligi. (−4; 2] bolsa $-4 < x \le 2$ ten'sizligin qanaatlandi'ri'wshi' sanlar ko'pligi.

Nur $x > a$ yaki $x < a$, yaki $x \ge a$, $x \le a$ ten'sizligin qanaatlandi'ri'wshi' x sanlari'ni'n' ko'pligi boladi'. Mi'sali', $x \ge 5$ nuri' 5 ten u'lken sanlar ko'pligi boladi'.

3. ***a*** **sani'ni'n' moduli** ($|a|$ tu'rinde belgilenedi) to'mendegi formula menen ta'riyiplenedi:

$$|a| = \begin{cases} a, \text{ yeger } a \geq 0 \text{ bolsa,} \\ -a, \text{ yeger } a < 0 \text{ bolsa.} \end{cases}$$

Geometriyali'q ko'zqarastan $|a|$ — bul 0 noqati'nan a sani'n bildiriwshi noqatqa shekemgi bolg'an arali'q; $|a - b|$ – bul a ha'm b noqatlari' arasi'ndag'i' arali'q boladi'.

Qa'legen a sani' ushi'n $|a| \geq 0$ ten'sizligi wori'nlanadi', bunda tek $a = 0$ bolg'anda g'ana $|a| = 0$ boladi'.

$|x| \leq a$ ten'sizligin (bunda $a > 0$) $[-a; a]$ kesindisindegi x noqatlari', yag'ni'y $-a \leq x \leq a$ ten'sizligin qanaatlandi'ri'wshi' x sanlari' qanaatlandi'radi'. $|x| < a$ ten'sizligin (bunda $a > 0$) $(-a; a)$ interval (arali'q)dag'i' x sanlari', yag'ni'y $-a < x < a$ ten'sizligin qanaatlandi'ri'wshi' x sanlari' qanaatlandi'radi'.

$|x| \geq a$ ten'sizligin (bunda $a > 0$) barli'q $x \leq -a$ ha'm $x \geq a$ sanlar qanaatlandi'radi'. $|x| > a$ ten'sizligin (bunda $a > 0$) barli'q $x < -a$ ha'm $x > a$ sanlar qanaatlandi'radi'.

4. Sanli' an'latpalar — a'meller belgileri menen birlestirilgen sanlardan du'zilgen jazi'w.

Mi'sali', 1,2·(–3)–9:0,5— sanli' an'latpa.

Sanli' an'latpani'n' ma'nisi – usi' an'latpada ko'rsetilgen a'mellerdi wori'nlaw na'tiyjesinde payda bolg'an san. Mi'sali', –21,6 sani' $1{,}2 \cdot (-3) - 9 : 0{,}5$ an'latpasi'ni'n' ma'nisi.

5. A'mellerdi wori'nlaw ta'rtibi.

Birinshi basqi'sh a'melleri – qosi'w ha'm ali'w.

Yekinshi basqi'sh a'melleri – ko'beytiw ha'm bo'liw.

U'shinshi basqi'sh a'melleri – da'rejege ko'teriw.

1) yeger an'latpada qawsi'rmalar bolmasa, wonda da'slep u'shinshi basqi'sh a'melleri wori'nlanadi', keyin yekinshi basqi'sh ha'm aqi'ri'nda birinshi basqi'sh a'melleri wori'nlanadi': bunda sol bir qi'yli' basqi'shlarg'a tiyisli a'meller wolar qanday ta'rtipte jazi'lg'an bolsa, da'l sonday ta'rtipte wori'nlanadi'.

3) yeger bo'lshek an'latpani'n' ma'nisi yesaplani'p ati'rg'an bolsa, wonda bo'lshektin' ali'mi' ha'm bo'limindegi a'meller dara jag'dayda wori'nlanadi' ha'm birinshi na'tiyje yekinshisine bo'linedi;

4) yeger an'latpa basqa qawsi'rmalar ishinde jaylasqan qawsi'rmalardan quralg'an bolsa, wonda da'slep ishki qawsi'rmalardag'i' a'meller wori'nlanadi'.

6. Sanni'n' standart tu'ri, bul $a \cdot 10^n$ si'yaqli' ko'rinistegi jazi'w, bunda $1 \le |a| < 10$, n – pu'tin san, a – sanni'n' mantissasi', n – sanni'n' ta'rtibi. Mi'sali', $345{,}4 = 3{,}454 \cdot 10^2$, $0{,}003 = 3 \cdot 10^{-3}$, $-0{,}12 = -1{,}2 \cdot 10^{-1}$.

7. Juwi'qlasi'w qa'teligi.

Juwi'qlasi'wdi'n' absolyut qa'teligi — shamani'n' ani'q ma'nisi menen woni'n' juwi'q ma'nisi arasi'ndag'i' ayi'rmani'n' moduli. Yeger a — juwi'q san, x ani'q san bolsa, wonda absolyut qa'telik $|x - a|$ g'a ten'. $x = a \pm h$ jazi'wi' juwi'qlasi'wdi'n' absolyut qa'teligi h tan arti'p ketpewin bildiredi, yag'ni'y $|x - a| \le h$ yaki $a - h \le x \le a + h$. Bunda x sani' a g'a h qa shekemgi da'llik penen ten' delinedi. Mi'sali', $\pi = 3{,}14 \pm 0{,}01$ jazi'wi' $|\pi - 3{,}14| \le 0{,}01$, yag'ni'y π sani' 3,14 ke 0,01 ge shekemgi da'llik penen ten' yekenligin bildiredi.

Sandi' kemisi menen 10^{-n} ge shekemgi da'llikte do'n'geleklewde u'tirden keyingi da'slepki n belgi saqlani'p qaladi', keyingileri bolsa tu'sirip qaldi'ri'ladi'. Mi'sali', 17,2397 sani'n kemisi menen mi'n'li'qlarg'a shekem, yag'ni'y 10^{-3} shekemgi da'llikte do'n'geleklewde 17,239, ju'zliklerge shekemgi do'n'geleklewde 17,23, wonli'qlarg'a shekemgi do'n'geleklewde 17,2 payda boladi'.

Sandi' arti'g'i' menen 10^{-n} ge shekem do'n'geleklewde u'tirden keyingi da'slepki n- belgi (cifr) bir birlikke artti'ri'ladi', keyingi barli'q belgiler bolsa taslap ketiledi. Mi'sali', 2,5143 sani'n arti'g'i' menen mi'n'li'qlarg'a shekemgi da'llikte do'n'geleklewde 2,515, ju'zliklerge shekemgi do'n'geleklewde 2,52, wonli'qlarg'a shekemgi do'n'geleklewde 2,6 payda yetiledi.

Yeki jag'dayda da do'n'geleklew qa'teligi 10^{-n} nen artpaydi'.

Yen' kishi qa'telik penen do'n'geleklew: yeger berilgen sanni'n' birinshi taslap ketilgen cifri' 5 ten kishi bolsa, wonda kemisi menen do'n'geleklenedi, yegerde bul san 5 ten u'lken yaki wog'an ten' bolsa, arti'g'i' menen do'n'geleklenedi. Mi'sali', 8,351 sani'n ju'zliklerge shekem do'n'geleklewde 8,35 ti, wonli'qlarg'a shekem do'n'geleklewde bolsa 8,4 ti payda yetemiz. $x \approx a$ jazi'wi' a sani' x sani'ni'n' juwi'q ma'nisi yekenligin bildiredi. Mi'sali', $\sqrt{2} \approx 1{,}4$.

Sali'sti'rmali' qa'telik absolyut qa'telikti shamani'n' juwi'q ma'nisinin' moduline qatnasi' (tiyindisi) boladi'. Yeger x – ani'q ma'nis, a – juwi'q ma'nis bolsa, wonda sali'sti'rmali' qa'telik $\frac{|x-a|}{|a|}$ ge ten' boladi'. Sali'sti'rmali' qa'telik a'dette procentlerde an'lati'ladi'. Ma'selen, yeger shamani'n' ani'q ma'nisi 1,95 ke ten', juwi'q ma'nisi 2 ge ten' bolsa, wonda juwi'qlasi'wdi'n' sali'sti'rmali' qa'teligi

$$\frac{|2-1{,}95|}{|2|}=\frac{|0{,}05|}{|2|}=0{,}25 \text{ yaki } 2{,}5\%.$$

Algebrali'q an'latpalar

8. Algebrali'q an'latpa — a'meller belgileri menen birlestirilgen sanlar ha'm ha'riplerden du'zilgen an'latpa. Algebrali'q an'latpalarg'a mi'sallar: $2(m+n)$; $3a+2ab-1$; $(a-b)^2$; $\frac{2x+y}{z}$.

Algebrali'q an'latpani'n' ma'nisi — bul an'latpadag'i' ha'ripler sanlar menen almasti'ri'lg'annan keyingi yesaplaw na'tiyjesindegi san. Mi'sali', $a=2$ ha'm $b=3$ bolg'anda $3a+2ab-1$ an'latpani'n' san ma'nisi $3\cdot 2+2\cdot 2\cdot 3-1=17$ boladi'.

9. Algebrali'q qosi'ndi' – «+» yaki «–» belgileri menen birlestirilgen birneshe algebrali'q an'latpalardan du'zilgen jazi'w.

Qawsi'rmalardi' ashi'w ta'rtibi.

1) Yeger algebrali'q an'latpag'a qawsi'rma ishine ali'ng'an algebrali'q qosi'ndi' qosi'lsa, wonda bul algebrali'q qosi'ndi'dag'i' ha'rbir qosi'li'wshi'ni'n' belgisin saqlag'an halda qawsi'rmalardi' taslap ketiw mu'mkin, mi'sali',

$$14+(7-23+21)=14+7-23+21,$$
$$a+(b-c-d)=a+b-c-d.$$

2) Yeger algebrali'q an'latpadan qawsi'rma ishine ali'ng'an algebrali'q qosi'ndi' ali'nsa, wonda usi' algebrali'q qosi'ndi'dag'i' ha'rbir qosi'li'wshi'ni'n' belgisin qarama-qarsi'si'na wo'zgertip, qawsi'rmalardi' taslap jiberiwge boladi', mi'sali',

$$14-(7-23+21)=14-7+23-21,$$
$$a-(b-c-d)=a-b+c+d.$$

10. Bir ag'zali' — sanli' ha'm ha'ripli ko'beytiwshilerdin' ko'beymesinen ibarat algebrali'q an'latpa.

Bir ag'zali'larg'a mi'sallar: $3ab$, $-2ab^2c^3$, a^2, a, $0{,}6xy^5y^2$, $-t^4$.

Mi'sali' $3a^2(0,4) \cdot b(-5)c^3$ bir ag'zali'ni'n' sanli' ko'beytiwshileri 3; 0,4; −5, ha'ripli ko'beytiwshileri bolsa a^2, b, c^3.

Standart tu'rindegi bir ag'zali' — birinshi wori'nda turg'an tek bir sanli' ko'beytiwshiden ha'm ha'rqi'yli' ha'ripli da'rejelerden du'zilgen bir ag'zali'. Bir ag'zali'ni' standart tu'rinde jazi'w ushi'n woni'n' barli'q sanli' ko'beytiwshilerin wo'z ara ko'beytiw ha'm na'tiyjeni birinshi wori'ng'a qoyi'w, son' bir qi'yli' ha'ripli ko'beytiwshilerdin' ko'beymesin da'reje tu'rinde jazi'w kerek.

Bir ag'zali'ni'n' koefficienti — standart tu'rinde jazi'lg'an bir ag'zali'ni'n' san ko'beytiwshisi.

Mi'sali', $\frac{3}{4}abc^2$ bir ag'zali'ni'n' koefficienti $\frac{3}{4}$ ke ten', $-7a^3b$ bir ag'zali'ni'n' koefficienti −7 ge ten', a^2bc bir ag'zali'ni'n' koefficienti 1 ge ten'. $-ab^2$ bir ag'zali'ni'n' koefficienti −1 ge ten'.

11. Ko'p ag'zali' — birneshe bir ag'zali'ni'n' algebrali'q qosi'ndi'si'.

Ko'p ag'zali'g'a mi'sallar: $4ab^2c^3$ – bir ag'zali', $2ab-3bc$ — yeki ag'zali', $4ab+3ac-bc$ — u'sh ag'zali'.

Ko'p ag'zali'ni'n' ag'zalari' — ko'p ag'zali'ni' payda yetiwshi bir ag'zali'lar. Mi'sali', $2ab^2-3a^2c+7bc-4bc$ ko'p ag'zali'ni'n' ag'zalari' $2ab^2$, $-3a^2c$, $7bc$, $-4bc$ boladi'.

Uqsas ag'zalar — tek koefficientleri menen pari'qlani'wshi' bir ag'zali'lar yaki bir qi'yli' bir ag'zali'lar.

Uqsas ag'zalari'n ji'ynaw — ko'p ag'zali'ni' a'piwayi'lasti'ri'w, bunda uqsas bir ag'zali'lardi'n' algebrali'q qosi'ndi'si' bir ag'zali' menen almasti'ri'ladi'. Mi'sali':

$$2ab-4bc+ac+3ab+bc=5ab-3bc+ac.$$

Ko'p ag'zali'ni'n' standart tu'ri — ko'p ag'zali'ni'n' barli'q ag'zalari' standart tu'rinde jazi'lg'an ha'm wolardi'n' arasi'nda uqsas ag'zalar bolmag'an jazi'li'wi'.

Bir ag'zali'lar ha'm ko'p ag'zali'lar u'stinde a'meller:

1) birneshe ko'pag'zali'lardi'n' algebrali'q qosi'ndi'si'n standart tu'rindegi ko'p ag'zali' ko'rinisinde jazi'w ushi'n qawsi'rmalardi' ashi'w ha'm uqsas ag'zalari'n ji'ynaw kerek, mi'sali':

$$(2a^2b-3bc)+(a^2b+5bc)-(3a^2b-bc)=$$
$$=2a^2b-3bc+a^2b+5bc-3a^2b+bc=3bc;$$

2) ko’p ag’zali’ni’ bir ag’zali’g’a ko’beytiw ushi’n ko’p ag’zali’ni’n’ ha’rbir ag’zasi’n usi’ bir ag’zali’g’a ko’beytiw ha’m payda bolg’an ko’beymelerdi qosi’w kerek. Mi’sali’:

$$(2ab-3bc)(4ac)=(2ab)(4ac)+(-3bc)(4ac)=8a^2bc-12abc^2;$$

3) ko’p ag’zali’ni’ ko’p ag’zali’g’a ko’beytiw ushi’n birinshi ko’p ag’zali’ni’n’ ha’rbir ag’zasi’n yekinshi ko’p ag’zali’ni’n’ ha’rbir ag’zasi’na ko’beytiw ha’m payda bolg’an ko’beymelerdi qosi’w kerek.

Mi’sali’:

$$(5a-2b)(3a+4b)=(5a)(3a)+(5a)(4b)+$$
$$+(-2b)(3a)+(-2b)(4b)=15a^2+14ab-8b^2;$$

4) ko’p ag’zali’ni’ bir ag’zali’g’a bo’liw ushi’n ko’p ag’zali’ni’n’ ha’r bir ag’zasi’n usi’ bir ag’zali’g’a bo’liw ha’m payda bolg’an na’tiyjelerdi qosi’w kerek, mi’sali’,

$$(4a^3b^2-12a^2b^3):(2ab)=(4a^3b^2):(2ab)+(-12a^2b^3):(2ab)=2a^2b-6ab^2.$$

12. Qi’sqasha ko’beytiw formulalari’.

1) $(a+b)^2=a^2+2ab+b^2;$

2) $(a-b)^2=a^2-2ab+b^2;$

3) $(a+b)^3=a^3+3a^2b+3b^2a+b^3;$

4) $(a-b)^3=a^3-3a^2b+3ab^2-b^3;$

5) $a^2-b^2=(a+b)(a-b);$

6) $a^3+b^3=(a+b)(a^2-ab+b^2);$

7) $a^3-b^3=(a-b)(a^2+ab+b^2).$

13. Ko’p ag’zali’ni’ ko’beytiwshilerge jiklew — ko’p ag’zali’ni’ yeki yaki birneshe ko’p ag’zali’lardi’n’ ko’beymesi tu’rinde an’lati’w, mi’sali’, $4x^2-9y^2=(2x+3y)(2x-3y)$. Ko’p ag’zali’ni’ ko’beytiwshilerge jiklewde to’mendegi usi’llardan paydalani’ladi’.

1) *Uluwma ko’beytiwshini qawsi’rmadan si’rtqa shi’g’ari’w.* Mi’sali’, $3ax+6ay=3a(x+2y)$.

2) *Gruppalaw usi’li’.* Mi’sali’,

$$a^3-2a^2-2a+4=(a^3-2a^2)-(2a-4)=$$
$$=a^2(a-2)-2(a-2)=(a-2)(a^2-2).$$

3) *Qi’sqasha ko’beytiw formulalari’n qollani’w.* Mi’sali’,

$$9x^2-\frac{1}{16}y^2=(3x+\frac{1}{4}y)(3x-\frac{1}{4}y);$$

$$27x^3+8y^6=(3x+2y^2)(9x^2-6xy^2+4y^4);$$

$$z^2-14z+49=(z-7)^2.$$

Kvadrat u'sh ag'zali'ni' ko'beytiwshilerge jiklew — woni' $ax^2 + bx + c = a(x - x_1)(x - x_2)$ si'yaqli' ko'rinisinde ko'rsetiw, bunda x_1 ha'm x_2 lar $ax^2 + bx + c = 0$ kvadrat ten'lemenin' korenleri. Mi'sali':

$$2x^2 + 3x - 2 = 2\left(x - \frac{1}{2}\right)(x + 2).$$

14. Algebrali'q bo'lshek – ali'mi' ha'm bo'limi algebrali'q an'latpalardan ibarat bo'lshek.

Algebrali'q bo'lsheklerge mi'sallar: $\frac{a^2+b}{c}$, $\frac{3x-2y}{a+1}$. Algebrali'q bo'lshekti jazi'wda qollani'lg'an ha'ripler tek usi' bo'lshektin' bo'limi nolge ten' bolmaytug'i'n ma'nislerdi qabi'l yetiwi mu'mkin, dep woylayi'q.

Bo'lshektin' tiykarg'i' qa'siyeti: ali'mi' ha'm bo'limin sol bir algebrali'q an'latpag'a ko'beytkende wog'an ten' bolg'an bo'lshek payda boladi'. Mi'sali':

$$\frac{a-b}{a+b} = \frac{(a-b)(a-b)}{(a+b)(a-b)} = \frac{(a-b)^2}{a^2-b^2}.$$

Bo'lshektin' tiykarg'i' qa'siyetinen paydalani'p, algebrali'q bo'lshekti woni'n' ali'mi' ha'm bo'liminin' ko'beytiwshisine qi'sqarti'w mu'mkin. Mi'sali':

$$\frac{x^2-1}{x^3-1} = \frac{(x-1)(x+1)}{(x-1)(x^2+x+1)} = \frac{x+1}{x^2+x+1}.$$

Algebrali'q bo'lsheklerdi qosi'w ha'm ali'w sanli' bo'lshekler ushi'n qollani'latug'i'n qag'i'ydalar boyi'nsha ali'p bari'ladi'.

Yeki yaki birneshe bo'lsheklerdin' algebrali'q qosi'ndi'si'n tabi'w ushi'n bul bo'lshekler uluwma bo'limge keltiriledi ha'm bir qi'yli' bo'limge iye bo'lsheklerdi qosi'w qag'i'ydasi'nan paydalani'ladi'.

Mi'sali', $\frac{1}{a^2b}$ ha'm $\frac{1}{ab^2}$ bo'lsheklerdin' uluwma bo'limi a^2b^2 qa ten', soni'n' ushi'n $\frac{1}{a^2b} + \frac{1}{ab^2} = \frac{b}{a^2b^2} + \frac{a}{a^2b^2} = \frac{b+a}{a^2b^2}$.

Algebrali'q bo'lsheklerdi ko'beytiw ha'm bo'liw sanli' bo'lshekler ushi'n qollani'lg'an qag'i'ydalar boyi'nsha ali'p bari'ladi', mi'sali',

$$\frac{2a}{3b} \cdot \frac{b^2}{4a} = \frac{2ab^2}{3b \cdot 4a} = \frac{1}{6}b; \quad \frac{x^2-y^2}{2xy} : \frac{x+y}{4x} = \frac{(x^2-y^2) \cdot 4x}{2xy(x+y)} = \frac{2(x-y)}{y}.$$

15. Birdeylik — wog'an kirgen ha'riplerdin' mu'mkin bolg'an ma'nislerinde duri's bolg'an ten'lik. Mi'sali', to'mendegi ten'likler birdeylik boladi':

$$a^2 - b^2 = (a-b)(a+b); \quad \sqrt{a^2} = |a|,$$

$$\sin^2\alpha + \cos^2\alpha = 1, \qquad \frac{a^2-1}{a-1} = a+1.$$

DA'REJELER HA'M KORENLER

16. *a* sani'ni'n' 1 den u'lken bolg'an *n* natural ko'rsetkishli da'rejesi, bul *a* g'a ten' *n* ko'beytiwshinin' ko'beymesi, yag'ni'y,

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ ma'rte}}.$$

Mi'sali', $2^3 = 2 \cdot 2 \cdot 2$, $m^5 = \underbrace{m \cdot m \cdot m \cdot m \cdot m}_{5 \text{ ma'rte}}$.

Da'rejenin' a^n jazi'wi'nda a sani' — da'rejenin' tiykari', n – da'reje ko'rsetkishi. Mi'sali', 2^3 jazi'wi'nda 2 sani' – da'rejenin' tiykari', 3 sani' – da'reje ko'rsetkishi.

Sanni'n' birinshi da'rejesi — sanni'n' wo'zi: $a^1=a$. Mi'sali',

$$3^1 = 3, \quad \left(\frac{1}{13}\right)^1 = \frac{1}{13}; \qquad (0,7)^1 = 0,7; \qquad 0^1 = 0; \qquad (-5)^1 = -5.$$

Da'rejege ko'teriw a'meli sanni'n' da'rejesin tabi'w boladi'.

Da'rejenin' tiykarg'i' qa'siyetleri:

1) ten' tiykarli' da'rejelerdi ko'beytiwde tiykar buri'ng'i'si'nsha qaladi', da'reje ko'rsetkishleri bolsa qosi'ladi':

$$a^n \cdot a^m = a^{n+m};$$

2) ten' tiykarli' da'rejelerdi bo'liwde tiykar buri'ng'i'si'nsha qaladi', da'reje ko'rsetkishleri bolsa ali'nadi':

$$a^n : a^m = a^{n-m};$$

3) da'rejeni da'rejege ko'teriwde tiykar buri'ng'i'si'nsha qaladi', da'reje ko'rsetkishleri bolsa wo'z ara ko'beytiledi:

$$(a^n)^m = a^{nm};$$

4) ko'beymeni da'rejege ko'teriwde ha'rbir ko'beytiwshi usi' da'rejege ko'teriledi:

$$(a \cdot b)^n = a^n \cdot b^n;$$

5) bo'lshekti da'rejege ko'teriwde woni'n' ali'mi' ha'm bo'limi usi' da'rejege ko'teriledi:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

17. *a* sani'ni'n' kvadrat koreni — kvadrati' a g'a ten' bolg'an san. Mi'sali', 6 – bul 36 sani'ni'n' kvadrat koreni; –6 sani' da 36 sani'ni'n' kvadrat koreni.

Kvadrat koren shi'g'ari'w — kvadrat korendi tabi'w a'meli. Tek teris yemes sannan kvadrat koren shi'g'ari'w mu'mkin.

a sani'nan ali'ng'an (shi'g'ari'lg'an) *arifmetikali'q kvadrat koren* — kvadrati' a g'a ten' bolg'an teris yemes san. Bul san bi'lay belgilenedi: $\sqrt{a}$.

Mi'sali', $\sqrt{16} = 4$, $\sqrt{144} = 12$; $\sqrt{1,69} = 1,3$; $\sqrt{6,25} = 2,5$.

$\sqrt{a}$ an'latpasi' tek $a \geq 0$ bolg'anda ma'niske iye, bunda

$$\sqrt{a} \geq 0, \ (\sqrt{a})^2 = a.$$

Kvadrat korenlerdin' qa'siyetleri:

1) yeger $a \geq 0$, $b \geq 0$ bolsa, wonda $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ boladi'. Ma'selen, $\sqrt{144 \cdot 196} = \sqrt{144} \cdot \sqrt{196} = 12 \cdot 14 = 168$;

2) yeger $a \geq 0$, $b > 0$ bolsa, wonda $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ boladi'. Ma'selen,

$$\sqrt{\frac{169}{225}} = \frac{\sqrt{169}}{\sqrt{225}} = \frac{13}{15}; \qquad \sqrt{5\frac{1}{16}} = \sqrt{\frac{81}{16}} = \frac{\sqrt{81}}{\sqrt{16}} = \frac{9}{4};$$

3) yeger $a \geq 0$, n – natural san bolsa, $\sqrt{a^{2n}} = a^n$ boladi'. Ma'selen,

$$\sqrt{3^6} = 3^3 = 27; \qquad \sqrt{5^8} = 5^4 = 625; \qquad \sqrt{(0,4)^2} = 0,4.$$

Bul qa'siyetlerden kvadrat korenler qatnasqan an'latpalardi' almasti'ri'wda paydalani'ladi'. Bul almasti'ri'wlardi'n' tiykarg'i'lari':

ko'beytiwshini koren belgisi asti'nan shi'g'ari'w:

yeger $a \geq 0$, $b \geq 0$ bolsa, wonda $\sqrt{a^2 b} = a\sqrt{b}$ boladi';

ko'beytiwshini koren belgisi asti'na kirgiziw:

yeger $a \geq 0$, $b \geq 0$ bolsa, wonda $a\sqrt{b} = \sqrt{a^2 b}$ boladi'.

TEN'LEMELER

18. Bir belgisizli ten'leme — ha'rip penen belgilengen belgisizdi wo'z ishine alg'an ten'lik.

Ten'lemege mi'sal: $2x + 3 = 3x + 2$, bunda x tabi'li'wi' kerek bolg'an belgisiz san.

Ten'lemenin' koreni — belgisizdin' ten'lemeni duri's ten'likke aylandi'ri'wshi' ma'nisi.

Mi'sali', 3 sani' $x + 1 = 7 - x$ ten'lemenin' koreni, sebebi $3 + 1 = 7 - 3$.

Ten'lemeni sheshiw — woni'n' barli'q korenlerin tabi'w yaki wolardi'n' joq yekenligin da'lillew boli'p tabi'ladi'.

Ten'lemelerdin' tiykarg'i' qa'siyetleri:

1) ten'lemenin' qa'legen ag'zasi'n woni'n' bir bo'leginen yekinshi bo'legine qarama-qarsi' belgi menen ali'p wo'tiw mu'mkin.

2) ten'lemenin' yeki bo'legin de nolge ten' bolmag'an ten'dey bir sang'a ko'beytiw yaki bo'liw mu'mkin.

19. Kvadrat ten'leme, bul $ax^2 + bx + c = 0$ tu'rindegi ten'leme, bunda a, b ha'm c – berilgen sanlar, soni'n' menen birge, $a \neq 0$, x — belgisiz san.

Kvadrat ten'lemenin' koefficientleri to'mendegishe ataladi': a — birinshi yaki bas koefficient, b – 2-koefficient, c – saltan' ag'za.

Kvadrat ten'lemege mi'sallar: $2x^2 - x - 1 = 0$, $3x^2 + 7x = 0$.

Toli'q yemes kvadrat ten'leme de $ax^2 + bx + c = 0$ tu'rindegi kvadrat ten'leme, biraq wondag'i' b yaki c koefficientlerinen birewi nolge ten' boladi'.

Toli'q yemes kvadrat ten'lemelerge mi'sallar:

$$x^2 = 0,\ 5x^2 + 4 = 0,\ 8x^2 + x = 0.$$

Kvadrat ten'lemenin' korenlerinin' formulasi': $x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Mi'sali', $3x^2 + 5x - 2 = 0$ ten'lemesi yeki korenge iye:

$$x_{1,2} = \frac{-5 \pm \sqrt{25 + 24}}{6} = \frac{-5 \pm 7}{6}, \text{ yag'ni'y } x_1 = \frac{1}{3},\ x_2 = -2.$$

Keltirilgen kvadrat ten'leme $x^2 + px + q = 0$ tu'rindegi ten'leme.

Keltirilgen kvadrat ten'lemenin' korenlerinin' formulasi':

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}.$$

Mi'sali', $x^2 - 6x - 7 = 0$ ten'lemesinin' korenleri:

$$x_{1,2} = 3 \pm \sqrt{9+7} = 3 \pm 4, \text{ yag'ni'y } x_1 = 7,\ x_2 = -1.$$

Viet teoremasi'. Keltirilgen kvadrat ten'lemenin' korenlerinin' qosi'ndi'si' qarama-qarsi' belgi menen ali'ng'an yekinshi koefficientke, wolardi'n' ko'beymesi bolsa saltan' ag'zag'a ten'.

Solay yetip, yeger x_1 ha'm x_2 sanlari' $x^2 + px + q = 0$ kvadrat ten'lemenin' korenleri bolsa, wonda $x_1 + x_2 = -p$, $x_1 \cdot x_2 = q$ boladi'.

Viet teoremasi'na keri teorema. Yeger p, q, x_1, x_2 sanlari' ushi'n $x_1 + x_2 = -p$, $x_1x_2 = q$ ten'likleri wori'nli' bolsa, wonda x_1 ha'm x_2 sanlari' $x^2 + px + q = 0$ ten'lemesinin' korenleri boladi'.

20. Yeki belgisizli ten'lemeler sistemasi' — birgelikte qaralatug'i'n x ha'm y belgisizli yeki ten'leme.

Yeki belgisizli ten'lemeler sistemasi'na mi'sal:

$$\begin{cases} 3x - y = 5, \\ 2x + y = 7; \end{cases} \qquad \begin{cases} x - 2y = 7, \\ x^2 - 4y^2 = -35; \end{cases} \qquad \begin{cases} x^2 + y^2 = 25, \\ xy = 12. \end{cases}$$

Sistemani'n' sheshimi — usi' sistemag'a qoyg'anda woni'n' ha'rbir ten'lemesin duri's ten'likke aylandi'ratug'i'n x ha'm y sanlar jubi'.

Mi'sali', mi'na $\begin{cases} 4x - y = 2, \\ 5x + y = 7 \end{cases}$ sistemani'n' sheshimi $x = 1$, $y = 2$ sanlar jubi' boladi'.

Sistemani' sheshiw — woni'n' barli'q sheshimlerin tabi'w yaki wolardi'n' joq yekenligin da'lillew boli'p tabi'ladi'.

Ten'lemeler sistemasi'n sheshkende to'mendegi *usi'llar* qollani'ladi':

1) *Worni'na qoyi'w usi'li'*.

Ten'lemelerdin' birewinen bir belgisiz yekinshisi arqali' an'lati'ladi' ha'm sistemani'n' basqa ten'lemesine qoyi'ladi'.

2) *Algebrali'q qosi'w usi'li'*. Mi'na $\begin{cases} a_1x + b_1y = c_1, \\ a_2x + b_2y = c_2 \end{cases}$ tu'rindegi sistemani' sheshiw ushi'n belgisizlerden birinin' koefficientlerin

modulleri boyi'nsha ten'lestirip, sistema ten'lemelerin ag'zama-ag'za qosi'w yaki ali'w arqali' bul belgisiz joq yetiledi.

3) *Grafikali'q usi'l.* Sistema ten'lemelerinin' grafikleri jasaladi' ha'm wolardi'n' kesilisiw noqatlari'ni'n' koordinatalari' tabi'ladi'.

TEN'SIZLIKLER

21. Sanli' ten'sizlikler.

$a > b$ ten'sizligi $a - b$ ayri'masi'ni'n' won' yekenligin bildiredi.

$a < b$ ten'sizligi $a - b$ ayi'rmasi' teris yekenligin bildiredi.

Yeger $a > b$ bolsa, wonda $b < a$ boladi'.

Ten'sizlik $>$ yaki $<$ belgileri menen birlestirilgen yeki sanli' yaki algebrali'q an'latpa boladi'.

Ten'sizliklerge mi'sallar: $4 > 7 - 5$; $2a + b < a^2 + b^2$.

Qa'legen yeki a ha'm b sani' ushi'n to'mendegi u'sh qatnastan tek birewi duri's boladi': $a > b$, $a = b$, $a < b$.

Sanli' ten'sizliktin' tiykarg'i' qa'siyetleri:

1) Yeger $a > b$ ha'm $b > c$ bolsa, wonda $a > c$ boladi'.

2) Yeger ten'sizliktin' yeki jag'i'na da birdey san qosi'lsa, yaki ali'nsa, wonda ten'sizlik belgisi wo'zgermeydi: yeger $a > b$ bolsa, wonda qa'legen $a + c > b + c$ ha'm $a - c > b - c$ boladi'.

Qa'legen sandi' ten'sizliktin' bir jag'i'nan yekinshi jag'i'na, woni'n' belgisin qarama-qarsi'si'na wo'zgertip ali'p wo'tiw mu'mkin.

3) Ten'sizliktin' yeki jag'i'nda nolge ten' bolmag'an sang'a ko'beytiw ha'm bo'liw mu'mkin, bunda, yeger bul san won' bolsa, ten'sizlik belgisi $a > b$ bolsa, wonda

$$c > 0 \text{ bolg'anda } ac > bc \text{ ha'm } \frac{a}{c} > \frac{b}{c},$$

$$c < 0 \text{ bolg'anda } ac < bc \text{ ha'm } \frac{a}{c} < \frac{b}{c}.$$

Ten'sizliklerdi qosi'w. Birdey belgige iye ten'sizliklerdi qosi'w mu'mkin, bunda da'l usi' belgige iye ten'sizlik payda boladi': yeger $a > b$ ha'm $c > d$ bolsa, wonda $a + c > b + d$ boladi'. Mi'sali':

$$+\begin{array}{r} 4 > 3{,}5 \\ -2 > -5 \\ \hline 2 > -1{,}5 \end{array} \qquad +\begin{array}{r} 2{,}3 < 3{,}5 \\ -4 < -3 \\ \hline -1{,}7 < 0{,}5 \end{array}$$

Ten’sizliklerdi ko’beytiw. Shep ha’m won’ bo’limleri won’ bolg’an birdey belgige iye ten’sizliklerdi ag’zama-ag’za ko’beytiw mu’mkin, bunda da’l usi’ belgige iye ten’sizlik payda boladi’: yeger $a > b$, $c > d$ ha’m a, b, c, d won’ sanlar bolsa, wonda $ac > bd$ boladi’. Mi’sali’:

$$\times\begin{array}{r} 2,4 > 2,1 \\ 4 > 3 \\ \hline 9,6 > 6,3 \end{array} \qquad \times\begin{array}{r} 1,7 < 2,3 \\ 2 < 3 \\ \hline 3,4 < 6,9 \end{array}$$

Yeger $a > b$ ha’m a, b won’ sanlar bolsa, wonda $a^2 > b^2$, $a^3 > b^3$ ha’m, uluwma, qa’legen natural n ushi’n $a^n > b^n$ ten’sizligi wori’nlanadi’. Mi’sali’, $6^2 > 5^2$, $6^3 > 5^3$, $6^{12} > 5^{12}$.

Qatan’ ten’sizlikler > (u’lken) ha’m < (kishi) belgisine iye ten’sizlikler. Mi’sali’, $5 > 3$, $x < 1$.

Qatan’ yemes ten’sizlikler ≥ (u’lken yaki ten’) ha’m ≤ (kishi yaki ten’) belgisine iye ten’sizlikler. Mi’sali’, $a^2 + b^2 \geq 2ab$, $x \leq 3$.

$a \geq b$ qatan’ yemes ten’sizlik $a > b$ yaki $a = b$ yekenligin bildiredi.

Qatan’ yemes ten’sizliklerdin’ qa’siyetleri, qatan’ ten’sizliklerdin’ qa’siyetleri si’yaqli’ boladi’. Bunda qatan’ ten’sizliklerdin’ qa’siyetlerinde > ha’m < belgileri, qatan’ yemes ten’sizliklerdin’ qa’siyetlerinde bolsa, ≥ ha’m ≤ belgileri *qarama-qarsi’ belgiler* delinedi.

Yeki a ha’m b sani’ni’n’ *arifmetikali’q wortasi’:* $\frac{a+b}{2}$.

Yeki a ha’m b sani’ni’n’ *geometriyali’q wortasi’:* $\sqrt{ab}$.

Yeger $a \geq 0$, $b \geq 0$ bolsa, wonda $\frac{a+b}{2} \geq \sqrt{ab}$ boladi’.

22. Bir belgisizli ten’sizlik — ha’rip penen belgilengen belgisiz sandi’ wo’z ishine alg’an ten’sizlik.

Bir belgisizli birinshi da’rejeli ten’sizliklerge mi’sallar:

$$3x + 4 < 5x - 2; \qquad \frac{1}{3}x - 1 \geq \frac{3-x}{4}; \qquad -2,7x + 1 \leq 4,5 - 0,7x.$$

Bir belgisizli ten’sizliktin’ sheshimi — belgisizdin’ berilgen ten’sizlikti duri’s sanli’ ten’sizlikke aylandi’ri’wshi’ ma’nisi.

Mi’sali’, 3 sani’ $x + 1 > 2 - x$ ten’sizliginin’ sheshimi boladi’, sebebi $3 + 1 > 2 - 3$.

Ten’sizlikti sheshiw — woni’n’ barli’q sheshimlerin tabi’w yaki wolardi’n’ joq yekenligin da’lillew boli’p tabi’ladi’.

Bir belgisizli ten’sizliklerdin’ tiykarg’i’ qa’siyetleri:

1) ten’sizliktin’ qa’legen ag’zasi’n woni’n’ bir jag’i’nan yekinshi jag’i’na belgisin qarama-qarsi’g’a wo’zgertken halda ali’p wo’tiwge boladi’, bunda ten’sizlik belgisi wo’zgermeydi;

2) Ten’sizliktin’ yeki jag’i’n nolge ten’ bolmag’an birdey sang’a ko’beytiw yaki bo’liw mu’mkin: yeger bul san won’ bolsa, ten’sizlik belgisi wo’zgermeydi, yegerde, bul san teris bolsa, wonda ten’sizlik belgisi qarama-qarsi’g’a wo’zgeredi.

Bir belgisizli birinshi da’rejeli ten’sizlikler sistemasi’ — bir belgisiz sanni’n’ birinshi da’rejesin wo’z ishine alg’an ha’m birgelikte qaralatug’i’n yeki yaki birneshe ten’sizlikler.

Ten’sizlikler sistemasi’ni’n’ sheshimi — belgisizdin’ sistemani’n’ barli’q ten’sizliklerin duri’s sanli’ ten’sizlikke aylandi’ri’wshi’ ma’nisi.

Ten’sizlikler sistemasi’n sheshiw — woni’n’ barli’q sheshimlerin tabi’w yaki wolardi’n’ joq yekenligin da’lillewden ibarat.

FUNKCIYALAR HA’M GRAFIKLER

23. Funkciya. Yeger qandayda bir sanlar ko’pliginen ali’ng’an ha’rbir x sani’na y sani’ sa’ykes qoyi’lg’an bolsa, wonda bul ko’plikte $y(x)$ funkciyasi’ berilgen delinedi. Bunda x ti’ *yerkli wo’zgeriwshi* (yaki *argument*), al y bolsa *yerksiz wo’zgeriwshi* delinedi.

Funkciyani’n’ ani’qlani’w oblasti’ — woni’n’ argumenti qabi’l yetiwi mu’mkin bolg’an barli’q ma’nislerinin’ ko’pligi.

Yeger funkciya formula menen berilgen bolsa, wonda woni’n’ ani’qlani’w oblasti’ — argumenttin’ sol formula ma’niske iye bolatug’i’n ma’nislerinin’ ko’pligi boladi’.

Mi’sali’, $y = \sqrt{x-2}$ funkciya $x \geq 2$ bolg’anda ani’qlang’an.

Yeger bir arali’qta argumenttin’ u’lken ma’nisine funkciyani’n’ u’lken ma’nisi sa’ykes kelse, $y(x)$ funkciyasi’ usi’ arali’qta wo’siwshi delinedi, yag’ni’y bul arali’qqa tiyisli qa’legen x_1, x_2 ushi’n $x_2 > x_1$ bolsa, wonda $y(x_2) > y(x_1)$. Mi’sali’, $y = x^3$ funkciyasi’ san ko’sheri $\boldsymbol{R}$ de wo’sedi. $y = x^2$ funkciya $x > 0$ arali’qta wo’sedi. Yeger qaysi’ bir arali’qta argumenttin’ u’lken ma’nisine funkciyani’n’ kishi ma’nisi sa’ykes kelse, wonda $y(x)$ funkciyasi’ usi’ arali’qta kemeyiwshi delinedi, yag’ni’y usi’ arali’qqa tiyisli qa’legen x_1, x_2 ushi’n $x_2 > x_1$ bolsa, wonda $y(x_2) < y(x_1)$ boladi’. Mi’sali’, $y = -2x$ funkciyasi’ san ko’sheri $\boldsymbol{R}$ de kemeyiwshi boladi’; $y = x^2$ funkciya $x \leq 0$ arali’qta kemeyedi; $y = \frac{1}{x}$ funkciyasi’ barli’q $x \neq 0$ de kemeyedi.

$y(x)$ funkciyasi'ni'n' grafigi — koordinatalar tegisliginin' $(x;\ y(x))$ koordinatali' barli'q noqatlari'ni'n' ko'pligi.

Jup funkciya — woni'n' ani'qlani'w oblasti'nan ali'ng'an ha'rbir x ushi'n $y(-x) = y(x)$ qa'siyetine iye bolg'an $y(x)$ funkciyasi'. Mi'sali', $y = x^4$ jup funkciya. *Jup funkciyani'n' grafigi ordinata ko'sherine sali'sti'rg'anda simmetriyali'* boladi'.

Taq funkciya — woni'n' ani'qlani'w oblasti'nan ali'ng'an ha'rbir x ushi'n $y(-x) = -y(x)$ qa'siyetine iye bolg'an $y(x)$ funkciyasi'.

Mi'sali', $y = x^3$ – taq funkciya.

Taq funkciyani'n' grafigi koordinatalar basi'na sali'sti'rg'anda simmetriyali' boladi'.

24. Si'zi'qli' funkciya – $y = kx + b$ tu'rindegi funkciya, bunda k ha'm b – berilgen sanlar. $y = kx + b$ si'zi'qli' funkciyani'n' grafigi – tuwri' si'zi'q, $b = 0$ bolg'anda funkciya $y = kx$ ko'rinisine iye boladi', woni'n' grafigi koordinatalar basi'nan wo'tedi.

25. Tuwra proporcional baylani's – $y = kx$ formulasi' menen an'lati'lg'an baylani's, bunda $k > 0$, $x > 0$.

26. Keri proporcional baylani's, bul $y = \frac{k}{x}$ formulasi' menen an'lati'lg'an baylani's, bunda $k > 0$, $x > 0$, k – proporcionalli'q koefficienti.

$y = \frac{k}{x}$ $(k \neq 0)$ funkciya $x \neq 0$ bolg'anda ani'qlang'an, nolden basqa barli'q haqi'yqi'y ma'nislerdi qabi'l yetedi.

Yeger $k > 0$ bolsa, wonda $y = \frac{k}{x}$ funkciyasi' (mi'sali', $y = \frac{2}{x}$, $y = \frac{1}{2x}$):

a) $x > 0$ bolg'anda won' ma'nislerdi, $x < 0$ bolg'anda teris ma'nislerdi qabi'l yetedi; b) $x < 0$ ha'm $x > 0$ arali'qlarda kemeyiwshi.

Yeger $k < 0$ bolsa, $y = -\frac{k}{x}$ funkciyasi' (mi'sali', $y = -\frac{1}{x}$, $y = -\frac{2}{x}$, $y = -\frac{1}{3x}$). a) $x < 0$ bolg'anda won' ha'm $x > 0$ bolg'anda teris ma'nislerdi qabi'l yetedi; b) $x < 0$ ha'm $x > 0$ arali'qlarda wo'sedi.

$y = \frac{k}{x}$ funkciyasi'ni'n' grafigi *giperbola* delinedi. Wol koordinatalar basi'na sali'sti'rg'anda simmetriyali' jaylasqan yeki tarmaqqa iye. $k > 0$ bolg'anda grafik birinshi ha'm u'shinshi shereklerde, $k < 0$ bolg'anda bolsa yekinshi ha'm to'rtinshi shereklerde jaylasadi'.

JUWAPLAR

2. 2) $x_1 = 0$, $x_2 = 1$; 4) x ti'n' berilgen funkciyani'n' ma'nisi -5 ke ten' bolatug'i'n ma'nisleri joq. **3.** 2) $x_1 = 1\frac{3}{4}$, $x_2 = -1$; 4) $x_1 = 0$, $x_2 = \frac{3}{4}$. **4.** 2) 0; 4) 1. **5.** 2) nolleri joq; 4) $x_1 = \frac{2}{3}$, $x_2 = \frac{1}{2}$; 6) nolleri joq; 8) $x = 1$. **6.** 2) $p = 3$, $q = -4$; 4) $p = -2$, $q = -15$. **7.** $x_{1,2} = \pm 2$. **9.** B ha'm C. **12.** 2) $(\sqrt{5}; 5)$, $(-\sqrt{5}; 5)$; 4) (0; 0), (2; 4); 6) (1; 1). **13.** 2) Awa. **14.** 2) Awa; 4) yaq; **16.** 1) $x < -3$, $x > 3$; 2) $-5 \le x \le 5$; 3) $x \le -4$, $x \ge 4$; 4) $-6 < x < 6$. **20.** 2) (−3; −4,5), (2; −2). **21.** 2) Awa; 4) yaq. **22.** 1) Wo'siwshi; 2) kemeyiwshi; 3) wo'siwshi; 4) wo'siwshi de, kemeyiwshi de bolmaydi'. **23.** 3m/s^2. **26.** 2) (0; −5); 4) $\left(\frac{1}{8}; \frac{1}{16}\right)$. **27.** 2) $x=-2$; 4) $x=2$; 6) $x = \frac{3}{4}$. **28.** 2) Yaq; 4) yaq. **29.** 2) (1; 0), (0,5; 0), (0; −1); 4) (0; 0), $\left(\frac{4}{3}; 0\right)$. **30.** $y = x^2 - 2x + 3$. **32.** 2) $k = -10$. **34.** 1) $y = 2(x - 3)^2$; 2) $y = 2x^2 + 4$; 3) $y = 2(x + 2)^2 - 1$; 4) $y = 2(x - 1{,}5)^2 + 3{,}5$. **35.** 2) $\left(-\frac{3}{2}; \frac{11}{4}\right)$; 4) $\left(\frac{5}{2}; \frac{21}{4}\right)$. **36.** 2) (1; 0), (−5; 0), (0; 10); 4) (0; 14). **40.** 7,5+7,5. **41.** 5 ha'm 5. **42.** Diy-walg'a parallel ta'repi 6 m; qalg'an ta'repleri 3 m den. **43.** Joq. **44.** 2) $x = 1$ da $y = -5$ yen' kishi ma'nis; 4) $x = 1$ de $y = -2$ yen' kishi ma'nis. **45.** 1) $a > 0$, $b > 0$, $c > 0$; 2) $a < 0$, $b > 0$, $c < 0$. **46.** 1) 5 s tan keyin yen' u'lken biyiklik 130 m ge ten'; 2) $(5+\sqrt{26})s$. **47.** 2) $x_1 = 2$, $x_2 = 0{,}5$; 4) x ti'n' bunday ma'nisi joq. **48.** 2) (1; 1), (2; 4); 4) (−5; 18). **49.** 2) $x < -6$, $x > 6$. **50.** 2) (5; 0), (−2; 0) , (0; 10); 4) (1; 0), $\left(-\frac{11}{7}; 0\right)$, (0; −11). **51.** 2) (−1; 4); 4) $\left(-\frac{1}{2}; 1\right)$; 6) $\left(-\frac{1}{2}; -6\frac{1}{4}\right)$. **53.** 2) Yen' u'lken ma'nis 4 ke ten'; 4) yen' kishi ma'nis $3\frac{2}{3}$ ge ten'. **54.** 150 m ha'm 150 m. **55.** 200 m ha'm 400 m. **56.** 2) $p = 1$, $q = 0$. **57.** 2) $p = -4$, $q = 3$. **58.** 1) $x_1 = 1$, $x_2 = -5$; 2) $x_1 = 0$, $x_2 = 1$, $x_3 = 2$. **59.** 1) $a = 1$, $b = -2$, $c = 0$; 2) $a = 1$, $b = -2$, $c = 4$; 3) $a = -2$, $b = 8$, $c = -6$. **61.** 2) $3x^2 - x - 1 > 0$; 4) $2x^2 + x - 5 < 0$. **63.** 2) $3 < x < 11$; 4) $x < -7$, $x > -1$. **64.** 2) $x < -3$, $x > 3$; 4) $x < 0$, $x > 2$. **65.** 2) $-2 < x < 1$; 4) $x < -3$, $x > 1$; 6) $x < -1$, $x > \frac{1}{3}$. **66.** 2) $x = \frac{1}{6}$; 4) $x < -4$, $x > 2$. **69.** Won' ma'nisler $x < -3$, $x > 2$

arali'g'i'nda, teris ma'nis-leri $-3 < x < 2$ intervalda. **71.** 2) $x \le -1$, $x \ge 4$; 4) $-1 < x < 4$. **72.** 2) $x < -\frac{1}{3}$, $x > 2$; 4) $x \le -0{,}25$; $x \ge 1$. **73.** 2) $x = 7$; 4) sheshimleri joq; 6) $x-$ qa'legen haqi'yqi'y san. **74.** 2) Sheshimleri joq; 4) sheshimleri joq; 6) x – qa'legen haqi'yqi'y san. **75.** 2) $x < -\sqrt{7}$, $x > \sqrt{7}$; 4) $x < -2$; $x > 0$; 6) $x < -5$; $x > 3$; 8) $-2 < x < 1$. **77.** 2) $x < -\frac{5}{3}$, $x > \frac{5}{3}$; 4) $-1 < < x < 4$; 6) x – qa'legen haqi'yqi'y san; 8) $x = -3$. **78.** 2) x – qa'legen haqi'yqi'y san; 4) $x \ne \frac{1}{4}$; 6) $-\frac{1}{3} \le x \le 0$; 8) sheshimleri joq. **79.** 2) Sheshimleri joq; 4) $-0{,}5 < x < 3$; 6) x – qa'legen haqi'yqi'y san. **80.** 2) $x = 1$; 4) x – qa'legen haqi'yqi'y san. **82.** $-6 < r < 2$. **84.** 2) $-5 < x < 8$; 4) $x < -5$, $x > 3\frac{1}{2}$. **85.** 2) $x < 0$, $x > > 9$; 4) $-3 < x < 0$; 6) $x < -1$, $x > 3$. **86.** 2) $-\frac{1}{2} < x < 0$, $x > \frac{1}{2}$; 4) $-2 < x < 2$, $x > 5$. **87.** 2) $-7 < x < 7$; 4) $-4 < x < 4$, $x > 4$; 6) $x = -2$; $2 \le x \le 5$. **88.** 2) $-3 < x < 4$; 4) $-3{,}5 \le x < 7$; 6) $-2 \le x < -1$, $x \ge 3$. **89.** 2) $x < 0{,}5$, $x > 1$; 4) $x < -\frac{2}{3}$, $0 < x < \frac{1}{2}$, $x > \frac{2}{3}$. **90.** 2) $-4 < < x < -2$, $x > 3$; 4) $-3 \le x \le -1$, $4 \le x \le 5$. **91.** 2) $x < -2$, $2 < x < 6$; 4) $x < -3$, $-1 \le x < 2$, $x \ge 4$. **92.** 2) $-\sqrt{15} < x < -3$, $0 < x < \sqrt{15}$. **93.** 1) $-8 < x < -1$; 2) $x < -5$, $x > 2$; 3) $-1 < x \le -\frac{2}{5}$; 4) $x < -4$, $--4 < x < \frac{3}{2}$, $x > 4$. **94.** 2) $x < 2$, $x > 4$; 4) $x < 3$, $x > 4$. **95.** 2) $x < -6$, $x > 6$; 4) $-\frac{3}{4} \le x \le \frac{3}{4}$. **96.** 2) $-\frac{1}{2} \le x \le \frac{1}{2}$; 4) $x \le 0$, $x \ge \frac{1}{3}$. **97.** 2) $x < \frac{1}{2}$, $x > 4$; 4) $-2 < < x < \frac{1}{2}$; 6) $x < \frac{4}{5}$, $x > 1$. **98.** 2) $x \ne -5$; 2) $x \ne -\frac{3}{2}$; 6) $x \ne \frac{1}{2}$. **99.** 2) Sheshimleri joq; 4) sheshimleri joq; 6) sheshimleri joq. **100.** 2) $x < -1$, $1 < x < 4$; 4) $x < -\frac{1}{2}$, $4 < x \le 7$; 6) $x \ge 2$, $-\frac{1}{2} \le x < 1$. **101.** 2) $-1 < x < 5$; 4) $-5 \le x \le 2$; 6) $x \le \frac{3}{2}$, $x \ge \frac{1}{3}$. **102.** 2) x – qa'legen haqi'yqi'y san; 4) sheshimleri joq; 6) $\frac{1}{2} < x < 1$; 8) x – qa'legen haqi'yqi'y san. **103.** 2) $x \le -\frac{3}{2}$, $x \ge -1$; 4) $x = \frac{2}{3}$; 6) sheshimleri joq. **104.** 2) $x < -\sqrt{3}$; $-\frac{\sqrt{3}}{2} < x < \sqrt{3}$; 4) $x < -4$, $-1 < x \le 1$, $x > 1$. **105.** 2) $-1 < x < -\frac{1}{5}$, $\frac{3}{4} < x < 2$; 4) $-\frac{1}{3} < x \le -\frac{1}{5}$, $\frac{1}{2} < x \le 2$. **106.** 12 km/saattan kem yemes. **108.** 2) $x < -3$, $-2 < x < 1$, $x \ge 3$; 2) $-3 < x < -2$, $-1 \le x \le 1$; 3) $-\sqrt{2} < x < < -1$, $1 < x < \sqrt{2}$; 4) $x < -2$, $-\sqrt{3} < x < -3$, $x > 2$. **109.** 2) 32; 4) 0. **110.** 2) $\left(\frac{1}{5}\right)^5$; 4) $\left(\frac{c}{d}\right)^2$. **112.** 2) 21^{-3}; 4) a^{-9}. **113.** 2) $\frac{121}{81}$; 4) 32; 6) $-\frac{1}{169}$. **114.** 2) $\frac{53}{16}$; 4) -875. **116.**

2) $\frac{1}{(x+y)^3}$; 4) $\frac{9a^3}{b^4}$; 6) $\frac{a^2}{bc^4}$. **117.** 2) -125; 4) $\frac{1}{17}$. **118.** 2) 0,0016; 4) $\frac{16}{625}$. **119.** 2) b^8; 4) b^{-28}. **120.** 2) a^8b^{-4}; 4) $3^{-4}a^{-12}$; **121.** 2) $m^{12}n^{-15}$; 4) $-64x^{-15}y^3z^{-9}$. **122.** 2) $\frac{97}{9}$. **123.** 2) $2{,}7\cdot10^{-8}$; 4) $8\cdot10^{-9}$. **124.** 2) $5{,}086\cdot10^{-8}$; 4) $1{,}6\cdot10^{-3}$. **125.** 0,003. **126.** 10^{-11}. **127.** 0,0001 mm. **128.** 2) a^5, $\frac{1}{32}$. **129.** 2) 0. **130.** 2) $b-a$. **132.** 2) 2; 4) 15. **133.** 2) 81; 4) $\frac{1}{81}$. **134.** 2) -1; 4) -4; 6) -8. **135.** 2) $x=-\frac{1}{2}$; 4) $x_1=-2$, $x_2=2$. **136.** 2) x – qa'legen san; 4) $\frac{2}{3}\le x<2$. **137.** 2) 5; 4) -11; 6) $\frac{1}{30}$. **138.** 2) 2; 4) $4\sqrt{6}$. **139.** 1) $x-2$; 2) $(3-x)^3$, $x\le3$, $(x-3)^3$, $x>3$ da. **140.** 3974. **141.** 2) $36\sqrt[3]{4}$; 4) 20. **142.** 2) 33; 4) 7. **143.** 2) 0,2; 4) 2. **144.** 2) 50; 4) 16. **145.** 2) a^2b^3; 4) a^2b^3. **146.** 2) $3ab$; 4) $\frac{2}{b}$. **147.** 2) $\frac{2}{3}$; 4) $\frac{3}{2}$. **148.** 2) $\frac{2}{5}$; 4) 2; 6) 4. **149.** 2) $3x$; 4) $2\frac{b}{a}$. **150.** 2) $\frac{1}{3}$; 4) $\frac{1}{4}$. **151.** 2) $4\sqrt[4]{4}$; 4) 5. **152.** 2) y^2; 4) a^8b^9; 6) $3a$. **153.** 2) $\frac{3}{2}$; 4) $\frac{3}{2}$; 6) 4. **154.** 2) $\frac{2a^2}{b}$; 4) $\frac{a}{b}$; 6) a^2b. **155.** 2) 6; 4) $\frac{1}{2}$; 6) 4. **156.** 2) ab^2c; 4) $2xy$. **157.** 2) $3x$; 4) 0. **158.** 2) 7; 4) 1. **162.** 2) 3; 4) 27; 6) $\frac{1}{27}$. **163.** 2) 5; 4) $\frac{1}{2}$; 6) $\frac{1}{2}$. **164.** 2) 49; 4) 125. **165.** 2) 121; 4) 150. **166.** 2) 3; 4) 2,7. **167.** 2) b; 4) a; 6) 1. **168.** 2) a^2b. **169.** 2) 1. **170.** 2) 3. **171.** 2) $b^{\frac{1}{2}}$; 4) $a+b$; 6) $a^{\frac{1}{4}}+b^{\frac{1}{4}}$; 8) $\sqrt{c}-1$. **172.** 2) $\frac{a^{\frac{1}{3}}\cdot b^{\frac{1}{3}}}{a^{\frac{1}{3}}+b^{\frac{1}{3}}}$; 4) $2\sqrt{b}$. **173.** 2) $2y$; 4) $2\sqrt[3]{b}$. **174.** 2) $2\sqrt[3]{b}$; 4) $\frac{2\sqrt[3]{a}}{a+b}$. **176.** 2) $\left(\frac{5}{12}\right)^{-\frac{1}{4}}<(0{,}41)^{-\frac{1}{4}}$; 4) $\left(\frac{11}{12}\right)^{-\sqrt{5}}>\left(\frac{12}{13}\right)^{-\sqrt{5}}$. **177.** 2) $x=3$; 4) $x=2$; 6) $x=\frac{1}{2}$. **178.** $\sqrt{\left(1\frac{1}{4}-1\frac{1}{5}\right)^3}>\sqrt{\left(1\frac{1}{6}-1\frac{1}{7}\right)^3}$. **179.** 2) $x=\frac{5}{2}$; 4) $y=5$. **180.** 2) $x=2{,}6$; 4) $x=4$. **181.** 2) $x=-\frac{1}{3}$; 4) $x=1$. **182.** 2) 6; 4) -3. **183.** 2) 1,8; 4) $\frac{1}{16}$. **184.** 2) 51; 4) 0,04; 6) $-0{,}1$. **185.** 2) 1000. **186.** 2) $\sqrt[4]{x}$; 4) $\frac{1}{\sqrt{x^2-y^2}}$. **187.** 2) $x=-1$; 4) $x=1$. **188.** 2) $\frac{95}{16}$; 4) $-609\frac{8}{27}$. **189.** 2) x – qa'legen san; 4) $x\le2$, $x\ge3$; 6) $0\le x\le2$, $x\ge3$. **190.** 2) $a+1$; 4) $a^{\frac{1}{3}}+b^{\frac{1}{3}}$; 6) $a^{\frac{1}{2}}-b^{\frac{1}{2}}$. **191.** 2) $x=2$ de $y=1$; $x=0$ ha'm $x=4$ de $y=5$; $x=-1$ ha'm $x=5$ da $y=10$; $x=-2$ ha'm $x=6$ de $y=17$. **192.** 1) $y(-2)=-1$, $y(0)=-5$,

$y\left(\frac{1}{2}\right) = -11,\ y(3) = 4$; 2) $x = -\frac{1}{2}$ de $y = -3$; $x = -1$ de $y = -2$; $x = \frac{3}{2}$ de ыy $= 13$; $x = \frac{4}{3}$ de $y = 19$. **194.** 2) $x \le 2$, $x \ge 5$; 4) $-2 \le x < 3$. **195.** 1) $y(-3) = 3$, $y(-1) = 1$, $y(1) = -1$, $y(3) = -1$; 2) $x = 2$ de $y = -2$; $x = 0$ ha'm $x = 4$ da $y = 0$; $x = -2$ ha'm $x = 6$ de $y = 2$; $x = -4$ ha'm $x = 8$ de $y = 4$. **196.** 2) $x \ne -1$; 5) $-1 \le x \le 1$, $x \ge 4$; 6) $-5 \le x \le 1$, $x > 2$; 8) $x \ge 0$. **197.** 2) Awa; 4) awa. **203.** 2) $x = 16$; 4) $x = \frac{1}{16}$; 6) $x = \frac{1}{243}$. **205.** 2) $x = 32$; 4) $x = 8$. **208.** 2) taq; 4) jup ha'm, taq ha'm bolmaydi'. **209.** 2) taq; 4) taq; 6) jup ha'm, taq ha'm bolmaydi'. **218.** 2) $x = 0$. **219.** 2) $(-1;\ 0)$. **220.** 2) $x = -4$ da $y = -\frac{1}{2}$; 4) $x < 0$ ha'm $x \ge 2$ de $y \le 1$. **222.** 2) $(-2;\ 4)$ ha'm $(2;\ -4)$; 4) $(-4;\ -2)$ ha'm $(1;\ 3)$. **228.** 2) $x \le 3$; 4) $y < 5$; 6) $x < -5$, $x > 5$. **229.** 2) Kubti'n' qabi'rg'asi' 7 dm den arti'q. **232.** 2) $x = 10$; 4) $x = 5$. **233.** 2) $x = 2$; 4) $x = 2$; $x = -7$. **234.** 2) $x = 4$; 4) $x = 0{,}2$. **235.** $x = \frac{7}{3}$. **236.** 2) $x > -3$; 4) $x < 2$; 6) $x < 1$, $x > 7$. **238.** 2) $x = -2$; 4) $x_1 = 1$; $x_2 = 3$. **239.** 2) $x = 2{,}25$. **240.** 2) $x = 1$; 4) $x = 5$. **241.** 2) $x = 4$. **242.** 2) $2 \le x \le 3$; 4) $1 < x \le 2$; 6) $x \ge 1$. **243.** 2) $x \ne \frac{3}{2}$; 4) x – qa'legen san. **246.** 2) $\left(-\frac{1}{\sqrt{2}};\ -\sqrt{2}\right)$, $\left(-\frac{1}{\sqrt{2}};\ \sqrt{2}\right)$. **247.** 2) $(-1;\ -1)$; $(1;\ 1)$. **248.** 2) $x > 2$; 4) $x \le -2$. **249.** 2) $x = 16$. **250.** 2) $x_1 = \frac{1}{2}$, $x_2 = \frac{1}{3}$; **251.** 2) x – qa'legen san; 4) $2 \le x \le 11$; 6) $x < -7, -3 \le x < -1$, $x \ge 3$. **252.** 2) kemeyedi; 4) kemeyedi. **253.** 2) taq; 4) jup ha'm, taq ha'm bolmaydi'. **255.** 2) $-2 \le x \le \frac{1}{3}$. **256.** 2) $x_1 = -1$, $x_2 = 7$; 4) $x = 81$; 6) $x_1 = 3$, $x_2 = 7$. **257.** 1) $x < -1$, $x > 9$; 2) $-1 < x \le 0$, $3 \le$ $\le x < 4$; 3) $\frac{2}{3} \le x < 6$; 4) $x \ge 4$. **258.** 2) $\frac{2\pi}{3}$; 4) $\frac{5\pi}{6}$; 6) $\frac{8\pi}{45}$; 8) $\frac{7\pi}{9}$. **259.** 2) $20°$; 4) $135°$; 6) $\left(\frac{720}{\pi}\right)^{\circ}$; 8) $\left(\frac{324}{5\pi}\right)^{\circ}$. **260.** 2) 4,71; 4) 2,09. **261.** 2) $2\pi < 6{,}7$; 4) $\frac{3\pi}{2} < 4{,}8$; 6) $-\frac{3\pi}{2} < -\sqrt{10}$. **263.** 0,4 m. **264.** 2 rad. **265.** $\frac{3\pi}{8}$ sm^2. **266.** 2 rad. **267.** 2) $(-1;\ 0)$; 4) $(0;\ -1)$; 6) $(1;\ 0)$. **269.** 2) yekinshi sherek; 4) to'rtinshi sherek; 6) yekinshi sherek. **270.** 2) $(0;\ 1)$; 4) $(-1;\ 0)$; 6) $(0;\ 1)$. **271.** 2) $2\pi k$, $k = 0, \pm1, \pm2, \ldots$; 4) $\frac{\pi}{2} + 2\pi k$, $k = 0, \pm1, \pm2, \ldots$. **272.** 2) yekinshi sherek; 4) to'rtinshi sherek. **273.** 2) $x = 1{,}8\pi$, $k = 4$; 4) $x = \frac{4}{3}\pi$, $k = 3$; 6) $x = \frac{5}{3}\pi$, $k = 2$. **275.** 2) $(0;\ 1)$; 4) $(0;\ -1)$. **276.** 2) $\frac{\pi}{6} + 2\pi k$, $k = 0, \pm1, \pm2, \ldots$; 4) $\frac{3\pi}{4} + 2\pi k$, $k = 0, \pm1, \pm2, \ldots$. **277.** 2) $-\frac{1}{2}$; 4) -1; 6) -1; 8) $\frac{1}{\sqrt{2}}$. **279.** 2) -1; 4) -1; 6) 1. **280.** 2) 0; 4) -1. **281.** 2) $\frac{-\sqrt{2}-9}{2}$; 4) $-\frac{1}{4}$.

282. 2) $x=\frac{\pi}{2}+\pi k$, $k=0, \pm1, \pm2, \dots$; 4) $x=\frac{\pi}{2}+2\pi k$, $k=0, \pm1, \pm2, \dots$. **284.** 2) $-\frac{5}{4}$; 4) $\frac{1+\sqrt{2}}{2}$. **285.** 2) $x=\pi+2\pi k$, $k=0, \pm1, \pm2, \dots$; 4) $x=\pi+2\pi k$, $k=0, \pm1, \pm2, \dots$; 6) $x=\frac{2}{3}k\pi$, $k=0, \pm1, \pm2, \dots$. **286.** 2) $x=2\pi k-1$, $k=0, \pm1, \pm2, \dots$; 4) $x=k\pi-1$, $k=0, \pm1, \pm2, \dots$; 6) $x=\frac{2\pi k}{3}+1$, $k=0, \pm1, \pm2, \dots$. **287.** 2) yekinshi sherek; 4) yekinshi sherek; 6) yekinshi sherek. **288.** 2) won'; 4) won'; 6) won'. **289.** 2) teris; 4) teris; 6) won'. **290.** 2) won', won'; 4) teris, teris; 6) teris, teris; 8) won', won'. **291.** 2) $\sin\alpha<0$, $\cos\alpha>0$, $\mathrm{tg}\alpha<0$, $\mathrm{ctg}\alpha<0$; 4) $\sin\alpha>0$, $\cos\alpha>0$, $\mathrm{tg}\alpha>0$, $\mathrm{ctg}\alpha>0$. **292.** 2) $\sin 3>$ >0, $\cos 3<0$, $\mathrm{tg}3<0$; 4) $\sin(-1,3)<0$, $\cos(-1,3)>0$, $\mathrm{tg}(-1,3)<0$. **293.** 2) teris; 4) won'; 6) won'; 8) teris. **294.** Yeger $0<\alpha<\frac{\pi}{2}$ yaki $\pi<\alpha<\frac{3\pi}{2}$ bolsa, $\sin\alpha$ ha'm $\cos\alpha$ sanlari'ni'n' belgileri sa'ykes keledi; yeger $\frac{\pi}{2}<\alpha<\pi$ yaki $\frac{3\pi}{2}<\alpha<2\pi$ bolsa, $\sin\alpha$ ha'm $\cos\alpha$ sanlari' qarama-qarsi' belgilerge iye. **295.** 2) teris; 4) won'. **296.** 2) $\cos 1{,}3>\cos 2{,}3$; **297.** 2) $x=\frac{\pi}{2}+k\pi$, $k=0, \pm1, \pm2, \dots$; 4) $x=\pi+2k\pi$, $k=0, \pm1, \pm2, \dots$. **298.** 2) yekinshi sherek. **299.** $\frac{h\cos\alpha}{1-\cos\alpha}$. **300.** 2) $\cos\alpha=-\frac{3}{5}$, $\mathrm{tg}\alpha=-\frac{4}{3}$; 4) $\cos\alpha=-\frac{\sqrt{21}}{5}$, $\mathrm{tg}\alpha=\frac{2}{\sqrt{21}}$, $\mathrm{ctg}\alpha=\frac{\sqrt{21}}{2}$; 6) $\sin\alpha=-\frac{1}{\sqrt{10}}$, $\cos\alpha=\frac{3}{\sqrt{10}}$. **301.** 2) wori'nlanadi'; 4) wori'nlanbaydi'. **302.** 2) wori'nlanbaydi'. **303.** $\cos\alpha=\frac{9}{11}$, $\mathrm{tg}\alpha=\frac{2\sqrt{10}}{9}$. **304.** $\frac{1}{3}$. **305.** $\cos\alpha=\pm\frac{3}{4}$. **306.** $\sin\alpha=\pm\frac{2}{\sqrt{5}}$. **307.** 2) $\frac{1}{3}$; 4) 2. **308.** 1) $-\frac{3}{8}$; 2) $\frac{11}{16}$. **309.** 1) $x=$ $=\pi k$, $k=0, \pm1, \pm2, \dots$; 2) $x=-\frac{\pi}{2}+2\pi k$, $k=0, \pm1, \pm2, \dots$; 3) $x=2\pi k$, $k=0, \pm1, \pm2, \dots$; 4) $\frac{\pi}{2}+\pi k$, $k=0, \pm1, \pm2, \dots$. **311.** 1) 0; 4) $1+\sin\alpha$. **312.** 2) 3; 4) 4. **316.** 2) $\frac{2}{\sqrt{3}}$. **317.** $\frac{8}{25}$. **318.** $\frac{37}{125}$. **319.** 1) $x=\pi k$, $k=0, \pm1, \pm2, \dots$; 2) $x=\frac{\pi}{2}+2\pi k$, $k=0, \pm1, \pm2, \dots$. **320.** 2) $\frac{1}{3}$; 4) -3. **321.** 2) $2\cos\alpha$; 4) 2. **323.** 2) 2. **324.** 2) $-2\cos\alpha$. **325.** 2) $-\frac{1}{2}$; 4) $-\frac{1}{2}$. **326.** 2) $\frac{1}{\sqrt{2}}$; 4) -1. **327.** 2) $\frac{4-\sqrt{2}}{6}$. **328.** 2) $\cos 3\beta$; 4) -1. **329.** $-\sin\alpha\cdot\sin\beta$. **330.** 2) $\frac{\sqrt{3}}{2}$; 4) 1. **331.** 2) $-\frac{2+\sqrt{14}}{6}$. **332.** 2) $-\sin\alpha\cdot\cos\beta$; 4) $\sin\alpha\cdot\cos\beta$. **333.** $\cos(\alpha+\beta)=\frac{84}{85}$; $\cos(\alpha-\beta)=\frac{36}{85}$. **334.** $-\frac{63}{65}$. **335.** 2) 0; 4) $\mathrm{tg}\alpha\cdot\mathrm{tg}\beta$. **338.** 2) $\frac{\sqrt{3}}{2}$; 4) $\frac{3}{2}$. **339.** 2) $\frac{1}{\sqrt{2}}$; 4) -1. **340.** 2) $\frac{24}{25}$. **341.** 2) $\frac{7}{25}$. **342.** 2) $\frac{1}{2}\sin 2\alpha$; 4) 1. **343.** 2) $2\mathrm{ctg}\alpha$; 4) $\mathrm{ctg}^2\alpha$. **345.** 2) $\frac{8}{9}$. **347.** 2) $\frac{1}{\sqrt{2}}$; 4) $\frac{\sqrt{3}}{2}$. **348.** 2) $\cos 6\alpha$; 4) $\frac{1}{2\sin\alpha}$. **350.** $\frac{15}{8}$. **351.** 2) $\sqrt{3}$. **352.** 2) 0; 4) 0; 6) -1. **353.** 2) $\frac{1}{\sqrt{3}}$; 4) $\frac{1}{\sqrt{3}}$;

6) $-\frac{1}{\sqrt{3}}$. **354.** 2) $\frac{1}{\sqrt{2}}$; 4) $-\frac{1}{\sqrt{2}}$. **355.** 2) $-\frac{1}{2}$; 4) $\frac{1}{2}$; 6) $\sqrt{3}$. **356.** 2) $-\sqrt{2}$; 4) –1. **357.** 2) cos2α. **358.** 2) $-\frac{5\sqrt{3}}{6}$; 4) $\frac{1}{2}$; 6) $\frac{5-3\sqrt{3}}{4}$. **359.** 2) 1; 4) $-\frac{1}{\cos\alpha}$. **362.** $x=\frac{\pi}{2}+2\pi k$, $k=0, \pm1, \pm2, \ldots$; 4) $x=\pi+2\pi k$, $k=0, \pm1, \pm2, \ldots$. **363.** 2) $\sqrt{2}\sin\beta$; 4) sin2α. **364.** 2) 0; 4) $-\frac{\sqrt{6}}{2}$; 6) $\frac{\sqrt{6}}{2}$. **365.** 2) $4\sin\left(\frac{\pi}{12}-\frac{\alpha}{2}\right)\cos\left(\frac{\pi}{12}+\frac{\alpha}{2}\right)$; 4) $2\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$. **367.** 2) 2sinα. **370.** 2) $2\sqrt{3}\sin\frac{5\pi}{24}\sin\frac{\pi}{8}$. **371.** 2) 0. **372.** 2) $2\cos\alpha(\cos\alpha-1)$; 4) $(\sin\alpha+\cos\alpha)\cdot\left(1+\frac{1}{\cos\alpha}\right)$. **373.** 2) u'shinshi sherek; 4) yekinshi sherek; 6) yekinshi sherek. **374.** 2) 0; 1; 4) 1; 0; 6) 0; –1. **375.** 2) 2; 4) –1. **376.** 2) $\frac{2}{\sqrt{5}}$; 4) $-\frac{1}{\sqrt{3}}$. **378.** 2) 3; 4) $\mathrm{tg}^2\alpha$. **379.** 2) $-\frac{1}{3}$. **380.** 2) $-\frac{\sqrt{3}}{2}$; 4) $\frac{\sqrt{2}(\sqrt{3}+1)}{4}$. **381.** 2) sin2α; 4) tg2α. **382.** 2) 1; 4) $-\frac{1}{\sqrt{2}}$. **383.** 2) $-\frac{\sqrt{3}}{2}$; 4) $-1-\frac{1}{\sqrt{2}}$. **384.** 2) cos 0 > > sin5. **385.** 2) teris; 4) won'. **386.** 2) $\frac{\sqrt{2}(\sqrt{3}-1)}{4}$; 4) $\frac{\sqrt{6}-\sqrt{2}}{4}$; 6) $-\frac{1}{\sqrt{2}}$. **387.** 2) $\frac{1}{\sin\alpha}$. **388.** $\cos\alpha=-\frac{2}{3}$; $\mathrm{tg}\alpha=-\frac{\sqrt{5}}{2}$; $\mathrm{ctg}\alpha=-\frac{2}{\sqrt{5}}$; $\sin 2\alpha=-\frac{4\sqrt{5}}{9}$; $\cos 2\alpha=-\frac{1}{9}$. **389.** 2) tgα. **390.** 2) $\frac{1}{\sin 4\alpha}$; 4) $-\frac{1}{\cos 2\alpha}$. **391.** 2) 1; 4) 1. **392.** 2) −7. **393.** 2) cos4α. **395.** 2) –3, –1, 1, 3, 5. **397.** 2) 79; 4) −42. **398.** 2) $a_n=29-4n$; 4) $a_n=6-5n$. **399.** 12. **400.** Awa, $n=11$. **401.** $n=11$, yaq. **402.** 2) 0,5. **403.** 2) –13. **404.** 2) –100. **405.** 2) $a_n=$ $=5n-17$. **406.** $n\geq 9$. **407.** $n<25$. **408.** 2) $a_9=-57$, $d=7$; 4) $a_9=-1$, $d=-15$; **409.** 44,1 m. **410.** 10 kun. **411.** 30. **412.** 60. **413.** 2) 10050; 4) 2550. **414.** 4850. **415.** 4480. **416.** 2) –192. **417.** 2) 204. **418.** 2) 240. **419.** 4905; 494550. **420.** 2) 2900. **421.** 10. **422.** 2) $a_{10}=15\frac{5}{6}$, $d=\frac{3}{2}$. **423.** 2) $a_1=-88$, $d=18$. **424.** 78 tosi'n. **425.** 44. **426.** $a_1=5$, $d=4$. **429.** 2) –3, 12, –48, 192, –768. **431.** 2) $\frac{1}{16}$; 4) $\frac{1}{81}$. **432.** 2) $b_n=3\cdot\left(\frac{1}{3}\right)^{n-1}$; 4) $b_n=3\cdot\left(-\frac{4}{3}\right)^{n-1}$. **433.** 2) 5; 4) 8. **434.** 2) 3; 4) $-\frac{1}{5}$. **435.** $b_8=2374$, $n=5$. **436.** $b_7=3\sqrt{3}$, $q=\frac{1}{\sqrt{3}}$. **437.** $b_5=6, b_1=30\frac{3}{8}$ yaki $b_5=-6$, $b_1=-30\frac{3}{8}$. **438.** 659100 swm. **439.** 0,25 sm². **440.** 2) $-\frac{31}{8}$; 4) $-\frac{275}{81}$; 6) –400. **441.** 2) 2186. **442.** 2) $b_1=-1$, $b_8=128$. **443.** 2) $n=7$; 4) $n=5$. **444.** 2) $n=9$, $b_9=2048$; 4) $n=5$, $q=7$. **445.** 2) 364; 4) 305. **446.** 2) $b_5==4802$, $S_4=800$. **447.** 2) $-1\frac{31}{32}$. **449.** 2) $q=5$, $b_3=300$ yaki $q=-6$, $b_3=432$. **450.** 2) $q=2$ yaki $q=-2$;

4) $S_5 = 781$ yaki $S_5 = 521$. **452.** 2) awa; 4) awa. **453.** 2) 7,2; 4) $-8\frac{1}{6}$. **454.** 2) $\frac{27}{4}$; 4) $\frac{2}{3}$. **455.** 2) yaq; 4) awa. **456.** 2) $90\frac{10}{11}$. **457.** 2) $6+4\sqrt{3}$. **458.** 2) $\frac{1}{2}$; **459.** $2a$. **460.** $R_n = \frac{1}{3^{n-1}} \cdot R_1$. **461.** 2) 1; 4) $\frac{7}{30}$. **462.** 2) $d=-\frac{1}{2}$, $a_4 = 2$, $a_5 = 1\frac{1}{2}$; 4) $d=-3$, $a_4 = \sqrt{2}-9$, $a_5 = \sqrt{2}-12$. **464.** $-5\frac{1}{3}$. **465.** 2) –1080. **466.** 143. **467.** 2) –22. **468.** 2) $q=-\frac{1}{2}$, $b_4 = -\frac{1}{32}$, $b_5 = \frac{1}{64}$; 4) $q=-\sqrt{2}$, $b_4 = -10\sqrt{2}$, $b_5 = 20$. **469.** 2) $b_n = -0{,}5\cdot(-2)^{n-1}$. **470.** 2) $b_n = \frac{125}{8}$. **471.** 2) $S_{10} = 1\frac{85}{256}$; 4) $S_9 = 5$. **472.** 2) 242; 4) $\frac{65}{36}$. **473.** 2) $-\frac{4}{5}$. **474.** $24\frac{41}{74}$. **475.** 2) 14, 11, 8, 5, 2. **476.** $-\frac{5}{2}$. **477.** 2) $a_{19} = 0$, $a_1 = -108$. **478.** 2) $x_1 = \frac{1}{3}$; 4) $x_2 = -4$. **480.** 14. **481.** 2) $a_{16} = -1\frac{2}{3}$, $d=-\frac{2}{15}$. **482.** 2) 27. **483.** 2) –27; 4) $\pm\frac{1}{25}$. **484.** 6. **485.** 2) Yaq; 4) Awa. **487.** Sa'rshembi ku'ni. **488.** $a_1 = 8$, $d=-3$ yaki $a_1 = 2$, $d=3$. **489.** $a_1 = 5$, $d=-5$ yaki $a_1 = -5$, $d=5$. **490.** 180 ma'rte. **495.** 2) $-15 < x < 2$; 4) $x \le 12$, $x \ge 12$; **496.** 2) $0 < x < \sqrt{5}$; 4) $x < -\sqrt{3}$; $x > \sqrt{3}$. **497.** 2) $-9 < x < 6$; 4) $-2 < x < 0{,}1$; 6) $x \le \frac{1}{8}$, $x \ge 2$. **498.** 2) $x = -12$; 4) x – qa'legen haqi'yqi'y san; 6) sheshimleri joq. **499.** 2) x – qa'legen haqi'yqi'y san; 4) x – qa'legen haqi'yqi'y san; 6) x – qa'legen haqi'yqi'y san. **500.** 2) $-0{,}7 < x < \frac{1}{2}$; 2) $-2 \le x \le 1$. **501.** 2) $x \le -2$, $x = 1$; 4) $x \le -\frac{1}{3}$, $0 \le x \le 2$. **502.** 2) $-0{,}5 \le x < 2$; 4) $-3 < x < 0$, $x > 1$. **503.** Biyiklik 3,1 sm den arti'q, worta si'zi'q 6,2 sm den arti'q. **504.** 8 s den arti'q. **505.** 5 sm arti'q. **506.** 2) $x < -7$, $-1 \le x \le 2$; 4) $-1 \le x < -\frac{1}{3}$, $x > \frac{1}{3}$. **507.** $p = 5$, $q = -14$. **508.** 2) $p = 14$, $q = 49$. **509.** $y = -2x^2 + 11x - 5$. **510.** $y = \frac{n}{r^2}x^2$. **511.** 2) $a=-1$, $b=-1$, $c=2$. **512.** *Ko'rsetpe.* 1) $\frac{a}{b} = A^3$, $\frac{b}{c} = B^3$, $\frac{c}{a} = C^3$ tu'rinde belgilep ha'm $ABC = 1$ ten'likti yesapqa ali'p, berilgen ten'sizlikti $A^3 + B^3 + C^3 \ge 3ABC$ ko'riniste jazi'n', woni' $(A+B+C)(A^2 + B^2 + C^2 - AB - AC - BC) \ge 0$ ko'riniste tu'rlendirin'. ($A^2 + B^2 + C^2 \ge AB+AC+BC$ ten'sizliklerdi qosi'w menen payda yetiledi; 2) worta arifmetikali'q ha'm worta geometriyali'q shamalarg'a tiyisli ten'sizliklerdi qosi'n': $A^2 + B^2 \ge 2AB$, $A^2 + C^2 \ge 2AC$, $B^2 + C^2 \ge 2BC$ $\frac{bc}{a} + \frac{ac}{b} \ge 2c$, $\frac{ac}{b} + \frac{ab}{c} \ge 2a$, $\frac{ab}{c} + \frac{bc}{a} \ge 2b$; 3) ten'sizliktin' shep bo'liminen won' bo'limin ali'n' ha'm payda bolg'an bo'lshektin' ali'mi'n mi'na ko'riniste jazi'n': $(a+b)(a-b)^2 + (b+c)(b-c)^2 + (a+c)(a-c)^2$; 1) $x_{1,2} = \pm 2$; 2)

$x_{1,2} = \pm 1$; 3) $x_{3,4} = \pm 3$; 3) $x_1 = -1$, $x_2 = 2$; 4) $x_{1,2} = \frac{-1 \pm \sqrt{5}}{2}$; 5) $x_1 = 0$, $x_{2,3} = \pm 2$; 6) $x_{1,2} = \pm 4$, $x_{3,4} = \pm 6$. **516.** 2) $2\frac{1}{3}$; 4) $\frac{2x^2}{3y}$. **517.** 2) $3 - \sqrt[3]{2}$; 4) $6\sqrt{7}$. **518.** 2) $\left(2\sqrt{0,5}\right)^{0,3} < \left(2\sqrt{0,5}\right)^{0,37}$. **519.** 2) $\sqrt{x}$; 4) $9b^{-4}$. **520.** 2) $5ab\sqrt{b}$; 4) $2ab\sqrt{ab}$. **521.** 2) $-\sqrt{3x^2}$; 4) $\sqrt{5a^2}$. **522.** 2) $-8\frac{1}{8}$. **523.** 2) $-1\frac{5}{6}$. **524.** 2) $x = \frac{1}{9}$; 2) $x = 0$. **525.** 2) Joq; **526.** 2) Joq. **527.** 2) $1,5 < x \le 7$; 4) $x \ge -7,5$; 6) $0 \le x < 2$, $x > 2$. **530.** -1. **531.** 2) Teris. **532.** 2) $-0,8$. **533.** 2) $2\sin\frac{3\alpha}{4}\cos\frac{\alpha}{4}$; 4) $\sin\alpha(\sin\alpha - 2\cos\alpha)$. **534.** $\sin\alpha = \frac{240}{289}$, $\cos\alpha = -\frac{161}{289}$, $\operatorname{tg}\alpha = -\frac{240}{161}$. **535.** 2) $a_{12} = 47,5$, $S_{12} = 537$; 4) $a_{18} = 11\frac{2}{3}$, $S_{18} = 108$. **536.** 1220. **538.** 2) $b_1 = 5$. **539.** 2) $b_4 = 125$, $S_4 = 156$; 4) $b_4 = 81$, $S_5 = 61$. **540.** $15\frac{3}{4}$. **541.** 2) $4\frac{1}{6}$; 4) 1; 6) $-\frac{5}{4}(1+\sqrt{5})$. **542.** 2) $2ab\sqrt[3]{b}$; 4) $a + 3$. **543.** 2) -1; 4) $-\frac{1}{x}$. **544.** Birinshisi. **545.** 2) $\frac{(a+\sqrt{b})(\sqrt{a}+\sqrt[4]{b})}{a^2-b}$; 4) $0,1(5-\sqrt{5})5+\sqrt{5}$. **546.** 2) $-\frac{\sqrt{a}}{b}$; 4) $\sqrt{a}+\sqrt{b}$. **547.** Kemeyedi. **548.** 2) $x \le 0$, $x \ge 6$; 4) $x \ne \sqrt{3}$; 6) $x \le -3$, $0 < x < 2$, $x \ge 3$. **550.** 2) $x = 61$; 4) $x = 0,5$; 6) $x_1 = 0$, $x_2 = -3$, $x_3 = 2$. **551.** 2) $\frac{1}{\cos^2\alpha}$; 4) 4. **552.** 2) $\cos^2 x$. **553.** 2) $x = \frac{\pi}{2} + \pi n$, $x = \pi + 2n$, $n \in \boldsymbol{Z}$. **556.** $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$. **557.** $39\frac{2}{3}$. **558.** 7, −28, 112, −448 yaki $-11\frac{2}{3}; -46\frac{2}{3}; -186\frac{2}{3}; -746\frac{2}{3}$. **559.** $b_1 = 5$, $b_5 = 405$. **560.** $\frac{1}{8}$. 561. 8, 13, 18 yaki 20, 13, 6. **562.** 1) $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{2}}$; 2) $\frac{1+\sqrt{7}}{\sqrt{2}}$. **563.** 1) $1-\sqrt{a}$; 2) $a^{\frac{2}{3}} + b^{\frac{2}{3}}$. **565.** $\sin\alpha = -\frac{120}{169}$, $\cos\alpha = -\frac{119}{169}$. **567.** 10, 4, −2, 1 yaki $-\frac{5}{4}, \frac{1}{4}, \frac{7}{4}, \frac{49}{4}$.

VII–IX KLASLARDA ALGEBRA KURSI'N TA'KIRARLAW USHI'N SHI'NI'G'I'WLARDI'N' JUWAPLARI'

569. 2) 4; 4) $4\frac{3}{4}$. **570.** 2) 5,8; 4) $-\frac{1}{11}$. **571.** 2) $x = 7$; 4) $x = 0{,}5$; $x = 2{,}25$. **572.** 2) 3; 4) 0,1125. **573.** 2) 300; 4) 3600. **574.** 2) 5%; 4) $16\frac{2}{3}$%. **575.** 2) $5a^4b$; 4) $4a^8b^7$. **576.** 2) $35 - 2x - 2x^3 - x^5$; 4) $8a^2 + 4b^2 + 36a + 36$. **577.** 2) 4,9; 2) 2. **578.** 2) $b^2 - 7a^2b^3$. **579.** 2) $\left(\frac{b}{3}-1\right)\left(\frac{b}{3}+1\right)$; 4) $(b-\sqrt{3})(b+\sqrt{3})(b^2+3)$. **580.** 2) $\left(\frac{b}{2}+1\right)^2$; 4) $(1 + 9b)^2$. **581.** 2) $(a + 1)(a - x)$; 4) $(a - x)(5a - 7)$. **582.** 2) $2a^3b(a - 1)^2$; 4) $(a - b)^2(a + b)^2$. **583.** 2) $2(x - 3)^2$; 4) $(x - 1)(x + 2)$. **584.** 2) $\frac{b+3}{3b}$; 4) $\frac{3y}{4x}$; 6) $\frac{x+4}{x+5}$; 8) $\frac{x-1}{x+2}$. **585.** 2) $\frac{3}{2}m^2$; 4) $\frac{3c^2}{k^3}$; 6) $\frac{15a}{4c}$; 8) $(x + 1)(x - 2)$. **586.** 2) $\frac{6-5a}{a^2-4}$; 4) $\frac{3b-a^2}{a(b^2-a^2)}$. **587.** 2) $\frac{1}{2a+3}$; 4) $b + a - 1$. **588.** 2) $\frac{2}{a(a+2)}$; 4) $\frac{1}{a+1}$. **589.** 3) $\frac{x}{y}$; 4) $\frac{10}{2a+1}$. **590.** 2) $-0{,}25$; 4) $1\frac{9}{16}$. **591.** 2) 3. **592.** 2) $\frac{1}{x+\sqrt{2}}$; 4) $\frac{\sqrt{x}}{\sqrt{x}-1}$. **593.** 2) 4; 4) 8. **594.** 2) -2; 4) 0; 6) 7. **595.** 2) 2; 4) 14. **596.** 2) $\frac{\sqrt{3}}{11}$; 4) $6\sqrt{2}$. **597.** 2) $2\cdot10^{-3}$; 4) $1{,}2\cdot10^{-3}$. **598.** 2) 1,25. **599.** 2) 3,5. **600.** 2) $-x^2y^2$; 4) xy^2. **601.** 2) -1; 4) $1+\sqrt{m}$. **602.** 2) $x = 1$; 4) $x = -0{,}5$. **603.** 2) $x = 12\frac{1}{14}$; 4) $x = -13{,}5$. **604.** 2) $x = 3$; 4) $x = -9$. **605.** 2) $x_1 = -2$, $x_2 = 3$; 4) $x_1 = 5$, $x_2 = -1$. **606.** 2) $x_1 = 0$, $x_2 = 5$; 4) $x_1 = 0$, $x_2 = -\frac{1}{6}$; 6) $x_{1,2} = \pm2$; 8) $x_{1,2} = \pm2\sqrt{2}$. **607.** 2) $x_1 = -1$, $x_2 = 1{,}5$. **608.** 2) $x_1 = 5$, $x_2 = -\frac{3}{4}$. **609.** 2) $x_1 = 1$, $x_2 = 4{,}5$; 4) $x_1 = -5$, $x_2 = 0{,}5$. **610.** 2) $x_1 = -3$, $x_2 = 5$; 4) $x = -1$; 6) $x_1 = 4{,}3$, $x_2 = -11{,}7$. **611.** 2) $x = 3$; 4) $x = -4$. **612.** 2) $x_{1,2} = \pm1$, $x_{3,4} = \pm6$. **613.** 2) $x = 33$ 4) $x = 9$; 6) $x_1 = 1$, $x_2 = 4$. **614.** 2) $x = -2$; 4) $x = 1{,}5$. **615.** 2) $x = -1$; 4) $x_1 = 1$, $x_2 = -0{,}5$; 6) $x = 4$. **616.** 2) (3; 7); 4) (2; 3); 6) (−2; −3). **617.** 2) (14; 10); 4) (2; 2). **618.** 2) (5; 3), (−3; −5); 4) (4; −9), (−9; 4); 6) (4; 5), (−4; −5), (5; 4), (−5; −4). **619.** 2) $x \le \frac{22}{27}$; 4) $x > 1$. **620.** 2) $x \le 1$; 4) $x < 3\frac{1}{6}$; 6) $x < 2$. **621.** 2) $x \ge 1{,}5$; 4) $x \ge 3$. **622.** 2) 1; 2; 3; 4. **623.** 2) −1; 0; 1; 2; 4) −1; 0; 1; 2; 3. **624.** −4; 3; −2; − 1. **625.** 2) $-1 \le x \le 3$; 4) $\frac{3-\sqrt{5}}{2} \le x \le \frac{3+\sqrt{5}}{2}$;

6) sheshimleri joq; 8) $x < -1\frac{1}{3}$, $x > 1$. **626.** 2) $-1\frac{1}{3} < x < 3\frac{1}{3}$; 4) $-1 \le x \le 3$. **627.** 1) $-4 < x < 2$; 4) $0 < x < 7$, $x > 8$; 6) $x \le -3$, $-0,5 \le x \le 0,5$. **628.** 2) $9 > 4\sqrt{5}$; 4) $5\sqrt{6} < 6\sqrt{5}$; 6) $2\sqrt[6]{3} < \sqrt{2}\cdot\sqrt[3]{5}$. **629.** 62,5 ha'm 57,5. **630.** 5 km/saat. **631.** 4 km/saat. **632.** 12,5 km/saat, 2,5 km/saat. **633.** 26 sm, 2sm. **634.** 48 min. **635.** 20 min. **636.** 35 sr. **637.** 5 saat, 7 saat. **638.** 2) Awa; (0; −4), (8; 0), $y(-2)=$ $= -5$; 4) joq; (0; 6), (4; 0), $y(-2) = 9$. **641.** 2) 5; −10); 4) $\left(\frac{5}{4}; -\frac{1}{8}\right)$. **642.** 2) 23; 4) $6\frac{1}{4}$. **643.** 2) $x_1 = -2$, $x_2 = -5$. **645.** $\sqrt[5]{-\frac{2}{9}} < \sqrt[5]{-\frac{1}{7}}$. **647.** 2) $\frac{5\pi}{4} + 2\pi n$, $n\in \boldsymbol{Z}$; 4) $7\pi +$ $+2\pi n$, $n\in \boldsymbol{Z}$. **648.** 2. **650.** 2) $2\cos^2\alpha$. **651.** $-\mathrm{tg}2\alpha$. **653.** 2) $\frac{7}{9}$. **654.** 2) 0,5. **655.** 2) $\frac{3}{8}$. **656.** 7. **657.** 1) 0; 2) 0. **658.** $-\sin\alpha - \cos\alpha$. **659.** −2. **660.** −0,5. **661.** 2) $a_1 = 201$, $S_{17} = 2737$. **662.** $n = 39$. **663.** 682. **664.** 2) 0,5; 4) 1. **665.** 189. **666.** 2) $a_1 = 1$, $d = 3$; 4) $a_1 = 2$, $d = 3$ yaki $a_1 = 14$, $d = -3$; **671.** $b_n = 3\left(-\frac{1}{6}\right)^{n-1}$ yaki $b_n = -3\left(\frac{1}{6}\right)^{n-1}$. **672.** $b_4 = 12$, $q = -2$ yaki $b_4 = -12$, $q = 2$. **673.** $\frac{1}{3}$; 1; 3; 9; 27 yaki $\frac{1}{3}$; −1; 3; −9; 27. **674.** 1) $b_1 = 0,384$, $q = 0,25$ yaki $b_1 = 0,6$, $q = -0,2$. **675.** 2) $b_1 = 37,5$, $q = 0,6$ yaki $b_1 = 48$, $q = 0,25$. **676.** 2) 11; 4) 341 yaki 121.

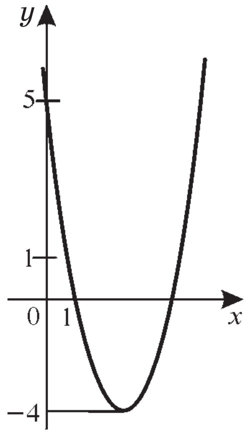
«Wo'zin'izdi tekserip ko'rin'» tapsi'rmalari'na juwaplar

I bap. 1. 84- su'wret. **2.** $x_1 = 0$, $x_2 = 2$. **3.** $-1 < x < 1$ bolg'anda $y > 0$; $x < -1$ bolg'anda $y < 0$; $x > 1$. **4.** 1) $x > 0$ bolg'anda funkciya wo'sedi; $x < 0$ bolg'anda funkciya kemeyedi. **5.** 1) (3; 0); 85- su'wret.

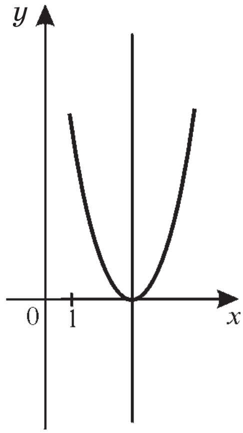
II bap. 1. 1) $-1 < x < 4$; 2) x − qa'legen haqi'yqi'y san; 3) sheshimleri joq; 4) $x = -10$. **2.** 1) $x \ge 1$; $-2 \le x \le 0$.

III bap. 1. 1) $8\frac{3}{8}$; 2) 16. **2.** $8,6 \cdot 10^3$; $7,8 \cdot 10^{-3}$; $6,708 \cdot 10^1$; $\approx 1,1 \cdot 10^6$. **3.** 1) 6; 2) $(y + x)xy$. **4.** $a^{\frac{3}{4}}$; 27. **5.** 1) $(0,78)^{\frac{2}{3}} > (0,67)^{\frac{2}{3}}$; 2) $(3,09)^{\frac{1}{3}} < (3,08)^{\frac{1}{3}}$.

IV bap. 1. 1) $x \ne 1$; 2) $-3 \le x \le 3$. **2.** a) 1) $y \approx 1,4$; 2) $y = 3$; 3) $y = -2,5$; 4) $y = 8$; b) 1) $x = 9$; 2) $x = 2$; 3) $x = -\frac{5}{3}$; 4) $x = \sqrt[3]{3}$; d) $y(x) > 0$ mi'na jag'daylarda boladi': 1) $x > 0$; 2) $x > 0$; 3) $x < 0$; 4) $x > 0$; $y(x) < 0$ mi'na jag'daylarda boladi': 1) bunday



84- su'wret.



85- su'wret.

arali'qlar joq; 2) $x < 0$; 3) $x > 0$; 4) $x < 0$; e) funkciya mi'na jag'daylarda wo'sedi: 1) $x \geq 0$; 2) bunday arali'qlar joq; 3) $x > 0$, $x < 0$; 4) $x \in \boldsymbol{R}$; funkciya mi'na jag'daylarda kemeyedi: 1) bunday arali'qlar joq; 2) $x > 0$, $x < 0$; 3) arali'qlar joq; 4) bunday arali'qlar joq; **3.** 1) jup; 2) taq. **4.** 1) $x = 28$; 2) $x = 1$.

V bap. 1. 1) $\cos\alpha = -\frac{3}{5}$, $\operatorname{tg}\alpha = -\frac{4}{3}$, $\sin 2\alpha = -\frac{24}{25}$. **2.** 1) 1; 2) $-\frac{\sqrt{3}}{2}$; 3) $\frac{\sqrt{3}}{2}$; 4) $-\sqrt{3}$; 5) $\frac{\sqrt{2}}{2}$. **5.** 1) $\sin\alpha\cos\beta$; 2) $\cos^2\alpha$; 3) $2\sin\alpha$.

VI bap. 1. 1) $a_{10} = -25$, $S_{10} = -115$. **2.** 1) $b_6 = \frac{1}{8}$, $S_6 = 7\frac{7}{8}$. **3.** 1) $q = \frac{1}{3}$, $S = 1{,}5$.

I–VI baplar si'naq (test) shi'ni'g'i'wlari'na juwaplar gilti

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	A	A	B	C	D	A	A	B	C	D	A	A	B	C	D	A	A	B	C	D	A

Tu'sinikler ko'rsetkishi

Ismler ko'setkishi

MAZMUNI'

V bap. TRIGONOMETRIYA ELEMENTLERI

VI bap. PROGRESSIYALAR

Alimov Sh.A.

22.14
A.50

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Mirfazil Abdilxakovich Mirzaxmedov

ALGEBRA

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3-nashri

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Ijarag'a berilgen sabaqli'qti'n' jag'dayi'n ko'rsetetug'i'n keste

№	Woqi'wshi'ni'n' ati' familiyasi'	Woqi'w ji'li'	Sabaqli'qti'n' ali'ng'andag'i' jag'dayi'	Klass basshi'si'ni'n' qoli'	Sabaqli'qti'n' tapsi'ri'lg'andag'i' jag'dayi'	Klass basshi'si'ni'n' qoli'
1						
2						
3						
4						
5						
6						

Sabaqli'q ijarag'a berilip, woqi'w ji'li' aqi'ri'nda qaytari'p ali'ng'anda joqari'dag'i' keste klass basshi'si' ta'repinen to'mendegi bahalaw wo'lshemlerine tiykarlani'p tolti'ri'ladi'

Jan'a	Sabaqli'qti'n' birinshi ret paydalani'wg'a berilgendegi jag'dayi'
Jaqsi'	Muqabasi' pu'tin, sabaqli'qti'n' tiykarg'i' bo'liminen aji'ralmag'an. Barli'q betleri bar. Ji'rti'lmag'an, betleri almasti'ri'lmag'an, betlerinde jazi'w ha'm si'zi'qlar joq.
Qanaatlandi'rarli'	Muqaba jelingen, bir qansha si'zi'li'p shetleri qayri'lg'an, sabaqli'qti'n' tiykarg'i' bo'liminen ali'ni'p qali'w jag'dayi' bar, paydalani'wshi' ta'repinen qanaatlandi'rarli' qa'lpine keltirilgen. Ali'ng'an betleri qayta jelimlengen, ayi'ri'm betlerine si'zi'lg'an.
Qanaatlandi'rarsi'z	Muqabag'a si'zi'lg'an, ji'rti'lg'an, tiykarg'i' bo'limnen aji'ralg'an yamasa pu'tkilley joq, qanaatlandi'rarsi'z remontlang'an. Betleri ji'rti'lg'an, betleri toli'q yemes, si'zi'p, boyap taslang'an. Sabaqli'qti' qayta tiklew mu'mkin yemes.