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GEOMETRI'YA 9

O'zbeki'stan Respubli'kasi' Xali'q bi'li'mlendi'ri'w mi'ni'strli'gi' uli'uwma bi'li'm beretug'i'n mekteplerinin' 9-klasi' ushi'n sabaqli'q si'pati'nda usi'ni'lg'an

«O'zbekistan»
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9-klasta geometriyani’n’ planimetriya bo’limin — tegis geometriyali’q figuralardi’n’ qa’siyetlerin u’yreniw dawam yettiriledi. Bunda siz geometriyali’q figuralardi’n’ uqsasli’g’i’, u’shmu’yeshliklerdin’ ta’repleri ha’m mu’yeshleri arasi’ndag’i’ qatnaslar, shen’ber uzi’nli’g’i’ ha’m do’n’gelektin’ maydani’’, u’shmu’yeshlik ha’m shen’berdegi metrikaliq qatnaslar menen tani’sasi’z.

Bul “Geometriya” sabaqli’g’i’ni’n’ mazmuni’ qatan’ aksiomatikali’q sistema tiykari’nda quri’lg’an. Bunda teoriyali’q materiallar mu’mkinshiligi bolg’ansha a’piwayi’ ha’m ani’q tilde bayan yetilgen. Barli’q tema ha’m tu’siniklerdi ha’r tu’rli turmi’sta ushi’rasatug’i’n mi’sallar arqali’ ashi’p beriwge ha’reket yetilgen. Ha’r bir temadan son’ berilgen sorawlar, da’liyllewler, yesaplawlarg’a ha’m jasawlarg’a tiyisli ma’sele ha’m mi’sallar woqi’wshi’ni’ do’retiwshilik pikirlewge jeteleydi, wog’an wozlestirilgen bilimlerdi teren’lestiriwge ha’m bekkemlep bari’wg’a ja’rdem beredi. Sabaqli’q wo’zini’n’ wo’zgeshe dizayn ha’m sabaq materiali’ni’n’ ko’rgizbeli yetip usi’ni’li’wi’ menen de aji’rali’p turadi’. Sabaqli’qta keltirilgen su’wret ha’m si’zi’lma sabaqli’q materiali’n jaqsi’lap wo’zlestiriwge xi’zmet yetedi.

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MAZMUNI'

SO’Z BASI

A’ziz oqiwshilar!

Bizler informatsion texnologiyalar a’sirinde jasap atirmiz. Zamanago’y rawajlaniw da’wirinde boli’p atirg’an du’nya ju’zilik o’zgerisler tiykarinda a’lbette pa’n ha’m texnika rawajlaniwi jatir. Bunday sharayatta siz jaslardin’ tiykarg’i’ waziypasi ulli babalarimizg’a mu’na’sip a’wlad boli’p, zaman menen ten’dey qa’dem taslaw, ilim-pa’n shin’larin qunt penen u’yreniwden ibarat. Bul orinda matematikanin’ orni ayriqsha boladi.

Matematika siz jaslardin’ kamalg’a jetisiwin’izde oqiw pa’ni sipatinda ken’ imkaniyatlarg’a iye ekeni ma’lim. Ol pikirlewin’izdi rawajlandirip, aqilin’izdi toliqtiradi, logikaliq pikirlew, tapqirliq qa’siyetlerin qa’liplestiredi ha’m tu’rli jag’daylarda aqillilig penen qarar qabil etiw, analizlew ha’m juwmaq shig’ariw ko’nlikpelerin qa’liplestiredi.

Qolin’izdag’i 9-klass “Geometriya” sabaqlig’inin’ tiykarg’i’ waziypasi — tegis geometriyaliq figuralardi u’yreniw menen bir qatarda sizde izbe-iz logikaliq pikirlewdi o’sirip bariw na’tiyjesinde aqilin’izdi rawajlandiriwdan ibarat. Ol o’zlestirilgen bilim, ta’jiriybe ha’m ko’nlikpelerdi ku’ndelik turmista qollaniwin’izg’a ko’meklesedi.

Sabaqliqti jaratiwda du’nya ju’zinde toplang’an ag’la ta’jiriybe u’lgilerinen paydalandiq. Sonin’ menen bir qatarda elimizge ta’n bolg’an shig’is ha’m o’mirbaqiy qa’diriyatlarimizg’a, ulli babalarimiz miyrasinan da paydalaniwg’a ha’reket ettik.

Bul sabaqliqtan bilim alar ekensiz, sizge bul juwapkershilikli, sonin’ menen birge zawiqli jolda qunt ha’m qatan’liq tilep qalamiz. Geometriya tiykarlari boyi’nsha alg’an sabaqlarin’iz sizdi keleshekke jetelep, Watanimizdin’ rawajlaniwi jolinda xizmet etiwge sizge ko’mekshi boladi dep, isenim bildiremiz!

Sabaqliqta qollanilg’an belgiler ha’m olardin’ tu’sindirmesi:

— taza kirgizilgen geometriyaliq tu’siniktin’ aniqlamasi

— teoremanin’ sipatlamasi

— u’lgi retinde sheship ko’rsetilgen ma’sele.

— soraw, ma’sele ha’m tapsirmalar

— oqiwshilardin’ belsendiligin asiriwshi o’z betinshe yamasa toparlarda talqilanatug’in tapsirmalar

— jeke ta’rtipte yamasa toparlarda orinlanatug’in a’meliy jumis

— tariyxiy mag’liwmatlar ha’m ma’seleler

— qiziqli ma’seleler ha’m basqatirmalar

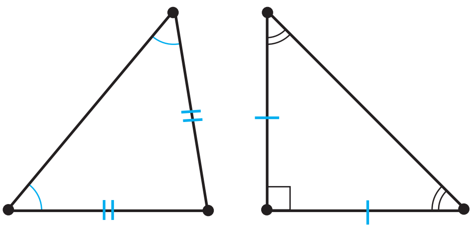
— internetten usinis etiletug’in mag’liwmatlar ma’nzili.

8. — u’yge sheshiwge usinis etiletug’in ma’seleler basqa ren’de berilgen

Teorema yamasa ma’selelerdin’ sxematikaliq talqilaniwi

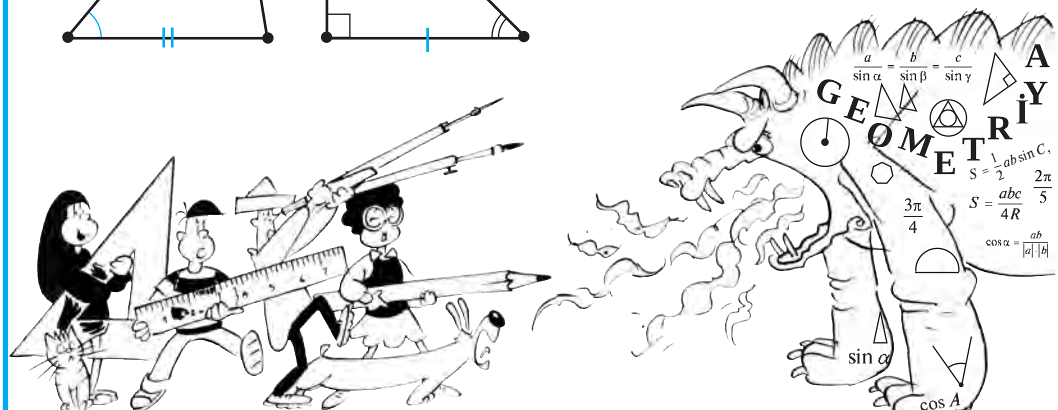
Teorema yamasa ma’selenin’ sha’rtinde berilgen mag’liwmatlar ⇒ Da’lillew kerek bolg’an qa’siyet yamasa tabiliw talap etiletug’in elementler

Si’zi’lmalarda qabil etilgen o’z aldina belgiler



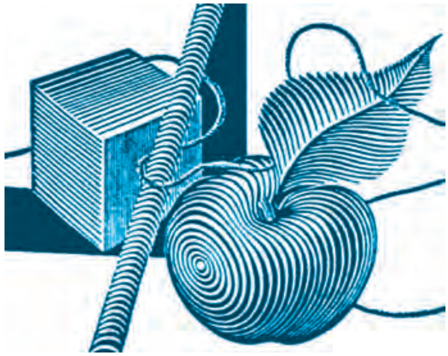
Sizilmalarda ten’ mu’yeshler bir qiyli sandag’i dog’ashalar menen ajiratiladi.

Sizilmalarda uzinlig’i ten’ bolg’an kesindiler bir qiyli sandag’i siziqshalar menen ajiratiladi.



Geometriyani iyelep aliw ushi’n alg’a!

TA'KIRARLAW



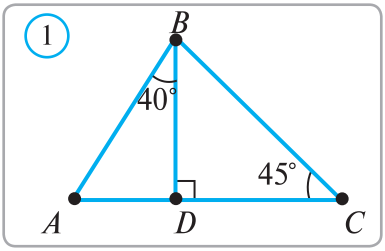
7 — 8-KLASLARDA WO'TILGENLERDI TA'KIRARLAW

√ *7— 8-klaslarda geometriyadan wo'tilgen temalardi' ta'kirarlap, alg'an bilimlerin'izdi yeske alasi'z ha'm yerisken ko'nlikpelerin'izdi bekkemleysiz.*

√ *Bul sizge 9-klasta geometriyani' u'yreniwdi tabi'sli' dawam yettiriwin'izge qolayli'li'q jaratadi'.*

1 U'SHMU'YESHLIKLER

Bul bo'limdegi ma'seleler 7-8 klaslarda u'yrenilgen geometrik figuralar ha'm wolardi'n' qa'siyetlerin yadqa ali'w ushi'n berilmekte. Maselelerdi sheshiw ushi'n sabaqli'qti'n' aqi'ri'nda keltirilgin tiykarg'i' geometriyali'q figuralarg'a tiyisli mag'liwmatlar ha'mde wolardi'n qa'siyetlerin ani'qlawshi' figuralardan paydalani'wi'mi'z mu'mkin



1-ma'sele. ABC u'shmu'yeshliktin' BD biyikligi ju'rgizilgen (*1-su'wret*). Yeger $\angle ABD = 40°$, $\angle BCD = 45°$ bolsa u'shmu'yeshliktin' A ha'm B to'besindegi mu'yeshlerin tabi'n'.

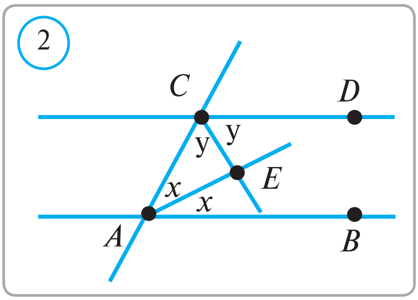
Sheshiliwi. 1) Tuwri' mu'yeshli ABD u'shmu'yeshlikte $\angle ABD = 40°$ ha'm u'shmu'yeshlik ishki mu'yeshlerinin' qosi'ndi'si 180° ga ten' bolg'ani'' ushi'n $\angle A = 180° - (90° + 40°) = 50°$.

2) Tuwri' mu'yeshli BCD u'shmu'yeshlikte $\angle BCD = 45°$ bolg'ani' ushi'n

$$\angle DBC = 180° - (90° + 45°) = 45°.$$

$$\angle ABC = \angle ABD + \angle DBC \text{ bolg'ani' ushi'n } \angle B = 40° + 45° = 85°.$$

Juwabi': 50°, 85°.



2-ma'sele. Yeki parallel tuwri' si'zi'qti' kesiwshi menen keskende payda bolg'an ishki bir ta'repleme mu'yeshlerdin' bissektrisalari' arasi'ndag'i mu'yeshti tabi'n'.

Sheshiliwi. AC tuwri' si'zi'q AB ha'm CD-parallel tuwri' si'zi'qlardi' 2-su'wrette su'wretlengendey kesip wotken bolsi'n. Ishki bir ta'replemen BAC ha'm ACD mu'yeshlerdin' bissektrisalari' E noqatta kesilisken boli'p $\angle EAC = x$, $\angle ECA = y$ bolsi'n wonda mu'yesh bissektrisa ani'qlamasi boyi'nsha,

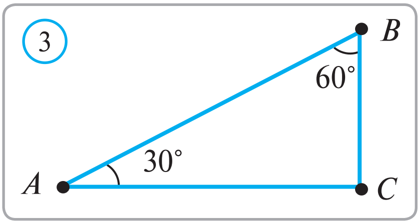
$$\angle BAC = x + x = 2x, \qquad \angle ACD = y + y = 2y.$$

$AB || CD$ bolg'ani' ushi'n ishki ta'replemeli mu'yeshlerdin' qa'siyeti boyi'nsha,

$$2x + 2y = 180°, \qquad x + y = 90°.$$

Yendi, ACE u'shmu'yeshlik ishki mu'yeshlerinin' qosi'ndi'si' 180° qa ten' bolg'ani' ushi'n $\angle AEC = 180° - (x + y) = 180° - 90° = 90°$. ***Juwabi':*** 90°.

3-ma'sele. ABC u'shmu'yeshliktin' AB ta'repi 6 *sm* A ha'm B mu'yeshlerine saykes tu'rde 30° ha'm 60° bolsa, ABC u'shmu'yeshliktin' maydani'n tabi'n'.



Sheshiliwi. U'shmu'yeshliktin' C mu'yeshin tabi'n'

$\angle C = 180° - (\angle A + \angle B) = 180° - (60° + 30°) = 90°$.

Demek, tuwri' mu'yeshli ABC u'shmu'yeshliktin' AB gepotenuzasi' 6 *sm* ha'm A mu'yeshi 30° yeken. Tuwri' mu'yeshli u'shmu'yeshlikte 30° li mu'yesh qarsi'si'ndag'i katet gepotenuzani'n' yari'mi'na ten' bolg'ani' ushi'n $BC = 3$ *sm* (*3-su'wret*).

Pifagor teoremasi'nan paydalani'p AC katetin tabami'z:

$$AC^2 = AB^2 - BC^2 = 6^2 - 3^2 = 27,\ AC = 3\sqrt{3}\ sm.$$

Yendi u'shmu'yeshliktin' maydani'n tabami'z:

$$S_{ABC} = \frac{1}{2} AC \cdot BC = \frac{1}{2} \cdot 3\sqrt{3} \cdot 3 = \frac{9\sqrt{3}}{2}\ (sm^2).$$

Juwabi': $\frac{9\sqrt{3}}{2}\ sm^2$.

Soraw, ma'sele ha'm tapsi'rmalar

1. ABC u'shmu'yeshlikte $\angle A = 47°$, $\angle C = 83°$ bolsa, u'shmu'yeshliktin' u'shinshi ishki mu'yeshin ha'm si'rtqi' mu'yeshlerin tabi'n'.

2. Katetleri 15 *sm* ha'm 20 *sm* bolg'an tuwri' mu'yeshli u'shmu'yeshlik gepotenuzasi'na tu'sirilgen biyikligin tabi'n'.

3. ABC u'shmu'yeshliktin' AC ta'repine parallel tuwri' si'zi'q AB ha'm BC ta'replerin saykes tu'rde E ha'm F noqatlarinda kesip wotedi. Yeger $\angle BEF = 65°$ ha'm $\angle EFC = 135°$ bolsa, ABC u'shmu'yeshliginin' mu'yeshlerin tabi'n'.

4. ABC u'shmu'yeshlik bissektrisalari' I noqatta kesilisedi. Yeger $\angle A = 80°$ ha'm $\angle B = 70°$ bolsa, AIB, BIC ha'm CIA mu'yeshlerin tabi'n'.

5. Ten' ta'repli u'shmu'yeshliktin' bir si'rtqi' mu'yeshi 70° qa ten'. U'shmu'yeshliktin' mu'yeshlerin tabi'n'.

6. ABC u'shmu'yeshliktin' AK bissektrisasi' ju'rgizilgen. Yeger $\angle BAK = 47°$ ha'm $\angle AKC = 103°$ bolsa, u'shmu'yeshlik mu'yeshlerin tabi'n'.

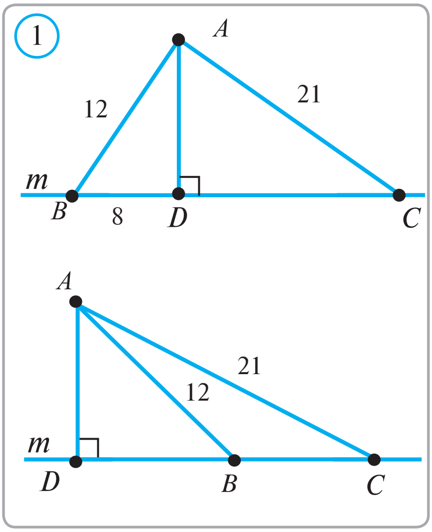
7*. ABC u'shmu'yeshlik biyiklikleri H noqatta kesilisedi. Yeger $\angle A = 50°$, $\angle B = 60°$ bolsa, AHB, BHC ha'm CHA mu'yeshlerin tabi'n'.

8. U'shmu'yeshliktin' worta si'zi'qlari woni ten'dey to'rtmu'yeshliklerge aji'ratatugi'ni'n da'liyllen'.

9*. ABC u'shmu'yeshlikte CD mediana dawam yettirilip bul medianag'a ten' DE kesindisi qoyi'ladi'. AF mediana dawam yettirilip AF medianag'a ten' FH kesindisi qoyi'lg'an. B, H, E noqatlari bir tuwri'da jatatug'i'nli'g'i'n da'liyllen'.

10. ABC ten' qaptalli' u'shmu'yeshlikte $(AB = BC)$ AN ha'm CK bissektrisalar ju'rgizilgen. a) KN kesindisi AC ta'repke parallel yekenligin ko'rsetin'.
b) $AK = KN = NC$ ten'lik wori'nli' boli'wi'n da'liyllen'.

U’SHMU’YESHLIKLER (dawami’)



1-ma’sele. A noqattan m tuwri’ si’zi’qqa uzi’nli’qlari’ 12 sm ha’m 21 sm bolg’an yeki qi’ya tu’sirilgan. Yeher birinshi qi’yani’n’ m tuwri’si’na proekciyasi’ 8 *sm* bolsa, yekinshi qi’yani’n’ proekciyasi’n tabi’n’.

Sheshiliwi. m tuwri’ si’zi’qti’n si’rti’ndag’i A noqattan usi’ tuwri’ si’zi’qqa AB ha’m AC qi’yalar ha’mde AD perpendikulyar tu’sirilgen boli’p, AB=12 *sm* ha’m AC=21 *sm* bolsi’n (*1-su’wret*). Wonda ma’sele sha’rti boyi’nsha BD=8 *sm* boladi’ ha’m CD kesindi uzi’nli’g’i’n tabi’w kerek.

1) Pifagor teoremasi’nan paydalani’p tuwri’ mu’yewli ABD u’shmu’yeshliktin’ AD katetin tabami’z.

$AD^2=AB^2-BD^2=12^2-8^2=80,\ AD=\sqrt{80}$ *sm*.

2) Tuwri’ mu’yewli ACD u’shmu’yeshlikten Pifagor teoremasi’nan paydalani’p CD kesindi uzi’nli’g’i’n tabami’z.

$$CD^2=AC^2-AD^2=21^2-(\sqrt{80})^2=441-80=361,\ CD=19\ sm.$$

Juwabi’: 19 *sm*.

2-ma’sele. Tarepleri 13, 14 ha’m 15 ke ten’ bolg’an u’shmu’yeshliktin’ maydani’n ha’m biyiklikleri tabi’n’.

Sheshiliwi. Yeron formulasinan paydalani’p, ta’repleri $a=13$, $b=14$, $c=15$ bolg’an u’shmu’yeshliktin’ maydan’n tabami’z:

$$p=\frac{a+b+c}{2}=\frac{13+14+15}{2}=\frac{42}{2}=21,$$

$$S=\sqrt{p(p-a)(p-b)(p-c)}=\sqrt{21\cdot(21-13)\cdot(21-14)\cdot(21-15)}=$$

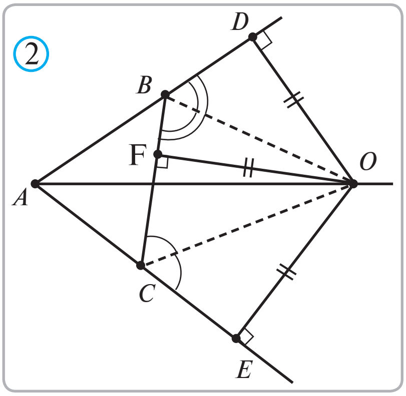
$$=\sqrt{21\cdot8\cdot7\cdot6}=\sqrt{7\cdot3\cdot8\cdot7\cdot2\cdot3}=3\cdot4\cdot7=84.$$

Yendi, u’sgmu’yeshliktin’ maydani’n yesaplaw formulasi’ $S=\frac{1}{2}a\cdot h_a$ dan paydalni’p, u’shmu’yeshliktin’ h_a biyikligin tabami’z:

$$h_a=\frac{2S}{a}=\frac{2\cdot84}{13}=\frac{168}{13}=12\frac{12}{13}.$$

Tap usi’ jol menen h_b ha’m h_c biyikliklerin.

Juwabi’: 84; $12\frac{12}{13}$; 12; $11\frac{1}{5}$.



3-ma'sele. ABC u'shmu'yeshliginin' B ha'm C ushlarindag'i si'rtqi' mu'yeshlerinin' bissektrisalari O noqatta kesilisedi. O noqattin' BAC mu'yesh bissektrisasi'nda jati'wi'n da'liyllen'.

Da'lillew. O noqattin' AB, AC ha'm BC tuwri' si'zi'qlardag'i' proekciyalari' sa'ykes tu'rde D, E ha'm F noqatlar bolsi'n *(2-su'wret)*. Onda, birinshiden O noqat DBC mu'yeshtin' bissektrisasi'nda jatqani' ushi'n $OD=OF$ boladi'. Yekinshiden, O noqat BCE mu'yeshinin' bissektrisasi'nda jatqani' ushi'n $OF=OE$ boladi. Sonli'qtan, $OD=OF=OE$. Demek, O noqat BAC mu'yesh ta'replerinen ten' uzaqli'qta jaylasqan yeken. Soni'n' ushi'n O noqat BAC mu'yesh bissektrisasi'nda jatadi'.

? *Soraw, ma'sele ha'm tapsi'rmalar*

1. Ta'repleri 5, 6 ha'm 7 bolg'an u'shmu'yeshliktin' maydani'n tabi'n'.

2. Berilgen noqattan a tuwri'ga uzi'nli'qlari'ni'n' ayi'rmasi' 6 g'a ten' bolg'an yeki qi'ya tu'sirilgen. Qi'yalardi'n'a a tuwri'dag'i' proekciyalari' 27 ha'm 15 ge ten'. Berilgen noqqattan a tuwri'g'a shekemgi arali'qti' tabi'n'.

3*. ABC u'shmu'yeshliktin' A ha'm B ushlari'ndag'i' si'rtqi' mu'yeshlerinin' bissektrisalari' D noqatta kesilisedi. Yeger $ADB=75°$ bolsa, u'shmu'yeshliktin' ACB mu'yeshin tabi'n'.

4. Ultani' AC bolg'an ABC ten' ta'repli u'shmu'yeshlikte CD bissektrisa ju'rgizilgen. ADC mu'yesh: a) 60°; b) 75° ge ten' bolsa, u'shmu'yeshliktin' mu'yeshlerin tabi'n'.

5. Bir kateti 7 *sm* ge, gipotenuzasi' 25 *sm* ge ten' bolg'an tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi'na tu'sirilgen biyiklikti tabi'n'.

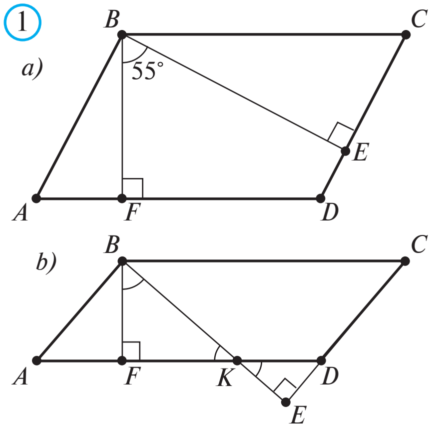
6. ABC u'shmu'yeshliktin' BD biyikligi wo'tkerilgen ($D \in AC$). Yeger $BD=12$, $AD=5$ ha'm $DC=16$ bolsa, u'shmu'yeshliktin' perimetrin ha'm maydani'n' tabi'n'.

7. Ten' qaptalli' u'shmu'yeshliktin' qaptal ta'repi 10 *sm*, ultani' bolsa $10\sqrt{3}$ *sm*. U'shmu'yeshliktin' ultanina tu'sirilgen biyiklikti, maydani'n ha'm mu'yeshlerin tabi'n'.

8. Su'yir mu'yeshli ABC u'shmuyeshlikke si'rtlay si'zi'lg'an shen'ber orayi' O noqatta boli'p $\angle AOB = 120°$, $\angle BOC = 110°$ bolsa, ABC u'shmu'yeshlik mu'yeshlerin tabi'n'.

9. Yeger ABC u'shmu'yeshliginin' CD medianasi AB ta'repten yeki yese kishi bolsa, ACB mu'yeshin tabi'n'.

10. ABC u'shmu'yeshliktin' biyiklikleri O noqqatta kesilisedi. Yeger $\angle A = 60°$, $\angle B = 80°$ bolsa, AOB mu'yeshin tabi'n'.



1-ma'sele. Yeger parallelogrammni'n' bir to'besinen woni'n' yekinshi ta'repine tu'sirilgen biyikliklerinin' arasi'ndag'i' mu'yesh 55° qa ten' bolsa, parallelogrammni'n' mu'yeshlerin tabi'n'.

Sheshiliwi. Parallelogrammni'n' *BF* ha'm *BE* biyikliklerinin' arasi'ndag'i' mu'yesh 55° bolsi'n (1-*su'wret*). Su'wrette su'wretlengen yeki jag'day: a) *BE* biyikligi *CD* ta'repine; b) *BE* biyikligi *CD* ta'repinin' dawami'na tu'sken boli'wi' mu'mkin.

a) jag'dayda *BEDF* to'rtmu'yeshliktin' mu'yeshliklerinin' qosi'ndi'si' 360° bolg'ani' ushi'n, $55^{\circ} + 90^{\circ} + \angle D + 90^{\circ} = 360^{\circ}$.

Bunnan $\angle D = 125^{\circ}$.

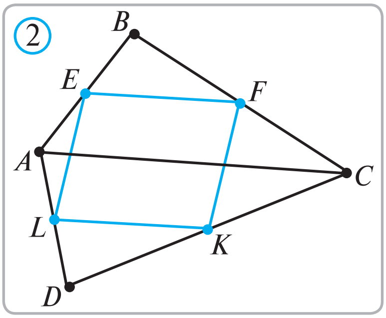
b) jag'dayda *BE* biyikligi *AD* ta'repi menen kesilisken noqat *K* bolsi'n. Wonda,

$$\angle DKE = \angle BKF = 90^{\circ} - 55^{\circ} = 35^{\circ}.$$

U'shmu'yeshliktin' si'rtqi' mu'yeshlerinin' qa'siyetleri boyi'nsha,

$$\angle ADC = \angle DKE + \angle KED = 35^{\circ} + 90^{\circ} = 125^{\circ}.$$

Demek, ha'r yeki jag'dayda da $D=125^{\circ}$. Wonda, $\angle A=\angle C=180^{\circ}-\angle D=55^{\circ}$, $\angle B=\angle D=125^{\circ}$. ***Juwabi':*** 55°, 125°, 55°, 125°.



2-ma'sele. To'rtmu'yeshliktin' ta'replerinin' wortalari' parallelogrammni'n' to'beleri bolatug'i'ni'n da'liyllen'.

Sheshiliwi. *ABCD* to'rtmu'yeshliginin' *AB, BC, CD* ha'm *DA* ta'replerinin' wortalari' sa'ykes tu'rde *E, F, K* ha'm *L* noqatlari' bolsi'n (2-*su'wret*). *EFKL* — parallelogramm yekenligin ko'rsetemiz.

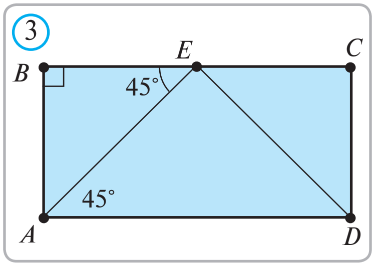
EF kesindisi *ABC* u'shmu'yeshliginin', *KL* kesindisi *ACD* u'shmu'yeshliginin' worta si'zi'g'i' boladi'. U'shmu'yeshliktin' worta si'zi'g'i'ni'n' qa'siyetlerinen,

$$EF \| AC, \; KL \| AC, \qquad EF = \tfrac{1}{2} AC, \; KL = \tfrac{1}{2} AC.$$

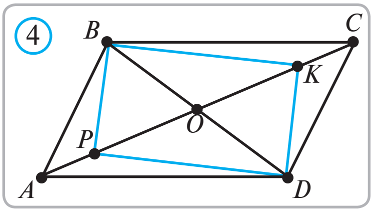
Bunnan $EF \| KL$ ha'm $EF = LK$. Soni'n' ushi'n, parallelogrammni'n' belgileri boyi'nsha *EFKL* — parallelogramm boladi'.

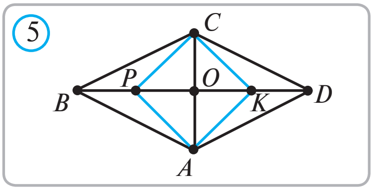
3-ma'sele. $ABCD$ tuwri'mu'yeshliginin' A ha'm D to'besindegi mu'yeshlerinin' bissektrisalari' BC ta'repinde kesilisedi. Yeger $AB = 4$ *sm* bolsa, tuwri'mu'yeshliktin' maydani'n tabi'n'.

Sheshiliwi. Tuwri'mu'yeshliktin' A ha'm D mu'yeshlerinin' bissektrisalari' kesilisken noqat E bolsi'n (*3-su'wret*). Wonda, $\angle B = 90°$, $\angle BAE = 45°$ bolg'ani' ushi'n, $\angle AEB = 180° - 90° - 45° = 45°$. Yag'ni'y, ABE — ten' qaptalli' u'shmu'yeshlik Bunda, $AB = BE = 4$ (*sm*). Da'l usi'g'an uqsas $EC = CD = 4$ (*sm*) yekenligin ko'rsetiw mu'mkin. Bunnan $BC = BE + EC = 8$ (*sm*) ha'm $S_{ABCD} = AB \cdot BC = 4 \cdot 8 = 32$ (sm^2). ***Juwabi':*** 32 sm^2.



Soraw, ma'sele ha'm tapsi'rmalar



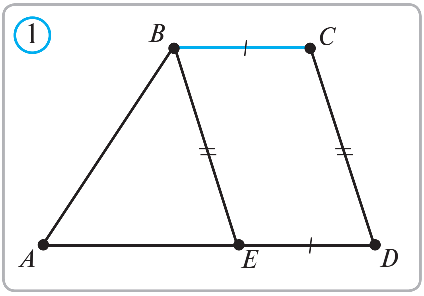


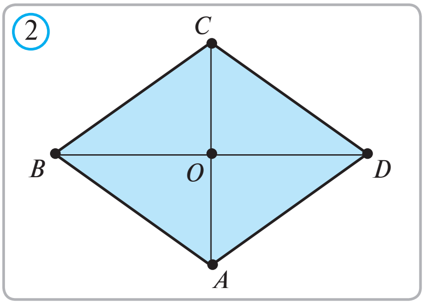
1. To'rtmu'yeshliktin' u'sh mu'yeshi: 47°, 83° ha'm 120° qa ten'ligi belgili. Woni'n' to'rtinshi mu'yeshin tabi'n'.
2. Parallelogrammni'n' yeki mu'yeshinin' qosi'ndi'si' 156° qa ten'. Woni'n' mu'yeshlerin tabi'n'.
3. Tuwri'mu'yeshliktin' diagonallari'ni'n' arasi'ndag'i mu'yesh 74°. Woni'n' bir diagonali menen ta'repleri arasi'ndag'i mu'yeshlerin tabi'n'.
4. Ten' qaptalli trapeciyani'n' yeki mu'yeshinin' ayi'rmasi' 40° qa ten'. Woni'n' mu'yeshlerin tabi'n'.
5. Rombi'ni'n' mu'yeshlerinen biri yekinshisinen u'sh ma'rte u'lken. Rombi'ni'n' mu'yeshlerin tabi'n'.
6. $ABCD$ tuwri'mu'yeshliginin' A mu'yeshinin' bissektrisasi' BC ta'repin 2 *sm* ha'm 6 *sm* ge ten' kesindilerge aji'ratadi'. Tuwri'mu'yeshliktin' perimetrin tabi'n'.
7. Ta'repleri 3 *sm* ha'm 6 *sm*, u'lken ta'repleri arasi'ndag'i arali'q 2 *sm* bolg'an parallelogramm jasan'.
8. $ABCD$ parallelogrammni'n' AC diagonali'nda P ha'm K noqatlari' belgilep aling'an (*4-su'wret*). Yeger $OP=OB=OK$ bolsa, $BKDP$ tuwri'mu'yeshlik yekenligin da'liyllen'.

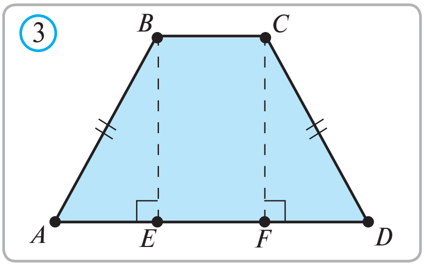
9*. $ABCD$ rombi'ni'n' BD u'lken diagonali'nda P ha'm K noqatlari' belgilep ali'ng'an (*5-su'wret*). Yeger $OA=OP= OK$ bolsa, $APCK$ to'rtmu'yeshligi kvadrat yekenligin da'liyllen'.

10*. $ABCD$ parallelogrammni'n' BD diagonali'nda P ha'm K noqatlari' belgilep ali'ng'an (*6-su'wret*). Yeger $BP=KD$ bolsa, $APCK$ to'rtmu'yeshligi parallelogramm yekenligin da'liyllen'.

4 TO'RTMU'YESHLIKLER (dawami')







1-ma'sele. *ABCD* trapeciyasi'ndag'i' *BC* kishi ultanni'n' *B* to'besinen *CD* ta'repke parallel tuwri' ju'rgizilgen. Na'tiyjede payda bolg'an u'shmu'yeshliktin' perimetri 24 *sm* ge ten'. Yeger trapeciyanin' perimetri 36 *sm* bolsa, *BC* ta'repinin' uzi'nli'g'i'n tabi'n'.

Sheshiliwi. Ma'selenin' sha'rti boyi'nsha ju'rgizilgen tuwri' kesindisi *BE* bolsi'n, *E* noqati' *AD* ta'repinde jatadi' (*1-su'wret*). *BE* kesindisi trapeciyani *ABE* u'shmu'yeshlikke ha'm *BCDE* parallelogrammg'a aji'ratadi'. Sonday aq, $BC = ED$ ha'm $CD = BE$. Ma'selenin' sha'rti boyi'nsha,

$$P_{ABCD} = AB + BC + CD + DA = AB + BC + CD + DE + EA = AB + BE + EA + 2BC = P_{ABE} + 2BC = 24 + 2BC = 36 \ (sm).$$

Bunnan, $2BC = 12$, yamasa $BC = 6$ *sm* yekenligin tabami'z. ***Juwabi':*** 6 *sm*.

2-ma'sele. Rombi'ni'n' diagonallari'nan biri 14 *sm*, ta'repi bolsa 25 *sm*. Rombi'ni'n' maydani'n tabi'n'.

ABCD — romb,
AC = 14 sm, AB = 25 sm.
⇒
S_{ABCD}=?

Sheshiliwi. Romb diagonallari'ni'n' kesilisiw noqati' *O* bolsi'n (*2-su'wret*). Wonda rombi'ni'n' qa'siyeti boyi'nsha,

$$AO = \frac{1}{2}AC = \frac{1}{2}\cdot 14 = 7 \ (sm), \qquad \angle AOB = 90^\circ.$$

Pifagor teoremasi'nan paydalani'p *OB* kesindisin tabami'z:

$$OB^2 = AB^2 - AO^2 = 25^2 - 7^2 = 576 \ \text{yoki} \ OB = 24 \ sm.$$

Wonda $BD = 2, OB = 2, 24 = 48$ (*sm*). Rombi'ni'n' maydani'n yesaplaw formulasi' boyi'nsha, $S_{ABCD} = \frac{1}{2}AC \cdot BD = \frac{1}{2}\cdot 14 \cdot 48 = 7 \cdot 48 = 336 \ (sm^2)$. ***Juwabi':*** 336 sm^2.

3-ma'sele. Ten' qaptalli trapeciyanin' qaptal ta'repi 20 *sm*, ultanlari' bolsa 12 *sm* ha'm 36 *sm*. Trapeciyanin' maydani'n tabi'n'.

Sheshiliwi. $ABCD$ trapeciyada $AB = CD = 20$ *sm*, $BC = 12$ *sm*, $AD = 36$ *sm* bolsi'n. Trapeciyanin' BE ha'm CF biyikliklerin ju'rgizemiz (*3-su'wret*).

Wonda,

$$EF = BC = 12\ (sm),\quad AE = FD = \frac{AD - EF}{2} = \frac{36 - 12}{2} = 12\ (sm).$$

Tuwri' mu'yeshli ABE u'shmu'yeshlikke Pifagor teoremasin qollanip, BE - biyikligin tabami'z: $BE^2 = AB^2 - AE^2 = 20^2 - 12^2 = 256$ yaki $BE = 16$ *sm*.

Trapeciyanin' maydani'n tabami'z:

$$S_{ABCD} = \frac{BC + AD}{2} \cdot BE = \frac{12 + 36}{2} \cdot 16 = 24 \cdot 16 = 384\ (sm^2).$$

Juwabi': 384 sm^2.

? Soraw, ma'sele ha'm tapsi'rmalar

1. $ABCD$ trapeciyasi'ni'n' kishi BC ultani' 7 *sm* ge ten'. Wo'ni'n' B to'besinen CD ta'repine parallel tuwri' ju'rgizilgen. Payda bolg'an u'shmu'yeshliktin' perimetri 16 *sm* ge ten'. Trapeciyani'n' perimetrin tabi'n'.

2. Tuwri'ni' kesip wo'tpeytug'i'n kesindinin' ushlari' bul tuwri'dan 8 *sm* ha'm 18 *sm* qashiqli'qta jaylasqan. Kesindi wortasi'nan tuwri'g'a shekem bolg'an arali'qti' tabi'n'.

3. Ta'repleri 4 *sm* ha'm 5 *sm*, maydani' bolsa 10 sm^2 bolg'an parallelogramm jasan'.

4. Rombi'ni'n' diagonallari'ni'n' biri 80 *sm*, ta'repi bolsa 81 *sm*. Rombi'ni'n' maydani'n tabi'n'.

5. Parallelogrammni'n' 135° qa ten' bolg'an dog'al mu'yeshinin' to'besinen tu'sirilgen biyikligi 4 *sm* ge ten' boli'p, wol wo'zi tu'sken ta'repin ten' yekige bo'ledi.

a) Usi ta'repti tabi'n'.

b) Parallelogrammni'n' dog'al mu'yeshlerinin' to'belerin tutasti'ri'wshi diagonali' menen ta'repleri arasi'ndag'i mu'yeshlerin tabi'n'.

d) Parallelogrammni'n' perimetrin ha'm maydani'n tabi'n'.

6. Rombi'ni'n' dog'al mu'yeshi to'besinen tu'sirilgen biyiklik rombi'ni'n' ta'repin ten' yekige bo'ledi. Yeger rombi'ni'n' ta'repi 6 *sm* bolsa, wonda rombi'ni'n' maydani'n tabi'n'.

7. Tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi' 13 *sm*, katetlerinin' qosi'ndi'si' bolsa 17 *sm*. U'shmu'yeshliktin' maydani'n tabi'n'.

8*. Tuwri' mu'yeshli trapeciyanin' bir mu'yeshi 135° qa, worta si'zi'g'i bolsa 18 sm ge ten'. Yeger trapeciyani'n' ultanlari'ni'n' qatnasi' 1:8 ge ten' bolsa, trapeciyanin' qaptal ta'replerin tabi'n'.

9*. $ABCD$ $(AB \| CD)$ trapeciyasi O worayi'na iye bolg'an shen'berge si'rtlay si'zi'lg'an. $\angle AOD = 90^\circ$ yekenligin da'liyllen'.

MATEMATIKALIQ MA'SELELER G'A'ZIYNESI

Keyingi waqitlarda informacion kommunikaciya texnologiyalari ju'da' tez pa't penen rawajlanip barmaqta. Internette barg'an sayin alis awillardi da qamtip almaqta. Usi ku'nge kelip, internettin' World-Wide-Web — Ja'ha'n informatciya tarmag'inda sonshelli ko'p informaciya derekleri jaylastirilg'an boli'p, bul g'a'ziyneden paydalaniw zamanimizdin' ha'r bir puqarasi ushi'n ha'm qariz, ham pariz yesaplanadi. Sonday-aq, bir-birinen qiziq sonday web-betler bar boli'p, wolardan qa'legen pa'ndi, sonin' ishinde geometriyani u'yreniw barisinda boli'p paydalaniw mu'mkin. To'mende bul informaciya dereklerinin' jaylasiw worinlarin beriwdi lazim taptiq. Bul web-betlerden siz wo'zbek, rus, ingliz ha'm basqa tillerde matematika a'lemindegi yen' son'g'i jan'aliqlar, yelektron kitapxanalar bazasinda saqlanip atirg'an ko'plep sabaqliqlar, arali'qtan turip matematikadan bilim aliw kurslari ha'm wolardi'n' materiallari, bul sabaqliq betlerine kirgen ha'm de kirmegen tu'rli teoriyaliq materiallar, matematikadan sabaq berip atirg'an ta'jiriybeli woqitiwshilardin' sabaq usillari ha'm metodikaliq usinislari, yesap-sansiz ma'seleler, misallar ha'm wolardi'n' sheshimleri, tu'rli ma'mleketlerde wo'tkerilip atirg'an matematikaliq ko'rik tan'lawlar ha'm olimpiadalar haqqindag'i mag'liwmatlar ha'm wolarda keltirilgen ma'seleler ha'm de wolardi'n' sheshimleri, qi'zi'qli' matematikaliq ma'seleler ha'm wolardi'n' sheshimleri menen tanisiwin'iz mu'mkin.

http://www.eduportal.uz — Xali'q bilimlendiriw ministrliginin' bilimlendiriw portali'
http://www.multimedia.uz — Xali'q bilimlendiriw ministirligi qasindag'i Multimedia worayi' sayti'
http://www.uzedu.uz — Xaliq bilimlendiriw sayti'
http://www.edu.uz — Joqari' ha'm worta arnawli bilimlendiriw ministrliginin' bilimleniriw portali'
http://www.pedagog.uz — Pedagogika bilimlendiriw sho'lkemleri portali'
http://ziyo.edu.uz — Bilimlendiriw sho'lkemlerinin' portali'
http://www.matematika.uz — Matematikadan qosi'msha materiallar sayti'
http://ziyonet.uz — Informacion-bilimlendiriw resurslari' tarmag'i'
http://cde.sakha.ru — Arali'qtan turi'p woqi'ti'w sayti' (rus tilinde)
http://www.iro.sakha.ru — Bilimin jetilistiriw instituti' sayti' (rus tilinde)
http://www.school.edu.ru — Uli'wma bilimlendiriw portali' (rus tilinde)
http://www.alledu.ru — "Internetten bilim" portali' (rus tilinde)
http://www.rsl.ru — Rossiya ma'mleketlik kitapxanasi' portali' (rus tilinde)
http://www.rostest.runnet.ru — Test ali'w worayi' servari (rus tilinde)
http://www.allbest.ru — Internet resurslari elektron kitapxanasi' (rus tilinde)
http://int-edu.ru/soft/base_geom.html — «Живая геометрия» bag'darlamasin qollap-quwatlaw sayti'
http://matematica.mgdtd.ru/ — Matematika ha'm informatikadan si'rtqi' tan'law (rus tilinde)
http://www.mathtype.narod.ru/ — Online-sabaqli'qlar (rus tilinde)
http://www.e-pi.narod.ru/ — Ha'mmesi *e* ha'm ϖ sanlari' haqqi'nda (rus tilinde)
http://mschool.kubsu.ru/ — Elektron qollanbalar kitapxanasi. Si'rtqi' matematikaliq olimpiadalar
http://matematika.agava.ru/ — Matematikadan 2000 nan arti'q ma'seleler (rus tilinde)
http://mat-game.narod.ru/ — Matematikali'q gimnastika. Matematikali'q ma'seleler ha'm basqatirmalar

I BAP

UQSAS GEOMETRIYALIQ FIGURALAR

Bul bapti' u'yreniw na'tiyjesinde siz to'mendegi bilim ha'm a'meliy ko'nlikpelerge iye bolasi'z:

Bilimler:

√ *uqsas figuralardi'n' ani'qlamasi'n ha'm belgileniwin biliw;*

√ *u'shmu'yeshliklerdin' uqsasli'q belgilerin biliw;*

√ *gomotetiya tu'sinigin biliw.*

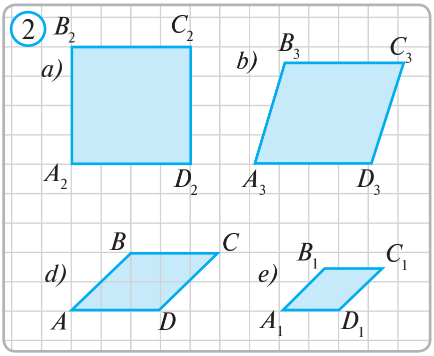
A'meliy ko'nlikpeler:

√ *yeki uqsas u'shmu'yeshliklerden sa'ykes yelementlerin taba ali'w;*

√ *u'shmu'yeshliklerdin' uqsasli'q belgilerin da'liyllewge ha'm yesaplawg'a tiyisli ma'selelerdi sheshiwde qollana ali'w;*

√ *gomotetiyadan paydalani'p uqsas ko'pmu'yeshliklerdi jasay ali'w.*

KO'PMU'YESHLIKLERDIN' UQSASLI'G'I'



Ku'ndelikli turmi'sta ten' figuralardan basqa formasi' (ko'rinisi) bir qi'yli', lekin wo'lshemleri tu'rlishe bolg'an figuralarg'a da dus kelemiz. Tariyx ha'm geografiya pa'nlerinde tu'rli masshtabta islengen kartalardan paydalang'anbi'z. Klass taxtasi'na ildirilgen ha'm sabaqli'qta su'wretlengen respublikami'zdin' kartalari' tu'rli wo'lshemde, lekin wolar bir qi'yli' formada (ko'riniste). Sonday-aq, bir fotoplyonkadan tu'rli wo'lshemdegi fotosu'wretler tayarlanadi'. Bul su'wretlerdin' wo'lshemleri tu'rlishe bolsa da, bir qi'yli' ko'riniste, yag'niy wolar bir-birine uqsaydi (*1-su'wret*).

Shinig'iw. 3-su'wrette to'rt romb su'wretlengen. Wo'lardan tek *d*) ha'm *e*) romblar bir qi'yli' ko'riniske iye. Bul romblar nesi menen basqa romblardan ajiralip turadi'? Kelin', birgelikte ani'qlayi'q.

1. Su'wretten ko'rinip turg'ani'nday, $AD=3$, $A_1D_1=2$. Rombi'ni'n' ta'repleri ten' bolg'ani' ushi'n,

$$\frac{AD}{A_1D_1}=\frac{BC}{B_1C_1}=\frac{CD}{C_1D_1}=\frac{AD}{A_1D_1}=\frac{3}{2}=1,5$$

ten'ligin payda yetemiz. Bul jag'dayda romblardi'n' sa'ykes ta'repleri *proporcional* dep ayti'ladi'.

2. $ABCD$ ha'm $A_1B_1C_1D_1$ romblardi'n' sa'ykes mu'yeshleri wo'z ara ten'. Haqi'yqattan da $\angle A=\angle A_1=45°$, $\angle B=\angle B_1=135°$, $\angle C=\angle C_1=45°$, $\angle D=\angle D_1=135°$.

Solay yetip, bul romblardi'n' bir-birine uqsasli'g'i'ni'n' sebebi — sa'ykes ta'replerinin' proporcionalli'g'i' ha'm sa'ykes mu'yeshlerinin' ten'ligi dep ayta alami'z. Qa'legen ko'pmu'yeshliklerdin' uqsasli'g'i tu'sinigi de usi' tiykari'nda kiritiledi. Mu'yeshlerinin' sani bir qi'yli' (demek, ta'replerinin' sani' da bir qi'yli') bolg'an ko'pmu'yeshlikler **bir qi'yli' atamadag'i ko'pmu'yeshlikler** dep ayti'ladi'.

Yeki bir qi'yli' atamali $ABCDE$ ha'm $A_1B_1C_1D_1E_1$ ko'pmu'yeshliklerinin' mu'yeshleri sa'ykes tu'rde ten' bolsi'n: $\angle A=\angle A_1$, $\angle B=\angle B_1$, $\angle C=\angle C_1$, $\angle D=\angle D_1$,

$\angle E=\angle E_1$. Bunday mu'yeshler **sa'ykes mu'yeshler dep ayti'ladi'**. Wonda, AB ha'm A_1B_1, BC ha'm B_1C_1, CD ha'm C_1D_1, DE ha'm D_1E_1, EA ha'm E_1A_1 ta'repleri **sa'ykes ta'repler** delinedi.

✔ ***Ani'qlama.*** Bir qi'yli' atamali' ko'pmu'yeshliklerdin' birinin' mu'yeshleri yekinshisinin' mu'yeshlerine sa'ykes tu'rde ten', sa'ykes ta'repleri bolsa proporcional bolsa, bunday ko'pmu'yeshlikler **uqsas ko'pmu'yeshlikler** dep ataladi' (*3-su'wret*).

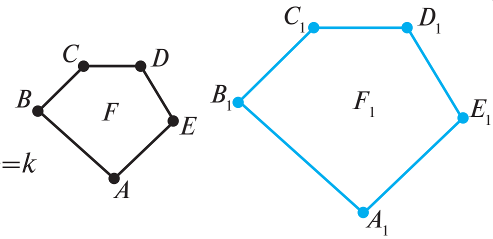
Ko'pmu'yeshliklerdin' uqsasli'g'i' $\backsim$ belgisi menen belgilenedi.

3

Sa'ykes mu'yeshler ten'

$$F \backsim F_1 \begin{cases} \angle A=\angle A_1,\ \angle B=\angle B_1,\ \angle C=\angle C_1, \\ \angle D=\angle D_1,\ \angle E=\angle E_1 \\ \dfrac{A_1B_1}{AB}=\dfrac{B_1C_1}{BC}=\dfrac{C_1D_1}{CD}=\dfrac{D_1E_1}{DE}=\dfrac{E_1A_1}{EA}=k \end{cases}$$

Sa'ykes ta'repler proporcional

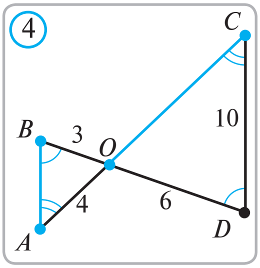


Uqsas ko'pmu'yeshliklerdin' sa'ykes ta'replerinin' qatnasi'na ten' bolg'an san **uqsasli'q koefficienti** delinedi.

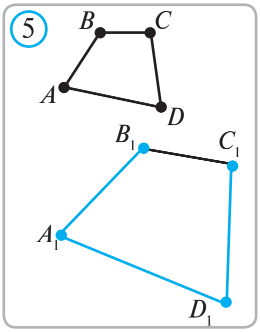
? Soraw, ma'sele ha'm tapsi'rmalar

1. Uqsas ko'pmu'yeshlikerdin' ani'qlamasi'n ayti'n'.
2. Uqsasli'q koefficienti degen ne ha'm wol qanday ani'qlanadi?
3. Yeger ABC ha'm DEF u'shmu'yeshliklerinde $\angle A=105°$, $\angle B=35°$, $\angle E=105°$, $\angle F=40°$, $AC=4{,}4$ *sm*, $AB=5{,}2$ *sm*, $BC=7{,}6$ *sm*, $DE=15{,}6$ *sm*, $DF=22{,}8$ *sm*, $EF=13{,}2$ *sm* bolsa, wolar uqsas bolama?
4. 2-su'wrettegi su'wretlengen a) ha'm b) romblar ne sebepten uqsas yemes? b) ha'm d) romblar-she?
5. 4-su'wrettegi ABO ha'm CDO u'shmu'yeshlikleri uqsas bolsa, AB, OC ta'repleri uzi'nli'g'i'n ha'm uqsasli'q koefficientin tabi'n'.
6. 5-su'wrette $ABCD \backsim A_1B_1C_1D_1$. $AB = 24$, $BC = 18$, $CD = 30$, $AD = 54$, $B_1C_1 = 54$. A_1B_1, D_1A_1 ha'm C_1D_1 lardi' tabi'n'.
7*. ABC u'shmu'yeshliginin' AB ha'm AC ta'replerinin' wortalari' sa'ykes tu'rde P ha'm Q bolsi'n. $\Delta ABC \backsim \Delta APQ$ yekenligin da'liyllen'.

4



5



6 UQSAS U'SHMU'YESHLIKLER HA'M WOLARDI'N' QA'SIYETLERI

Yen' a'piwayi' ko'pmu'yeshliklerden bolg'an u'shmu'yeshliklerdin' uqsasli'g'i'n u'yrenemiz.

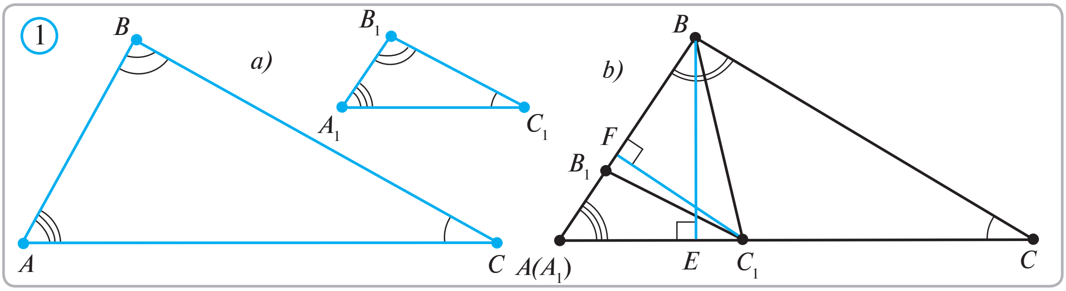
Teorema. **Yeki uqsas ushmu'yeshliktin' perimetrlerinin' qatnasi' uqsasli'q koefficentine ten'.**

Bul teoremani wo'z betin'izshe da'liyllen'.

Teorema. **Yeki uqsas u'shmu'yeshlik maydanlari'ni'n' qatnasi uqsasli'q koefficientinin' kvadratina ten'.**

$\Delta ABC \backsim \Delta A_1B_1C_1$ *(1-su'wret)*, k— uqsasli'q koefficienti $\Rightarrow$ $S_{ABC}:S_{A_1B_1C_1}=k^2$

Da'liyllew: Teoremani'n' sha'rti boyi'nsha, $\Delta ABC \backsim \Delta A_1B_1C_1$. Demek, ko'p-mu'yeshliklerdin' uqsasli'g'i'ni'n' ani'qlamasi' boyi'nsha, $\angle A=\angle A_1$, $\angle B=\angle B_1$, $\angle C=\angle C_1$ ha'm $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=\frac{CA}{C_1A_1}=k$.



$\angle A=\angle A_1$ yekenliginen paydalani'p, wolardi' 1-b, su'wrettegidey u'stpe-u'st qoyami'z ha'm tiyisli jasaw ha'mde belgilewlerdi a'melge asi'rami'z.

To'mendegi u'shmu'yeshliklerdin' maydanlari'n tabami'z ha'm wolardi'n' qatnaslari'n qaraymiz:

$$\left.\begin{aligned} S_{ABC_1}&=\frac{A_1C_1\cdot BE}{2};\\ S_{ABC}&=\frac{AC\cdot BE}{2}; \end{aligned}\right] \Rightarrow \frac{S_{ABC}}{S_{ABC_1}}=\frac{AC}{A_1C_1} \qquad (1),$$

$$\left.\begin{aligned} S_{A_1B_1C_1}&=\frac{A_1B_1\cdot C_1F}{2};\\ S_{ABC_1}&=\frac{AB\cdot C_1F}{2}; \end{aligned}\right] \Rightarrow \frac{S_{A_1B_1C_1}}{S_{ABC_1}}=\frac{A_1B_1}{AB} \qquad (2).$$

$$\frac{S_{ABC}}{S_{A_1B_1C_1}}=\frac{AB\cdot AC}{A_1B_1\cdot A_1C_1} \qquad (3)$$

(1) ten'likti (2) ten'likke bo'lsek, ten' mu'yeshke iye bolg'an u'shmu'yeshliklerdin' maydani'nin' qatnasi' ushi'n (3) ten'likti payda yetemiz.

Bul jerde sha'rt boyi'nsha, $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=\frac{CA}{C_1A_1}=k$ yekenligin yesapqa alsaq,

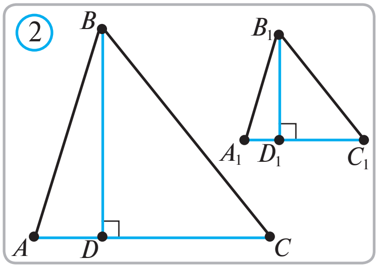
$\frac{S_{ABC}}{S_{A_1B_1C_1}}=\frac{AB\cdot AC}{A_1B_1\cdot A_1C_1}=\frac{AB}{A_1B_1}\cdot\frac{AC}{A_1C_1}=k\cdot k=k^2$ kelip shig'adi. ***Teorema da'liyllendi.***

1-ma'sele. Uqsas u'shmu'yeshliklerdin' sa'ykes ta'replerinin' qatnasi' usi' ta'replerge tu'sirilgen biyikliklerdin' qatnasi'na ten' yekenligin da'liyllen' (*2-su'wret*).

$\Delta ABC \sim \Delta A_1B_1C_1$, *BD*, B_1D_1 — *biyiklikler* ⇒ $\frac{AC}{A_1C_1}=\frac{BD}{B_1D_1}$

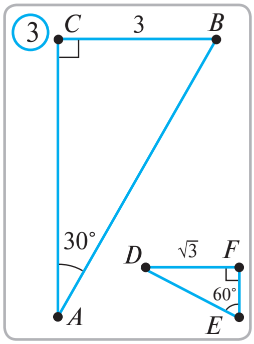
Sheshiliwi. Berilgen u'shmu'yeshliklerdin' uqsasli'q koefficenti *k* bolsi'n. Wonda, $AC : A_1C_1 = k$; $S_{ABC} : S_{A_1B_1C_1} = k^2$ (1) boladi'. Yekinshi ta'repten,

$$\frac{S_{ABC}}{S_{A_1B_1C_1}}=\frac{\frac{1}{2}AC\cdot BD}{\frac{1}{2}A_1C_1\cdot B_1D_1}=\frac{AC}{A_1C_1}\cdot\frac{BD}{B_1D_1}=k\cdot\frac{BD}{B_1D_1}. \quad (2)$$



(1) ha'm (2) ten'liklerden $k\cdot\frac{BD}{B_1D_1}=k^2$ yaki $\frac{BD}{B_1D_1}=k$. Solay yetip, $\frac{BD}{B_1D_1}$ qatnasi' da, $\frac{AC}{A_1C_1}$ qatnasi' da *k* g'a ten', yag'niy $\frac{AC}{A_1C_1}=\frac{BD}{B_1D_1}$.

Soraw, ma'sele ha'm tapsi'rmalar

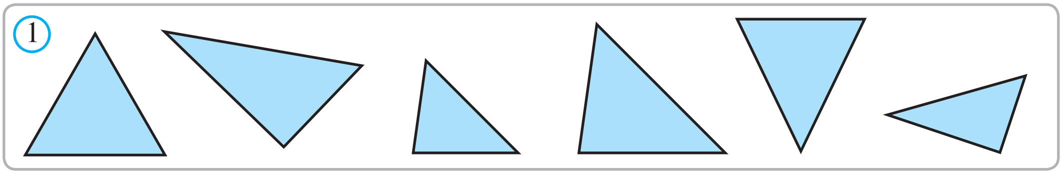


1. Uqsas u'shmu'yeshliklerdin' maydanlarinin' qatnasi haqqindag'i teoremani aytin' ha'm da'liyllen'.
2. Yeki *ABC* ha'm $A_1B_1C_1$ uqsas u'shmu'yeshlikleri berilgen. Yeger $S_{ABC}= 25$ sm^2 ha'm $S_{ABC}= 81$ sm^2 bolsa, uqsasli'q koefficntin tabi'n'.
3. Yeki uqsas u'shmu'yeshliktin' maydanlari 65m^2 ha'm 260 m^2.Birinshi u'shmu'yeshliktin' bir ta'repi 6 m bolsa, yekinshi u'shmu'yeshliktin' wog'an sa'ykes ta'repin tabi'n'.
4. Berilgen u'shmu'yeshliktin' ta'repleri 15 sm, 25 sm ha'm 30 sm. Yeger perimetri 35 sm bolg'an u'shmu'yeshlik berilgen u'shmu'yeshlikke uqsas bolsa, woni'n' ta'replerin tabi'n'.
5. $\Delta ABC \sim \Delta A_1B_1C_1$ ha'm bul u'shmu'yeshliklerdin' sa'ykes ta'replerinin' qatnasi 7:5 ke ten'.Yeger *ABC* u'shmu'yeshli-ginin' maydani' $A_1B_1C_1$ u'shmu'yeshliginin' maydani'nan 36 sm^2 ge artiq bolsa, bul u'shmu'yeshliklerdin' maydanlarin tabi'n'.
6. 3-su'wrette berilgenlerden paydalani'p, u'shmu'yeshliklerdin' uqsas yaki uqsas yemesligin aniqlan'.

7 U'SHMU'YESHLIKLERDIN' UQSASLIG'ININ' BIRINSHI BELGISI

Jedellestiriwshi shinig'iw

1-su'wrette su'wretlengen u'shmu'yeshliklerdin' ishinen uqsaslari'n ani'qlan'. Wolardi'n' uqsasli'g'i'n qalay ani'qladi'n'i'z?



Ani'qlama boyi'nsha yeki u'shmu'yeshliktin' uqsasli'g'i'n ani'qlaw ushi'n wolardi'n' mu'yeshlerinin' ten'ligin ha'm sa'ykes ta'replerinin' proporsional yekenligin tekseriw kerek boladi.' U'shmu'yeshlikler ushi'n bul is an'satlasadi yeken. To'mende keltirilgen teoremalar usi' tuwrali' boli'p, olar "u'shmu'yeshliklerdin' uqsasli'g'i'ni'n' belgileri" dep ataladi.'

Teorema. (***U'shmu'yeshliklerdin' uqsasli'g'i'ni'n' MM belgisi***). **Yeger bir u'shmu'yeshliktin' yeki mu'yeshi yekinshi u'shmu'yeshliktin' yeki mu'yeshine sa'ykes tu'rde ten' bolsa, wonda bunday u'shmu'yeshlikler uqsas boladi'** (*2-su'wret*).

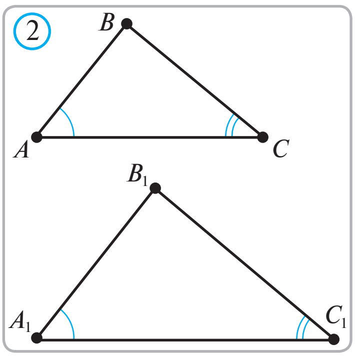
$\Delta ABC, \Delta A_1B_1C_1, \angle A=\angle A_1, \angle C=\angle C_1 \Rightarrow \Delta ABC \backsim \Delta A_1B_1C_1$

Da'liyllew. 1) U'shmu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si' haqqindag'i teorema boyi'nsha,

$$\angle B=180°-(\angle A+\angle C),\ \angle B_1=180°-(\angle A_1+\angle C_1) \quad \Rightarrow \quad \angle B=\angle B$$

Demek, ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklerinin' mu'yeshleri sa'ykes tu'rde ten'.

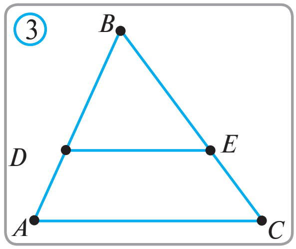
2) Sha'rt boyi'nsha, $\angle A=\angle A_1$, $\angle C=\angle C_1$. Ten'dey mu'yeshke iye bolg'an u'shmu'yeshliklerdin' maydanlari'ni'n' qatnasi haqqindag'i teorema boyi'nsha,



$$\frac{S_{ABC}}{S_{A_1B_1C_1}}=\frac{AB\cdot AC}{A_1B_1\cdot A_1C_1} \quad \text{ha'm} \quad \frac{S_{A_1B_1C_1}}{S_{A_1B_1C_1}}=\frac{CA\cdot CB}{C_1A_1\cdot C_1B_1}.$$

Bul ten'liklerdin' won' bo'limlerin ten'lestirip, bir qi'yli' ag'zalar qisqartilsa, $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}$ ten'ligi payda boladi. Sonday-aq, $\angle A=\angle A_1$ ha'm $\angle B=\angle B_1$ ten'liklerinen paydalani'p, $\frac{BC}{B_1C_1}=\frac{CA}{C_1A_1}$ ten'ligine iye bolami'z. Solay yetip, ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklerinin' mu'yeshleri ten' ha'm sa'ykes ta'repleri proporcional, yag'niy bul u'shmu'yeshlikler uqsas boladi.' ***Teorema da'liyllendi.***

Ma'sele. ABC u'shmu'yeshliginin' yeki ta'repin kesip wo'tiwshi ha'm u'shinshi ta'repine parallel bolg'an DE tuwri'si u'shmu'yeshlikten wog'an uqsas u'shmu'yeshlik aji'ratatug'i'ni'n da'liyllen' (*3-su'wret*).



Da'liyllew. ABC ha'm DBE u'shmu'yeshliklerde $\angle B$ — uli'wma, $\angle CAB = \angle EDB$ (AC ha'm DE parallel tuwri'lari'n AB kesiwshi menen keskende payda bolg'an sa'ykes mu'yeshler bolg'ani' ushi'n) (*3-su'wret*).

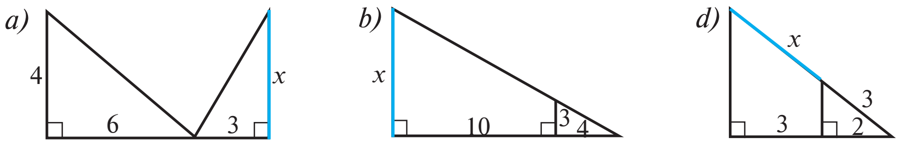
Demek, u'shmu'yeshliklerdin' uqsaslig'i'ni'n' MM belgisi boyi'nsha, $ABC \backsim DBE$.

Soraw, ma'sele ha'm tapsi'rmalar

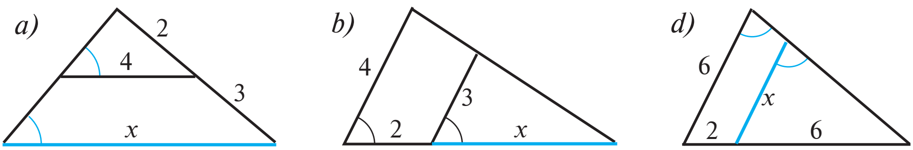
1. U'shmu'yeshliklerdin' uqsasli'q ani'qlamasi' ha'm MM belgisin wo'z ara salistirin'.

2. U'shmu'yeshliklerdin' uqsaslig'inin' MM belgisin da'liyllen'.

3. Su'wrettegi mag'liwmatlarg'a tiykarlanip x ti tabin.



4. Su'wrettegi mag'liwmatlarg'a tiykarlani'p x ti tabi'n'.

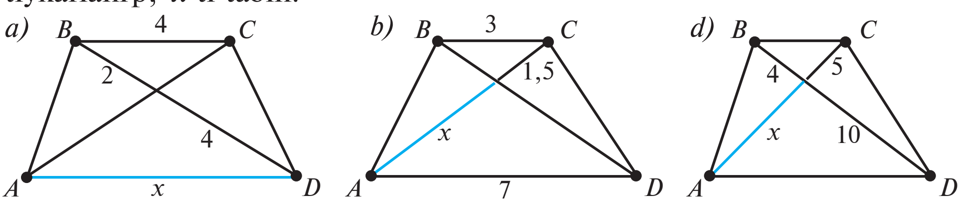


5. $ABCD$ parallelogrammnin' CD ta'repinen E noqati aling'an. AE ha'm BC nurlari F noqatinda kesilisedi.

a) Yeger $DE = 8$ *sm*, $EC = 4$ *sm*, $BC = 7$ *sm*, $AE = 10$ *sm* bolsa, EF ha'm FC ni;

b) Yeger $AB = 8$ *sm*, $AD = 5$ *sm*, $CF = 2$ *sm* bolsa, DE ha'm EC ni tabin.

6. Su'wrette $ABCD$— trapeciyasi' berilgen. Su'wrettegi mag'liwmatlarg'a tiykarlanip, x ti tabin.

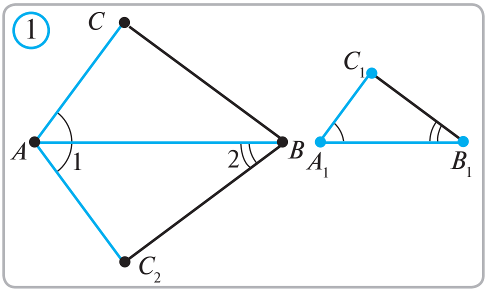


7*. Bir su'yir mu'yeshleri ten' bolg'an yeki tuwri' mu'yeshli u'shmu'yeshlikler uqsas yekenligin da'liyllen'.

8 U'SHMU'YESHLIKLERDIN' UQSASLIG'ININ' EKINSHI BELGISI

Teorema. (U'shmu'yeshliklerdin' uqsaslig'i'ni'n' TMT belgisi). **Yeger bir u'shmu'yeshliktin' yeki ta'repi yekinshi u'shmu'yeshliktin' yeki ta'repine proporcional ha'm bul ta'repler payda yetken mu'yeshler ten' bolsa, onda bunday u'shmu'yeshlikler uqsas boladi'** (*1-su'wret*).

$\Delta ABC,\ \Delta A_1B_1C_1,\ \angle A=\angle A_1,\ \frac{AB}{A_1B_1}=\frac{AC}{A_1C_1}$ $\Rightarrow$ $\Delta ABC \sim \Delta A_1B_1C_1$



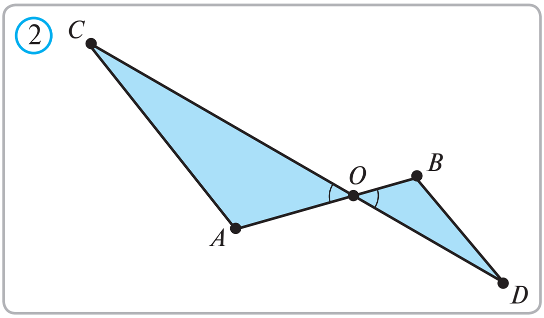
Da'liyllew. $\angle 1=\angle A_1$, $\angle 2=\angle B_1$ bolatug'i'nday yetip ABC_2 u'shmu'yeshligin jasaymi'z (1-su'wret). Ol MM belgisi boyi'nsha $A_1B_1C_1$ u'shmu'yeshligine uqsas boladi'.

$$\frac{AB}{A_1B_1}=\frac{AC_2}{A_1C_1} \Leftarrow (\Delta A_1B_1C_1 \sim \Delta ABC_2)$$

$$\frac{AB}{A_1B_1}=\frac{AC}{A_1C_1} \Leftarrow \text{(sha'rtke ko're).}$$

Bul yeki ten'likten, $AC_2=AC$ yekenligin ani'qlaymi'z. Wonda u'shmu'yeshliklerdin' ten'liginin' TMT belgisi boyi'nsha $\Delta ABC=\Delta ABC_2$. Sonliqtan, $\angle 2=\angle B$. Lekin, jasaw boyi'nsha $\angle 2=\angle B_1$ yedi. Demek, $\angle B=\angle B_1$. Onda, $\angle A=\angle A_1$ ha'm $\angle B=\angle B_1$ bolg'ani ushi'n, u'shmu'yeshliklerdin' uqsaslig'inin' MM belgisi boyi'nsha,, $\Delta ABC \sim \Delta A_1B_1C_1$. ***Teorema da'liyllendi.***

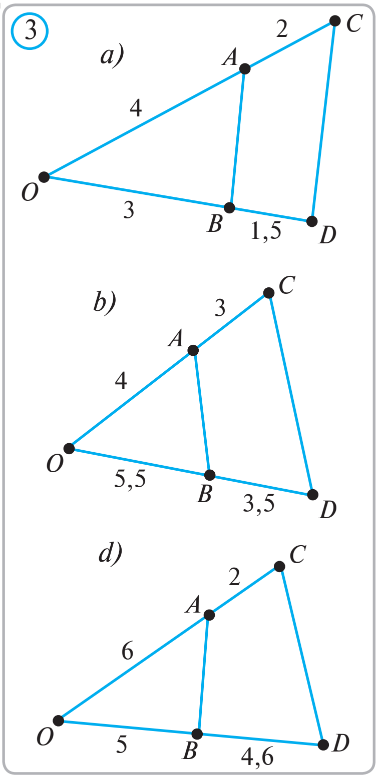
Ma'sele. AB ha'm CD kesindileri O noqatinda kesilisedi, $AO=12sm$, $BO=4sm$, $CO=30sm$, $DO=10sm$ bolsa, AOC, BOD u'shmu'yeshliklerinin' maydanlari'ni'n' qatnasi'n tabi'n'.



Sheshiliwi. Sha'rt boyi'nsha

$$\left.\begin{array}{l}\frac{OA}{OB}=\frac{12}{4}=3\\ \frac{OC}{OD}=\frac{30}{10}=3\end{array}\right\} \Rightarrow \frac{OA}{OB}=\frac{OC}{OD}=3.$$

Demek, AOC u'shmu'yeshliginin' yeki ta'repi BOD u'shmu'yeshliginin' yeki ta'repine proporcional ha'm bul ta'replerdin' arasi'ndag'i' sa'ykes mu'yeshler vertikal mu'yeshler bolg'ani' ushi'n: $\angle AOC=\angle BOD$. Soni'n' ushi'n, u'shmu'yeshliklerdin' uqsasli'g'i'ni'n' TMT belgisi boyi'nsha, $\Delta AOC \sim \Delta BOD$ ha'm uqsasli'q koefficienti $k=\frac{OA}{OB}=3$. Yendi uqsas u'shmu'yeshliklerdin' maydanlari'ni'n' qatnasi haqqindag'i teoremani qollanamiz. $\frac{S_{AOC}}{S_{BOD}}=k^2=9$. ***Juwabi':*** 9.



Soraw, ma'sele ha'm tapsi'rmalar

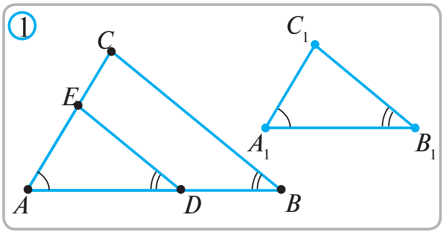
1. U'shmu'yeshliklerdin' uqsasli'g'i'nin' ani'qlamasi' ha'm TMT belgilerin wo'z ara salisti'ri'n'.
2. U'shmu'yeshliklerdin' uqsasli'g'inin' TMT belgisin da'liyllen'.
3. To'besindegi mu'yeshleri ten' bolg'an ten' qaptalli' u'shmu'yeshliklerdin' uqsaslig'in a) MM, b) TMT belgisinen paydalani'p da'liyllen'.
4. 3-su'wrette su'wretlengen OAB ha'm OCD u'shmu'yeshlikleri uqsas pa?
5. AC ha'm BD nurlari' O noqati'nda kesilisedi. Yeger $AO:CO=BO:DO=3$, $AB=7$ *sm* bolsa, CD kesindisin ha'm de AOB ha'm COD u'shmu'yeshliklerinin' maydanlati'ni'n' qatnasi'n tabi'n'.
6. ABC ha'm $A_1B_1C_1$ u'shmuyeshliklerde $\angle A=\angle A_1$, $AB:A_1B_1=AC:A_1C_1=4:3$.
   a) Yeger AB kesindisi A_1B_1 den 5 *sm* arti'q bolsa, AB ha'm A_1B_1 ta'replerin tabi'n';
   b) Yeger A_1B_1 kesindi AB dan 6 *sm* qi'sqa bolsa, AB ha'm A_1B_1 ta'replerin tabi'n';
   d) Yeger berilgen u'shmu'yeshliklerdin' maydanlari'ni'n' qosi'ndi'si' 400 sm^2 bolsa, onda ha'r bir u'shmu'yeshliktin' maydani'n tabi'n'.
7. Yeger bir tuwri' mu'yeshli u'shmu'yeshliktin' katetleri yekinshi tuwri' mu'yeshli u'shmu'yeshliktin' sa'ykes katetlerine proporcional bolsa, onda u'shmu'yeshliklerdin' uqsas ekenligin da'liyllen'.
8. ABC u'shmu'yeshliginde $AB=15$ *m*, $AC=20$ *m*, $BC=32$ *m*. U'shmu'yeshliktin' AB ta'repine $AD=9$ *m* kesindi, AC ta'repine $AE=12$ *m* kesindi qoyi'ldi. DE kesindisin tabi'n.
9. Katetleri 3 *dm* ha'm 4 *dm* bolg'an tuwri'mu'yeshli u'shmu'yeshlik penen bir kateti 8 *dm* ha'm gipotenuzasi 10 *dm* bolg'an tuwri'mu'yeshli u'shmu'yeshliktin' uqsas yekenligin da'liyllen'.

10*. AB kesindisi ha'm l tuwri'si O noqatinda kesilisedi. l tuwri'sina AA_1 ha'm BB_1 perpendikulyarlari' tu'sirilgen. Yeger $AA_1=2$ *sm*, $OA_1=4$ *sm* ha'm $OB_1=3$ *sm* bolsa, BB_1, OA ha'm AB kesindilerin tabin.

9 U'SHMU'YESHLIKLERDIN' UQSASLIG'ININ' U'SHINSHI BELGISI

Teorema. (U'shmu'yeshliklerdin' uqsasli'g'i'ni'n' TTT belgisi). **Yeger bir u'shmu'yeshliktin' u'sh ta'repi yekinshi u'shmu'yeshliktin' u'sh ta'repine proporcional bolsa, wonda bunday u'shmu'yeshlikler uqsas boladi.**

$\Delta ABC, \Delta A_1B_1C_1, \dfrac{AB}{A_1B_1}=\dfrac{BC}{B_1C_1}=\dfrac{CA}{C_1A_1}$ (*1-su'wret*). ⇨ $\Delta ABC \backsim \Delta A_1B_1C_1$



Da'liyllew. ABC u'shmu'yeshliginin' AB ta'repinde $AD=A_1B_1$ bolatug'i'nday yetip D noqatin belgileymiz. D noqati'nan BC ta'repine parallel yetip ju'rgizilgen tuwri' AC ta'repin E noqati'nda kesip wo'tsin. Wonda u'shmu'yeshliklerdin' MM belgisi boyi'nsha ΔADE ha'm ΔABC uqsas boladi'. Bul jag'dayda ani'qlama boyi'nsha: $\dfrac{AB}{A_1B_1}=\dfrac{BC}{B_1C_1}$ ha'm $\dfrac{AB}{AD}=\dfrac{BC}{DE}$.

Biraq jasaw boyi'nsha $A_1B_1=AD$. Wonda joqari'dag'i' ten'liklerden, $B_1C_1=DE$ ten'ligi payda boladi'. Solay yetip, u'shmu'yeshliklerdin' ten'liginin' TMT belgisi boyi'nsha ΔADE ha'm $\Delta A_1B_1C_1$ ten' ha'm $\Delta ADE \backsim \Delta ABC$. Demek, $\Delta ABC \backsim \Delta A_1B_1C_1$. ***Teorema da'liyllendi.***

Ma'sele. Yeger yeki ten' qaptalli' u'shmu'yeshliktin' birewinin' ultani' ha'm qaptal ta'repi yekinshisinin' ultani ha'm qaptal ta'repine proporcional bolsa, wonda bul u'shmu'yeshliklerdin' uqsas yekenligin da'liyllen'.

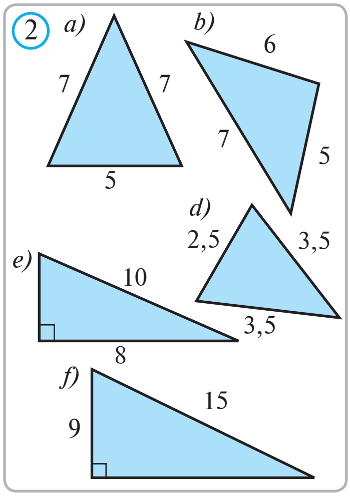
ΔABC, $AB=BC$, $\Delta A_1B_1C_1$, $A_1B_1=B_1C_1$, $\dfrac{AB}{A_1B_1}=\dfrac{AC}{A_1C_1}$ ⇨ $\Delta ABC \backsim \Delta A_1B_1C_1$

Da'liyllew. Berilgen $AB=BC$, $A_1B_1=B_1C_1$ ten'likler ha'm $\dfrac{AB}{A_1B_1}=\dfrac{AC}{A_1C_1}$ qatnastan $\dfrac{AB}{A_1B_1}=\dfrac{AC}{A_1C_1}=\dfrac{BC}{B_1C_1}$ ten'liklerin payda yetemiz. Demek, u'shmu'yeshliklerdin' uqsasli'g'i'ni'n' TTT belgisi boyi'nsha, $\Delta ABC \backsim \Delta A_1B_1C_1$.

Soraw, ma'sele ha'm tapsi'rmalar

1. U'shmu'yeshliklerdin' uqsasli'g'i'ni'n' TTT belgisin ayti'n' ha'm da'liyllewin tu'sindirip berin'.
2. $AC=14$ *sm*, $AB=11$ *sm*, $BC=13$ *sm*, $A_1C_1=28$ *sm*, $A_1B_1=22$ *sm*, $B_1C_1=26$ *sm* yekenligi belgili. ABC ha'm $A_1B_1C_1$ u'shmu'yeshlikleri uqsas bolama?

3. 2-su'wrettegi uqsas u'shmu'yeshlikler jupli'qlari'n ko'rsetin'.
4. *BCD* trapeciyani'n' *AB* ha'm *CD* qaptal ta'repleri dawam ettirilse, *E* noqati'nda kesilisedi. Yeger $AB=5$ *sm*, $BC=10$ *sm*, $CD=6$ *sm*, $AD=15$ *sm* bolsa, *AED* u'shmu'yeshliginin' maydani'n tabi'n.
5. Trapeciyani'n' ultanlari' 6 *sm* ha'm 9 *sm*, biyikligi 10 *sm*. Trapeciyani'n' diagonallari' kesisken noqati'nan ultanlari'na shekemgi arali'qlari'n tabi'n.
6. Qa'legen yeki ten' ta'repli u'shmu'yeshliktin' uqsas yekenligin da'liyllen'.
7. Ultani 12 sm, biyikligi 8 sm bolg'an ten' qaptalli' u'shmu'yeshliktin' ishine kvadrat sonday yetip ishley si'zi'lg'an, kvadrattin' yeki to'besi u'shmu'yeshliktin' ultani'nda, al qalgan yeki to'besi bolsa qaptal ta'replerde jatadi'. Kvadratti'n' ta'repin tabi'n.



8*. Su'yir mu'yeshli *ABC* u'shmu'yeshliginin' AA_1 ha'm BB_1 biyiklikleri ju'rgizilgen. $\Delta ABC \backsim \Delta A_1B_1C$ yekenligin da'liyllen'.
9. Yeki uqsas u'shmu'yeshliktin' maydanlari 6 ha'm 24 ge ten'. Wolardi'n birewinin' perimetri yekinshisinen 6 g'a artiq. U'lken u'shmu'yeshliktin' perimetrin tabi'n.

Tariyxi'y waqi'yalar. Bul waqi'ya erami'zdan aldi'n'g'i' VI a'sirde bolg'an yedi. Bul waqi'tta grekler geometriya menen derlik shug'i'llanbaytug'in yedi. Grek filosofi' Fales mi'si'r pa'ni menen tani'si'w ushi'n wol jerge barg'an. Mi'si'rli'lar og'an qi'yi'n ma'sele bergen yeken: u'lken piramidalardan birinin' biyikligin qalay tabi'w mu'mkin? Fales bul ma'selenin' a'piwayi' ha'm jen'il sheshimin tapti'. Wol tayaqshani' jerge qaqti' ha'm sonday dedi: "Qashan bul tayaqsha sayasi'ni'n' uzi'nli'g'i' tayaqshani'n' uzi'nli'g'i' menen ten' bolsa, piramida sayasi'ni'n' uzi'nlig'i piramida biyikligi menen ten' boladi'". Falestin' pikirin tiykarlawg'a ha'reket yetin'!

10 TUWRI' MU'YESHLI U'SHMU'YESHLIKLERDIN' UQSASLI'Q BELGILERI

Tuwri' mu'yeshli u'shmu'yeshliklerdin' bir-birden mu'yeshleri tuwri' mu'yeshten ibarat boladi. Soni'n' ushi'n bunday u'shmu'yeshlikler ushi'n uqsasli'q belgileri a'piwayilastiriladi.

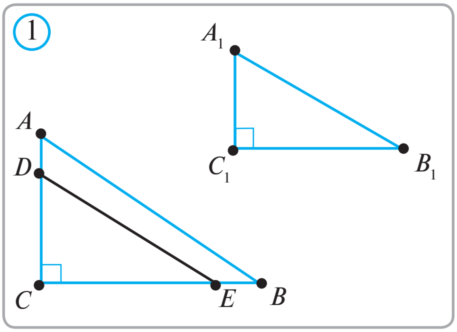
1-Teorema. **Tuwri' mu'yeshli u'shmu'yeshliklerdin' bir-birden su'yir mu'yeshi sa'ykes tu'rde ten' bolsa, wonda wolar uqsas boladi'.**

2-Teorema. **Tuwri' mu'yeshli u'shmu'yeshliklerdin' katetleri sa'ykes tu'rde proporcional bolsa, wonda wolar uqsas boladi'.**

3-Teorema. **Tuwri' mu'yeshli u'shmu'yeshliklerdin' birinin' gipotenuzasi' ha'm kateti, yekinshisinin' gipotenuzasi ha'm katetine sa'ykes tu'rde proportcional bolsa, wonda wolar uqsas boladi'.**

Bul belgilerdin' birinshi yekewinin' duri's yekenligi wo'z-wo'zinen ayqi'n. U'shinshi belgisin bolsa da'liyllewimiz kerek.

$\Delta ABC,\ \Delta A_1B_1C_1,\ \angle C=90°,\ \angle C_1=90°,\ \frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}$ ⇒ $\Delta ABC \backsim \Delta A_1B_1C_1$



Da'liyllew. ABC u'shmu'yeshliginin' BC ta'repinde $CE=C_1B_1$ bolatug'inday yetip C_1B_1 kesindisin qoyamiz ha'm $DE \parallel AB$ ni ju'rgizemiz (*1-su'wret*). Wonda u'shmu'yeshliklerdin' uqsasli'g'i'ni'n' MM belgisi boyi'nsha, ΔDEC ha'm ΔABC uqsas boladi'. Uqsas u'shmu'yeshliklerdin' sa'ykes ta'replerinin' proporcionallig'inan:

$$\frac{AB}{DE}=\frac{CB}{CE}.$$

Jasaliwi boyi'nsha, $CE = C_1B_1$. Demek, $\frac{AB}{DE}=\frac{CB}{C_1B_1}$ (1) ten'ligi ori'nli boladi.

Basqa ta'repten, teorama sha'rti boyi'nsha, $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}$ (2)

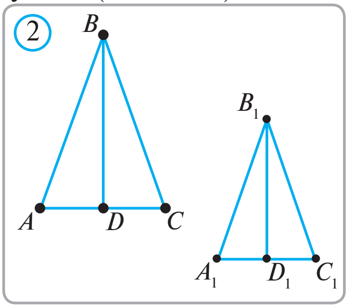
(1) ha'm (2) ten'liklerden $DE=A_1B_1$ yekenligin ani'qlaymiz.

$A_1B_1C_1$ ha'm DEC u'shmu'yeshliklerdi qaraymiz. Wolarda:

1. $CE=C_1B_1$ (jasaw boyi'nsha); 2. $DE=A_1B_1$ (da'lillengen ten'lik boyi'nsha).

Demek, tuwri' mu'yeshli u'shmu'yeshliklerdin' bir-birden kateti ha'm de gipotenuzasinin' ten'lik belgisi boyi'nsha, $\Delta A_1B_1C_1 = \Delta DEC$. Yekinshi ta'repten bolsa $\Delta ABC \backsim \Delta DEC$. Wonda $\Delta ABC \backsim \Delta A_1B_1C_1$ boladi. ***Teorema da'liyllendi.***

Ma'sele. Yeger yeki ten' qaptalli u'shmu'yeshlikten birewinin' qaptal ta'repi ha'm biyikligi yekinshisinin' qaptal ta'repi ha'm biyikligine proporcional bolsa, wonda bul u'shmu'yeshliklerdin' uqsas yekenligin da'liyllen' (*2-su'wret*).



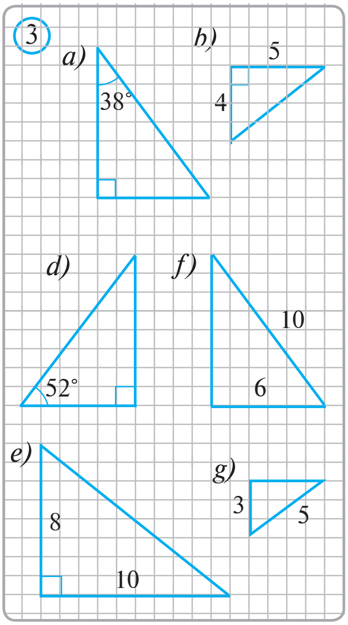
Da'liyllew. Tuwri' mu'yeshli ABD ha'm $A_1B_1D_1$ u'shmu'yeshliklerin qaraymi'z. Sha'rt boyi'nsha wolardi'n' birewden kateti ha'm gipotenuzasi wo'z ara proporcional. Demek, 3-teoremag'a tiykarlanip $\Delta ABD \backsim \Delta A_1B_1D_1$. Wonda $\angle DBA \backsim \angle D_1B_1A_1$.

Ten' qaptalli u'shmu'yeshliktin' ultaninatu'sirilgen biyikliktin' bissektrisasi da boli'wi'n yesapqa alsaq, $\angle B = 2\angle DBA = 2\angle D_1B_1A_1 = \angle B_1$ boladi.

Na'tiyjede, ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklerde $\angle B = \angle B_1$ ha'm $\frac{AB}{A_1B_1} = \frac{BC}{B_1C_1}$ ten'liklerine iye bolami'z.

Demek, u'shmu'yeshliklerdin' uqsasli'g'i'ni'n' TMT belgisi boyi'nsha, $\Delta ABC \backsim \Delta A_1B_1C_1$.

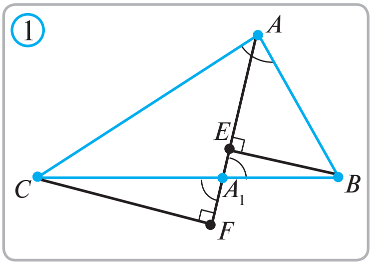
Teorema da'liyllendi.



Soraw, ma'sele ha'm tapsi'rmalar

1. Tuwri' mu'yeshli u'shmu'yeshliklerdin' uqsasli'q belgilerin ayti'n' ha'm da'liyllen'.
2. 3-su'wrettegi uqsas u'shmu'yeshliklerdi tabi'n'.
3. Katetleri 3 *m* ha'm 4 *m* bolg'an tuwri' mu'yeshli u'shmu'yeshlikke uqsas u'shmu'yeshliktin' bir kateti 27 *m* bolsa, yekinshi kateti neshe *m* boladi?
4. Maydanlari' 21 m^2 ha'm 84 m^2 bolg'an yeki tuwri' mu'yeshli u'shmu'yeshlikler wo'z ara uqsas. Yeger birinshi u'shmu'yeshliktin' bir kateti 6 *m* bolsa, yekinshi u'shmu'yeshliktin' katetlerin tabi'n'.
5. Bir shen'berge yeki uqsas tuwri' mu'yeshli u'shmu'yeshlik ishley si'zi'lg'an. Bul u'shmu'yeshliklerdin' ten'ligin da'liyllen'.
6*. Katetleri 10 *sm* ha'm 12 *sm* bolg'an tuwri' mu'yeshli u'shmu'yeshlikke bir mu'yeshi uliwma bolg'an kvadrat ishley si'zi'lg'an. Yeger kvadrattin' bir to'besi gipotenuzada yekenligi belgili bolsa, kvadrattin' ta'repin tabi'n'.
7*. ABC u'shmu'yeshlik berilgen. Wog'an $ADEF$ sonday etip ishley si'zi'lg'an D, E ha'm F noqatlar sa'ykes tarizde u'shmu'yeshliktin' AB, BC ha'm CA tareplerinde jatadi? Yeger $AB=c$, $AC=b$, bolsa, rombni'n ta'repin tabi'n'.

11 UQSASLI’Q BELGILERININ’ DA’LIYLLEWGE BAYLANISLI MA’SELELERGE QOLLANIWLARI



1-ma’sele. U’shmu’yeshliktin’ bissektrisasi’ wo’zi tu’sken ta’repti qalgan yeki ta’repke proporcional bolgan kesindilerge aji’ratatug’i’ni’n da’liyllen’.

ΔABC, AA_1 — *bissektrisa* (*1-su’wret*). $\Rightarrow$ $\dfrac{AB}{A_1B}=\dfrac{AC}{A_1C}$

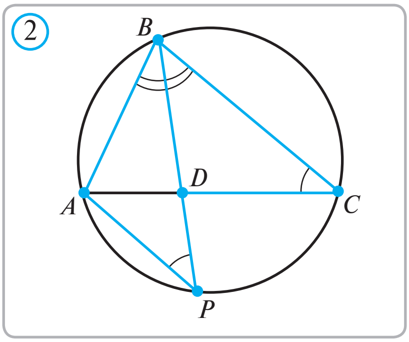
Da’liyllew. AA_1 tuwri’si’na BE ha’m CF perpendikulyari’n tu’siremiz. Wonda $\angle CAF=\angle BAE$ bolg’ani’ ushi’n tuwri’ mu’yeshli CAF ha’m BAE u’shmu’yeshlikler uqsas boladi’. Uqsas u’shmu’yeshliklerdin’ sa’ykes ta’replerinin’ proporcionalli’g’i’nan

$$\Delta CAF \backsim \Delta BAE \Rightarrow \frac{AC}{AB}=\frac{CF}{BE}. \qquad (1)$$

Bug’an uqsas,

$$\Delta CA_1F \backsim \Delta BA_1E \Rightarrow \frac{CA_1}{BA_1}=\frac{CF}{BE}. \qquad (2)$$

(1) ha’m (2) ten’liklerdi sali’sti’rsaq, $\frac{AC}{AB}=\frac{CA_1}{BA_1}$ yaki $\frac{AB}{A_1B}=\frac{AC}{A_1C}$ boladi’. Bul A_1B ha’m A_1C kesindileri AB ha’m AC kesindilerge proporcional yekenligin an’latadi’.



2-ma’sele. ABC u’shmu’yeshliginin’ BD bissektrisasi’ u’shmu’yeshlikke si’rtlay si’zi’lg’an shen’berdi B ha’m P noqati’nda kesip wo’tedi. $\Delta ABP \backsim \Delta BDC$ li’gi’n da’liyllen’ *(2-su’wret)*.

Sheshiliwi. ΔABP ha’m $\angle BDC$ da:

1. $\angle DBC=\angle ABP \Leftarrow$ sha’rt boyi’nsha;

2. $\angle DCB=\angle APB \Leftarrow$ sebebi wolar bir dog’ag’a tirelgen. Demek, u’shmu’yeshliklerdin’ uqsasli-g’i’ni’n’ MM belgisi boyi’nsha, $\Delta ABP \backsim \Delta BDC$.

Soraw, ma’sele ha’m tapsi’rmalar

1. U’shmu’yeshliktin’ bissektrisasi’ wo’zi tu’sken ta’repte aji’ratqan kesindileri ha’m u’shmu’yeshliktin’ qalg’an ta’repleri arasi’ndag’i’ proporcionalli’qti jazi’p ko’rsetin’.

2. Tuwri’ mu’yeshli ABC u’shmu’yeshliginin’ C tuwri’ mu’yeshinen CD biyikligi ju’rgizilgen. $\angle ACD=\angle CBD$ yekenligin da’liyllen’. Paydabolg’an figuradaneshe wo'z ara uqsas u'shmu'yeshliklerin ko'rsete alasiz?

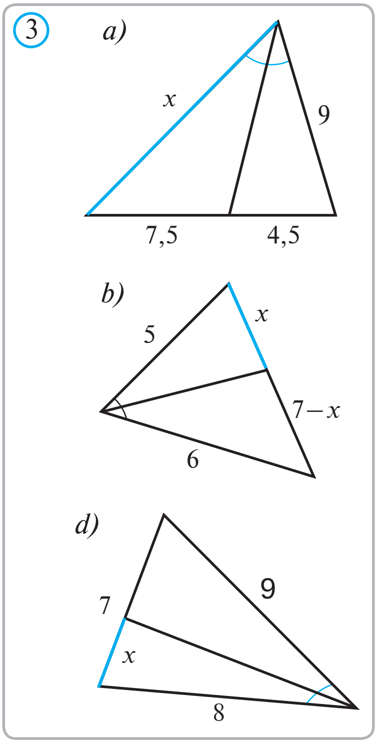
3. 3-su'wrettegi mag'liwmatlarg'a tiykarlanip x ti tabi'n'.

4. *ABC* u'shmu'yeshliginin' *AD* bissektrisasi' ju'rgizilgen. Yeger $CD = 4{,}5$ *m*; $BD = 13{,}5$ *m* ha'm *ABC* u'shmu'yeshliktin' perimetri 42 *sm* bolsa, *AB* ha'm *AC* ta'replerin tabi'n'.

5. *ABC* u'shmu'yeshliginin' medianalari' *N* noqatinda kesilisedi. Yeger *ABC* u'shmu'yeshliginin' maydani' 87 dm^2 bolsa, *ANB* u'shmu'yeshliginin' maydani'n tabi'n'.

6. *ABC* u'shmu'yeshliginin' medianalari' kesilisken *N* noqati'nan *AB* ha'm *BC* ta'replerine shekem bolg'an arali'qlar sa'ykes tu'rde 3 *dm* ha'm 4 *dm* ge ten'. Eger $AB = 8$ *dm* bolsa, *BC* ta'repin tabi'n'.

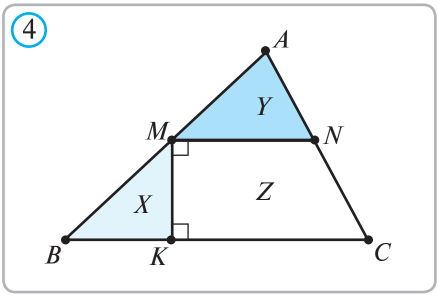
7*. Trapeciyani'n' ultani'na parallel tuwri' qaptal ta'replerinin' birewin $m : n$ qatnasta bo'liwi mu'mkin. Bul tuwri' woni'n' yekinshi qaptal ta'repin qanday qatnasta bo'ledi?



Qi'zi'qli' ma'seleler

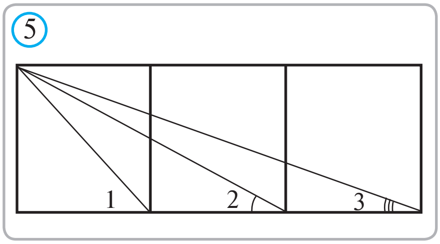
Geometriya ha'm ingliz tili. To'mendegi ingliz tilinde berilgen geometriyali'q ma'seleni sheship ko'rin'shi! Buni'n' menen ha'm ingliz tili, ha'm geometriyadan bilimin'izdi bilip alasi'z.

1) ***Dissection Puzzle:*** Let *M* be the midpoint of the side *AB* of a given triangle *ABC*. The triangle has been dissected into parts *X*, *Y*, *Z* along the lines *MN* and *MK* passing through *M* such that *MN* is parallel while *MK* is perpendicular to the base *BC* (*picture 4*). Show how the three pieces can be fitted together to make a rectangle, respectively two different parallelograms.



2) Look at the picture 5 and proof

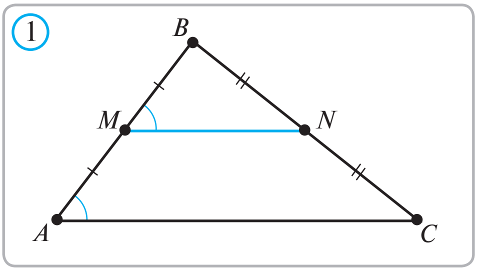
$$\angle 1 + \angle 2 + \angle 3 = 90°.$$



12 MA'SELELERDI SHESHIW

1-ma'sele. U'shmu'yeshliktin' uqsasli'g'i'nan paydalani'p, u'shmu'yeshliktin' worta sizig'i u'shmu'yeshliktin' bir ta'repine parallel ha'm usi' ta'reptin' yari'mi'na ten' ekenligin da'liyllen'.

ΔABC, MN — worta si'zi'q (1-su'wret): $MA=MB$, $NC=NB$ ⇒ $MN \parallel AC$, $MN=\frac{1}{2}AC$



Sheshiliwi. ΔABC ha'm ΔMBN ushi'n:

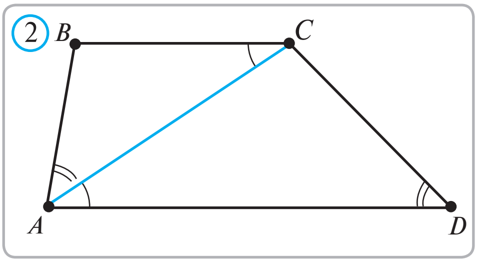
$\angle B$ — uliwma, $\frac{BM}{AB}=\frac{BN}{BC}=\frac{1}{2}$.

Soni'n' ushi'n, u'shmu'yeshliktin' uqsasli'g'inin' TMT belgisi boyi'nsha, bul yeki u'shmu'yeshlik uqsas. Yendi talqi'lawdi' usilay yetip dawam yettiremiz:

$$\Delta MBN \sim \Delta ABC \Rightarrow \begin{cases} \angle BMN = \angle A, \\ \frac{MN}{AC}=\frac{1}{2} \end{cases} \Rightarrow \begin{cases} MN \parallel AC, \\ MN=\frac{1}{2}AC. \end{cases}$$

2-ma'sele. Ultanlari BC ha'm AD bolg'an $ABCD$ trapeciyasi'ni'n' AC diagonali' oni' yeki uqsas u'shmu'yeshlikke ajiratadi. $AC^2=BC\cdot AD$ yekenligin da'liyllen'.

$ABCD$ — trapetsiya, $BC \parallel AD$, $\Delta ABC \sim \Delta DCA$ (2-su'wret) ⇒ $AC^2=BC\cdot AD$



Sheshiliwi. **1-qa'dem.** ABC ha'm ACD u'shmu'yeshliklerinin' mu'yeshlerin sali'sti'ramiz. $\angle ACB=\angle CAD$, sebebi bul mu'yeshler — ishki ayqi'sh mu'yeshler. $\angle B\neq\angle D$, sebebi $ABCD$ — trapeciya (keri jag'dayda,

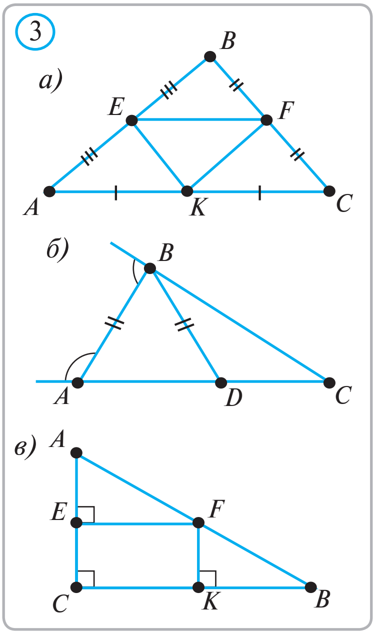
$$\angle D+\angle A=\angle B+\angle A=180°,$$

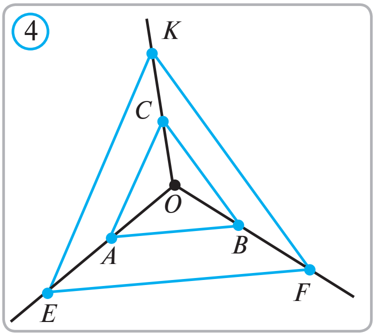
yag'ni'y $AB \parallel CD$ boli'p, $ABCD$ trapeciya bolmay qalar yedi). Solay etip, $\angle D=\angle BAC$ ha'm $\angle ACD=\angle B$.

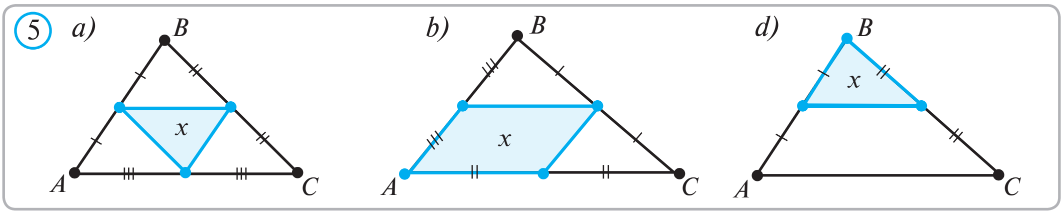
2-qa'dem. Yendi ABC ha'm DCA u'shmu'yeshliktin' sa'ykes ta'replerinin' qatnasi'n jazamiz: $\frac{AC}{BC}=\frac{AD}{AC}$, bunnan $AC^2=BC\cdot AD$.

Soraw, ma'sele ha'm tapsi'rmalar

1. a) Boyi' 170 *sm* bolg'an adam sayasi'ni'n' uzi'nli'g'i 1 *m* bolsa, biyikligi 5,4 *m* bolg'an terek sayasi'ni'n' uzi'nli'g'i'n tabi'n'.
b) Yeki ten' qaptalli u'shmu'yeshliktin' to'besindegi mu'yeshleri ten'. Birinshi u'shmu'yeshliktin' qaptal ta'repi 17 *sm*, ultani' 10 *sm* ge, yekinshi u'shmu'yeshliktin' ultani' 8 *sm* ge ten'. Yekinshi u'shmu'yeshliktin' qaptal ta'repin tabi'n'.
2. 3-su'wrettegi ha'r bir si'zi'lmadan uqsas u'shmu'yeshliklerdi ko'rsetin'.
3. *ABC* u'shmu'yeshliginin' *AP* medianasi' *BC* ta'repine parallel ha'm to'beleri *AB* ha'm *AC* ta'replerinde jatqan qa'legen kesindini ten' yekige bo'letug'i'ni'n da'liyllen'.
4. U'shmu'yeshliktin' to'beleri woni'n' worta si'zi'g'i'n wo'z ishine alg'an tuwri'dan ten'dey arali'qta jatatug'i'ni'n da'liyllen'.
5. Shen'berge ishley si'zi'lg'an *ABCD* to'rtmu'yeshliktin' diagonallari *O* noqatta kesilisedi. $\Delta AOB \backsim \Delta COD$ yekenligin da'liyllen'.
6. *ABC* u'shmu'yeshliktin' ishinen *O* noqati' ha'm *OA, OB, OC* nurlari'ndasa'ykes tu'rde *E, F, K* noqatlari ali'ng'an (*4-su'wret*). Yeger $AB \| EF$ ha'm $BC \| FK$ bolsa, *ABC* ha'm *EFK* u'shmu'yeshliklerinin' uqsas yekenligin da'liyllen'.
7*. Trapeciyani'n' diagonallari' kesilisiw noqati'nan wo'tiwshi tuwri' trapeciya ultanlari'nan birin $m:n$ qatnasta bo'ledi. Bul tuwri' yekinshi ultani'n qanday qatnasta bo'ledi?
8. Yeger *ABC* u'shmu'yeshliklerdin' maydani' *S* g'a ten' bolsa, 5-su'wrettegi *x* penen belgilengen maydandi' tabi'n'.







13 BILIMIN'IZDI SINAP KO'RIN'

I. Testler

1. 1. To'mendegi tasti'yi'qlawlardan qaysi biri duri's?

A) Yeki u'shmu'yeshliktin' mu'yeshleri sa'ykes tu'rde ten' bolsa, wolar uqsas delinedi;

B) Yeki u'shmu'yeshliktin' ta'repleri sa'ykes tu'rde ten' bolsa, wolar uqsas delinedi;

D) Yeki u'shmu'yeshliktin' sa'ykes ta'repleri proporcional ha'm sa'ykes mu'yeshleri ten' bolsa, wolar uqsas delinedi;

E) Yeki u'shmu'yeshliktin' sa'ykes ta'repleri ha'm sa'ykes mu'yeshleri ten' bolsa, wolar uqsas delinedi.

2. Yeki uqsas u'shmu'yeshliktin' maydanlari'ni'n' qatnasi' nege ten'?

A) Uqsasli'q koefficientine;

B) Wolardi'n' sa'ykes ta'replerinin' qatnasi'na;

D) Wolardin' perimetrlerinin' qatnasi'na;

E) Uqsasli'q koefficientinin' kvadrati'na.

3. To'mendegi tasti'yi'qlawdan qaysi' biri duri's?

A) U'shmu'yeshliklerdin' birinin' yeki mu'yeshi yekinshisinin' yeki mu'yeshine ten' bolsa, wolar uqsas boladi';

B) U'shmu'yeshliklerdin' birinin' yeki ta'repi yekinshisinin' yeki ta'repine ten' bolsa, wolar uqsas boladi';

D) Yeki u'shmu'yeshliktin' bir-birden mu'yeshleri ten' ha'm yeki ta'repleri proporcional bolsa, wolar uqsas boladi';

E) Yeki u'shmu'yeshliktin' bir-birden mu'yeshleri ten' ha'm bir-birden ta'repleri proportcional bolsa, wolar uqsas boladi'.

4. Duri'si'n tabi'n. Eger yeki u'shmu'yeshlik uqsas bolsa, olardi'n':

A) Biyiklikleri ten' boladi'; B) Ta'repleri proporcional boladi';
D) Ta'repleri ten' boladi'; E) Maydanlari ten' boladi.'

5.Uqsas u'shmu'yeshliklerdin' perimetrlerinin' qatnasi' nege ten'?

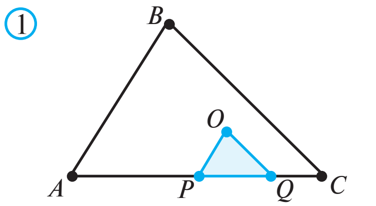
A) Sa'ykes ta'repleri qatnasi'ni'n' kvadrati'na; B) Uqsasli'q koefficientine;
D) Uqsasli'q koefficientinin' qatnasi'na; E) Maydanlari'ni'n' qatnasi'na.

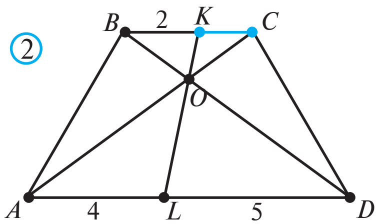
II. Ma'seleler

1. *ABC* u'shmu'yeshliginin' *AB* ha'm *AC* ta'replerinin' wortalari' sa'ykes tu'rde *E* ha'm *F* noqatlari bolsi'n. Yeger *AEF* u'shmu'yeshliginin' maydani' 3 sm^2 bolsa, *ABC* u'shmu'yeshliginin' maydani'n tabi'n'.
2. *ABC* u'shmu'yeshliginin' *AC* ta'repine parallel tuwri' *AB* ha'm *BC* ta'replerin

sa'ykes tu'rde N ha'm P noqatlarda kesip wo'tedi. Yeger $AN=4$, $NB=3$, $BP=3{,}6$ bolsa, BC ta'repin tabi'n'.

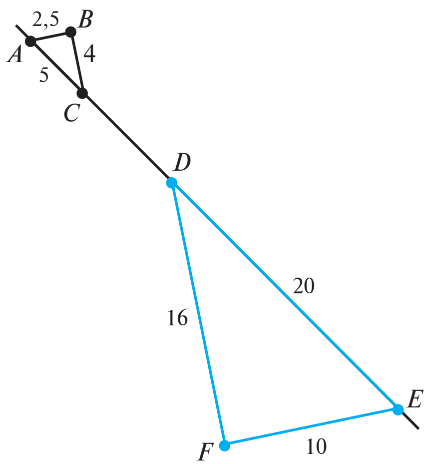
3. Su'yir mu'yeshli ABC u'shmu'yeshliginin' AB ta'repinde A noqati' ali'ng'an. Yeger $AK=3$, $BK=2$ ha'm u'shmu'yeshliktin' BD biyikligi 4 ke ten' bolsa, K noqati'nan AC kesindisine shekem bolg'an arali'qti' tabi'n'.
4. $ABCD$ parallelogrammni'n' BC ta'repinin' ortasi'ndag'i K noqati'nan ju'rgizilgen DK nuri' menen AB nuri F noqati'nda kesilisedi. Yeger $AD=4$, $DK=5$ ha'm $DC=5$ bolsa, AFD u'shmu'yeshliginin' perimetrin yesaplan'.
5. ABC u'shmu'yeshligi ishinde ali'ng'an O noqati'nan u'shmu'yeshliktin' AB ha'm BC ta'replerine parallel tuwri'lar ju'rgizilgen. Bul tuwri'lar AC ta'repin sa'ykes tu'rde P ha'm Q noqatlarda kesip wo'tedi. Yeger $PQ=2$, $AC=7$ ha'm ABC u'shmu'yeshliginin' maydani' 98 ge ten' bolsa, POQ u'shmu'yeshliginin' maydani'n ani'qlan'.
6. $ABCD$ trapesiyasi'ni'n' BC ha'm AD ultanlari'nda K ha'm L noqatlari' sa'ykes tu'rde ali'ng'an. KL kesindisi trapetsiyani'n' diagonallari' kesisken noqattan wo'tedi. Yeger $AL=4$, $LD=5$ ha'm $BK=2$ bolsa, KC kesindisin tabi'n'.(*2-su'wret*).





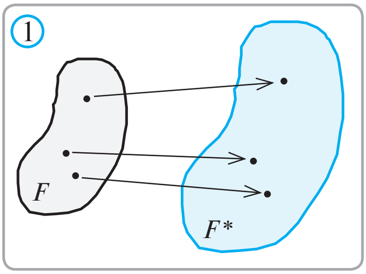
III. Wo'zin'izdi sinap ko'rin' (u'lgi ushi'n baqlaw jumi'si').

1. $ABCD$ trapeciyani'n' AC diagonali' woni' yeki uqsas ΔABC ha'm ΔACD u'shmu'yeshlikke aji'ratadi'. Bunda $BC=4$ *m*, $AD=9$ *m* bolsa, AC diagonali'ni'n' uzi'nli'g'i'n yesaplan'.
2. Yeki uqsas u'shmu'yeshliktin' maydanlari' 50 dm^2 ha'm 32 dm^2, wolardi'n' perimetrlerinin' qosi'ndi'si' 117 *dm* bolsa, ha'r bir u'shmu'yeshliktin' perimetrin tabi'n'.
3. Su'wrette su'wretlengen u'shmu'yeshliklerdin' uqsasli'g'i'n da'liyllen'. BC ha'm DF tuwri'lari'ni'n' wo'z ara jaylasi'wi' haqqi'nda ne ayta alasi'z.



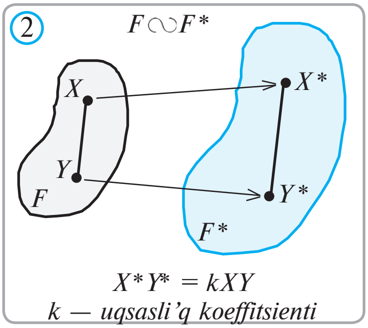
4. (*Qosi'msha*). Su'yir mu'yeshli ABC u'shmu'yeshliginin' BD ha'm AE biyiklikleri ju'rgizilgen. $DC\cdot AC=EC\cdot BC$ yekenligin da'liyllen'.

14 GEOMETRIYALIQ FIGURALARDIN' UQSASLIG'I



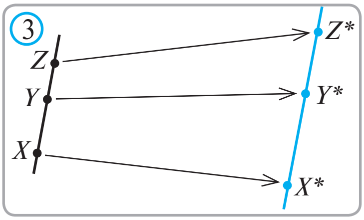
Wo'tken sabaqlarda ko'pmu'yeshliklerdin' uqsasli'g'i' tu'sinigi menen tani'sti'q. Uqsasli'q tu'sinigin tek ko'pmu'yeshlikler ushi'n yemes, ba'lki qa'legen geometriyaliq figuralar ushi'n da kiritiw mu'mkin. Yeger *F* ha'm *F** figuralari' berilgen boli'p, *F* figurasi'ni'n' ha'r bir noqati'nda *F** figurasi'ni'n' qaysi' bir noqati' sa'ykes qoyi'lg'an bolsa ha'm bunda *F** figurasi'ni'n' ha'r bir noqati' *F* figurasi'ni'n' tek bir noqati' sa'ykes kelse, ***F* figurasi *F* figurasina tu'rlendiriw** delinedi.

Ani'qlama. Eger *F* figurasi'n *F** figurasi'na tu'rlendiriwde noqatlar arasi'ndag'i' arali'qlar 0 den wo'zgeshe ani'q bir sang'a ko'beyse, bunday tu'rlendiriw **uqsasli'q tu'rlendiriw** delinedi (*2-su'wret*).



Bul ani'qlamani' to'mendegishe talqi'law mu'mkin: Aytayi'q, qandayda bir tu'rlendiriw na'tiyjesinde *F* figurasi'ni'n' qa'legen *X*, *Y* noqatlari'na *F** figurasinin' *X**, *Y** noqatlari sa'ykes qoyi'lg'an bolsi'n. Yeger $X^*Y^*=k\cdot XY$, $k>0$ bolsa, bunday tu'rlendiriwge **uqsasli'q tu'rlendiriw** delinedi. Bunda *k* — barli'q *X* ha'm *Y* noqatlari ushi'n bir qi'yli' san boli'p, ol **uqsasli'q koeffitsienti** delinedi. Yeger *F* ha'm *F** figuralari berilgen boli'p, bul figuralardin' birin ekinshisine o'tkeretug'in uqsasli'q tu'rlendiriwi bar bolsa, *F* ha'm *F** figuralari **o'z ara uqsas** delinedi. Figuralardin' uqsaslig'i $F \backsim F^*$ siyaqli jaziladi. Yeger uqsasli'q koeffitsienti *k* ni dako'rsetiw lazim bolsa, $F \backsim F^*$ tu'rinde belgilenedi. Yeger uqsasli'q tu'rlendiriwinde *X* noqatina' *X** noqati sa'ykes qoyi'lg'an bolsa, *X* noqati **X* noqatina tu'rlendi** yaki w**o'tti delinedi.** Uqsasli'q tu'rlendiriliwi to'mendegi qa'siyetlerge iye:

Teorema. **Uqsasli'q tu'rlendiriwi a) tuwri' si'zi'qti' tuwri' si'zi'qqa; b) nurdi nurg'a; d) mu'yeshti (woni'n' u'lkenligin saqlag'an halda) mu'yeshke; e) kesindini (uzi'nli'g'i' bul kesindiden *k* ma'rte uzi'n bolg'an) kesindige wo'tkeredi.**



Da'liyllew. a) Uqsasli'q koeffitsienti *k* bolg'an uqsas tu'rlendiriwde bir tuwri'da jatqan tu'rli *X*, *Y* ha'm *Z* noqatlari'na sa'ykes tu'rde *X**, *Y** ha'm *Z** noqatlarg'a tu'rlendiriw (*3-su'wret*). *X*, *Y*, *Z* noqatlari'nan biri, aytayi'q, Y qalg'an yekewinin' wortasi'nda jatsin. Wonda $XZ = XY + YZ$. Uqsasli'q tu'rlendiriwdin' ani'qlamasi' boyi'nsha:

$$X^*Z^* = k\cdot XZ = k\cdot(XY + YZ) = k\cdot XY + k\cdot YZ = X^*Y^* + Y^*Z^*.$$

Bul ten'likten X^*, Y^* ha'm Z^* noqatlari' bir tuwri' da jatatug'ini kelip shi'g'adi'. Bul teoremani'n' da'liyllewin tek: a) tasti'yi'qlaw ushi'n keltirdik. Qalg'an jag'daylarin wo'z betin'izshe da'liyllen'.

? *Soraw, ma'sele ha'm tapsi'rmalar*

1. Uqsasli'q tu'rlendiriw degen ne?

2. Qanday figuralar uqsas figuralar delinedi?

3. Yeni 3 sm, biyikligi 4 sm bolg'an tuwri' mu'yeshlikke uqsas, uqsasli'q koefficienti 2 ge ten' bolg'an to'rtmu'yeshlik jasan'.

4. 4-su'wrette mektep ha'wlisinin' sxemasi' 1:1000 masshtabta su'wretlengen wo'l-shew islerin wori'nlan'. a) ha'wlinin'; b) mektep imarati'ni'n'; d) gu'lzarlar-di'n'; e) sport maydani'ni'n'; f) bag'-di'n' haqi'yqi'y wo'lshemlerin tabi'n'.

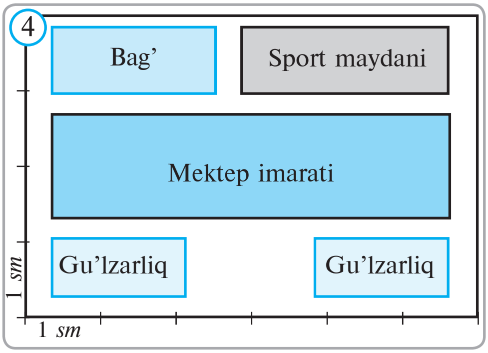
5. Yeger karta 1:50000 masshtabta su'w-retlengen bolsa, Abat ha'm Azat awi'llari' arasi'ndag'i' arali'qti tabi'n'.

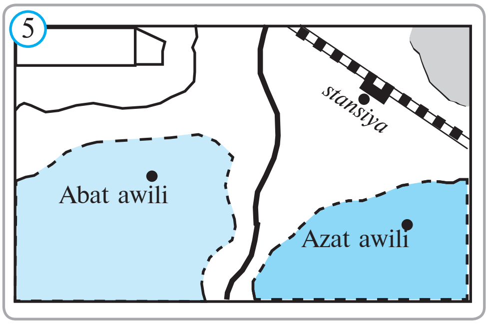
6. Uqsasli'q tu'rlendiriwde nurlar ara-si'ndag'i mu'yesh saqlanatug'i'ni'n da'liyllen'.

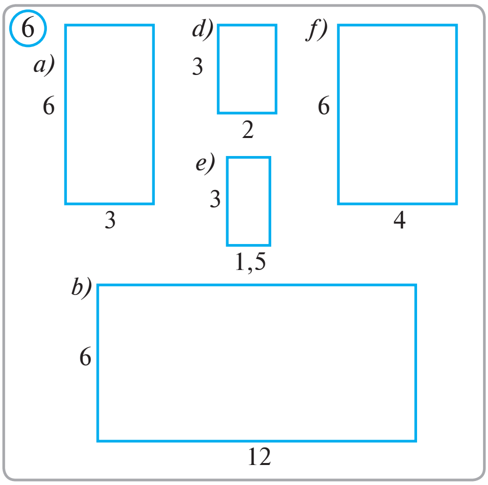
7*. Uqsasli'q tu'rlendiriwde nurlar ara-si'ndag'i mu'yesh saqlanatug'i'ni'n da'liyllen'.

8*. ABC u'shmu'yeshliginin' uqsasli'q tu'rlendiriwinde $A^*B^*C^*$ u'shmu'yesh-ligine tu'rlendiriledi. Yeger uqsasli'q koefficienti 0,6 g'a ha'm ABC u'sh-mu'yeshliginin' perimetri 12 *sm* ge ten' bolsa, $A^*B^*C^*$ u'shmu'yesh-liginin' perimetrin tabi'n'.

9. 6-su'wretten uqsas tuwri' mu'yeshlikler-din' jupli'qlari'n tabi'n' ha'm uqsasli'q koefficientlerin ani'qlan'.







15 UQSAS KO'PMU'YESHLIKLERDIN' QA'SIYETLERI

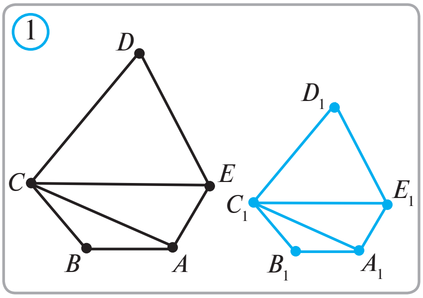
1-teorama. **Uqsas ko'pmu'yeshliklerdin' perimetrlerinin' qatnasi' uqsasli'q koefficientine ten'.**

Da'liyllew. Haqi'yqattan da, $A_1A_2...A_n$ ha'm $B_1B_2...B_n$ ko'pmu'yeshlikleri uqsas ha'm uqsasli'q koefficienti k bolsa, $B_1B_2=k\cdot A_1A_2$, $B_2B_3=k\cdot A_2A_3$, ... , $B_nB_1=k\cdot A_nA_1$ boladi.' Bunnan

$$P=B_1B_2+B_2B_3+...+B_nB_1=k\cdot A_1A_2+k\cdot A_2A_3+...+k\cdot A_nA_1=k\cdot(A_1A_2+A_2A_3+...+A_nA_1)=k\cdot P_1$$

yeken'ligin payda yetemiz. *Teorema da'liyllendi.*

2-teorema. **Uqsas ko'pmu'yeshliklerdi bir qi'yli' sandag'i uqsas u'shmu'yeshliklerge ajirati'w mu'mkin.**



Da'liyllew. Aytayi'q, $ABCDE$ ha'm $A_1B_1C_1D_1E_1$ ko'pmu'yeshlikleri uqsas boli'p, uqsasli'q koefficienti k bolsi'n.

Wo'z ara sa'ykes C ha'm C_1 to'belerinen CA, CE ha'm C_1A_1, C_1E_1 diagonallari'n ju'rgizemiz. Na'tiyjede ko'pmu'yeshlikler bir qi'yli' sandag'i' u'shmu'yeshliklerge aji'rati'ldi'. Payda bolg'an u'sh jup sa'ykes u'shmu'yeshliklerdin' uqsasli'g'i'n ko'rsetemiz.

1. $\Delta ABC \sim \Delta A_1B_1C_1$. Sebebi bul u'shmu'yeshliklerde, sha'rti boyi'nsha, $\angle B=\angle B_1$, $\dfrac{AB}{A_1B_1}=\dfrac{BC}{B_1C_1}=k$.

U'shmu'yeshliklerdin' uqsaslig'i'ni'n' TMT belgisi boyi'nsha $\Delta ABC \sim \Delta A_1B_1C_1$.

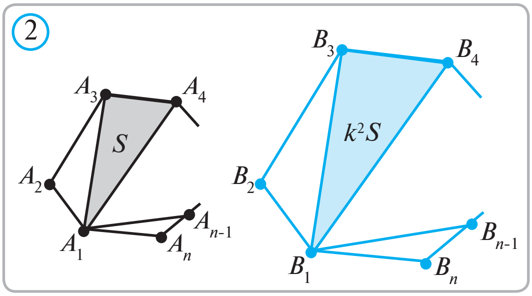
2. $\Delta CDE \sim \Delta C_1D_1E_1$. Bul uqsasli'q 1-ba'ntindegi siyaqli' da'liyllendi.

3. $\Delta ACE \sim \Delta A_1C_1E_1$. Haqi'yqattanda, $\angle CAE$ ha'm $\angle C_1A_1E_1$ mu'yeshlerin qaraymi'z. $\angle CAE=\angle BAE-\angle CAB$, $\angle C_1A_1E_1=\angle B_1A_1E_1-\angle C_1A_1B_1$.

Bul jerde, $\angle BAE=\angle B_1A_1E_1$ (berilgen uqsas besmu'yeshliklerdin' sa'ykes mu'yeshleri). $\angle CAB=\angle C_1A_1B_1$ (uqsas ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklerinin' sa'ykes mu'yeshleri). Demek, $\angle CAE=\angle C_1A_1E_1$.

AC ha'm AE ha'm de A_1C_1 ha'm A_1E_1 ta'replerin qaraymi'z: $AC=kA_1C_1$, sebebi wolar wo'z ara uqsas ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklerinin' sa'ykes ta'repleri, $AE=kA_1E_1$, sebebi wolarda berilgen uqsas besmu'yeshliklerinin' sa'ykes ta'repleri. Demek, u'shmu'yeshliklerdin' uqsaslig'i'ni'n' TMT belgisi boyi'nsha $\Delta ACE \sim \Delta A_1C_1E_1$. Qa'legen uqsas ko'pmu'yeshlikler ushi'n da usi siyaqli' talqi'lawlar paydali' boli'wi' ani'q. *Teorema da'liyllendi.*

3-teorema. **Uqsas ko'pmu'yeshliklerdin' maydanlari'ni'n' qatnasi' uqsasli'q koefficientinin' kvadrati'na ten'.**



Da'liyllew. Aytayi'q, $A_1A_2...A_n$ ha'm $B_1B_2...B_n$ ko'pmu'yeshlikleri uqsas ha'm k — uqsasli'q koefficienti bolsi'n. Wonda $A_1A_2A_3$, $A_1A_3A_4$, ..., $A_1A_{n-1}A_n$ u'shmu'yeshlikleri sa'ykes tu'rde $B_1B_2B_3$, $B_1B_3B_4$, ..., $B_1B_{n-1}B_n$ u'shmu'yeshliklerine uqsas boli'p, uqsas u'shmu'yeshlerinin' maydanlari'ni'n' qatnasi' k^2 qaten' boladi' (*2-su'wret*).

$S_{A_1A_2A_3}=k^2S_{B_1B_2B_3}$, $S_{A_1A_3A_4}=k^2S_{B_1B_3B_4}$, ... , $S_{A_1A_{n-1}A_n}=k^2S_{B_1B_{n-1}B_n}$. Bul ten'liklerdin' sa'ykes bo'limlerin qossaq, $S_{A_1A_2...A_n}=k^2\,S_{B_1B_2...B_n}$ boladi.' ***Teorema da'liyllendi.***

Ma'sele. Perimetrleri 18 *sm* ha'm 24 *sm* bolg'an yeki uqsas ko'pmu'yeshlik maydanlari'ni'n' qatnasi'n tabi'n'.

Sheshiliwi. 1) Uqsas ko'pmu'yeshlikler perimetrlerinin' qatnasi' uqsasli'q koefficientine ten' yekenliginen paydalani'p, $k=24:18=4:3$ yekenligin tabami'z.

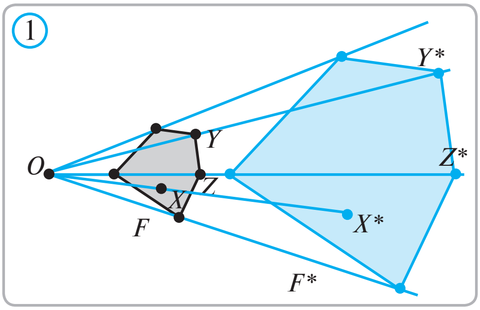
2) Uqsas ko'pmu'yeshliklerdin' maydanlari'ni'n' qatnasi uqsasli'q koefficientinin' kvadrati'na ten' bolg'ani' ushi'n izlengen qatnas $k^2=\frac{16}{9}$ g'aten'. ***Juwabi:*** $\frac{16}{9}$.

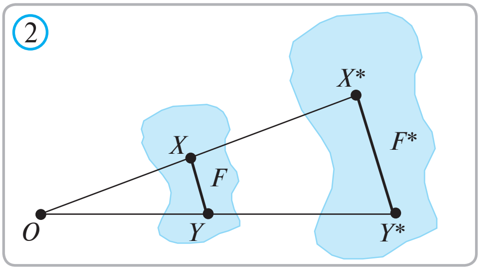
Soraw, ma'sele ha'm tapsi'rmalar

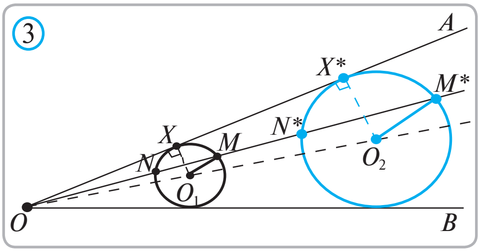
1. Uqsas ko'pmu'yeshliklerdin' perimetrlerinin' qatnasi' nege ten'?
2. Uqsas ko'pmu'yeshliklerdin' maydanlari'ni'n' qatnasi' haqqi'ndag'i' teoremani' tu'sindirin'.
3. U'shmu'yeshlik penen to'rtmu'yeshlik uqsas boli'wi' mu'mkin be?
4. Maydanlari' 6 m^2 ha'm 24m^2 bolg'an yeki to'rtmu'yeshlik uqsas. Uqsasli'q koefficientin tabi'n'.
5. Yeki ko'pmu'yeshliktin' perimetrleri 18 sm ha'm 36 *sm* ge, maydanlari'ni'n' qosi'ndi'si' bolsa 30 sm^2 qaten'. Ko'pmu'yeshliklerdin' maydanlari'n tabi'n'.
6. Perimetri 84 *sm* bolg'an u'shmu'yeshliktin' bir ta'repine parallel yetip ju'rgizilgen tuwri', wonnan perimetri 42 *sm* ge ha'm maydani' 26 sm^2 qaten' u'shmu'yeshlik ajiratadi. Berilgen u'shmu'yeshliktin' maydani'n tabi'n'.
7. *O* noqatina salistirg'anda simmetriyali figuralar uqsas bolama? Ko'sherge salistirg'anda simmetriyali' figuralar she? Wolardi'n' uqsasli'q koefficientin tabi'n?
8. To'rtmu'yeshlik formasindag'i paxta atizi kartada maydani' 12 sm^2 bolg'an u'shmu'yeshlik penen ko'rsetiledi. Yeger karta masshtabi 1:1000 bolsa, atizdin' maydani'n yesaplan'.

9*. Maydanlari' 8 sm^2 ha'm 32 sm^2 bolg'an yeki uqsas u'shmu'yeshlik perimetrlerinin' qosi'ndi'si' 48 *sm* ge ten'. U'shmu'yeshliklerdin' perimetrlerin tabi'n'.

GOMOTETIYA HA'M UQSASLI'Q







Yen' a'piwayi uqsas tu'rlendiriwlerden biri gomotetiya boladi'. Aytayi'q, F — figura, O— noqat ha'm k — on' sani' berilgen bolsi'n. F figurasi'ni'n' qa'legen X noqati' arqali' OX nuri'n ju'rgizemiz ha'm bul nurda uzi'nli'g'i' $k \cdot OX$ bolg'an OX^* kesindisin qoyamiz (*1-su'wret*). Bul usi'l menen F figurasi'ni'n' ha'r bir X noqati'na X^* noqati'n sa'ykes qoyatug'in tu'rlendiriw **gomotetiya** delinedi. Bunda, O noqati' gomotetiya worayi, k sani gomotetiya koefficenti, F ha'm gomotetiya na'tijesinde F figura almasatug'in F^* figuralar bolsa **gomotetiyaliq figuralar** delinedi.

Teorema. **Gomotetiya uqsasli'q tu'rlendiriwi boladi.**

Da'liyllew. Erkli O worayi'na iye, k koefficientli gomotetiyada F figurani'n' X ha'm Y noqatlari' X^* ha'm Y^* noqatlari'na wo'tsin (2-su'wret). Wonda, gomotetiya ani'qlamasi' boyi'nsha, XOY ha'm X^*OY^* u'shmu'yeshliklerinde $\angle O$ — uli'wma ha'm $\frac{OX^*}{OX}=\frac{OY^*}{OY}=k$ boladi.' Demek, XOY ha'm X^*OY^* u'shmu'yeshlikleri yeki ta'repi ha'm wolar arasi'ndag'i' mu'yeshi boyi'nsha uqsas. Soni‚n' ushi'n $\frac{X^*Y^*}{XY}=\frac{OX^*}{OX}=k$, sonli'qtan, $X^*Y^*=k \cdot XY$. ***Teorema da'liyllendi.***

Ma'sele. AOB mu'yeshinin' ta'replerine uriniwshi qa'legen yeki shen'ber gomotetiyaboli'wi'n ha'm O noqati bul gomotetiyaushi'n oray yekenligin da'liyllen'.

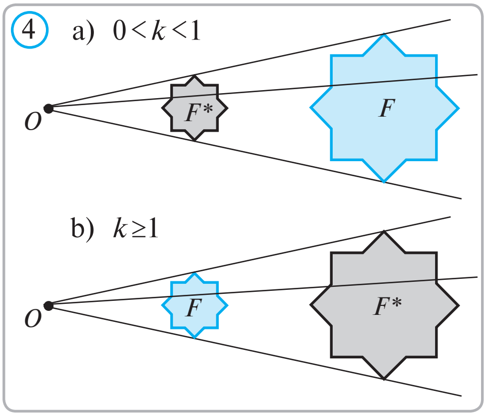
Sheshiliwi. Woraylari O_1 ha'm O_2 bolg'an shen'berler AOB mu'yeshinin' ta'replerine uri'nsi'n (*3-su'wret*). Bul shen'berlerdin' gomotetiyali'q ekenligin da'lilleymiz.

Shen'berler OA nurina sa'ykes tu'rde X ha'm X^* noqatlari'nda uri'ng'an bolsi'n (*3-su'wret).* Wonda, $\Delta OXO_1 \sim \Delta OX^*O_2$ (sebebi $\angle XOO_1=\angle X^*OO_2$ ha'm $\angle OXO_1=\angle OX^*O_2=90°$). Bunnan $\frac{OX^*}{OX}=\frac{OO_2}{OO_1}$.

Won' ta'reptegi qatnasti' k menen belgileymiz ha'm koefficienti $k=\frac{OO_2}{OO_1}$ worayi O bolg'an gomotetiyani' qaraymi'z. Aytayi'q, bul gomotetiyada O_1 worayi'na iye shen'berdin' qa'legen M noqati' M^* noqatina tu'rlendirilgen bolsi'n Wonda

$$O_2M^*=k.O_1M \text{ yaki } O_2M^*=\frac{O_2X^*}{O_1X}\cdot O_1M.$$

Bunnan, $O_1X=O_1M$ bolg'ani' ushi'n, $O_2M^*=O_2X^*$ ten'ligin payda yetemiz. Bul M^* noqati' orayi O_2 noqati'nda bolg'an radiusi' O_2X^* g'a ten' bolg'an shen'berde jatatug'i'ni'n bildiredi. Demek, qarali'p atirg'an shen'berler wo'z ara gomotetiyaliq yeken.



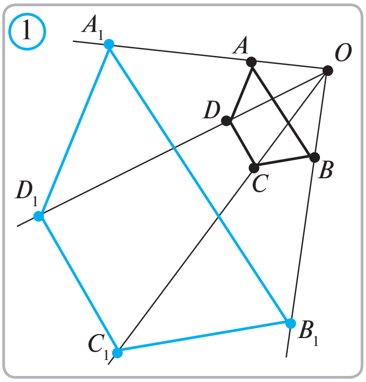
Jedellestiriwshi tapsi'rma

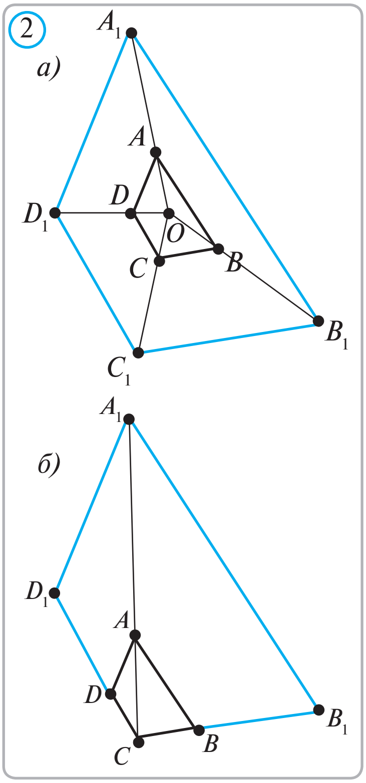
4-su'wrette gomotetiya koefficienti a) $0<k<1$; b) $k\geq 1$ bolg'an gomotetiyali'q figuralar su'wretlengen. Gomotetiya koefficientinin' shamasi'na qarap gomotetiyali'q figuralardi'n' "qi'si'li'wi'" yaki "sozi'li'wi'" haqqi'nda qanday juwmaq shi'g'ari'w mu'mkin?

Soraw, ma'sele ha'm tapsi'rmalar

1. Gomotetiya degen ne? Gomotetiya worayi', koefficienti she?
2. Gomotetiya uqsasli'q tu'rlendiriw yekenligin da'liyllen'.
3. U'shmu'yeshlik si'zi'n': a) U'shmu'yeshliktin' ishinde; b) U'shmu'yeshliktin' si'rti'nan O noqatin belgilen' ha'm koefficienti 2 ge ten' bolg'an O worayi'na iye gomotetiyani' qarap shi'g'i'p, berilgen u'shmu'yeshlikke gomotetiyali'q u'shmu'yeshlik jasan'.
4. Perimetrleri 18 *sm* ha'm 27 *sm* bolg'an yeki romb wo'z ara gomotetiyali'q boladi'. Bul rombi'lardi'n' ta'replerinin' ha'm maydanlari'ni'n' qatnaslari'n tabi'n'.
5. Gomotetiyada X noqati' X^* noqati'na, Y noqati' Y^* noqati'na wo'tedi. Yeger X, X^*, Y, Y^* noqatlari' bir tuwri'da jatpasa, bul gomotetiyani'n' worayin tabi'n'.
6. Koefficienti 2 ge ten' bolg'an gomotetiyada X noqati' X^* noqati'na wo'tetug'i'ni' belgili. Bul gomotetiyani'n' worayi'n jasan'.
7. Shen'berge gomotetiyali'q figura shen'ber bolatug'i'ni'n da'liyllen'.
8. Shen'ber si'zi'n'. Worayi' shen'ber worayi'nda ha'm koefficienti a) $\frac{1}{2}$; b) 2; d)3; e) $\frac{1}{3}$ ge ten' bolg'an gomotetiyada si'zi'lg'an shen'berge gomotetiyali'q bolg'an figuralardi' jasan'?
9. Mu'yesh ha'm woni'n' ishinde A noqati' berilgen. Mu'yesh ta'replerine uri'ni'wshi', A noqati'nan wo'tiwshi shen'ber jasan'.

17 UQSAS KO'PMU'YESHLIKLERDI JASAW





Usi waqitqa shekem teoremalardi' da'liyllewde ha'm ma'selelerdi sheshiwde tu'rli uqsas u'shmu'yeshliklerdi jasap keldik. Uqsas ko'pmu'yeshlikler qanday jasaladi'? To'mende soni'n' menen tani'sami'z.

Ma'sele. Berilgen $ABCD$ to'rtmu'yeshlikke uqsas, uqsasli'q koefficienti 3 ke ten' bolg'an $A_1B_1C_1D_1$ to'rtmu'yeshligin jasan' (*1-su'wret*).

Jasaliwi. Tegislikte qa'legen O noqati'n alami'z. Wonnan ha'm to'rtmu'yeshliktin' to'belerinen wo'tiwshi OA, OB, OC ha'm OD nurlari'n ju'rgizemiz. Bul nurlarda O noqati'nan shi'g'atug'i'n $OA_1=3OA$, $OB_1=3OB$, $OC_1=3OC$ ha'm $OD_1=3OD$ kesindilerin qoyami'z. Payda bolg'an $A_1B_1C_1D_1$ to'rtmu'yeshligi izlenip ati'rg'an to'rtmu'yeshlik boladi'.

Tiykarlaw. $ABCD \backsim A_1B_1C_1D_1$ yekenligin da'lilleymiz.

1. *Sa'ykes ta'replerinin' proportcionalli'g'i'.*

a) $\Delta AOD \backsim \Delta A_1OD_1 \Rightarrow \frac{A_1D_1}{AD} = \frac{O_1D_1}{OD} = \frac{OA_1}{OA} = 3;$ (1)

b) $\Delta DOC \backsim \Delta D_1OC_1 \Rightarrow \frac{OD_1}{OD} = \frac{D_1C_1}{DC} = \frac{OC_1}{OC} = 3.$ (2)

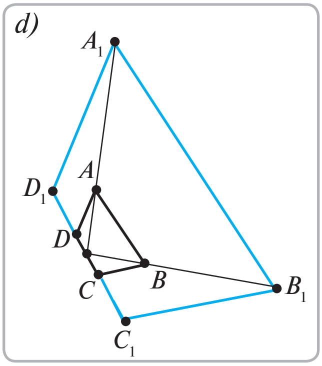
(1) ha'm (2) ten'likten $\frac{A_1D_1}{AD} = \frac{D_1C_1}{DC}$ yekenligi kelip shi'g'adi'. Da'l usi'g'an uqsas to'rtmu'yeshliklerdin' basqa sa'ykes ta'replerinin' proporcionalli'g'i'n da'liyllew mu'mkin.

2. *Sa'ykes mu'yeshlerdin' ten'ligi.*

Uqsas u'shmu'yeshliklerdin' sa'ykes mu'yeshleri ten' bolg'ani' ushi'n $\angle A_1D_1O=\angle ADO$, $\angle C_1D_1O=\angle CDO$. Wonda $\angle A_1D_1C_1=\angle A_1D_1O+\angle C_1D_1O=\angle ADO+$ $+\angle CDO=\angle ADC$, yag'ni'y to'rtmu'yeshliklerdin' sa'ykes $A_1D_1C_1$ ha'm ADC mu'yeshleri wo'z ara ten'. Da'l usi'g'an uqsas to'rtmu'yeshliklerdin' basqa sa'ykes mu'yeshlerinin' ten' yekenligi da'liyllenedi. Demek, $ABCD$ ha'm $A_1B_1C_1D_1$ to'rtmu'yeshlikleri uqsas yeken.

Ta'repleri qa'legen sanda bolg'an ko'pmu'yeshlikke uqsas ko'pmu'yeshlikte da'l usi' si'yaqli' jasaladi'.

Gomotetiya worayi'n bul ma'selede to'rtmu'yeshliktin' si'rti'nan tan'lap aldi'q. Uli'wma alg'anda gomotetiya worayi'n to'rtmu'yeshliktin' ishki oblasti'nda (*2, a-su'wret*), qaysi' bir to'besinde *(2-b su'wret)* yaki qaysi bir ta'repinde *(2-d su'wret)* jatatug'i'nday yetip tan'lap ali'wi'mi'zg'a da bolatug'i'n yedi. Gomotetiya worayi'n qay jerden almayi'q, berilgen *ABCD* to'rtmu'yeshlikke uqsas ha'm uqsasli'q koefficienti 3 ke ten' bolg'an to'rtmu'yeshlikler wo'z ara ten' boladi'.

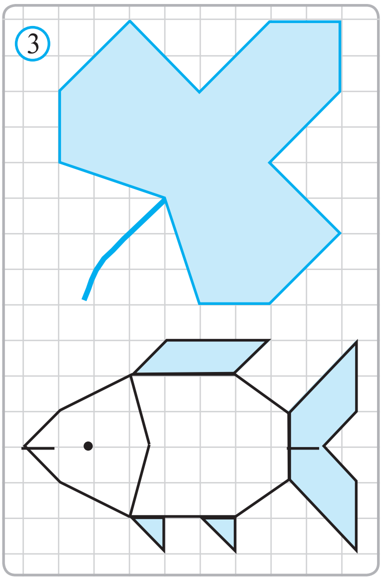


? Soraw, ma'sele ha'm tapsi'rmalar

1. Berilgen ko'pmu'yeshlikke uqsas ko'pmu'yeshlikti jasaw izbe-izligin ayti'p berin'.

2. Da'pterin'izge qanday da bir *ABCDE* besmu'yeshligin si'zi'n'. Gomotetiya ja'rdeminde bul besmu'yeshlikke uqsas, uqsasli'q koefficienti 0,5 ke ten' bolg'an besmu'yeshlik jasan'. Gomotetiya worayi' a) *C* noqati'nda; b) besmu'yeshliktin' ishinde; c) *AB* ta'repinde bolg'an jag'daylardi' wo'z aldi'na ko'rip shi'g'i'n'.

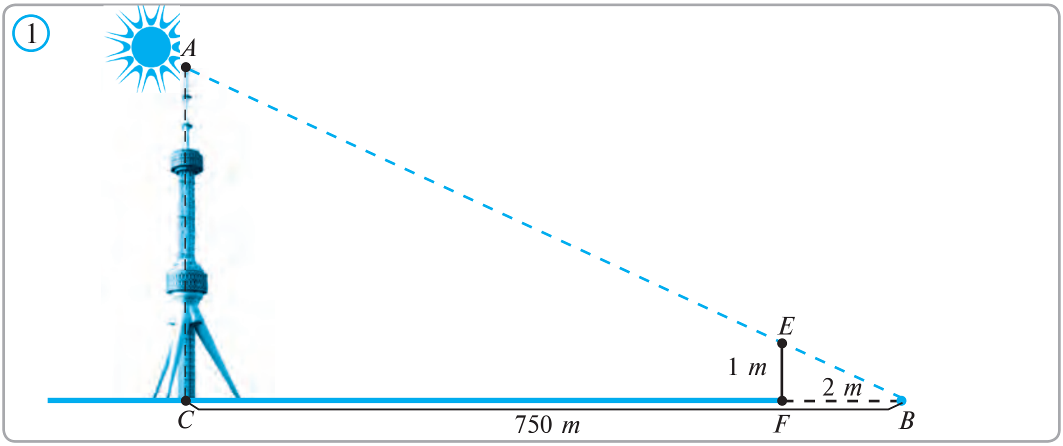
3. Keteklerdi yesapqa alg'an halda 3-su'wrette berilgen figuralardi' da'pterin'izge si'zi'n': a) japi'raqqa uqsasli'q koefficienti 3 ke ten' bolg'an japi'raq; b) bali'qshag'a uqsasli'q koefficienti 0,8 ge ten' bolg'an baliqshani' si'zi'n'.



4. F_1 ko'pmu'yeshligi F_2 ko'pmu'yeshligine uqsas, k—uqsasli'q koefficienti. P_1, P_2, S_1, S_2 ha'ripleri menen sa'ykes tu'rde bul ko'pmu'yeshliklerdin' perimetrleri ha'm maydanlari' belgilengen. To'mendegi kesteni da'pterin'izge ko'shirin' ha'm woni' tolti'ri'n'.

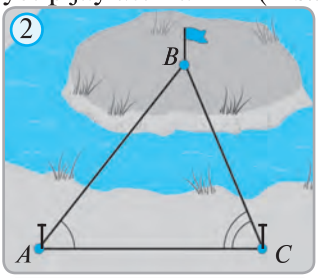
	P_1	P_2	S_1	S_2	k
a)	84		100	25	
b)	14	28		48	
d)		150	200	100	
e)		30	24		3

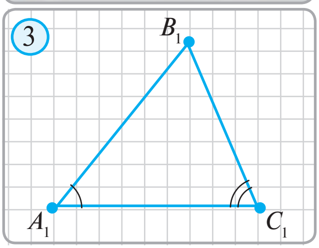
18 A'MELIY SHINIG'IW



1. Biyiklikti ani'qlaw.

Jerde turi'p, Tashkent teleminarasi'ni'n' biyikligin tabayi'q. Minarani'n' ushi' — A noqati'ni'n' sayasi' B noqati'nda bolsi'n. EF tayag'i'n vertikal hali'nda sonday yetip jaylasti'rami'z (*1-su'wret*), bunda tayaqtin' E ushi'ni'n' sayasi' da B noqati'nda bolsi'n. A noqati'ni'n' jerdegi proekciyasi'n C menen belgileymiz. Wonda, tuwri' mu'yeshli ABC ha'm EBF u'shmu'yeshlikleri wo'z - ara uqsas boladi. Soni'n' ushi'n





$$\frac{AC}{EF}=\frac{BC}{BF} \quad \text{yaki} \quad AC=\frac{BC\cdot EF}{BF}.$$

BC, BF arali'qlarin ha'm EF tayaqtin' uzi'nli'g'i'n wo'lshep, payda bolg'an formuladan teleminara biyikligi — AC kesindisinin' uzi'nli'g'i'n tabami'z. Mi'sali', yeger $EF=1$ *m*, $BC=750$ *m*, $FB=2$ *m* bolsa, wonda $AC=375$ *m* boladi'.

2. Bari'p bolmaytug'in jerge shekemgi bolg'an arali'qti' wo'lshew.

Aytayi'q, A noqati'nan bari'w mu'mkin bolmag'an B noqati'na shekemgi bolg'an arali'qti' ani'qlaw kerek bolsi'n (*2-su'wret*). A noqati'nan bari'wg'a bolatug'i'n jerge C noqati'n belgileymiz. Bunda C noqati'nan qarag'anda A ha'm B noqatlari' ko'rinip tursi'n ja'nede AC arali'qti' wo'lshep ali'w mu'mkin bolsi'n.

A'sbaplar ja'rdeminde BAC ha'm ACB mu'yeshlerin wo'lsheymiz. Aytayi'q, $\angle BAC=\alpha$ ha'm $\angle ACB=\beta$ bolsi'n. Qag'azg'a $\angle A_1=\alpha$, $\angle C_1=\beta$ bolg'an $A_1B_1C_1$

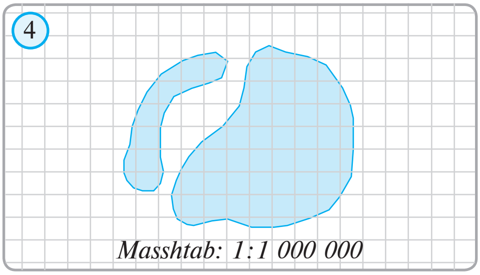
u'shmu'yeshligin jasaymi'z. Bunda ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklerinin' yeki mu'yeshi boyi'nsha uqsas boladi' (*2-ha'm 3-su'wretler*). Bunnan,

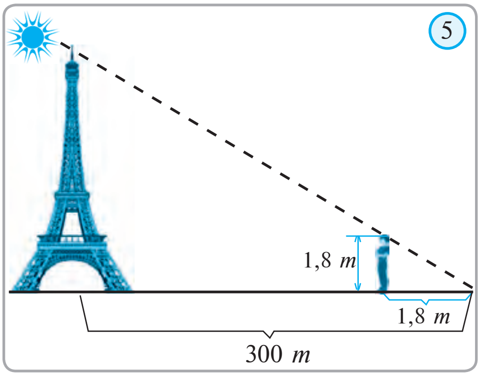
$$\frac{AB}{A_1B_1} = \frac{AC}{A_1C_1} \quad \text{yaki} \quad AB = \frac{AC \cdot A_1B_1}{A_1C_1}.$$

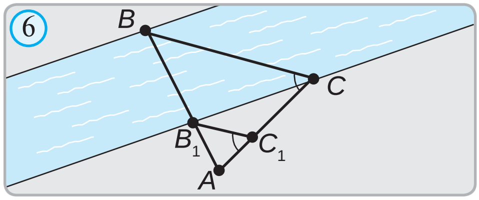
AC arali'g'i'n ha'm A_1B_1, A_1C_1 kesindilerin wo'lshep, na'tiyjede payda bolg'an formula ja'rdeminde AB kesindisi yesaplanadi'. Yesaplaw jollari'n an'satlasti'ri'w ushi'n $AC{:}A_1C_1$ qatnasi'n 100:1, 1000:1 siyaqli' qatnasta ali'w mu'mkin. Mi'sali', $\angle A=73°$, $\angle C=58°$ bolsa, qag'azda $A_1B_1C_1$ u'shmu'yeshligin $\angle A_1=73°$, $\angle C_1=58°$, $A_1C_1=130$ *mm* yetip si'zami'z. A_1B_1 kesindisin wo'lshep, woni 153 *mm* yekenligin tabami'z. Demek, izlenip ati'rg'an arali'q 153 *m* boladi'.

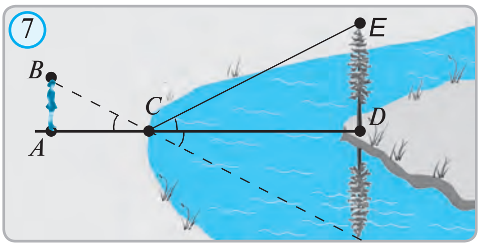
3. Aral ten'izi haqqinda a'meliy jumis.

4-su'wrette Aral ten'izinin' kosmik kemesinen ali'ng'an su'wreti ko'rsetilgen. Wol tiykari'nda wo'lshew ha'm yesaplawlardi' wori'nlap, suw saqlanip qalg'an maydani'ni'n' juwi'q ma'nisin tabi'n.'







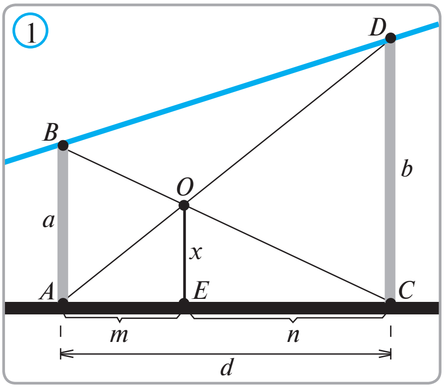


? *Soraw, ma'sele ha'm tapsi'rmalar*

1. Yeger boyi' 1,7 *m* bolg'an adam sayasi'ni'n' uzi'nli'g'i' 2,5 *m* bolsa, sayasinin' uzinlig'i 10,2 *m* bolg'an terektin' biyikligi qansha boladi'?
2. 5-su'wrette su'wretlengen minaralardin' biyikligin ani'qlan'.
3. 6-su'wrettegi yeki uqsas AB_1C_1 ha'm ABC u'shmu'yeshliklerdin' ja'rdeminde da'ryani'n' ken'ligin (yenin) ani'qlaw kerek. Yeger $AC=100m$, $AC_1=32$ *m* ha'm $AB_1=34$ *m* bolsa, da'ryani'n' yenin (BB_1) tabi'n'.
4. Jap jag'asi'ndag'i' DE tereginin' suwdag'i sa'wleleniwi A noqati'ndag'i' adamg'a ko'rinip tur. Yeger $AB=165$ *sm*, $AC=120$ *sm*, $CD=4,8$ *m* bolsa, terektin' biyikligin tabi'n (*7-su'wret*).
5. U'yin'izdin' jani'ndag'i bir terekti tan'lan' ha'm woni'n' biyikligin ani'qlan'. Bul jumisti' qalay ori'nlag'ani'n'i'z haqqi'nda yesabat tayarlan'.

MA'SELELERDI SHESHIW

1-ma'sele. Uzi'nli'qlari' sa'ykes tu'rde a ha'm b bolg'an AB ha'm CD bo'reneler bir-birinen d arali'qta vertikal ta'rizde wornati'lg'an. Wolardin' bekkemligin asi'ri'w ushi'n A ha'm D, B ha'm C ushlari'n O noqati'nda kesilisiwshi polat si'mlar menen bekkemlengen (*1-su'wret*). Su'wrette berilgen mag'liwmatlar boyi'nsha a) $\frac{m}{m+n}=\frac{x}{b}$ ha'm $\frac{n}{m+n}=\frac{x}{a}$ ten'liklerin da'liyllen'; b) $\frac{x}{a}+\frac{x}{b}=1$ ten'liktin' duri's yekenligin ko'rsetin' ha'm tu'sindirin'.



Sheshiliwi.

a) Ma'selenin' sha'rti boyi'nsha

1. $AOE \backsim ADC$. Sonin' ushi'n,

$$\frac{AE}{AC}=\frac{OE}{DC}, \quad \text{yag'ni'y} \quad \frac{m}{m+n}=\frac{x}{b}. \qquad (1)$$

2. $\Delta EOC \backsim \Delta ABC$. Sonin' ushi'n,

$$\frac{EC}{AC}=\frac{OE}{AB}, \quad \text{yag'ni'y} \quad \frac{n}{m+n}=\frac{x}{a}. \qquad (2)$$

b) (1) ha'm (2) ten'liklerdi ag'zama-ag'za qossaq, $\frac{m}{m+n}+\frac{n}{m+n}=\frac{x}{b}+\frac{x}{a}$ yaki $\frac{x}{a}+\frac{x}{b}=1$ ten'ligin payda yetemiz. Demek, bag'analar qanday wornatilmasin, polat simlar kesisken O noqat jerden birdey biyiklikte boladi yeken.

2-ma'sele. $ABCD$ trapeciyani'n' AB ha'm CD qaptal ta'replerinde M ha'm N noqatlari' belgilengen. Bunda MN kesindisi trapeciyanin' ultanlari'na parallel ha'm trapeciyanin' diagonallari' kesisken O noqati' arqali' wo'tedi. Yeger $BC=a$, $AD=b$ bolsa, a) MO; b) ON; d) MN kesindilerin tabi'n (*2-su'wret*).

Sheshiliwi. 1) AOD ha'm BOC u'shmu'yeshliklerdin' BB belgisi boyi'nsha uqsas, sebebi $\angle BOC=\angle AOD$, $\angle OBC=\angle ADO$. Bunnan,

$$\frac{OC}{OA}=\frac{BC}{AD} \quad \text{yaki} \quad \frac{OC}{OA}=\frac{a}{b}. \qquad (1)$$

2) ABC ha'm AOM u'shmu'yeshlikleri de BB belgisi boyi'nsha uqsas, sebebi $\angle AMO=\angle ABC$, $\angle ACB=\angle AOM$. Bunnan,

$$\frac{AC}{OA}=\frac{BC}{MO} \quad \text{yaki} \quad \frac{OA+OC}{OA}=\frac{a}{MO} \quad \Rightarrow \quad 1+\frac{OC}{OA}=\frac{a}{MO}, \quad \frac{OC}{OA}=\frac{a}{MO}-1. \qquad (2)$$

3) (1) ha'm (2) ten'liklerinin' won' bo'limin ten'lestirip, $\frac{a}{MO}-1=\frac{a}{b}$

ten'ligin ha'm wonnan

$$MO = \frac{ab}{a+b} \qquad (3)$$

yekenligin tabami'z.

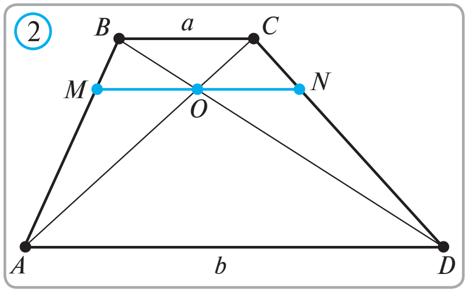
4) Joqaridag'iday jol tutip

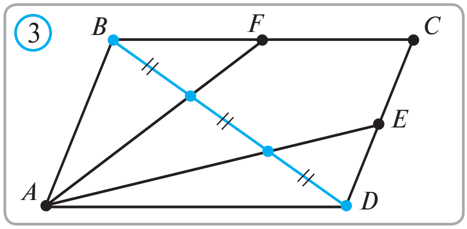
$$ON = \frac{ab}{a+b} \qquad (4)$$

ten'ligin, keyin bolsa (3) ha'm (4) ten'liklerinin' sa'ykes bo'limlerin qosi'p

$$MN = \frac{2ab}{a+b}$$

ten'ligin payda yetemiz.





Juwabi: a) $\frac{ab}{a+b}$; b) $\frac{ab}{a+b}$; d) $\frac{2ab}{a+b}$.

Yesletpe. *Bul ma'selenin' sheshiminen $MO = ON$ yekenligi kelip shi'g'adi'.*

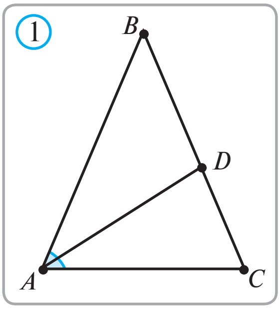
? *Soraw, ma'sele ha'm tapsi'rmalar*

1. ABC u'shmu'yeshliginin' AB ha'm BC qaptal ta'replerinde D ha'm E noqatlari' belgilengen. Yeger $AC \| DE$, $AC=6$, $DB=3$ ha'm $DE=2$ bolsa, AB ta'repin tabi'n'.
2. Yeki uqsas u'shmu'yeshliktin maydanlari' 8 dm^2 ha'm 72 dm^2 qa ten',wolardan birinin' perimetri yekinshisinen 26 dm ge kem. U'lken ko'pmu'yeshliginin' perimetrin tabi'n'.
3. Perimetri 1 m bolg'an $A_1B_1C_1$ u'shmu'yeshligi $A_2B_2C_2$ u'shmu'yeshliginin' ta'replerinin' wortalari'n, $A_2B_2C_2$ u'shmu'yeshlik $A_3B_3C_3$ u'shmu'yeshliginin' wortalari'n, $A_3B_3C_3$ u'shmu'yeshligi $A_4B_4C_4$ u'shmu'yeshliginin' ta'replerinin' wortalari'n tutasti'ri'wdan payda bolg'an bolsa, $A_4B_4C_4$ u'shmu'yeshliginin' perimetri qansha boladi'?
4. Yeki uqsas u'shmu'yeshliktin' perimetrleri 18 dm ha'm 36 dm ge , maydanlari'ni'n' qosi'ndi'si' 30 dm^2 qa ten'. U'lken u'shmu'yeshliktin' maydani'n tabi'n'.
5. Rombi'ni'n' ta'replerinin' wortalari' tuwri'mu'yeshliktin' to'beleri bolatug'i'ni'n da'liyllen'.
6. ABC u'shmu'yeshligin jasan'.Bul u'shmu'yeshlikke uqsas ha'm maydani' ABC u'shmu'yeshliginin' maydani'nan 9 ma'rte kishi bolg'an $A_1B_1C_1$ u'shmu'yeshlikti jasan'.

7*. E ha'm F noqatlari sa'ykes tu'rde $ABCD$ parallelogrammni'n' CD ha'm BC ta'replerinin' wortalari', AF ha'm AE tuwri'lari' BD diagonali'n ten'dey u'sh bo'lekke bo'letug'i'ni'n da'liyllen' (*3-su'wret*).

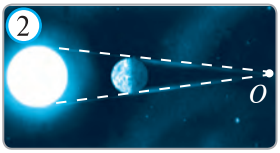
20 MA'SELELERDI SHESHIW

1. Ten' qaptalli u'shmu'yeshliktin' ultani'ndag'i' mu'yeshtin' bissektrisasi' bul u'shmu'yeshlikten wo'zine uqsas u'shmu'yeshlik ajiratadi'. U'shmu'yeshliktin' mu'yeshlerin ani'qlan' (*1-su'wret*, $AB=BC$, $\Delta ABC \sim \Delta CAD$).



2. Shen'ber jasan' ha'm wonnan *O* noqati'n belgilen'. Orayi' *O* noqatinda ha'm koeffitcienti 2 ge ten' bolg'an gomotetiyada berilgen shen'berge gomotetiyaliq bolg'an shen'ber jasan'.

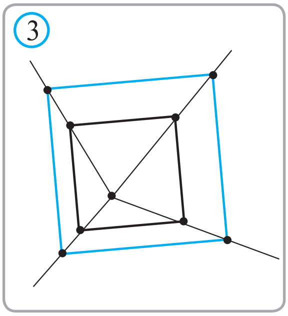
3. Yeki uqsas ko'pmu'yeshliktin' perimetrlerinin' qatnasi' 2:3 siyaqli' bolsi'n. U'lken ko'pmu'yeshliktin' maydani' 27, kishi ko'pmu'yeshliktin' maydani'n tabi'n'.



4. 2-su'wrette Quyashti'n' toli'q tuti'lg'an jag'dayi' su'wretlengen. Yeger Quyash radiusi' 686 784 *km*, Ay radiusi' 1760 *km* ha'm Jerden Ayg'a shekem bolg'an arali'q 384 400 *km* bolsa, Jerden Quyashqa shekem bolg'an arali'qti' tabi'n'.

5. a) Bir shen'berge yeki uqsas ko'pmu'yeshlik ishley si'zi'lg'an. Bul ko'pmu'yeshlikler wo'z ara ten' bolama?

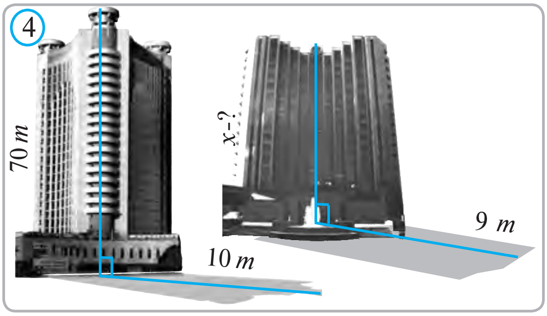
b) Bir shen'berge yeki uqsas ko'pmu'yeshlik si'rtlay si'zi'lg'an. Bul ko'pmu'yeshlikler o'z ara ten' bolama?



6*. Bir kvadratti'n' ta'repleri ekinshi kvadrattin' ta'replerine parallel. Yeger kvadratlar bir-birine ten' bolmasa, wonda wolar gomotetiyaliq bolatug'inin da'liyllen' (*3-su'wret*).

7. *ABC* u'shmu'yeshliginin' *AB* ha'm *BC* ta'repleri to'rt ten'dey kesindilerge bo'lindi ha'm bo'liniw noqatlari' *AC* ta'repine parallel bolg'an kesindiler menen tutasti'ri'ldi' (*4-su'wret*). Yeger $AC=24$ *sm* bolsa, payda bolg'an kesindilerdin' uzi'nli'qlari'n tabi'n'.

8. Yeger su'wretler da'l bir waqi'tti'n' wo'zinde su'wretke ali'ng'an bolsa, berilgen mag'li'wmatlar boyi'nsha yekinshi imaratti'n' biyikligin tabi'n' (*5-su'wret*).



21 I BAPQA TIYISLI QOSI'MSHA MA'SELELER HA'M MAG'LIWMATLAR

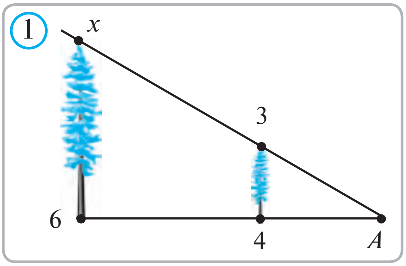
I. Testler

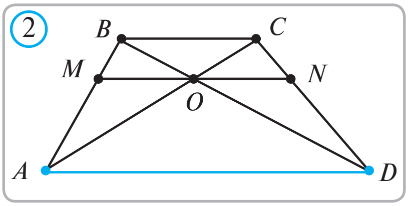
1. **Yeki uqsas u'shmu'yeshlikler ushi'n naduri's tasti'yi'qlawdi' tabi'n':**
   A. Maydanlari'ni'n' qatnasi'ni'n' uqsasli'q koefficientine ten';
   B. Sa'ykes medianalari'ni'n' qatnasi' uqsasli'q koefficientine ten';
   D. Sa'ykes bissektrisalari'ni'n' qatnasi' uqsasli'q koefficientine ten';
   E. Sa'ykes biyiklikleri'ni'n' qatnasi' uqsasli'q koefficientine ten'.
2. **Yeki gomotetiyaliq ko'pmu'yeshlik ushi'n duri's tasti'yi'qlawdi' tabi'n':**
   A. Wolar ten'; B. Wolar uqsas;
   D. Wolar ten' o'lshemli; E. Duri's juwap joq.
3. **U'shmu'yeshliktin' medianalari' ushi'n naduri's tasti'yi'qlawdi' ko'rsetin':**
   A. Bir noqatta kesilisedi; B. Kesilisiw noqati'nda 2:1 qatnasta bo'linedi;
   D. Bir-birine ten'; E. Ha'r biri u'shmu'yeshlikti yeki ten'dey bo'lekke bo'ledi.
4. **U'shmu'yeshliktin' bissektrisalari' ushi'n naduri's tasti'yi'qlawdi' ko'rsetin':**
   A. Bir noqatta kesilisedi; B. Kesilisiw noqati'nda 2:1 qatnasta bo'linedi;
   D. Wo'zi tu'sken ta'repti qalg'an yeki ta'repke proporcional kesindilerge aji'ratadi';
   E. Wo'zi shi'qqan to'bedegi mu'yeshti ten' yekige bo'ledi.
5. **Yeki uqsas ko'pmu'yeshlik ushi'n naduri's tasti'yi'qlawdi tabi'n':**
   A. Wolardi'n' ta'replerinin' sani' ten'; B. Wolardi'n' mu'yeshlerinin' sani' ten';
   D. Sa'ykes ta'repleri proporcional
   E.Maydanlari'ni'n' qatnasi' uqsasli'q koefficientine ten'.

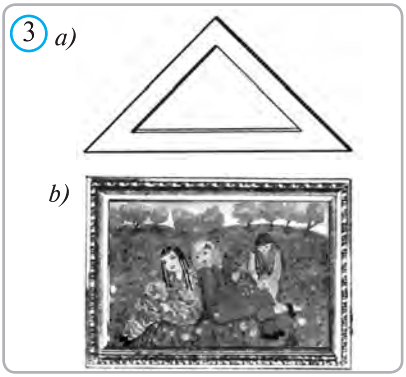
II. Ma'seleler

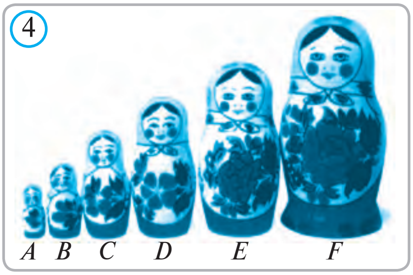
1. Ultanlari' 6 *m* ha'm 12 *m* bolg'an trapeciyani'n' diagonallari' kesilisken noqattan ultanlarg'a parallel tuwri' ju'rgizilgen. Tuwri'ni'n' trapeciya ishindegi bo'leginin' uzi'nli'g'i'n tabi'n'.
2. ABC u'shmu'yeshliginde $BC=BA=10$, $AC=8$. Yeger AA_1 ha'm CC_1 u'shmu'yeshliktin' bissektrisalari' bolsa, A_1C_1 kesindisin tabi'n'.
3. A noqati'nan bari'p bolmaytug'i'n B noqati'na shekemgi bolg'an arali'qti' ani'qlaw ushi'n tegis jerde C noqati' tan'lap ali'ndi'. Keyin AC arali'q, BAC ha'm ACB mu'yeshler wo'lshendi ha'm ABC u'shmu'yeshlikke uqsas $A_1B_1C_1$ u'shmu'yeshlik jasaldi. Yeger $AC=42$ *m*, $A_1C_1=6{,}3$ *sm*, $A_1B_1=7{,}2$ *sm* bolsa, AB arali'g'i'n tabi'n'.
4. Koefficienti $k=3$ bolg'an gomotetiyada F ko'pmu'yeshligi F_1 ko'pmu'yeshligine tu'rlendiriledi. Yeger F_1 ko'pmu'yeshliginin' perimetri 12 *sm* ha'm maydani' 4,5 sm^2 bolsa, F ko'pmu'yeshliginin' perimetrin ha'm maydani'n tabi'n'.

5. Boyi’ 180 *sm* bolg’an adam sayasi’ni’n’ uzi’nli’g’i’ 2,4 *m* bolg’an waqitta uzi’nli’g’i’ 4 *m* bolg’an si’m ag’ashti’n’ uzi’nli’g’i’ neshe metr boladi’?
6. Kartada Tashkent ha’m U’rgenish qalalari’ni’n’ ko’rinisleri arasi’ndag’i’ arali’q 8,67 *sm*. Yeger karta 1:100 000 bolsa, Tashkent ha’m U’rgenish qalalari’ arasi’ndag’i’ arali’qti’ tabi’n’.









III. Wo’zin’izdi si’nap ko’rin’ (u’lgi ushi’n baqlaw jumi’si’)

1. 1-su’wrette berilgen mag’li’wmatlar tiykari’nda terektin’ biyikligin tabi’n’.
2. *ABC* u’shmu’yeshliginin’ ta’repleri *AB*=5 *sm*, *AC*=6 *sm*, *BC*=7 *sm*. Bul u’shmu’yeshliktin’ *AC* ta’repine parallel tuwri’ *AB* ta’repin *P* noqati’nda, *BC* ta’repin bolsa *K* noqatinda kesip wo’tedi. Yeger *PK*=2 *sm* bolsa, *PBK* u’shmu’yeshliginin’ perimetrin tabi’n’.
3. 2-su’wrette $AD||BC||MN$. Yeger $BC = 6$ *sm*, $AD = 10$ *sm* bolsa, *MN* kesindisin tabi’n’.
4. *(Qosi’msha)*. Romb ta’replerinin’ wortalari’ tuwri’ to’rtmuyeshliktin’ to’leleri yekenligin daliyllen’.

Qi’zi’qli’ ma’seleler

1. 4 ma’rte u’lkeyttirilip ko’rsetilgen lupa ayna menen qaralg’anda 2^0 li mu’yesh shamasi’ qanshag’a wo’zgeredi (*3-su’wret*)?
2. a) u’shmu’yeshli sizg’ishtin’ su’wretinde ko’rsetilgen ishki ha’m si’rtqi’ u’shmu’yeshlikleri uqsaspa *(3-a-su’wret)*?
   b) 3 b-su’wrettegi ramani’n’ ishki ha’m si’rtqi’ qirlarin ko’rsetiwshi to’rtmu’yeshlikler uqsaspa?
3. To’mendegi rus tilinde berilgen ma’seleni sheship ko’rin’. Buni’n’ menen ha’m rus tilinen, ha’m geometriyadan nege tayar yekenligin’izdi bilip alasi’z.

   На 4-рисунке ижображена русская игрушка “матрёшка”. Выполнив соответствующие измерения, найти коэффициент подобия игрушек:
   a) *A* и *B*; b) *A* и *D*; d) *C* и *F*; e) *B* и *E*.

II BAP

U'SHMU'YESHLIKTIN' TA'REPLERI HA'M MU'YESHLERI ARASI'NDAG'I QATNASLAR

Bul bapti' u'yreniw na'tiyjesinde Siz to'mendegi bilim ha'm a'meliy ko'nlikpelerge iye bolasi'z:

Bilimler:

√ *qa'legen mu'yeshtin' sinusi', kosinusi', tangensi ha'm kotangensinin' ani'qlamalari'n biliw;*

√ *mu'yeshtin' radian wo'lshemin biliw;*

√ *tiykarg'i' trigonometriyali'q birdeyliklerdi biliw;*

√ *u'shmu'yeshliktin' maydani'n mu'yeshtin' sinusi' ja'rdeminde yesaplaw formulasi'n biliw;*

√ *sinuslar ha'm kosinuslardi'n' teoremasi'n biliw.*

A'meliy ko'nlikpeler:

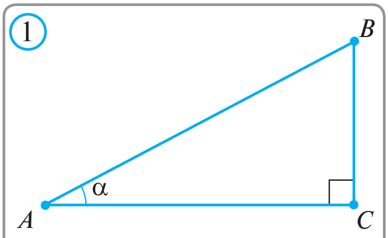
√ *bazi bir mu'yeshlerdin' sinusi', kosinusi', tangensi ha'm kotangensin yesaplay ali'w;*

√ *tiykarg'i' trigonometriyali'q birdeyliklerdi mi'sallar sheshiwde qollana ali'w;*

√ *u'shmu'yeshliktin' maydani'ni'n' yeki ta'repi ha'm wolar arasi'ndag'i' mu'yeshi boyi'nsha yesaplay ali'w;*

√ *sinuslar, kosinuslar teoremasi'nan paydalani'p yesaplawg'a ha'm da'liyllewge tiyisli ma'selelerdi sheshiw.*

22 SU'YIR MU'YESHTIN' SINUSI, KOSINUSI, TANGENSI HA'M KOTANGENSI



$\sin\alpha = \frac{BC}{AB}$; $\cos\alpha = \frac{AC}{AB}$;

$\mathrm{tg}\alpha = \frac{BC}{AC}$; $\mathrm{ctg}\alpha = \frac{AC}{BC}$.

Tuwri' mu'yeshli ABC u'shmu'yeshliginde $\angle C=90°$ bolsa AB ta'repi gipotenuza, BC ta'repi — A mu'yeshinin' qarsi'si'ndag'i' katet, AC ta'repi bolsa A mu'yeshine irgeles jatqan katet delinedi (*1-su'wret*).

Tuwri' mu'yeshli u'shmu'yeshliktin' su'yir mu'ye-shinin' **sinusi**' dep usi' mu'yeshtin' qarsi'si'ndag'i' katettin' gipotenuzasina qatnasi'na, **kosinusi**' dep, usi mu'yeshke irgeles jatqan katettin' gipotenuzag'a qatnasi'na ayti'ladi'.

Tuwri' mu'yeshli u'shmu'yeshliktin' su'yir mu'yeshinin' **tangensi** dep, usi mu'yeshtin' qarsi'si'ndag'i' katettin' irgeles jatqan katetke qatnasi'na, ***kotangensi*** dep, usi' mu'yeshke irgeles jatqan katettin' qarsi'si'ndag'i' katetke qatnasi'na ayti'ladi'.

α mu'yeshinin' sinusi', kosinusi', tangensi ha'm kotangensi sa'ykes tu'rde **sinα, cosα, tgα** ha'm **ctgα** tu'rinde belgilenedi (woqi'li'wi': ***«sinus alfa», «kosinus alfa», «tangens alfa», «kotangens alfa»***).

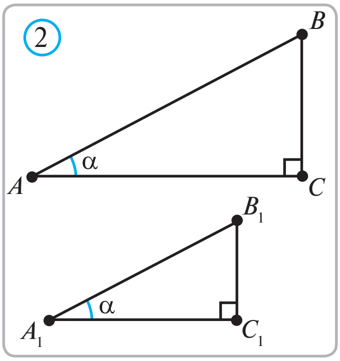
Joqari'dag'i' ani'qlamalardan to'mendegi formulalar kelip shig'adi':

1. $\frac{\sin A}{\cos A}=\frac{BC}{\cancel{AB}}\cdot\frac{\cancel{AB}}{AC}=\frac{BC}{AC}$; $\mathrm{tg}A=\frac{BC}{AC}$. $\Rightarrow$ $\boxed{\mathrm{tg}A=\frac{\sin A}{\cos A}}$.

2. $\frac{\cos A}{\sin A}=\frac{AC}{\cancel{AB}}\cdot\frac{\cancel{AB}}{BC}=\frac{AC}{BC}$; $\mathrm{ctg}A=\frac{AC}{BC}$. $\Rightarrow$ $\boxed{\mathrm{ctg}A=\frac{\cos A}{\sin A}}$.

3. $\mathrm{tg}A\cdot\mathrm{ctg}A=\frac{\cancel{BC}}{\cancel{AC}}\cdot\frac{\cancel{AC}}{\cancel{BC}}=1 \Rightarrow$ $\boxed{\mathrm{tg}A\cdot\mathrm{ctg}A=1}$.

Teorema. **Bir tuwri' mu'yeshli u'shmu'yeshliktin' su'yir mu'yeshi yekinshi tuwri' mu'yeshli u'shmu'yeshliktin' su'yir mu'yeshine ten' bolsa, wonda bul su'yir mu'yeshlerdin' sinuslari' (kosinusi', tangensi ha'm kotengensi) da ten' boladi.**



Da'liyllew. Tuwri' mu'yeshli ABC ha'm $A_1B_1C_1$ u'shmu'yeshliklerinde ($\angle C=\angle C_1=90°$) $\angle A=\angle A_1$ bolsi'n *(2-su'wret)*. Wonda, ABC ha'm $A_1B_1C_1$ u'shmu'yeshliktin MM belgisi boyi'nsha uqsas boladi'. Soni'n' ushi'n, $\frac{AB}{A_1B_1}=\frac{BC}{B_1C_1}=\frac{AC}{A_1C_1}$. Bul ten'liklerden $\frac{BC}{AB}=\frac{B_1C_1}{A_1C_1}$ yaki $\sin A=\sin A_1$ yekenligin tabami'z.

Bul su’yir mu’yeshlerdin’ kosinusi’, tangensi ha’m kotangensleri de ten’ boli’wi’ joqari’dag’ig’a uqsas da’liyllenedi. ***Teorema da’liyllendi.***

Ma’sele. *ABC* u’shmu’yeshlikte $\angle C=90°$, $AC=8$ *sm*, $BC=15$ *sm* bolsa, woni’n’ *B* mu’yeshinin’ sinusi’, kosinusi’, tangensi ha’m kotangensin tabi’n’.

Sheshiliwi. Pifagor teoremasi’nan paydalani’p, u’shmu’yeshliktin’ gipotenuzasin tabami’z:

$AB^2=AC^2+BC^2=8^2+15^2=289$, $AB=17$ (*sm*).

U’shmu’yeshliktin’ *B* mu’yeshi qarsi’si’ndag’i’ katet *AC*, *B* mu’yeshi qarsi’si’ndag’i’ katet *BC* (*1-su’wret*). Wonda, ani’qlamalar boyi’nsha

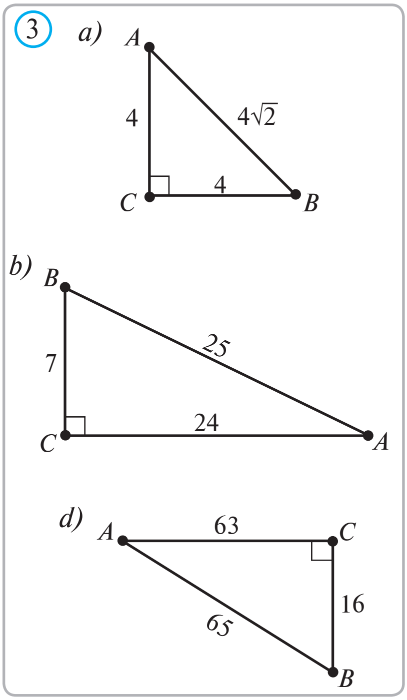
$$\sin B=\frac{AC}{AB}=\frac{8}{17};\qquad \cos B=\frac{BC}{AB}=\frac{15}{17};$$

$$\operatorname{tg}B=\frac{AC}{BC}=\frac{8}{15};\qquad \operatorname{ctg}B=\frac{BC}{AC}=\frac{15}{8}.$$

Yaki $$\operatorname{tg}B=\frac{\sin B}{\cos B}=\frac{8}{17}\cdot\frac{17}{15}=\frac{8}{15};$$

$$\operatorname{ctg}B=\frac{\cos B}{\sin B}=\frac{15}{17}\cdot\frac{17}{8}=\frac{15}{8}.$$

Juwabi’: $\frac{8}{17}$; $\frac{15}{17}$; $\frac{8}{15}$; $\frac{15}{8}$.



Soraw, ma’sele ha’m tapsi’rmalar

1. Tuwri’ mu’yeshli u’shmu’yeshliktin’ su’yir mu’yeshinin’ sinusi’, kosinusi’, tangensi ha’m kotangensi dep nege ayti’ladi’?

2. Su’yir mu’yeshtin’ sinusi’, kosinusi’, tangensi ha’m kotangensi nege baylani’sli’, nege baylani’sli’ yemes?

3. 4-su’wrettegi mag’li’wmatlar boyi’nsha sin*A*, cos*A*, sin*B*, cos*B* ni tabi’n’.

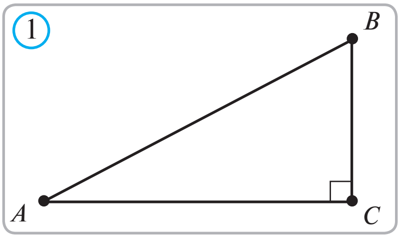
4. Tuwri’ mu’yeshli *ABC* u’shmu’yeshliginin’ *AB* gipotenuzasi 13 sm ge, *AC* kateti bolsa12 *sm* ge ten’.U’shmu’yeshliktin’ *A* mu’yeshinin’ sinusi’, kosinusi’, tangensi ha’m kotangensin tabi’n’.

5. Yeger tuwri’ mu’yeshli *ABC* ($\angle C=90°$) u’shmu’yeshliginde a) $AB=25$, $BC=7$; b) $AC=5$, $BC=12$; d) $AB=41$, $AC=40$; e) $AC=24$, $AB=25$ bolsa, *A* ha’m *B* mu’yeshlerinin’ sinusi’, kosinusi’, tangensi ha’m kotangenslerin tabi’n’.

6. Yeger *ABC* u’shmu’yeshliginde $\angle C=90°$, $\cos A=\frac{60}{61}$ ha’m $AC=3$ *sm* bolsa, u’shmu’yeshliktin’ qalg’an ta’replerin tabi’n’.

7. Yeger *ABC* u’shmu’yeshliginde $\angle C=90°$, $\sin A=\frac{8}{17}$ va $BC=16$ *sm* bolsa, u’shmu’yeshliktin’ qalg’an ta’replerin tabi’n’.

23 MA'SELELERDI SHESHIW



Ma'seleler sheshiwde ju'da' kerekli bolg'an ja'ne bir a'hmiyetli ten'liktin' duri'sli'g'i'n ko'rseteyik: tuwri' mu'yeshli ABC u'shmu'yeshlikte (*1-su'wret*) Pifagor teoremasi boyi'nsha $AB^2=BC^2+AC^2$. Wonda

$$\sin^2 A+\cos^2 A=\frac{BC^2}{AB^2}+\frac{AC^2}{AB^2}=\frac{BC^2+AC^2}{AB^2}=\frac{AB^2}{AB^2}=1.$$

$$\sin^2 A+\cos^2 A=1$$

ten'ligi trigonometriyani'n' tiykarg'i' birdeyligi dep ataladi' ("trigonometriya" so'zi grekshe "u'shmu'yeshliklerdi wo'lsheymen" degen ma'nisti an'latadi').

1-ma'sele. Yeger $\cos\alpha=\frac{1}{2}$ bo'lsa, $\sin\alpha$, $\operatorname{tg}\alpha$ ha'm $\operatorname{ctg}\alpha$ ni tabi'n'.

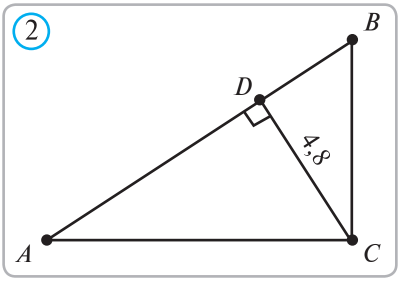
Sheshiliwi. Tiykarg'i' trigonometriyali'q birdeylik boyi'nsha:

$$\sin^2\alpha=1-\cos^2\alpha \;\Rightarrow\; \sin\alpha=\sqrt{1-\cos^2\alpha}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}.$$

Wonda, $\operatorname{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\sqrt{3}}{2}\cdot\frac{2}{1}=\sqrt{3}$, $\qquad \operatorname{ctg}\alpha=\frac{1}{\operatorname{tg}\alpha}=\frac{1}{\sqrt{3}}$.

2-ma'sele. ABC u'shmu'yeshlikte $\angle C=90°$ ha'm $\sin A=0{,}6$. Yeger u'shmu'yeshliktin' CD biyikligi 4,8 *sm* bolsa, woni'n' AC katetin ha'm usi' katettin' gipotenuzadag'i' proekciyasin tabi'n'.

Sheshiliwi. Tuwri' mu'yeshli ADC u'shmu'yeshligin qaraymi'z (2-su'wret).



Bunda, sinusti'n' ani'qlamasi boyi'nsha

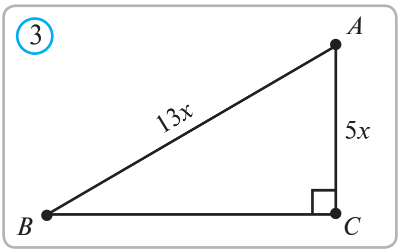
$\sin A=\frac{DC}{AC}$. Bunnan, $AC=\frac{DC}{\sin A}=\frac{4{,}8}{0{,}6}=8$ (*sm*).

Pifagor teoremasi'nan paydalani'p, AC katetinin' gipotenuzadag'i' proektsiyasi' AD ni tabami'z:

$$AD=\sqrt{AC^2-CD^2}=\sqrt{8^2-4{,}8^2}=6{,}4\ (sm).$$

Juwabi': 8 *sm*; 6,4 *sm*.

3-ma'sele. Yeger ABC u'shmu'yeshliginde $\angle C=90°$ ha'm $\cos A=\frac{5}{13}$ bolsa, u'shmu'yeshliktin' ta'repleri qanday qatnasta boladi' (*3-su'wret*).



Sheshiliwi. Mu'yeshtin' kosinusi'ni'n' ani'qlamasi' boyi'nsha

$$\cos A=\frac{AC}{AB}. \qquad \text{Demek, } \frac{AC}{AB}=\frac{5}{13}.$$

Yeger $AC=5x$ desek, wonda

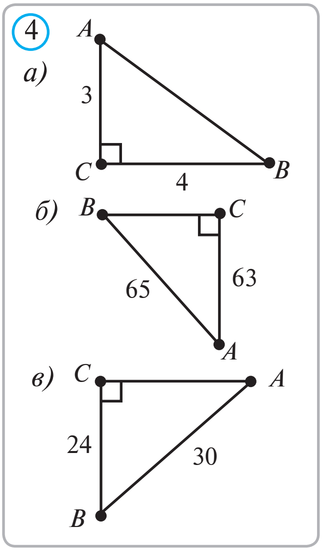
$$AB = \frac{13 \cdot AC}{5} = 13x.$$

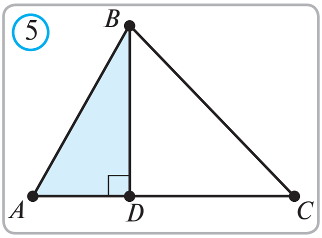
Pifagor teoremasi boyi'nsha,

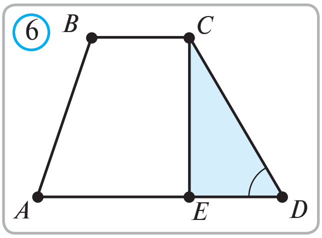
$$BC = \sqrt{AC^2 - BC^2} = \sqrt{169x^2 - 25x^2} = 12x.$$

Solay yetip, $AC : BC : AB = 5 : 12 : 13$.

Juwabi': $5 : 12 : 13$ siyaqli.







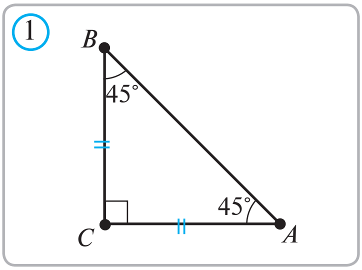
Soraw, ma'sele ha'm tapsi'rmalar

1. 4-su'wrettegi mag'li'wmatlar boyi'nsha to'mendegilerdi a) sin*A*, cos*A*, tg*A*, ctg*A*; b) sin*B*, cos*B*, tg*B*, ctg*B* ni tabi'n'.
2. Yeger $\sin\alpha = 0{,}5$ bolsa, $\cos\alpha$, $\text{tg}\,\alpha$ ha'm $\text{ctg}\,\alpha$ ni' tabi'n'.
3. Yeger $\cos\alpha = 0{,}6$ bolsa, $\sin\alpha$, $\text{tg}\,\alpha$ ha'm $\text{ctg}\,\alpha$ ni' tabi'n'.
4. Tuwri' mu'yeshli *ABC* ($\angle C = 90°$) u'shmu'yeshliginde $BC = 17$ *sm* ha'm $\sin B = \frac{15}{17}$ bolsa: a) u'shmu'yeshliktin' *CD* biyikligi; b) *BC* katetinin' gipotenuzadag'i' proekciyasi'n; c) gipotenuzasi'n; d) yekinshi katetin tabi'n'.
5. Yeger *ABC* u'shmu'yeshliginde $\angle C = 90°$, $\sin A = \frac{3}{8}$ ha'm $BC = 15$ *sm* bolsa, u'shmu'yeshliktin' gipotenuzasina tu'sirilgen biyiklikti tabi'n'.
6*. Yeger a) $\sin\alpha = \frac{2}{3}$; b) $\cos\alpha = \alpha$; d) $\text{tg}\,\alpha = \frac{3}{4}$; e) $\text{ctg}\,\alpha = \frac{4}{5}$ bolsa, α mu'yeshin jasan'.
7. *ABC* u'shmu'yeshliginde $AC = 12$ *sm*, $AB = 10$ *sm*, $\sin A = 0{,}7$ bolsa, u'shmu'yeshliktin' maydani'n tabi'n' (*5-su'wret*).
8. *ABC* u'shmu'yeshliginde *BD* — biyiklik, $AC = 7$ *sm*, $AD = 2$ *sm* ha'm $\text{tg}\,A = 3$ bolsa, u'shmu'yeshliktin' maydani'n tabi'n' (*5-su'wret*).
9. *ABCD* ($BC \| AD$) trapeciyada $\sin D = 0{,}5$; $CD = 8$, $BC = 6$, $AD = 10$ bolsa, trapeciya maydani'n tabi'n' (*6-su'wret*).
10. *ABCD* rombi'da $\sin A = 0{,}8$ ha'm $AB = 15$ *sm* bolsa, rombi'ni'n' maydani'n tabi'n'.
11*. Ten' qaptalli u'shmu'yeshliktin' ultanina tu'sirilgen biyikligi 5 *sm*, ultani' $10\sqrt{3}$ *sm* bolsa, u'shmu'yeshliktin' a) mu'yeshlerin; b) qaptal ta'repin; d) maydani'n tabi'n'.
12. Tuwri' mu'yeshli *ABC* u'shmu'yeshlikte $\sin A = \frac{3}{7}$ ha'm $\sin B = \frac{4}{7}$ boliwi mu'mkin be?

24 BAZI BIR MU'YESHLERDIN' SINUSI, KOSINUSI, TANGENSI HA'M KOTANGENSIN YESAPLAW

1. 45 gradusli' mu'yeshtin' sinusi', kosinusi', tangensi ha'm kotangensin yesaplan'.

Ten' qaptalli' tuwri' mu'yeshli ABC u'shmu'yeshligin qaraymi'z (*1-su'wret*).



$AC=BC$, $\angle A=\angle B=45°$ bolsi'n. Pifagor teoremasi' boyi'nsha $AB^2=AC^2+BC^2=2AC^2$ yaki $AB=AC\sqrt{2}$.

Bunnan $AC=BC=\dfrac{AB}{\sqrt{2}}=\dfrac{AB\sqrt{2}}{2}$ ni payda yetemiz.

Solay yetip,

$\sin 45° = \sin A = \dfrac{BC}{AB} = \dfrac{\sqrt{2}}{2}$; $\cos 45° = \cos A = \dfrac{AC}{AB} = \dfrac{\sqrt{2}}{2}$;

$\text{tg}45° = \text{tg}A = \dfrac{BC}{AC} = 1$; $\text{ctg}45° = \text{ctg}A = \dfrac{AC}{BC} = 1$.

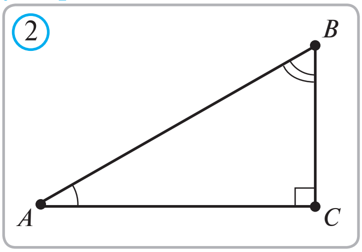
1-ma'sele. Tuwri' mu'yeshli ABC ($\angle C=90°$) u'shmu'yeshliginde $\angle A=45°$ ha'm $BC=6$ *sm*. U'shmu'yeshliktin' qalg'an ta'replerin tabi'n' (*1-su'wret*).

Sheshiliwi. $\dfrac{AC}{BC}=\text{ctg}45°$ yaki $\dfrac{AC}{BC}=1$, $AC=BC=6$ (*sm*);

$\dfrac{BC}{AB}=\sin 45°$ yaki $\dfrac{BC}{AB}=\dfrac{\sqrt{2}}{2}$, $AB=BC\sqrt{2}=6\sqrt{2}$ (*sm*).

Juwabi': 6 *sm*; $6\sqrt{2}$ *sm*.

2. 30° ha'm 60° mu'yeshlerdin' sinusi', kosinusi', tangensi ha'm kotangensin yesaplan'.



Mu'yeshleri $\angle A=30°$, $\angle B=60°$ ha'm $\angle C=90°$ bolg'an ABC u'shmu'yeshligin qaraymi'z (*2-su'wret*). 30 gradusli' mu'yeshtin' qarsi'si'nda jatqan katet gipotenuzani'n' yari'mi'na ten' bolg'ani' ushi'n AB yaki $BC=\dfrac{1}{2}$. Bunnan $\dfrac{BC}{AB}=\dfrac{1}{2}$

$\sin 30° = \sin A = \dfrac{BC}{AB} = \dfrac{1}{2}$; $\cos 60° = \cos B = \dfrac{BC}{AB} = \dfrac{1}{2}$

ten'liklerin tabamiz. Tiykarg'i trigonometriyaliq birdeylikten

$$\cos 30° = \sqrt{1-\sin^2 30°} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}, \quad \sin 60° = \sqrt{1-\cos^2 60°} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}.$$

Tabilg'anlar boyi'nsha $\text{tg}30° = \dfrac{\sin 30°}{\cos 30°} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$; $\text{tg}60° = \dfrac{\sin 60°}{\cos 60°} = \sqrt{3}$;

$$\text{ctg}30^\circ = \frac{\cos 30^\circ}{\sin 30^\circ} = \sqrt{3}; \qquad \text{ctg}60^\circ = \frac{\cos 60^\circ}{\sin 60^\circ} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

3. α nin' 30°, 45°, 60° qaten' ma'nislerde tabilg'an sinα, cosα, tgα ha'm ctgα ushi'n ma'nislerdi keste ko'rinisinde ji'ynaymi'z.

α	30°	45°	60°
sinα	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cosα	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tgα	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
ctgα	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$

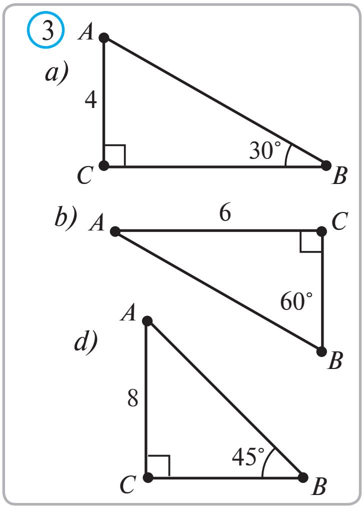
2-ma'sele. Tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi' 10 *sm* ha'm mu'yeshlerinen biri 60°. Woni'n' qalg'an ta'replerin tabi'n'.

Sheshiliwi. 2-su'wretten paydalanami'z. Bunda

$$BC = AB\sin A = 10 \cdot \sin 30^\circ = 10 \cdot \frac{1}{2} = 5\ (sm),$$

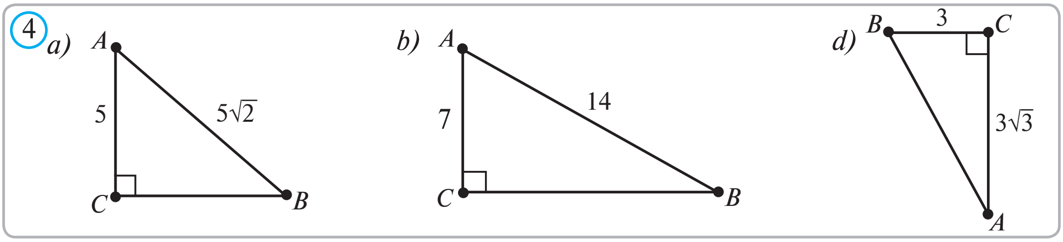
$$AC = AB\cos A = 10 \cdot \cos 30^\circ = 10 \cdot \frac{\sqrt{3}}{2} = 5\sqrt{3}\ (sm).$$

Juwabi': 5 *sm*; $5\sqrt{3}$ *sm*.



Soraw, ma'sele ha'm tapsi'rmalar

1. α mu'yesh 30°,45°, 60° qa ten' bolsa, sinα, cosα, tgα ha'm ctgα ma'nisleri nege ten'? Juwaplarin'izdi da'liyllen'.
2. 3-su'wrettegi u'shmu'yeshliklerdin' perimetrlerin tabi'n'.
3. 4-su'wrettegi u'shmu'yeshliklerdin' mu'yeshlerin tabi'n'.
4. α nin' 30°, 45°, 60° qaten' ma'nisinde sinα, cosα, tgα ha'm ctgα ushi'n ma'nisler kestesin yadlap ali'n'.
5. Tuwri' mu'yeshli u'shmu'yeshliktin' bir su'yir mu'yeshi 30° qa ten' boli'p, wog'an irgeles jatqan katet 6 *dm*. Woni'n' qalg'an ta'replerin tabi'n'.
6. Ten' qaptalli' u'shmu'yeshliktin' ultani' 10 *sm* ge, bir mu'yeshi bolsa120° qa ten'. Woni'n' maydani'n tabi'n'.
7. *ABC* u'shmu'yeshlikte $\angle C=90^\circ$, $AB=25$ *sm*, $\sin A= \frac{7}{25}$ .U'shmu'yeshliktin' qalg'an ta'replerin ha'm cos*A*, tg*A* ha'mde ctg*A* ni tabi'n'.
8. Diagonallari' $5\sqrt{3}$ *sm* ha'm 5 *sm* bolg'an rombi'ni'n' mu'yeshlerin tabi'n'.



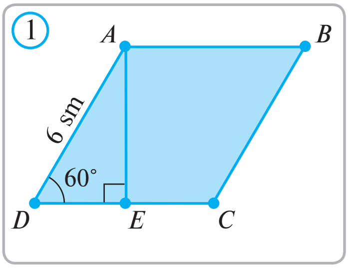
25 MA'SELELERDI SHESHIW

Jedellestiriwshi shinig'iw.

Kestenin' bos keteksheleriñ tolti'ri'n'.

α	sinα	cosα	tgα	ctgα
	$\frac{\sqrt{3}}{2}$			
			$\frac{\sqrt{3}}{3}$	
		$\frac{\sqrt{2}}{2}$		

1-ma'sele. Yeger *ABCD* rombida $\angle A=120°$ ha'm $AB=6$ *sm* bolsa, rombi'ni'n' biyikligin ha'm maydani'n tabi'n' (*1-su'wret*).



Sheshiliwi. 1) Rombi'ni'n' bir ta'repine irgeles jatqan mu'yeshlerinin' qosi'ndi'si' 180° qa ten' bolg'ani' ushi'n $\angle D=180°-\angle A=60°$. Rombi'ni'n' *AE* biyikligin ju'rgizip (*1-su'wret*), tuwri' mu'yeshli *AED* u'shmu'yeshligin paydayetemiz. Wonda

$$\frac{AE}{AD}=\sin D=\sin 60°=\frac{\sqrt{3}}{2} \quad \text{yaki} \quad AE=\frac{\sqrt{3}}{2}\cdot AD=3\sqrt{3}\ (sm).$$

2) Yendi rombi'ni'n' maydani'n tabami'z:

$$S_{ABCD}=DC\cdot AE=6\cdot 3\sqrt{3}=18\sqrt{3}\ (sm^2).$$

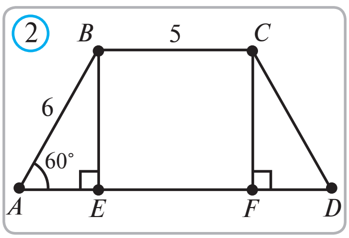
Juwabi': $h=3\sqrt{3}$ *sm*; $S_{ABCD}=18\sqrt{3}$ sm^2.

2-ma'sele. *ABCD* ten' qaptalli' trapeciyani'n' *BC* kishi ultani' 5 *sm*. Yeger $\angle A=60°$, $AB=6$ *sm* bolsa, trapeciyani'n' maydani'n tabi'n'.

Sheshiliwi. Trapeciyanin' *BE* ha'm *CF* biyikliklerin ju'rgizemiz (*2-su'wret*). Bunda tuwri' mu'yeshli *ABE* u'shmu'yeshliginen

$$AE=AB\cos 60°=6\cdot\frac{1}{2}=3\ (sm),$$

$$BE=AB\sin 60°=6\cdot\frac{\sqrt{3}}{2}=3\sqrt{3}\ (sm).$$



Bunnan ti'sqari' $AE=FD$, $EF=BC$ bolg'ani' ushi'n,

$$AD=AE+EF+FD=3+5+3=11\ (sm).$$

Trapeciyani'n' maydani'n tabi'w formulasi' boyi'nsha,

$$S_{ABCD} = \frac{BC + AD}{2} \cdot BE = \frac{5+11}{2} \cdot 3\sqrt{3} = 24\sqrt{3}\ (sm^2).$$

Juwabi': $24\sqrt{3}\ sm^2$.

? *Soraw, ma'sele ha'm tapsi'rmalar*

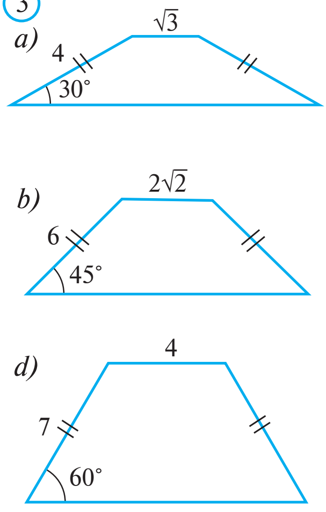
1. Ten' qaptalli' tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi 12 *sm*. Woni'n' maydanin yesaplan'.
2. Biyikligi $4\sqrt{3}$ *sm* bolg'an ten' ta'repli u'shmu'yeshliktin' perimetrin tabi'n'.
3. 3-su'wrette berilgenler boyi'nsha ten' qaptalli trapeciyalardi'n' maydani'n tabi'n'.
4. Tuwri' mu'yeshli trapeciyanin' su'yir mu'yeshi 30° qa, biyikligi 4 *sm* ge ha'm kishi ultani' 6 *sm* ge ten'. Trapeciyani'n' perimetrin ha'm maydani'n tabi'n'.
5. Shen'ber xordasi' 120 gradusli' dog'ani' kerip turadi'. Yeger shen'ber radiusi' 10 *sm* bolsa, xordani'n' uzi'nli'g'i'n tabi'n'.

6*. Ten' qaptalli' u'shmu'yeshliktin' to'besindegi mu'yeshi a) 120°, b) 90°, c) 60°. U'shmu'yeshlik biyikliginin' ultani'na qatnasi'n yesaplan'.

7*. 4-su'wrette su'wretlengen paxta qi'rmani'ni'n' qaptal jaqlari' ten' qaptalli' trapeciya, u'sti bolsa kvadrat formasi'nda. Su'wrette berilgenlerden paydalani'p, qi'rmandi' toli'q jabi'w ushi'n qansha material kerek yekenligin ani'qlan'.

8. Jen'il mashina shig'arli'qti'n' joqari'g'a ko'teriliw bo'liminde 340 m jol basti'. Yeger joldi'n' gorizontqa sali'sti'rg'anda ko'teriliw mu'yeshi 15° bolsa, jen'il mashina neshe metr biyiklikke ko'terilgen (*5-su'wret*)?

3



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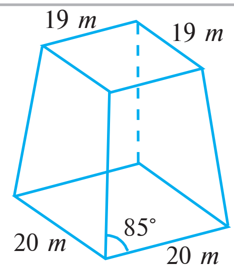

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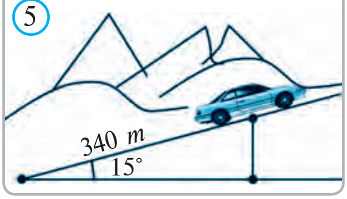


Arnawli kalkulyatorda trigonometriyali'q funkciyalardin' ma'nislerin tabi'w

[sin] ha'm [cos] tu'ymeleri bar arnawli' kalkulyatorda trigonometriyali'q funkciyalardi'n' ma'nisleri to'mendegishe saplanadi:

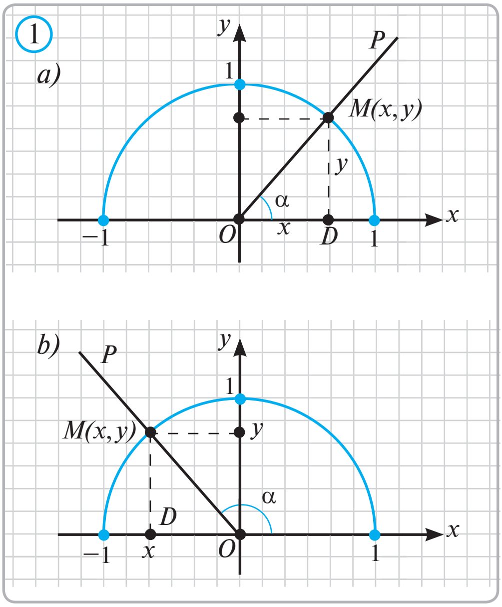
Mu'yeshlik graduslarda berilgen bolsi'n: mi'sali', sin30°:

Kalkulyator jalg'anip [DEG] (gradus) tu'ymesi basi'ladi'.

2. Keyin tu'ymeler [C] [3] [0] [Sin] ta'rtibinde basi'ladi' ha'm tiyisli juwap: 0,5 ali'nadi'. sin30°=0,5.

Yeger arnawli' kalkulyator bolmasa, sabaqli'qti'n' keynindegi qosi'mshada keltirilgen trigonometriyali'q funkciyalardin' ma'nislerinin' kestesinen paydalani'wi'mi'z mu'mkin.

26 0^o TAN 180^o QA SHEKEMGI BOLG'AN MU'YESHTIN' SINUSI, KOSINUSI, TANGENSI HA'M KOTANGENSI



Tuwri' mu'yeshli *Oxy* koordinatalar sistemasi'n ha'm woni'n' I ha'm de II shereklerinde jaylasqan, radiusi birlik kesindige ten', worayi' koordinatalar basi'nda bolg'an yari'm shen'ber jasaymi'z (*1-su'wret*). Shen'berdi M(*x, y*) noqati'nda kesip wo'tiwshi *OP* nuri'n ju'rgizemiz. Bul nurdi'n' *Ox* nuri' menen payda yetken mu'yeshin α menen belgileymiz. *OP* nuri'ni'n' *Ox* nuri' menen u'stpe-u'st tu'sken haldag'i mu'yeshin 0^o li mu'yesh si'pati'nda qabi'l yetemiz.

α su'yir mu'yesh bolg'anda (1.*a*-su'wret), tuwri' mu'yeshli *ODM* u'shmu'yeshlikten

$$\sin\alpha=\frac{DM}{MO};\qquad \cos\alpha=\frac{OD}{MO};$$
$$\mathrm{tg}\alpha=\frac{DM}{OD};\qquad \mathrm{ctg}\alpha=\frac{OD}{DM}$$

ten'likleri ja'rdeminde ani'qlanadi'. Yeger $MO=1$, $DM=y$, $OD=x$ yekenligin yesapqa alsaq,

$$\sin\alpha=y,\quad \cos\alpha=x,\quad \mathrm{tg}\alpha=\frac{y}{x},\quad \mathrm{ctg}\alpha=\frac{x}{y} \qquad (1)$$

ten'liklerine iye bolami'z.

Uli'wma jag'dayda, mu'yeshinin' 0^0 tan 180^0 qa shekemgi bolg'an barli'q ma'nislerinin' sinusi', tangensi ha'm kotangenslerin de (1) formula arqali ani'qlaymi'z.

Qa'legen α ($0°\leq\alpha\leq180°$) mu'yeshinin' **sinusi'** dep, *M* noqatinin' ordinatasi' — *y* ke aytiladi'. Qa'legen α ($0°\leq\alpha\leq180°$) mu'yeshinin' **kosinusi'** dep *M* noqati'ni'n' abcissasi — *x* qa ayti'ladi'. Qa'legen α ($0°\leq\alpha\leq180°$, $\alpha\neq90°$) mu'yeshinin' **tangensi** dep, *M* noqati'ni'n' ordinatasi abcissasi'na qatnasi'na ayti'ladi'. Qa'legen α ($0°<\alpha<180°$) mu'yeshinin' **kotangensi** dep, *M* noqati'ni'n' abcissasinin' ordinatasi'na qatnasi'na ayti'ladi'.

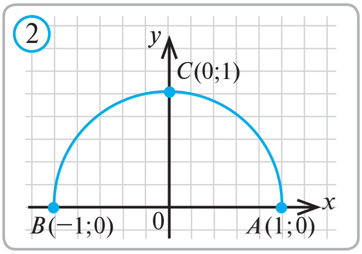
OMD u'shmu'yeshlik $OD^2+DM^2=MO^2$ yaki $x^2+y^2=1$. $\sin\alpha=y$ ha'm $\cos\alpha=x$ yekenligin yesapqa alsaq, qa'legen α ($0°\leq\alpha\leq180°$) mu'yesh ushi'n

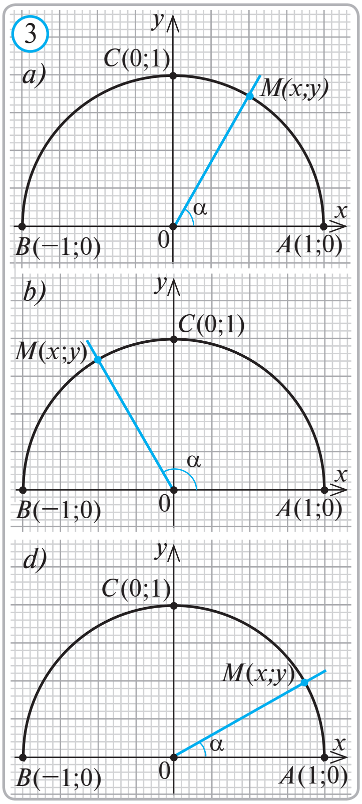
$$\sin^2\alpha+\cos^2\alpha=1 \qquad (2)$$

ten'lik payda boladi'. Bul ten'lik, **tiykarg'i' trigonometriyaliq birdeylik** dep ataladi', alding'i sabaqlarda su'yir mu'yeshler ushi'n da'liyllengen yedi.

A’meliy tapsirma

1. Birlik kesindini 5 *sm* ge ten’ dep, tuwri’ mu’yeshli koordinatalar sistemasi’n si’zi’n’.
2. Worayi’ koordinatalar basindaha’m radiusi’ birlik kesindige ten’, I ha’m II shereklerde jaylasqan yari’m shen’ber si’zi’n’.
3. Yari’m shen’berdi *M* noqatta kesip wo’tetug’in ha’m *Ox* ko’sherinin’ on’ bag’i’ti’ menen a) $\alpha=67°$; b) $\alpha=118°$; d) $\alpha=150°$ qaten’ mu’yesh payda yetetug’in *OM* nurin jasan’.
4. Wo’lshemler ja’rdeminde *M* noqati’ni’n’ koordinatalarin ha’m de $\sin\alpha$, $\cos\alpha$, $\mathrm{tg}\alpha$ ha’m $\mathrm{ctg}\alpha$ nin’ ma’nislerin tabi’n’.





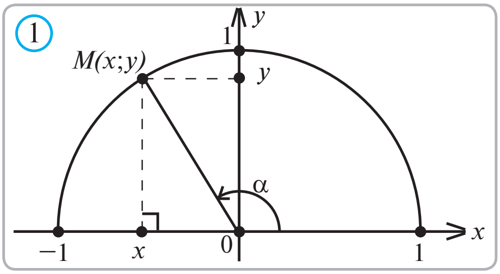
Ma’sele. 0°, 90° ha’m 180° li mu’yeshlerdin’ sinusin tabi’n’.

Sheshiliwi. 0° li’ mu’yesh *OA*, 90° li’ mu’yesh *OC*, 180° li mu’yesh *OB* nur ja’rdeminde ani’qlanadi’ *(2-su’wret)*. Ani’qlama boyi’nsha sin0° — *A*(1;0) noqati’ni’n’ ordinatasi’ si’pati’nda 0 ge, sin90° — *C*(0;1) noqati’ni’n’ ordinatasi’ si’pati’nda 1 ge, sin180° bolsa *B*(−1;0) noqati’ni’n’ ordinatasi’ si’pati’nda 0 ge ten’ boladi’. ***Juwabi’:*** $\sin0°=0$, $\sin90°=1$, $\sin180°=0$.

Soraw, ma’sele ha’m tapsi’rmalar

1. 0° tan 180° qa shekem bolg’an mu’yeshtin’ sinusi’ ha’m kosinusi’ degende ne tu’siniletug’i’ni’n ayti’p berin’.
2. α mu’yeshinin’ tangensi ha’m kotangensi degen ne? α mu’yeshinin’ tangensi ha’m kotangensi α nin’ qanday ma’nislerinde ani’qlanbag’an?
3. Yeger $90°<\alpha<180°$ bolsa, $\sin\alpha$, $\cos\alpha$, $\mathrm{tg}\alpha$ va $\mathrm{ctg}\alpha$ ma’nislerinin’ belgisin ani’qlan’.
4. Yeger $0°\le\alpha\le180°$ bolsa, $0\le\sin\alpha\le1$ va $-1\le\cos\alpha\le1$ ten’sizlikleri wori’nli’ bolatug’i’ni’n tu’sindirin’.
5. 3-su’wrettegi α mu’yeshin wo’lshen’ ha’m woni’n’ sinusi’, kosinusi’ ha’m kotangensin tiyisli wo’lshemler ja’rdeminde ani’qlan’.
6*. 1. *a* - su’wrette su’wretlengen yari’m shen’berdi si’zi’n’. *Ox* nuri’ menen 45° ha’m 135° li’ mu’yesh payda yetiwshi nurlardi’ jasan’. Si’zi’lg’an su’wretten paydalani’p, sin45° ti’ sin135° penen ha’m cos45° ti cos135° penen wo’z ara sali’sti’ri’n’.

27 TIYKARG'I' TRIGONOMETRIYALIQ BIRDEYLIKLER



Jedellestiriwshi shinig'iw

1-su'wretten paydalani'p noqatlardin' wornin toltirin':

$\sin\alpha = \ldots;$ $\cos\alpha = \ldots;$

$\operatorname{tg}\alpha = \frac{\ldots}{\ldots};$ $\operatorname{ctg}\alpha = \frac{\ldots}{\ldots}.$

Ani'qlama boyi'nsha, ha'r bir su'yir mu'yeshke bul mu'yeshtin' sinusi'ni'n' (kosinusi', tangensi ha'm kotangensinin') bir ma'nisi sa'ykes qoyi'li'p ati'r. Bul sa'ykeslikler su'yir mu'yeshtin' trigonometriyali'q funkciyalari': sinus, kosinus, tangens ha'm kotangens funkciyalarin ani'qlaydi'. Bul funkciyalar ko'binshe u'shmu'yeshliklerdi sheshiwde qollani'li'wi' sebepli, wolar trigonometriyaliq funkciyalar dep ataladi'.

"Trigonometriya" so'zi — grekshe "u'shmu'yeshliklerdi sheshiw" degen ma'nisti an'latadi.

Yendi α ($0°\le\alpha\le180°$) mu'yeshinin' sinusi', kosinusi', tangensi ha'm kotangensi arasi'ndag'i qatnaslardi' ani'qlayi'q.

1. **Trigonometriyani'n' tiykarg'i' birdeyligi** dep atali'wshi', α ni'n' $0°\le\alpha\le180°$ ma'nisleri ushi'n wori'nli' bolg'an mina

$$\sin^2\alpha + \cos^2\alpha = 1 \qquad (1)$$

formula menen aldin'g'i' sabaqlarda tani'sqan yedik.

2. Ani'qlama boyi'nsha, $\operatorname{tg}\alpha=\frac{y}{x}$, $\operatorname{ctg}\alpha=\frac{x}{y}$, $x=\cos\alpha$, $y=\sin\alpha$ bolg'ani' ushi'n

$$\operatorname{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}\ (\alpha\neq90°),\quad \operatorname{ctg}\alpha=\frac{\cos\alpha}{\sin\alpha}\ (\alpha\neq0,\ \alpha\neq180°),$$

$$\operatorname{tg}\alpha\cdot\operatorname{ctg}\alpha=1\ (\alpha\neq0,\ \alpha\neq90°,\ \alpha\neq180°) \qquad (2)$$

birdeylikleri wori'nli' boladi'.

3. (1) ten'liktin' ha'r yeki bo'limin aldi'n $\cos^2\alpha$ g'a, keyin bolsa $\sin^2\alpha$ g'a bo'lip

$$1+\operatorname{tg}^2\alpha=\frac{1}{\cos^2\alpha}\ (\alpha\neq90°),\qquad 1+\operatorname{ctg}^2\alpha=\frac{1}{\sin^2\alpha},\ (\alpha\neq0,\ \alpha\neq180°) \qquad (3)$$

birdeyliklerin payda yetemiz.

Ma'sele. Yeger $\sin\alpha=0{,}6$ ha'm $90°\neq\alpha\neq180°$ bolsa, $\cos\alpha$, $\operatorname{tg}\alpha$ ha'm $\operatorname{ctg}\alpha$ ma'nisin tabi'n'.

Sheshiliwi. Tiykarg'i' trigonometriyaliq birdeylikten paydalani'p $\cos\alpha$ ni yesaplaymiz:

$$\cos\alpha=-\sqrt{1-\sin^2\alpha}=-\sqrt{1-0{,}6^2}=-\sqrt{1-0{,}36}=-\sqrt{0{,}64}=-0{,}8.$$

$90^\circ\le\alpha\le180^\circ$, yag'ni'y α II sherekte bolg'anda, $\cos\alpha\le0$. Sol sebepli koren "–" belgisi menen ali'ndi'.

(2) formulag'a tiykarlani'p

$$\mathrm{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}=-\frac{0{,}6}{0{,}8}=-\frac{3}{4};\qquad \mathrm{ctg}\alpha=\frac{1}{\mathrm{tg}\alpha}=-\frac{4}{3}.$$

Juwabi': $\cos\alpha=-0{,}8$; $\mathrm{tg}\alpha=-\frac{3}{4}$; $\mathrm{ctg}\alpha=-\frac{4}{3}$.

? *Soraw, ma'sele ha'm tapsi'rmalar*

1. $\mathrm{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}$, $\mathrm{ctg}\alpha=\frac{\cos\alpha}{\sin\alpha}$, $\mathrm{tg}\alpha\cdot\mathrm{ctg}\alpha=1$ birdeylikleri α nin' qanday ma'nisleri ushi'n wori'nli'?

2. An'latpalardi' a'piwayi'lasti'ri'n':

1) $1-\cos^2\alpha$;
2) $(1-\sin\alpha)(1+\sin\alpha)$;
3) $\sin^4\alpha+2\sin^2\alpha\cdot\cos^2\alpha+\cos^4\alpha$;
4) $1-\sin^4\alpha-\sin^2\alpha\cdot\cos^2\alpha$;
5) $\mathrm{ctg}^2\alpha(2\sin^2\alpha+\cos^2\alpha-1)$;
6) $\mathrm{tg}^2\alpha-\sin^2\alpha\cdot\mathrm{tg}^2\alpha$.

3. Yeger a) $\sin\alpha=\frac{4}{5}$ ha'm $90^\circ<\alpha<180^\circ$ bolsa, $\cos\alpha$ nege ten' yekenligin tabi'n'; b) $\cos\beta=-\frac{2}{3}$ ha'm $90^\circ<\beta<180^\circ$ bolsa, $\sin\beta$ nege ten'; d) $\cos\alpha=1$ bolsa, $\sin\alpha$ ni'n' ma'nisin yesaplan'.

4. Su'yir mu'yeshi 60° qa, biyikligi bolsa 3 *sm* ge ten' rombi'ni'n' maydani'n tabi'n'.

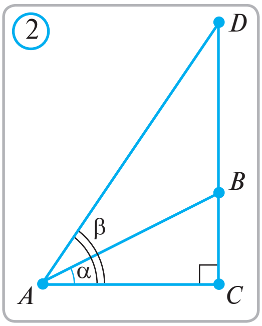
5. Ten' qaptalli' u'shmu'yeshliktin' ultani' 4,8 *sm,* ultani'ndag'i mu'yeshi bolsa 30°. U'shmu'yeshliktin' biyikligin ha'm qaptal ta'repin tabi'n'.

6. Yeger a) $\cos\alpha=\frac{1}{2}$; b) $\cos\alpha=-\frac{2}{3}$; d) $\cos\alpha=-1$ bolsa, $\sin\alpha$ nege ten'?

7. a) $\sin A=\frac{2}{3}$; b) $\cos A=\frac{3}{4}$; d) $\cos\alpha=\frac{2}{5}$ yekenligi belgili bolsa, A mu'yeshin jasan'.

8*. α ha'm β mu'yeshleri $0^\circ<\alpha<\beta<90^\circ$ sha'rtin qanaatlandi'ra-di'. 2-su'wretten paydalani'p yesaplan':

a) $\sin\alpha<\sin\beta$; b) $\cos\alpha>\cos\beta$;
d) $\mathrm{tg}\alpha<\mathrm{tg}\beta$; e) $\mathrm{ctg}\alpha>\mathrm{ctg}\beta$.

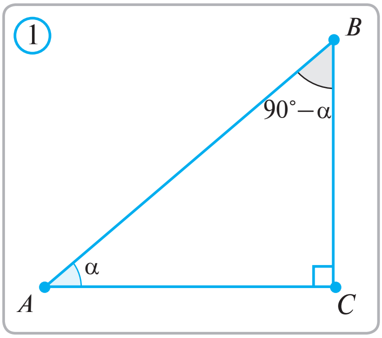


9*. OA nuri menen Ox nuri' arasi'ndag'i mu'yesh α g'aten'. Yeger
a) $OA=3$, $\alpha=45^\circ$; b) $OA=1{,}5$, $\alpha=90^\circ$; d) $OA=5$, $\alpha=150^\circ$;
e) $OA=2$, $\alpha=180^\circ$; f) $OA=4$, $\alpha=30^\circ$ bolsa, A noqati'ni'n' koordinatalari'n tabi'n'.

28 TIYKARG'I' TRIGONOMETRIYALIQ BIRDEYLIKLER (dawami)

1-teorema. Ha'r qanday su'yir α mu'yeshi ushi'n:

$$\sin(90^\circ-\alpha)=\cos\alpha, \qquad \cos(90^\circ-\alpha)=\sin\alpha. \qquad (1)$$



Da'liyllew. *A* to'besindegi su'yir mu'yeshi α g'a ten' bolg'an tuwri' mu'yeshli *ABC* u'shmu'yeshligin qaraymi'z (*1-su'wret*). Wondaoni'n' *B* to'besindegi su'yir mu'yeshi $\beta=90^\circ-\alpha$ ge ten'. Ani'qlama boyi'nsha

$$\sin(90^\circ-\alpha)=\sin\beta=\frac{AC}{AB}=\cos\alpha,$$

$$\cos(90^\circ-\alpha)=\cos\beta=\frac{BC}{AB}=\sin\alpha.$$

Teorema da'liyllendi.

1-ma'sele. To'mendegi sanlar ishinde wo'z-ara ten'lerin tabi'n': $\sin10^\circ$, $\cos10^\circ$, $\sin80^\circ$, $\cos80^\circ$.

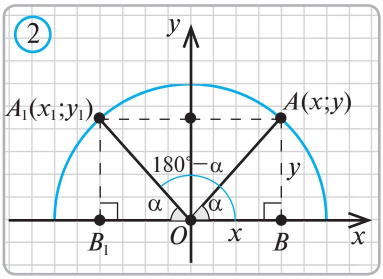
Sheshiliwi. $80^\circ=90^\circ-10^\circ$ $(\alpha=10^\circ)$ ha'm $50^\circ=90^\circ-40^\circ$ $(\alpha=40^\circ)$ bolg'ani' ushi'n 1-teorema boyi'nsha

$$\sin80^\circ=\sin(90^\circ-10^\circ)=\cos10^\circ, \qquad \cos80^\circ=\cos(90^\circ-10^\circ)=\sin10^\circ.$$

Juwabi': $\sin80^\circ=\cos10^\circ$, $\cos80^\circ=\sin10^\circ$.

2-teorema. **Ha'r qanday α $(0\le\alpha\le180^\circ)$** mu'yeshi ushi'n:

$$\sin(180^\circ-\alpha)=\sin\alpha, \qquad \cos(180^\circ-\alpha)=-\cos\alpha. \qquad (2)$$



Da'liyllew. Tuwri' mu'yeshli *Oxy* koordinatalar sistemasi'nda worayi' *O* noqati'nda, radiusi' 1 ge ten' bolg'an yari'm shen'berdi jasaymi'z (2-*su'wret*). Shen'berdin' *OA* radiusi' menen *Ox* nuri' arasi'ndag'i' mu'yesh $180^\circ-\alpha$ ge ten' mu'yesh payda yetiwshi OA_1 radiusi'n ju'rgizemiz. OAA_1 ha'm OBB_1 tuwri' mu'yeshli u'shmu'yeshlikleri ten'. Sonday-aq $OB=OB_1$ ha'm $AB=A_1B_1$ yaki $x_1=x$ ha'm $y_1=y$ ten'liklerine iye bolami'z. Solay yetip,

$$\sin(180^\circ-\alpha)=y_1=y=\sin\alpha; \qquad \cos(180^\circ-\alpha)=x_1=-x=-\cos\alpha.$$

Teorema da'liyllendi.

(1) ha'm (2) formulalari' ***keltiriw formulalari'*** delinedi.

2-ma'sele. $\alpha = 120^\circ$ bolsa, $\sin\alpha$, $\cos\alpha$, $\mathrm{tg}\,\alpha$ ha'm $\mathrm{ctg}\,\alpha$ ni'n' ma'nislerin yesaplan'.

Sheshiliwi. a) (2) formula boyi'nsha

$\sin 120^\circ = \sin(180^\circ - 60^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$; $\cos 120^\circ = \cos(180^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2}$.

Wonda

$$\mathrm{tg}\,120^\circ = \frac{\sin 120^\circ}{\cos 120^\circ} = -\sqrt{3};\quad \mathrm{ctg}\,120^\circ = \frac{\cos 120^\circ}{\sin 120^\circ} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}.$$

Juwabi': $\sin 120^\circ = \frac{\sqrt{3}}{2}$; $\cos 120^\circ = -\frac{1}{2}$; $\mathrm{tg}\,120^\circ = -\sqrt{3}$; $\mathrm{ctg}\,120^\circ = -\frac{\sqrt{3}}{3}$.

Soraw, ma'sele ha'm tapsi'rmalar

1. $\mathrm{tg}(90^\circ - \alpha) = \mathrm{ctg}\,\alpha$ $(\alpha \neq 0^\circ)$ ha'm $\mathrm{ctg}(90^\circ - \alpha) = \mathrm{tg}\,\alpha$ $(\alpha \neq 0^\circ)$ birdeyliklerin da'liyllen'.
2. $\mathrm{tg}(180^\circ - \alpha) = -\mathrm{tg}\,\alpha$ $(\alpha \neq 90^\circ)$ ha'm $\mathrm{ctg}(180^\circ - \alpha) = -\mathrm{ctg}\,\alpha$ $(\alpha \neq 0^\circ$ va $\alpha \neq 180^\circ)$ birdeyliklerin da'liyllen'.
3. Kesteni tolti'ri'n'.

α	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin\alpha$									
$\cos\alpha$									
$\mathrm{tg}\,\alpha$									
$\mathrm{ctg}\,\alpha$									

4. Yeger $90^\circ < \alpha < 180^\circ$ ha'm a) $\sin\alpha = \frac{1}{2}$; b) $\cos\alpha = -\frac{\sqrt{2}}{2}$; d) $\mathrm{tg}\,\alpha = -1$; e) $\mathrm{ctg}\,\alpha = -\sqrt{3}$ bolsa, α mu'yeshinin' shamasi'n tabi'n'.
5. Yesaplan':
   a) $\sin 180^\circ + 2\cos 90^\circ$; b) $4\sin 150^\circ + \sqrt{3}\,\mathrm{tg}\,150^\circ$;
   d) $\cos 40^\circ + \cos 50^\circ - \sin 40^\circ - \sin 50^\circ$; e) $3\cos 120^\circ - 2\sqrt{3}\,\mathrm{ctg}\,60^\circ$.
6. A'piwayi'lasti'ri'n':
   a) $\cos^2(180^\circ - \alpha) + \cos^2(90^\circ - \alpha)$; b) $\sin^2(180^\circ - \alpha) + \sin^2(90^\circ - \alpha)$;
   d) $\mathrm{tg}\,\alpha \cdot \mathrm{tg}(90^\circ - \alpha)$; e) $\mathrm{ctg}\,\alpha \cdot \mathrm{ctg}(90^\circ - \alpha)$.
7. ABC u'shmu'yeshliginde $\angle A = 150^\circ$ ha'm $AC = 7$ *sm* bolsa, u'shmu'yeshliktin' C to'besinen tu'sirilgen biyiklikti tabi'n'.
8. Tuwri'mu'yeshliktin' 12 *sm* ge ten' diagonali' bir ta'repi menen 30° qa ten' mu'yesh payda yetedi. Tuwri'mu'yeshliktin' maydani'n tabi'n'.
9. Yeger a) $\sin\alpha = \frac{\sqrt{3}}{2}$; b) $\sin\alpha = \frac{1}{4}$; d) $\sin\alpha = 1$ bo'lsa, $\cos\alpha$ ni tabi'n'.

10*. Yeger a) $\sin\alpha = \frac{1}{2}$; b) $\mathrm{tg}\,\alpha = -1$; d) $\cos\alpha = -\frac{\sqrt{3}}{2}$ bo'lsa, α ni tabi'n'.

29 BILIMIN’IZDI SI’NAP KO’RIN’

I. Shep bag’anada berilgen atamalarg’a won’ bag’anada berilgen ani’qlamalardan duri’si’n sa’ykes qoyi’n’.

1. α mu’yeshinin’ sinusi
2. α mu’yeshinin’ kosinusi
3. α mu’yeshinin’ tangensi
4. α mu’yeshinin’ kotangensi

a) α mu’yeshinin’ qarsi’si’ndag’i’ katettin’ gipotenuzag’a qatnasi’;
b) α mu’yeshinin’ qarsi’si’ndag’i’ katettin’ yekinshi katetke qatnasi’;
d) α mu’yeshinin’ qarsi’si’ndag’i’ katettin’ yekinshi katetke qatnasi’;
e) α mu’yeshke irgeles katettin’ yekinshi katetke qatnasi’.

II. Testler

1. Naduri’s formulani’ tabi’n’:
A. $\sin(90°-\alpha)=\cos\alpha$; B. $\cos(90°-\alpha)=\sin\alpha$;
D. $\sin(180°-\alpha)=\sin\alpha$; E. $\cos(180°-\alpha)=\cos\alpha$.

2. Yeger $90°<\alpha<180°$ bolsa, to’mendegilerden qaysi’ biri won’?
A. $\sin\alpha$; B. $\cos\alpha$; D. $\mathrm{tg}\alpha$; E. $\mathrm{ctg}\alpha$.

3. Duri’s ten’likti tabi’n’:
A. $\sin^2\alpha=1+\cos^2\alpha$; B. $\mathrm{tg}^2\alpha=1+\cos^2\alpha$;
D. $\frac{1}{\cos^2\alpha}=1+\mathrm{tg}^2\alpha$ $(\alpha\neq 90°)$; E. $\sin^2 x\cdot\cos^2 x=1$.

4. sin70° nege ten’:
A. sin20°; B. − sin20°; D. cos70°; E. cos 20°.

5. $\sin\alpha=\frac{1}{2}$ bolg’an α su’yir mu’yeshin ko’rsetin’:
A. 30°; B. 45°; D. 90°; E. 60°.

6. $\cos\alpha=\frac{1}{2}$ bolsa, α su’yir mu’yeshin tabi’n’:
A. 30°; B. 45°; D. 90°; E. 60°.

7. $\mathrm{tg}\alpha=1$ bolsa, α su’yir mu’yeshin tabi’n’:
A. 30°; B. 45°; D. 90°; E. 60°.

8. $\mathrm{ctg}\alpha=1$ bolsa, α su’yir mu’yeshin tabi’n’:
A. 30°; B. 45°; D. 90°; E. 60°.

9. Qaysi su’yir α mu’yesh ushin’ $\sin\alpha=\cos\alpha$ ten’lik wori’nli’?
A. 30°; B. 45°; D. 90°; E. 60°.

10. Yeger $\sin B=\frac{2}{5}$ bolsa, $\cos B$ ni’ tabi’n’.

A. $\frac{4}{25}$; B. $\frac{\sqrt{29}}{5}$; D. $\frac{\sqrt{21}}{5}$; E. $\frac{\sqrt{10}}{5}$.

11. Yeger cos*A*=0,2 bolsa, tg*A* ni' tabi'n'.

A. $\sqrt{96}$; B. $2\sqrt{6}$; D. $\sqrt{15}$; E. $\frac{\sqrt{6}}{12}$.

12. Tuwri' to'rtmu'yeshliktin' diagonali' woni'n' bir ta'repinen 2 ma'rte uzi'n. Tuwri' to'rtmu'yeshliktin' diagonallari' arasi'ndag'i mu'yeshin tabi'n'.

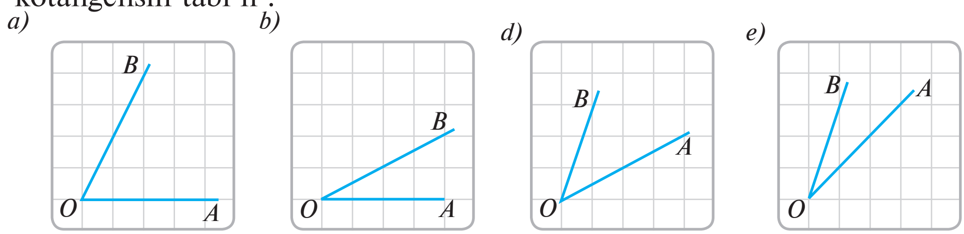
A. 30°; B. 60°; D. 90°; E. 150°.

13. Ten' qaptalli' u'shmu'yeshliktin' ultani'na tu'sirilgen biyikligi 3 *sm*, ultani' bolsa 8 *sm*. U'shmu'yeshliktin' ultani'na irgeles jatqan mu'yeshtin' sinusi'n tabi'n'.

A. $\frac{3}{5}$; B. $\frac{3}{4}$; D. $\frac{\sqrt{73}}{73}$; E. $\frac{4}{5}$.

III. Ma'seleler

1. Su'wrette sa'wlelengen mu'yeshlerdin' sinusi', kosinusi', tangensi ha'm kotangensin tabi'n'.



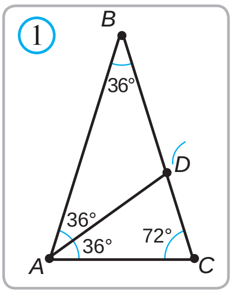
2. Ashirafjon u'yinen shi'g'i's ta'repke qarap 800 *m*, son' arqa ta'repke qarap 600 *m* jola ju'rdi. Wol u'yinen neshe metr uzaqliqqa keldi? Yendi wol u'yine tuwri' si'zi'q boylap jetip aliw ushi'n batisqa sali'sti'rg'anda qanday mu'yesh asti'nda ju'riwi kerek?
3. Poyezd ha'r 30 *m* jol ju'rgende 1 *m* biyiklikke ko'teriledi. Temir joldin' gorizontqa sali'stirg'andag'i' ko'teriliw mu'yeshin tabi'n'.
4. Yeger biyikligi 30 *m* bolg'an imarattin' sayasinan uzin'lig'i 45 *m* bolsa, quyash nuri'ni'n' sol imarat jaylasqan wori'ng'a tiyisli mu'yeshin tabi'n'.
5. Tuwri' mu'yeshli u'shmu'yeshliktin' bir mu'yeshi 60° qa, u'lken kateti bolsa 6 g'a ten'. Woni'n' kishi katetin ha'm gipotenuzasi'n tabi'n'.
6. *O* worayina iye shen'berdin' *A* noqati'nan ju'rgizilgen uri'nbadan *B* noqati' ali'ng'an. Yeger AB=9 *sm*, $\angle ABO$=30° bolsa, shen'ber radiusi'n ha'm *BC* kesindisi uzi'nli'g'i'n tabi'n'.
7. *m* tuwri'si' ha'm woni' kesip wo'tpeytug'i'n *AB* kesindisi berilgen. Bunda AB=10, *AB* ha'm *m* tuwrilari arasi'ndag'i' mu'yesh 60°. *AB* kesindisi ushlari'nan *m* tuwri'sina *AC* ha'm *BD* perpendikulyarlari' tu'sirilgen. *CD* kesindisin tabi'n'.
8. Rombi'ni'n' su'yir mu'yeshi 60° qa, biyikligi bolsa 6 g'a ten'. Rombi'ni'n' diagonali'n ha'm maydani'n tabi'n'.

9. Radiusi' 5 sm bolg'an shen'berge ten' qaptalli' trapeciya si'rtlay si'zi'lg'an. Yeger trapeciyani'n' su'yir mu'yeshi 30° bolsa, woni'n' qaptal ta'replerin ha'm maydani'n tabi'n'.
10. Yeger $ABCD$ tuwri'mu'yeshliginde $AB=4$, $\angle CAD=30°$ bolsa, wog'an si'rtlay si'zi'lg'an shen'ber radiusi'n ha'm tuwri'mu'yeshliktin' maydani'n yesaplan'.
11. Tuwri'mu'yeshliktin' ta'repleri 3 *sm* ha'm $\sqrt{3}$ *sm*. Woni'n' bir diagonali' menen ta'repleri payda yetken mu'yeshlerin tabi'n'.
12. Yeger a) $\sin A=\frac{4}{7}$; b) $\cos A=\frac{4}{7}$; d) $\cos A=-\frac{4}{7}$ bolsa, A mu'yeshin jasan'.
13. Tuwri' mu'yeshli u'shmu'yeshliktin' bir mu'yeshi 30°, gipotenuzasina tu'sirilgen biyikligi 6 *sm*. U'shmu'yeshliktin' ta'replerin tabi'n'.
14. Su'yir mu'yeshi 30° qa, biyikligi bolsa 4 *sm* ge ten' bolg'an rombi'ni'n' maydani'n yesaplan'.
15. Yeger $\sin A=\frac{8}{17}$ ha'm $90°<\alpha<180°$ bolsa, $\cos\alpha$, $\text{tg}\alpha$ ha'm $\text{ctg}\alpha$ nin' ma'nisin tabi'n'.
16. Tuwri' mu'yeshli ABC u'shmu'yeshliginin' AB gipotenuzasi'na CD biyikligi tu'sirilgen. Yeger $\angle A=60°$ ha'm $BD=2$ bolsa, BC katetin tabi'n'.
17. ABC u'shmu'yeshliginde $\angle A=30°$, $\angle C=45°$. Yeger u'shmu'yeshliktin' BD biyikligi 12 *sm* bolsa, woni'n' AC ta'repin ha'm maydani'n tabi'n'.

IV. Wo'zin'izdi sinap ko'rin' (u'lgi ushi'n baqlaw jumi'si')

1. Yeger $\cos\alpha=-\frac{8}{17}$ ha'm $90°<\alpha<180°$ bolsa, $\sin\alpha$, $\text{tg}\alpha$, $\text{ctg}\alpha$ nege ten'?
2. Tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi' $c=18$ *sm* ha'm kateti $a=4$ *sm* bolsa, woni'n' yekinshi kateti ha'm su'yir mu'yeshlerin tabi'n'.
3. Ten' ta'repli u'shmu'yeshliktin' medianasi' woni'n' ta'repinen kishi bolatug'i'ni'n da'liyllen'.
4. (*Qosi'msha*). To'rtmu'yeshliktin' ha'r bir ta'repi qalg'an ta'replerinin' qosi'ndi'si'nan kishi yekenin da'liyllen'.

Tariyx betlerinen. "Alti'n u'shmu'yeshlik"

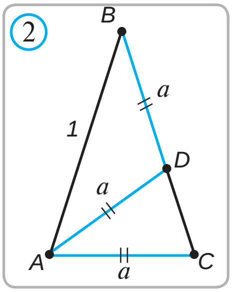


A'yyemgi grekler, mu'yeshleri 36°, 72°, 72° bolg'an ten' qaptalli' u'shmu'yeshlikti — "alti'n u'shmu'yeshlik" dep atag'an. Wo'ni'n' ultani'ndag'i' mu'yesh bissektrisasi AD woni' yeki ten' qaptalli' u'shmu'yeshlikke bo'ledi (*1-su'wret*).

Haqi'yqattan da, AD bissektrisa bolg'ani' ushi'n, BAD ha'm DAC mu'yeshleri de 36° tan. Demek, ABD u'shmu'yeshligi ten' qaptalli'. ADC u'shmu'yeshliginde ADC mu'yeshi $180°-36°-72°=72°$ boli'p, ACD mu'yeshine ten'. Demek, ADC u'shmu'yeshligi de ten' qaptalli'.

Na'tiyje. ABC u'shmu'yeshligi ACD u'shmu'yeshligine uqsas ha'm

$$\frac{AC}{AB}=\frac{CD}{AC}. \qquad (1)$$



Yeger ABC u'shmu'yeshliginin' qaptal ta'repleri $AB=BC=1$ dep alsaq, woni'n' ultani' to'mendegishe tabi'ladi' (*2-su'wret*). $AC=a$ bolsi'n. Wonda

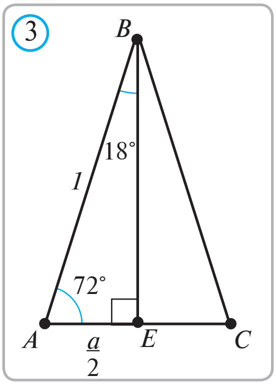
1. $AD=a$ boladi, sebebi ΔACD ten' qaptalli'.
2. $BD=a$ boladi, sebebi ΔABD ten' qaptalli'.
3. $CD=BC-BD=1-a$.

(1) ten'lik boyi'nsha: $\frac{a}{1}=\frac{1-a}{a}$

Bunnan $a^2+a-1=0$ yaki $a=\frac{\sqrt{5}-1}{2}$ yekenligin tabami'z.

Ma'sele. sin18°, cos18°, sin72°, cos72° ma'nislerin yesaplan'.

Sheshiliwi. Qaptal ta'repi $AB=BC=1$ ha'm ultani $AC=a=\frac{\sqrt{5}-1}{2}$ ten' bolg'an ABC "alti'n u'shmu'yeshlik" ti qaraymi'z (*3-su'wret*). Woni'n' BE biyikligin ju'rgizemiz.



Tuwri' mu'yeshli ABE u'shmu'yeshlikten

$$\sin 18^\circ=\frac{AE}{AB}=\frac{a}{2}=\frac{\sqrt{5}-1}{4}$$

Bunnan paydalani'p, tabi'li'wi' talap yetilgen basqa ma'nislerdi yesaplaymi'z:

$$\cos 18^\circ=\sqrt{1-\sin^2 18^\circ}=\frac{\sqrt{5}+1}{4};$$

$$\sin 72^\circ=\sin(90^\circ-18^\circ)=\cos 18^\circ=\frac{\sqrt{5}+1}{4};$$

$$\cos 72^\circ=\sin 18^\circ=\frac{\sqrt{5}-1}{2}.$$

Juwabi': $\sin 18^\circ=\cos 72^\circ=\frac{\sqrt{5}-1}{4}$; $\cos 18^\circ=\sin 72^\circ=\frac{\sqrt{5}+1}{4}$;

Tariyx betlerinen

Ulug'bek (1394 — 1449)

Ulug'bek (1394—1449) — ulli wo'zbek ali'mi' ha'm ma'mleket oyshi'li'. Negizgi ati Muxammed Tarag'ay. Wol sahibqiran Amir Temurdi'n' aqli'g'i'. Ulug'bektin' atasi' Shaxruxta ma'mleket oyshi'li' bolg'an.Ulug'bek shama menen 1425—1428-ji'llari' Samarqandti'n' a'tirapi'ndag'i' Obi Rahmat to'beshiginde wo'zinin' du'nyag'a belgili observatoriyasi'n quradi'.Observatoriyani'n' imarati' u'sh qabatli' boli'p, woni'n' tiykarg'i a'sbabi' — kvadrantti'n' biyikligi 50 metr yedi. Ulug'bektin' yen' du'nyag'a belgili miyneti "Zijiy kuragoniy" dep atali'wshi' astronomiyali'q keste boladi'. Wol 1018 juldi'zdi' wo'z ishine alg'an.

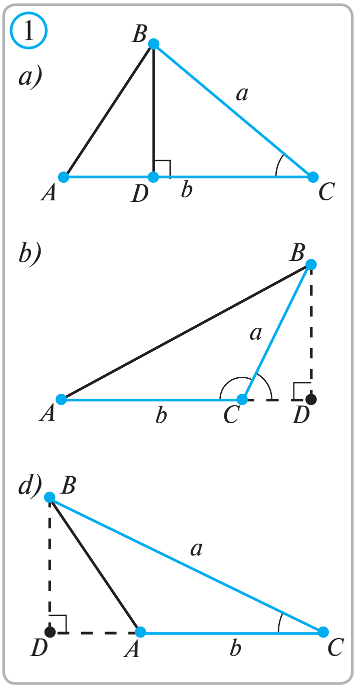
30 U'SHMU'YESHLIKTIN' MAYDANI'N MU'YESHTIN' SINUSI' JA'RDEMINDE YESAPLAW

1-teorema. U'shmu'yeshliktin' maydani' woni'n' eki ta'repi menen usi' yeki ta'rep arasi'ndag'i' mu'yeshtin' sinusi'ni'n' ko'beymesi'ni'n' yari'mi'na ten'.

ΔABC, $BC=a$, $AC=b$, $\angle C$ *(1-su'wret)* ⇒ $S_{ABC}=\frac{1}{2}ab\sin C$

Da'liyllew. *ABC* u'shmu'yeshliginin' *BD* biyikligin tu'siremiz. Wonda 1-su'wrette ko'rsetilgen u'sh jag'day boli'wi' mu'mkin.

Birinshi jag'daydi' qaraymi'z. *BCD* u'shmu'yeshliginde $\sin C=\frac{BD}{BC}$. Bunnan $BD=BC\cdot\sin C=a\cdot\sin C$. Solay yetip,



$$S_{ABC}=\frac{1}{2}\cdot AC\cdot BD=\frac{1}{2}\cdot b\cdot a\cdot\sin C=\frac{1}{2}ab\sin C.$$

Yekinshi ha'm u'shinshi jag'daylardin' da'liylleniwin wo'z betin'izshe worinlan'. ***Teorema da'liyllendi.***

1-teorema boyi'nsha, u'shmu'yeshliktin' maydani' ushi'n

$$S_{ABC}=\frac{1}{2}bc\sin A \quad \text{ha'm} \quad S_{ABC}=\frac{1}{2}ac\sin B$$

formulalari' da wori'nli' boladi'.

1-ma'sele. *ABC* u'shmu'yeshliginin' maydani' 24 sm^2. Yeger $AC=8$ *sm* ha'm $\angle A=30°$ bolsa, *BC* ta'repin tabi'n'.

Sheshiliwi. U'shmu'yeshliktin' maydani'n mu'yeshtin' sinusi' arqali' tabi'w formulasi boyi'nsha,

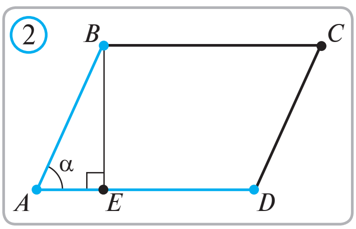
$$S_{ABC}=\frac{1}{2}\cdot AB\cdot AC\cdot\sin A$$

Bunnan,

$$AB=\frac{2S_{ABC}}{AC\cdot\sin A}=\frac{2\cdot 24}{8\cdot\sin 30°}=\frac{2\cdot 24}{8\cdot 0{,}5}=12\ (sm).$$

Juwabi': 12 *sm*.

2-ma'sele. Parallelogrammni'n' maydani' woni'n' yeki qon'si'las ta'repi ha'm bul ta'repler arasi'ndag'i' mu'yeshtin' sinusi'ni'n' ko'beymesine ten' yekenligin da'liyllen'.

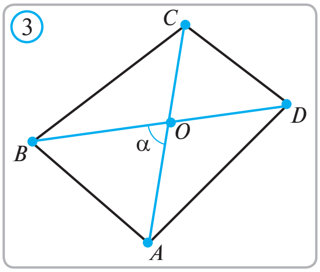


ABCD parallelogramm, $AB=a$, $AD=b$, $\angle A=\alpha$ *(2-su'wret)* ⇒ $S_{ABCD}=ab\sin\alpha$

Sheshiliwi. BE biyikligin tu'siremiz. ABE u'shmu'yeshliginde $\sin A = \dfrac{BE}{AB}$ yaki $BE = AB\sin A = a\sin\alpha$. Wonda $S_{ABCD} = AD \cdot BE = ab\sin\alpha$.

2-teorema. **To'rtmu'yeshliktin' maydani' woni'n' diagonallari' menen diagonallari' arasi'ndag'i' mu'yeshtin' sinusi' ko'beymesinin' yari'mi'na ten'.**

Da'liyllew. Diagonallardi'n' kesilisiwinen payda bolg'an mu'yeshlerdi qaraymi'z (*3-su'wret*):



$\angle AOB = \alpha$ $\Leftarrow$ sha'rt boyi'nsha,
$\angle COD = \alpha$ $\Leftarrow$ $\angle AOB$ g'a vertikal bolg'ani' ushi'n,
$\angle BOC = 180^\circ - \alpha$ $\Leftarrow$ $\angle AOB$ g'a qon'silas bolg'ani' ushi'n,
$\angle DOA = 180^\circ - \alpha$ $\Leftarrow$ $\angle BOC$ ke vertikal bolg'ani' ushi'n.

U'shmu'yeshliktin' maydani'n mu'yeshtin' sinusi' ja'rdeminde yesaplaw formulasi' boyi'nsha:

$S_{AOB} = \frac{1}{2} AO \cdot OB \sin\alpha$; $\quad S_{BOC} = \frac{1}{2} BO \cdot OC \sin(180^\circ - \alpha) = \frac{1}{2} BO \cdot OC \sin\alpha$;

$S_{COD} = \frac{1}{2} CO \cdot OD \sin\alpha$; $\quad S_{DOA} = \frac{1}{2} DO \cdot OA \sin(180^\circ - \alpha) = \frac{1}{2} DO \cdot OA \sin\alpha$.

Maydannin' qa'siyeti boyi'nsha: $S_{ABCD} = S_{AOB} + S_{BOC} + S_{COD} + S_{DOA} =$

$$= \frac{1}{2} AO \cdot OB \sin\alpha + \frac{1}{2} BO \cdot OC \sin\alpha + \frac{1}{2} CO \cdot OD \sin\alpha + \frac{1}{2} DO \cdot OA \sin\alpha =$$

$$= \frac{1}{2}(AO \cdot OB + BO \cdot OC + CO \cdot OD + DO \cdot OA)\sin\alpha = \frac{1}{2}\{(OB \cdot (AO + OC) +$$

$$+ OD \cdot (CO + OA)\}\sin\alpha = \frac{1}{2}(OB \cdot AC + OD \cdot AC)\sin\alpha = \frac{1}{2} AC \cdot BD \sin\alpha.$$

Teorema da'liyllendi.

Soraw, ma'sele ha'm tapsi'rmalar

1. 1-teoremani' 1.*b* - ha'm 1.*d* - su'wrette su'wretlengen halda da'liyllen'.

2. Yeger a) $AB = 6$ *sm*, $AC = 4$ *sm*, $\angle A = 30^\circ$; b) $AC = 14$ *sm*, $BC = 7\sqrt{3}$ *sm*, $\angle C = 60^\circ$; d) $BC = 3$ *sm*, $AB = 4\sqrt{2}$ *sm*, $\angle B = 45^\circ$ bolsa, ABC u'shmu'yeshliginin' maydani'n tabi'n'.

3. Diagonali' 12 *sm* ha'm diagonallari' arasi'ndag'i' mu'yeshi 30° bolg'an tuwri'-mu'yeshliktin' maydani'n tabi'n'.

4. Ta'repi $7\sqrt{2}$ *sm* ha'm dog'al mu'yeshi 135° bolg'an romb maydani'n tabi'n'.

5. Rombi'ni'n' u'lken diagonali' 18 *sm* ha'm dog'al mu'yeshi 120°. Rombi'ni'n' maydani'n tabi'n'.

6. Maydani' $6\sqrt{2}$ sm^2 qa ten' bolg'an ABC u'shmu'yeshliginde $\angle A = 45^\circ$. U'sh-mu'yeshliktin' AC ta'repin ha'm usi' ta'repke tu'sirilgen biyikligin tabi'n'.

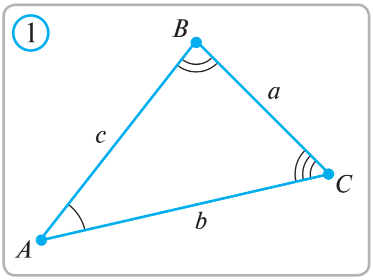
7*. ABC u'shmu'yeshliginde $\angle A = \alpha$, B ha'm C to'belerinen tu'sirilgen biyiklikleri bolsa sa'ykes tu'rde h_b ha'm h_c bolsa, u'shmu'yeshliktin' maydani'n tabi'n'.

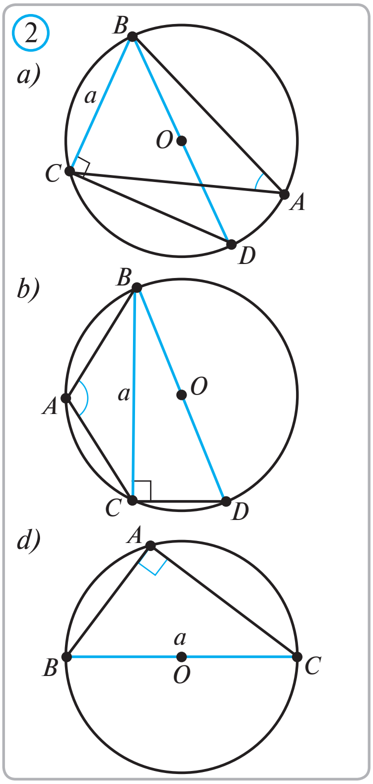
8*. ABC u'shmu'yeshliginde $AB = 8$ *sm*, $AC = 12$ *sm* ha'm $\angle A = 60^\circ$ bolsa, woni'n' AD bissektrisasi'n tabi'n' (*ko'rsetpe*: $S_{ABC} = S_{ABD} + S_{ADC}$).

31 SINUSLAR TEOREMASI'

Teorema. (Sinuslar teoremasi'). **U'shmu'yeshliktin' ta'repleri qarsisindag'i mu'yeshlerdin' sinuslarina proportcional.**

$\Delta ABC,\ AB=c,\ BC=a,\ CA=b$ (*1-su'wret*) $\Rightarrow$ $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$





Da'liyllew. U'shmu'yeshliktin' maydani'n mu'yeshtin' sinusi' arqali' tabi'w formulasi' boyi'nsha,

$$S=\frac{1}{2}ab\sin C,\quad S=\frac{1}{2}bc\sin A,\quad S=\frac{1}{2}ac\sin B.\quad (❖)$$

Bul ten'liklerdin' birinshi ekewi boyi'nsha

$$\frac{1}{2}ab\sin C=\frac{1}{2}bc\sin A,\ \text{demek}\ \frac{a}{\sin A}=\frac{c}{\sin C}.$$

Sonday-aq (❖) ten'liklerinin' birinshi ha'm u'shinshisinen $\dfrac{c}{\sin C}=\dfrac{b}{\sin B}$ ten'ligin paydayetemiz.

Solay yetip, $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$.

Teorema da'liyllendi.

1-ma'sele. ABC u'shmu'yeshliginde $AB=14\ dm$, $\angle A=30°$, $\angle C=65°$ (*1-su'wret*). BC ta'repin tabi'n'.

Sheshiliwi. Sinuslar teoremasi boyi'nsha

$\dfrac{AB}{\sin C}=\dfrac{BC}{\sin A}$. Wonnan

$$BC=\frac{AB\cdot\sin A}{\sin C}=\frac{14\cdot\sin 30°}{\sin 65°}\approx\frac{14\cdot 0{,}5}{0{,}9}\approx 7{,}78\ (dm).$$

Yesletpe: Trigonometriyali'q funkciyalardi'n' ma'nisleri arnawli' kalkulyator yaki kesteler ja'rdeminde tabi'ladi'. Bul jerde $\sin 65°\approx 0{,}9$ yekenligin sabaqli'qti'n' 153-betindegi kesteden ani'qladi'q.

Juwabi': 7,78 dm.

2-ma'sele. U'shmu'yeshliktin' ta'repinin' usi' ta'reptin' qarsi'si'ndag'i' mu'yeshinin' sinusi'na qatnasi', u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber diametrine

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$$

ten' yekenligin da'liyllen'. (*1-su'wret*).

Sheshiliwi: Ani'q, sinuslar teoremasi' boyi'nsha $\frac{a}{\sin A} = 2R$ ten'ligin da'liyllew jetkilikli yekenligi ko'rinip tur. U'sh jag'day boli'wi' mu'mkin:

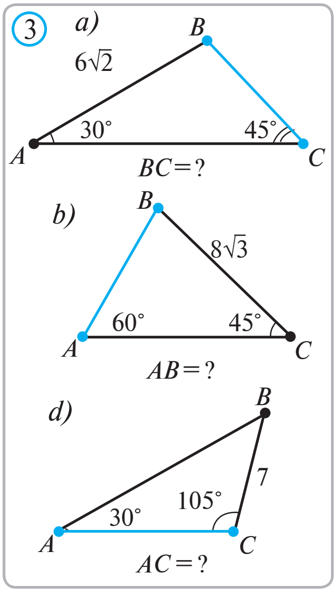
1-jag'day: $\angle A$ — su'yir mu'yesh (*2.a-su'wret*); 2-jag'day: $\angle A$ — dog'al mu'yesh (*2.b*-su'wret); 3-jag'day: $\angle A$ — tuwri' mu'yesh (*2.d-su'wret*).

1-jag'daydi' qaraymi'z: C ha'm D noqatlari'n tutasti'rami'z. BCD — tuwri' mu'yeshli u'shmu'yeshlik, sebebi $\angle BCD$ mu'yeshi BD diametrine uri'nadi'. ΔBCD da: $BC = BD \cdot \sin D = 2R\sin D$. Biraq, $\angle D = \angle A$, sebebi wolar bir BC dog'ag'a tirelgen ishley si'zi'lg'an mu'yeshler. Wonda

$$BC = 2R\sin A \qquad \text{yaki} \quad \frac{a}{\sin A} = 2R.$$

Qalg'an jag'daylardi' wo'z yerkin'izshe da'liyllen'. (*Ko'rsetpe*: $\angle D = 180° - \angle A$ yekenliginen, 3-jag'dayda $a = 2R$ yekenliginen paydalanin')

? Soraw, ma'sele ha'm tapsi'rmalar



1. U'shmu'yeshliktin' qa'legen ta'repinin' usi' ta'reptin' qarama-qarsi'si'ndag'i mu'yeshtin' sinusi'na qatnasi u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber diametrine ten' yekenligin 2-ma'selede keltirilgen 2-ha'm 3-jag'daylar ushi'n da'liyllen'.

2. 3-su'wrette berilgenler boyi'nsha, soralg'an kesindilerdi tabi'n'.

3. Yeger ABC u'shmu'yeshliginde:

a) $\sin A = 0{,}4$; $BC = 6$ *sm* ha'm $AB = 5$ *sm* bolsa, $\sin C$ ni

b) $\sin B = \frac{3}{5}$; $AC = 8$ *dm* ha'm $BC = 7$ *dm* bolsa, $\sin A$ ni

d) $\sin C = \frac{4}{7}$; $AB = 6$ *m* ha'm $AC = 8$ *m* bolsa, $\sin B$ ni tabi'n'.

4. U'shmu'yeshliktin' mu'yeshi 30° qa ten'. Woni'n' qarama-qarsisindag'i ta'rep 4,8 *dm*. U'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber radiusin yesaplan'.

5. U'shmu'yeshliktin' bir ta'repi u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber radiusina ten'. U'shmu'yeshliktin' usi' ta'repinin' qarama-qarsi'si'ndag'i mu'yeshin tabi'n'. Bunda yeki jag'daydi qarawg'a tuwra keliwine itibar berin'.

6. ABC u'shmu'yeshligi ushi'n $AB : BC : CA = \sin C : \sin A : \sin B$ ten'ligi orinli bolatug'inin tiykarlap berin'. $\sin A : \sin B : \sin C = 3 : 5 : 7$ ten'ligi duri's boliwi mu'mkin be?

7. Yeger ABC u'shmu'yeshliginde $BC = 20$ *m*, $AC = 13$ *m* ha'm $\angle A = 67°$ bolsa, u'shmu'yeshliktin' AB ta'repin, B ha'm C mu'yeshlerin tabi'n'.

8*. Yeger ABC u'shmu'yeshliginde $BC = 18$ *dm*, $\angle A = 42°$, $\angle B = 62°$ bolsa, u'shmu'yeshliktin' C mu'yeshin, AB ha'm AC ta'replerin tabi'n'.

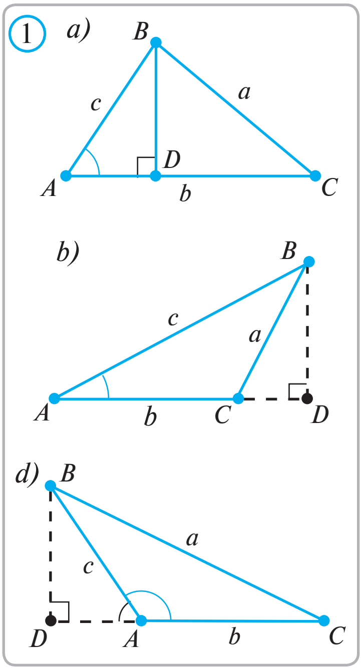
32 KOSINUSLAR TEOREMASI'

Tuwri' mu'yeshli u'shmu'yeshlikte tuwri' mu'yeshtin' qarsi'si'ndag'i' ta'rep (gipotenuza) tin' kvadrati' qalg'an ta'repler (katetler) din' kvadratlari'ni'n' qosi'ndi'si'na ten'.

Al, tuwri' bolmag'an mu'yesh ushi'n-she? To'mendegi teorema usi' tuwrali.

Teorema. (Kosinuslar teoremasi'). **U'shmu'yeshliktin' qa'legen ta'repinin' kvadrati', qalg'an yeki ta'repinin' kvadratlari'ni'n' qosi'ndi'si' usi yeki ta'rep penen wolar arasi'ndag'i' mu'yeshtin' kosinusi' ko'beymesinin' yeki yeselengen ayi'rmasi'na ten'.**

$\Delta ABC,\ AB=c,\ BC=a,\ CA=b$ (*1-su'wret*) ⇒ $a^2=b^2+c^2-2bc\cos A$



Da'liyllew. ABC u'shmu'yeshliginin' BD biyikligin ju'rgizemiz. D noqati' AC ta'repte (1.*a-su'wretler*) yaki woni'n' dawami'nda (1.*b*- ha'm 1.*d*-su'wretler) boli'wi' mu'mkin. Birinshi jag'daydi qaraymi'z. Tuwri' mu'yeshli BCD u'shmu'yeshlikte Pifagor teoremasi' boyi'nsha,

$$BC^2=BD^2+DC^2.$$

$DC=AC-AD$ bolg'ani' ushi'n:

$$BC^2=BD^2+(AC-AD)^2=BD^2+AC^2-2\cdot AC\cdot AD+AD^2.$$

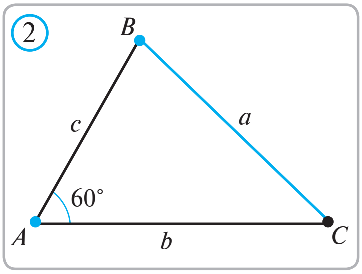
Tuwri' mu'yeshli ABD u'shmu'yeshliginde $BD^2+AD^2=AB^2$ ha'm $AD=AB\cos A$ yekenligin yesapqa ali'p, keyingi ten'likten

$$BC^2=AB^2+AC^2-2AB\cdot AC\cdot\cos A,$$

yag'niy $a^2=b^2+c^2-2bc\cos A$ ten'ligine iye bolami'z. ***Teorema da'liyllendi.***

1. b-su'wrette su'wretlengen jag'dayda $DC=AD-AC$, 1.*d*-su'wrette su'wretlengen jag'dayda $DC=AD+AC$ ha'm $\cos(180°-A)=-\cos A$ ten'liklerinen paydalani'p, kosinuslar teoremasi'n wo'z betin'izshe da'liyllen'.

Yeslertpe. Kosinuslar teoremasi' Pifagor teoremasi'ni'n' uliwmalasqan tu'ri boladi'. $\angle A=90°$ bolg'anda ($\cos 90°=0$ bolg'ani' ushi'n) kosinuslar teoremasi'nan Pifagor teoremasi' kelip shig'adi.

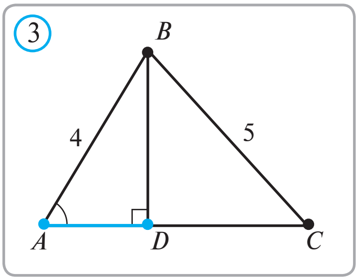


1-ma'sele. ABC u'shmu'yeshliginde $AB=6sm$, $AC=7sm$, $\angle A=60°$ (*2-su'wret*). BC ta'repin tabi'n'.

Sheshiliwi. Kosinuslar teoremasiboyi'nsha, $a^2 = b^2 + c^2 - 2bc\cos A$ yaki $BC^2 = AC^2 + AB^2 - 2AC \cdot AB \cdot \cos A$ bolg'anligi ushi'n,

$$BC^2 = 7^2 + 6^2 - 2 \cdot 7 \cdot 6 \cdot \cos 60° = 49 + 36 - 84 \cdot \frac{1}{2} = 43,$$

ya'gniy $BC = \sqrt{43}$ *sm*. ***Juwabi':*** $\sqrt{43}$ *sm*.



Sonday-aq, kosinuslar teoremasi'nan paydalani'p, ta'repleri belgili bolg'an u'shmu'yeshliktin' mu'yeshlerin tabiw mu'mkin:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}. \qquad (1)$$

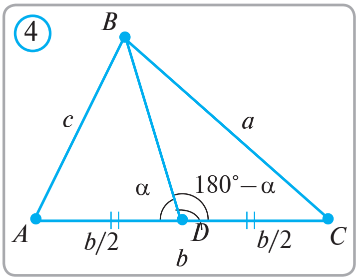
2-ma'sele. ABC u'shmu'yeshliginin' ta'repleri $a = 5$ *m*, $b = 6$ *m* ha'm $c = 4$ *m*. Kishi ta'reptin' u'lken ta'reptegi proekciyasin tabi'n' *(3-su'wret)*.

Sheshiliwi. (1) formula tiykarinda $\cos A$ ni' tabami'z:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{6^2 + 4^2 - 5^2}{2 \cdot 6 \cdot 4} = \frac{9}{16}.$$

Tuwri' mu'yeshli ABD u'shmu'yeshliginde $AD = AB \cdot \cos A$ bolg'ani' ushi'n $AD = 4 \cdot \frac{9}{16} = 2{,}25$ (m).

Juwabi': 2,25 *m*.



Soraw, ma'sele ha'm tapsi'rmalar

1. Kosinuslar teoremasi'n 1.b-ha'm 1.d-su'wrette su'wretlengen jag'daylarda da'liyllen'.
2. ABC u'shmu'yeshliginde: a) $AC = 3$ *sm*, $BC = 4$ *sm* ha'm $\angle C = 60°$ bolsa, AB ni'; b) $AB = 4$ *m*, $BC = 4\sqrt{2}$ *m* ha'm $\angle B = 45°$ bolsa AC ni'; d) $AB = 7$ *dm*, $AC = 6\sqrt{3}$ *dm* ha'm $\angle A = 150°$ bolsa BC ni tabi'n'.
3. Ta'repleri 5 *sm*, 6 *sm*, 7 *sm* bolg'an u'shmu'yeshliktin' mu'yeshlerinin' kosinuslari'n tabi'n'.
4. ABC u'shmu'yeshliginde $AB = 10$ *sm*, $BC = 12$ *m*, ha'm $\sin B = 0{,}6$ bolsa, AC ta'repin tabi'n'.
5. Parallelogrammni'n' diagonallari' 10 *sm*, 12 *sm* ha'm wolar arasi'ndag'i mu'yesh $60°$ qa ten'. Parallelogrammnin' ta'replerin tabi'n'.
6. Ta'repleri 5 *sm* ha'm 7 *sm* bolg'an parallelogrammni'n' bir mu'yeshi $120°$ qa ten'. Woni'n' diagonallari'n tabi'n'.

7*. Ta'repleri a, b, c bolg'an ABC u'shmu'yeshliginin' BD medianasi $BD = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}$ formulasi menen yesaplanatug'i'ni'n da'liyllen' (*4-su'wret*).

8*. Ta'repleri 6 *m*, 7 *m* ha'm 8 *m* bolg'an u'shmu'yeshliktin' medianalari'n tabi'n'.

9. 3-ma'seledegi u'shmu'yeshlik bissektrisalarin tabi'n'.
10. 3-ma'seledegi u'shmu'yeshliktin' biyikliklerin tabi'n'.

33 SINUSLAR HA'M KOSINUSLAR TEOREMALARI'NI'N' AYI'RI'M QOLLANI'WLARI'

Aldi'n'g'i' sabaqlarda da'liyllengen sinuslar ha'm kosinuslar teoremalari'nan u'shmu'yeshliklerge tiyisli ha'r qi'yli' ma'selelerdi sheshiwde na'tiyjeli paydalani'w mu'mkin. Bul sabaqta bul teoremalardi'n' ayi'ri'm qollani'wlari'na toqtap wo'temiz.

1. Kosinuslar teoremasi' u'shmu'yeshliktin' mu'yeshlerin tappastan, wonin' mu'yeshleri boyi'nsha tu'rin (su'yir, dog'al yaki tuwri' mu'yeshli yekenligin) ani'qlawg'a imkaniyat beredi. Haqiyqattan da,

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

formulasi'nda

1) yeger $b^2+c^2>a^2$ bolsa, $\cos A>0$. Demek, **A — su'yir mu'yesh;**

2) yeger $b^2+c^2=a^2$ bolsa, $\cos A=0$. Demek, **A — tuwri' mu'yesh;**

3) yeger $b^2+c^2<a^2$ bolsa, $\cos A<0$. Demek, **A — dog'al mu'yesh;**

$b^2+c^2=a^2$ ten'ligi yaki $b^2+c^2<a^2$ ten'sizligi a—u'shmu'yeshliktin' yen' u'lken ta'repi bolg'an jag'dayda g'ana ori'nlanadi. Demek, u'shmu'yeshliktin' tuwri' yaki dog'al mu'yeshi woni'n' yen' u'lken ta'repinin' qarama-qarsi'si'nda jatadi.

U'shmu'yeshtin' yen' u'lken ta'repinin' shamasi'na qarap, bul u'shmu'yeshliktin' qanday (su'yir, dog'al, tuwri' mu'yeshli) u'shmu'yeshlik yekenligi haqqindag'i' juwmaqqa keliw mu'mkin.

1-ma'sele. Ta'repleri 5 *m*, 6 *m* ha'm 7 *m* bolg'an u'shmu'yeshliktin' mu'yeshlerin tappastan, wonin' tu'rin ani'qlan'.

Sheshiliwi. Yen' u'lken mu'yesh qarsi'si'nda yen' u'lken ta'rep jatadi'. Soni'n' ushi'n, yeger $a=7$, $b=6$, $c=5$ bolsa, $\angle A$ yen' u'lken mu'yesh boladi'.

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{36+25-49}{2\cdot6\cdot5}=\frac{12}{60}=\frac{1}{5}>0.$$

Demek, A—su'yir mu'yesh, berilgen u'shmu'yeshlik bolsa su'yir mu'yeshli boladi'.

2. U'shmu'yeshliktin' maydani'n woni'n' yeki ta'repi ha'm wolar arasi'ndag'i' mu'yeshi arqali' yesaplaw formulasi',

$$S=\frac{1}{2}bc\sin A$$

ha'm $\sin A=\frac{a}{2R}$ formulalardan u'shmu'yeshlik maydani'n yesaplaw ushi'n

$$S=\frac{abc}{4R}$$

formulani ha'm u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusin yesaplaw ushi'n

$$R=\frac{abc}{4S}$$

formulasi'n payda yetemiz.

2-ma'sele. Ta'repleri $a=5$, $b=6$, $c=10$ bolg'an u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber radiusi'n tabi'n'.

Sheshiliwi. Geron formulasi'nan paydalani'p, u'shmu'yeshliktin' maydani'n tabami'z:

$$p=\frac{a+b+c}{2}=\frac{5+7+10}{2}=11,$$

$$S=\sqrt{p(p-a)(p-b)(p-c)}=\sqrt{11(11-5)(11-7)(11-10)}=\sqrt{11\cdot6\cdot4}=\sqrt{264}\approx16{,}3.$$

Wonda, $R=\dfrac{abc}{4S}\approx\dfrac{5\cdot7\cdot10}{4\cdot16{,}3}\approx5{,}4.$ ***Juwabi':*** $\approx5{,}4$.

Soraw, ma'sele ha'm tapsi'rmalar

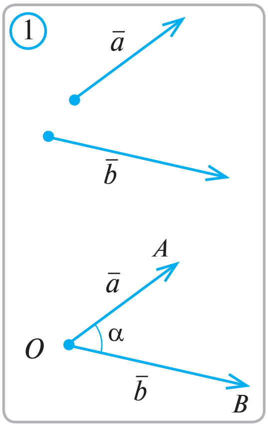
1. Yeger $AB=7$ *sm*, $BC=8$ *sm*, $CA=9$ *sm* bolsa, ABC u'shmu'yeshliginin' yen' u'lken ha'm yen' kishi mu'yeshin tabi'n'.
2. Yeger ABC u'shmu'yeshliginde $\angle A=47°$, $\angle B=58°$ bolsa, u'shmu'yeshliktin' yen' u'lken ha'm yen' kishi ta'replerin ani'qlan'.
3. U'shmu'yeshliktin' u'sh ta'repi berilgen:
   a) $a=5$, $b=4$, $c=4$; b) $a=17$, $b=8$, $c=15$; d) $a=9$, $b=5$, $c=6$.
   U'shmu'yeshliktin' su'yir mu'yeshli, tuwri' mu'yeshli yaki dog'al mu'yeshli yekenligin ani'qlan'.
4. Ta'repleri a) 13, 14, 15; b) 15, 13, 4; d) 35, 29, 8; e) 4, 5, 7 bolg'an u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi'n tabi'n'.
5. ABC u'shmu'yeshliginin' AB ta'repinde D noqati belgilengen. CD kesindisi AC ha'm BC kesindilerinin' keminde birewinen kishi yekenligin da'liyllen'.
6. U'shmu'yeshliktin' u'lken mu'yeshi qarsi'si'nda u'lken ta'repi jatatug'i'ni'n da'liyllen'.
7. U'shmu'yeshliktin' u'lken ta'repi qarsi'si'nda u'lken mu'yeshi jatatug'i'ni'n da'lillen'.

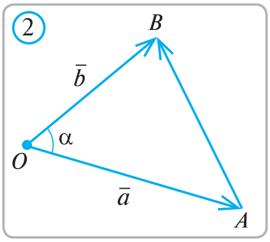
8*. ABC u'shmu'yeshliginin' CD medianasi' ju'rgizilgen. Yeger $AC>BC$ bolsa, ACD mu'yeshi BCD mu'yeshinen kishi bolatug'i'ni'n da'liyllen'.

9. To'mendegi su'wretke sa'ykes ma'sele du'zin'.

34 YEKI VEKTOR ARASI'NDAG'I MU'YESHTI YESAPLAW

Vektorlardin' skalyar ko'beymesi tu'sinigi ha'm qa'siyetleri menen 8-klasta tani'sqan yedin'iz. Yeki vektordi'n' skalyar ko'beymesi wolardi'n' koordinatalari arqali' an'lati'lg'an yedi. To'mende kosinuslar teoremasi' ja'rdeminde vektorlardi'n' skalyar ko'beymesi ushi'n ja'ne bir a'hmiyetli formula keltirip shi'g'ari'ladi. Bunda skalyar ko'beyme vektorlardi'n' uzi'nli'g'i' ha'm wolar arasi'ndag'i mu'yesh arqali' an'lati'ladi'.





Nol vektordan wo'zgeshe $\bar{a}$ ha'm $\bar{b}$ vektorlari' berilgen bolsi'n. Qa'legen O noqattan $\overline{OA}=\bar{a}$ ha'm $\overline{OB}=\bar{b}$ vektorlari'n qoyami'z. $\bar{a}$ ha'm $\bar{b}$ vektorlari' arasi'ndag'i' mu'yesh dep AOB mu'yeshke ayti'ladi' (*1-su'wret*). Bir qi'yli' bag'itlang'an vektorlar arasi'ndag'i' mu'yesh 0° qa ten' dep yesaplanadi'.

Yeger yeki vektor arasi'ndag'i mu'yesh 90° qaten' bolsa, wolar ***perpendikulyar*** delinedi.

Yesletip wo'temiz:

1. $\bar{a}(a_1;a_2)$ vektori'ni'n' uzi'nli'g'i':

$$|\bar{a}|=\sqrt{a_1^2+a_2^2}.$$

2. $\bar{a}(a_1;a_2)$ ha'm $\bar{b}(b_1;b_2)$ vektorlari'ni'n' skalyar ko'beymesi,

$$\bar{a}\bar{b}=a_1b_1+a_2b_2$$

formulalari' menen ani'qlanatug'in yedi.

Kollinear yemes $\bar{a}$ ha'm $\bar{b}$ vektorlari'n qaraymi'z. Qa'legen O noqatinan $\overline{OA}=\bar{a}$ ha'm $\overline{OB}=\bar{b}$ vektorlari'n qaraymi'z (*2-su'wret*). $\angle AOB=\alpha$ bolsi'n. Wonda, bir ta'repten kosinuslar teoremasi' boyi'nsha,

$$AB^2=OA^2+OB^2-2\cdot OA\cdot OB\cos\alpha. \qquad (1)$$

Yekinshi ta'repten

$$AB^2=|\overline{AB}|^2=|\overline{OB}-\overline{OA}|^2=(\overline{OA}-\overline{OB})^2=\overline{OA}^{\,2}+\overline{OB}^{\,2}-2\overline{OA}\cdot\overline{OB}. \qquad (2)$$

Demek, (1) ha'm (2) boyi'nsha, $\overline{OA}\cdot\overline{OB}=OA\cdot OB\cos\alpha$ yaki $\bar{a}\bar{b}=|\bar{a}|\cdot|\bar{b}|\cos\alpha$.

Na'tiyje. Nol vektordan wo'zgeshe $\bar{a}(a_1;a_2)$ ha'm $\bar{b}(b_1;b_2)$ vektorlari' arasi'ndag'i' α mu'yeshi ushi'n

$$\cos\alpha=\frac{\bar{a}\bar{b}}{|\bar{a}|\cdot|\bar{b}|} \qquad \text{yaki} \qquad \cos\alpha=\frac{a_1b_1+a_2b_2}{\sqrt{a_1^2+a_2^2}\sqrt{b_1^2+b_2^2}}$$

formulasi' orinli'.

Ma'sele. $\bar{a}(1;2)$ ha'm $\bar{b}(4;-2)$ vektorlari' arasi'ndag'i' mu'yeshti tabi'n'.

Sheshiliwi. Berilgen vektorlar arasi'ndag'i mu'yeshti α dep belgilesek, formula boyi'nsha,

$$\cos\alpha=\frac{1\cdot4+2\cdot(-2)}{\sqrt{1^2+2^2}\cdot\sqrt{4^2+(-2)^2}}=\frac{4-4}{\sqrt{5}\cdot\sqrt{20}}=0. \qquad \alpha=90^\circ.$$ ***Juwabi':*** 90°.

Soraw, ma'sele ha'm tapsi'rmalar

1. Yeger $\bar{a}$ ha'm $\bar{b}$ vektorlari ushi'n **a)** $a=4$, $b=5$, $\alpha=30^\circ$; **b)** $a=8$, $b=7$, $\alpha=45^\circ$; d) $a=2{,}4$, $b=10$, $\alpha=60^\circ$; e) $a=0{,}8$, $b=\frac{1}{4}$, $\alpha=40^\circ$ bolsa, bul vektorlardin' skalyar ko'beymesin tabi'n' (bul jerde α — $\bar{a}$ ha'm $\bar{b}$ vektorlari' arasi'ndag'i' mu'yesh).

2. a) $\bar{a}(\frac{1}{4};-1)$ ha'm $\bar{b}(2;3)$; b) $\bar{a}(-5;6)$ ha'm $\bar{b}(6;5)$; d) $\bar{a}(1{,}5;2)$ ha'm $\bar{b}(4;-2)$ vektorlari'ni'n' skalyar ko'beymesin yesaplan' ha'm wolar arasi'ndag'i' mu'yeshti tabi'n'.

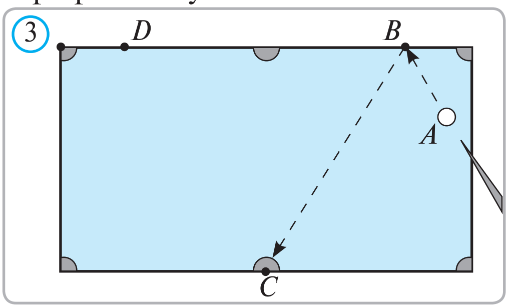
3. *ABCD* rombi'ni'n' diagonallari *O* noqatindakesilisedi ha'm bunda $BD=AB=4$ *sm*.

a) $\overline{AB}$ ha'm $\overline{AD}$; b) $\overline{AB}$ ha'm $\overline{AC}$; d) $\overline{AD}$ ha'm $\overline{DC}$; e) $\overline{AC}$ ha'm $\overline{OD}$ vektorlari'ni'n' skalyar ko'beymesin ha'm bul vektorlardi'n' arasi'ndag'i' mu'yeshti tabi'n'.

4. Nol vektordan wo'zgeshe $\bar{a}$ ha'm $\bar{b}$ vektorlari' berilgen bolsi'n. $\bar{a}\cdot\bar{b}=0$ bolg'anda bul vektorlar perpendikulyar bolatug'i'ni'n ha'm kerisinshe a ha'm b vektorlari' perpendikulyar bolsa, $a\cdot b=0$ bolatug'i'ni'n da'liyllen'.

5*. x tin' qanday ma'nisinde a) $\bar{a}(4;5)$ ha'm $\bar{b}(x;6)$; **b)** $\bar{a}(x;1)$ ha'm $\bar{b}(3;2)$; **d)** $\bar{a}(0;-3)$ ha'm $\bar{b}(5;x)$ vektorlari wo'z ara perpendiculyar boladi?

6. $\bar{a}(3;3)$, $\bar{b}(2;-2)$, $\bar{c}(-1;-4)$ ha'm $\bar{d}(-4;1)$ vektorlari arasi'nan wo'z-ara perpendiculyar jupli'qlari'n tabi'n'.

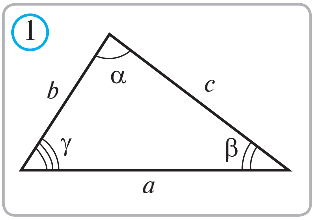


7. $a^2=|a|^2$ ten'ligin da'liyllen'.

8*. Bilyard oyi'ni'nda *A* noqatta turg'an shar soqqi'dan keyin bilyard stoli' jaqlawi'na *B* noqatta uri'ldi' ha'm bag'dari'n wo'zgertip *C* noqattag'i sebetshege tu'sti (*3-su'wret*). Yeger $AB=40$ *sm*, $BC=150$ *sm* ha'm $\angle ABD=120^\circ$ bolsa $\overline{AB}\cdot\overline{BC}$ skalyar ko'beymeni tabin.

9. $F(-3,\ 4)$ ku'sh ta'siri asti'nda noqat $A(5,-1)$ jag'daydan $B(2,\ 1)$ jag'dayg'a wo'tti. Bul processte qanday jumis orinlandi?

35 U'SHMU'YESHLIKLERDI SHESHIW



U'shmu'yeshliktin' ta'replerin *a, b, c* menen, al bul ta'replerdin' qarama-qarsi'si'ndag'i' mu'yeshlerdi sa'ykes tu'rde α, β, γ menen belgileymiz (*1-su'wret*). U'shmu'yeshliktin' ta'replerin ha'm mu'yeshlerin bir atama menen woni'n' **elementleri** dep ataydi'.

U'shmu'yeshlikti ani'qlawshi' beri'lgen elementi boyi'nsha, woni'n' qalg'an elementin tabiw **u'shmu'yeshlikti sheshiw** dep ayti'ladi'.

1-ma'sele. (U'shmu'yeshlikti berilgen bir ta'repi ha'm wog'an irgeles jatqan mu'yeshleri boyi'nsha sheshiw). Yeger u'shmu'yeshlikte $a=6$, $\beta=60^\circ$ ha'm $\gamma=45^\circ$ bolsa, woni'n' u'shinshi mu'yeshin ha'm qalg'an yeki ta'repin tabi'n'.

Sheshiliwi. **1.** U'shmu'yeshliktin' mu'yeshlerinin' qosi'ndi'si' 180° bolg'ani' ushi'n

$$\alpha=180^\circ-\beta-\gamma=180^\circ-60^\circ-45^\circ=75^\circ.$$

Sinuslar teoremasi'nan payadalani'p, qalg'an yeki ta'repin tabami'z:

2. $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}$ ten'likten $b=a\cdot\frac{\sin\beta}{\sin\alpha}=6\cdot\frac{\sin 60^\circ}{\sin 75^\circ}\approx 6\cdot\frac{0,8660}{0,9659}\approx 5,3794\approx 5,4.$

(sin60° ha'm sin75° ma'nisleri mikrokalkulyatordayaki sabaqli'qti'n' 153-betindegi kesteden tawi'p qoyi'ldi').

3. $\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}$ ten'liginen $c=a\cdot\frac{\sin\gamma}{\sin\alpha}=6\cdot\frac{\sin 45^\circ}{\sin 75^\circ}\approx 6\cdot\frac{0,7071}{0,9659}\approx 4,3924\approx 4,4.$

Juwabi': $\alpha=75^\circ$; $\beta\approx 5,4$; $c\approx 4,4$.

2-ma'sele. (U'shmu'yeshlikti berilgen yeki ta'repi ha'm wolar arasi'ndag'i mu'yeshi boyi'nsha sheshiw). Yeger u'shmu'yeshlikte $a=6$, $b=4$ ha'm $\gamma=120^\circ$ bolsa, woni'n' u'shinshi *c* ta'repin ha'm qalg'an mu'yeshlerin tabi'n'.

Sheshiliwi. 1. Kosinuslar teoremasi'nan paydalani'p, u'shmu'yeshliktin' u'shinshi *c* ta'repin tabami'z:

$$c=\sqrt{a^2+b^2-2ab\cos\gamma}=\sqrt{36+16-2\cdot 6\cdot 4\cdot(-0,5)}=\sqrt{76}\approx 8,7.$$

2. Yendi, u'shmu'yeshliktin' u'sh ta'repin bilgen halda, kosinuslar teoremasi'nan paydalani'p, u'shmu'yeshliktin' qalg'an mu'yeshlerin tabami'z:

$$\cos\alpha=\frac{b^2+c^2-a^2}{2bc}=\frac{4^2+76-6^2}{2\cdot 4\cdot\sqrt{76}}\approx 0,8046.$$

$\cos\alpha\approx 0,8046$ ten'ligi tiykarinda α mu'yeshinin' ma'nisin153-bettegi kesteden ani'qlaymi'z (α — su'yir mu'yesh): $\alpha\approx 36^\circ$.

3. $\beta=180^\circ-\alpha-\gamma\approx 180^\circ-(36^\circ+120^\circ)=24^\circ$. ***Juwabi':*** $c\approx 8,7$; $\alpha\approx 36^\circ$, $\beta\approx 24^\circ$.

***3-ma'sele*. (U'shmu'yeshlikti berilgen u'sh ta'repi boyi'nsha sheshiw).** Yeger u'shmu'yeshlikte $a=10$, $b=6$ ha'm $c=13$ bolsa, woni'n' mu'yeshlerin tabi'n'.

Sheshiliwi. **1.** U'shmu'yeshliksu'yir mu'yeshli boliwi' yaki bolmasli'g'i'n u'lken ta'reptin' qarama-qarsi'si'ndag'i' mu'yeshtin' kosinusi'ni'n' belgisine qarap ani'qlaymi'z:

$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{100+36-169}{2\cdot 10\cdot 6}=-\frac{33}{120}\approx -0{,}275<0.$$

Demek, C — dog'al mu'yesh yeken. Buni' 153-bettegi kesteden C mu'yeshinin' shamasi'n ani'qlawda yesapqa alami'z. Kesteden kosinusi' 0,275 ge ten' mu'yesh $\angle C_1=74°$ yekenligin tabami'z. Wonda $\cos(180°-\alpha)=-\cos\alpha$ formulasi' boyi'nsha,

$$\angle C=180°-\angle C_1=180°-74°=106°.$$

2. Sinuslar teoremasi' boyi'nsha, $\frac{a}{\sin A}=\frac{c}{\sin C}$. Bunnan

$\sin A=\frac{a\cdot\sin C}{c}=\frac{10\cdot\sin 106°}{13}=\frac{10\cdot\sin 74°}{13}\approx\frac{10\cdot 0{,}9615}{13}\approx 0{,}7396.$ A — su'yir mu'yesh bolg'ani' ushi'n 153-bettegi kesteden $\angle A\approx 47°$ yekenligin ani'qlaymiz.

3. $\angle B\approx 180°-(106°+47°)=26°$. ***Juwabi':*** $\angle A\approx 47°$, $\angle B\approx 26°$, $\angle C\approx 106°$.

Soraw, ma'sele ha'm tapsi'rmalar

1. U'shmu'yeshliktin' bir ta'repi ha'm wog'an irgeles jatqan yeki mu'yeshi berilgen:
a) $a=5$ *sm*, $\beta=45°$, $\gamma=45°$; b) $a=20$ *sm*, $\alpha=75°$, $\beta=60°$;
d) $a=35$ *sm*, $\beta=40°$, $\gamma=120°$; **e)** $b=12$ *sm*, $\alpha=36°$, $\beta=25°$.
U'shmu'yeshliktin' to'besinin' mu'yeshin ha'm qalg'an yeki ta'repin tabi'n'.

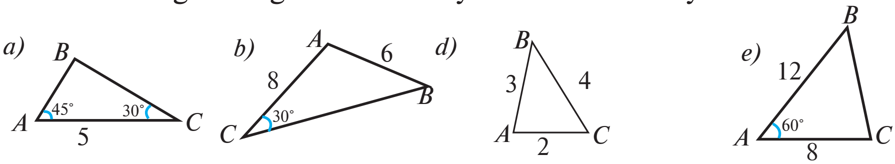
2. U'shmu'yeshliktin' yeki ta'repi ha'm wolar arasi'ndag'i' mu'yeshi berilgen:
a) $a=6$, $b=4$, $\gamma=60°$; b) $a=14$, $b=43$, $\gamma=130°$;
d) $b=17$, $c=9$, $\alpha=85°$; **e)** $b=14$, $c=10$, $\alpha=145°$.
U'shmu'yeshliktin' qalg'an mu'yeshlerin ha'm u'shinshi ta'repin tabi'n'.

3. U'shmu'yeshliktin' u'sh ta'repi berilgen: a) $a=2$, $b=3$, $c=4$; b) $a=7$, $b=2$, $c=8$; d) $a=4$, $b=5$, $c=7$; **e)** $a=15$, $b=24$, $c=18$.
U'shmu'yeshliktin' mu'yeshlerin tabi'n'.

4. U'shmu'yeshliktin' yeki ta'repi ha'm bul ta'replerden birinin' qarama-qarsisindag'i mu'yeshi berilgen. U'shmu'yeshliktin' qalg'an ta'repi ha'm mu'yeshlerin tabi'n': a) $a=12$, $b=5$, $\alpha=120°$; b) $a=27$, $b=9$, $\alpha=138°$; d) $b=2$, $c=2$, $\alpha=60°$; **e)** $b=6$, $c=8$, $\alpha=30°$.

5. 2-su'wrette berilgen mag'li'wmatlar tiykarinda u'shmu'yeshlikti sheshin'.

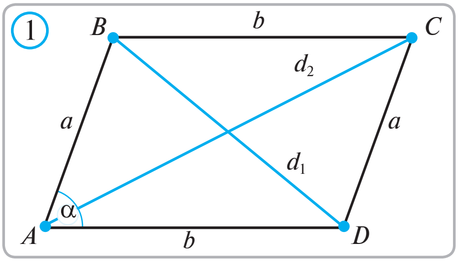
2



36 MA'SELELERDI SHESHIW

1-ma'sele. Parallelogramm diagonallari'ni'n' kvadratlari'ni'n' qosi'ndi'si' ta'replerinin' kvadratlari'ni'n' qosi'ndi'si'nin' ten' yekenligin da'liyllen'.

ABCD — parallelogramm, $AB=a$, $AD=b$, $BD=d_1$, $AC=d_2$ (1-su'wret). ⇒ $d_1^2+d_2^2=2(a^2+b^2)$



Sheshiliwi. *ABCD* parallelogrammni'n' *A* mu'yeshi α g'a ten' bolsi'n. Wonda $\angle B=180°-\alpha$. *ABD* ha'm *ABC* u'shmu'yeshliklerine kosinuslar teoremasi'n qollanami'z (*1-su'wret*):

$$d_1^2=a^2+b^2-2ab\cos\alpha, \qquad (1)$$

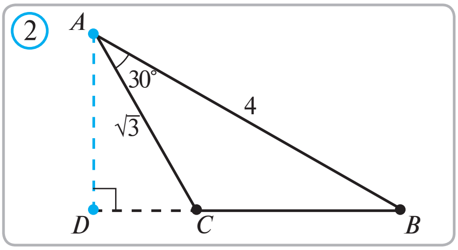
$$d_2^2=a^2+b^2-2ab\cos(180°-\alpha).$$

$\cos(180°-\alpha)=-\cos\alpha$ ten'ligin yesapqa alsaq,

$$d_2^2=a^2+b^2+2ab\cos\alpha. \qquad (2)$$

(1) ha'm (2) ten'liklerinin' sa'ykes ma'nislerin qosip $d_1^2+d_2^2=2(a^2+b^2)$ ten'ligin payda yetemiz.

2-ma'sele. *ABC* u'shmu'yeshliginde $\angle A=30°$, $AB=4$, $AC=\sqrt{3}$ bolsa, u'shmu'yeshliktin' *A* to'besinen tu'sirilgen *AD* biyikligin tabi'n' (*2-su'wret*).



Sheshiliwi. 1) Kosinuslar teoremasi'nan paydalani'p, u'shmu'yeshliktin' *BC* ta'repin tabami'z:

$$BC^2=AB^2+AC^2-2AB\cdot AC\cdot\cos A=$$
$$=4^2+(\sqrt{3})^2-2\cdot4\cdot\sqrt{3}\cdot\cos30°=7, \qquad BC=\sqrt{7}.$$

2) Yendi u'shmu'yeshliktin' maydani'n tabami'z:

$$S=\frac{1}{2}\cdot AB\cdot AC\cdot\sin A=\frac{1}{2}\cdot\sqrt{3}\cdot4\cdot\sin30°=\sqrt{3}.$$

3)Tabilg'anlardan paydalani'p, u'shmu'yeshliktin' *AD* biyikligin tabami'z:

$S=\frac{1}{2}\cdot BC\cdot AD$ formuladan $AD=\frac{2S}{BC}=\frac{2\sqrt{3}}{\sqrt{7}}=\frac{2\sqrt{21}}{7}$. ***Juwabi':*** $\frac{2\sqrt{21}}{7}$.

3-ma'sele. Aydawshi' jol ha'reketi qag'iydalari'n buzi'p, saat 12^{00} de ko'shenin' *A* noqati'nan Almazar ko'shesine qarap buri'ldi' ha'm 140 *km/saat* tezlikte ha'reketin dawam yetti *(3-su'wret)*. Saat 12^{00} de MAI xizmetkeri *B* noqati'nan taslaq jol boylap 70 *km/saat* tezlikte qag'i'yda buzg'an aydawshinin' jolin kesip shi'g'i'w ushi'n jolg'a shi'qti'. MAI xizmetkeri kesilispede yag'ni'y C noqati'nda qag'iyda buzi'wshi'ni' toqtati'p qala ala ma?

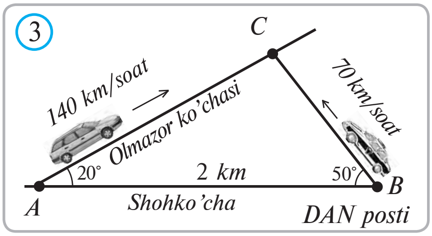
Sheshiliwi: ABC u'shmu'yeshliginde

$$\angle C=180^\circ-(\angle A+\angle B)=180^\circ-(20^\circ+50^\circ)=180^\circ-70^\circ=110^\circ.$$

1. Almazar ko'shesindegi joldin' AC bo'liminin' uzi'nli'g'i'n tabami'z: Sinuslar teoremasi boyi'nsha, $\frac{AC}{\sin B}=\frac{AB}{\sin C}$. Bul ten'likten $AC=\frac{AB\cdot\sin B}{\sin C}=\frac{2\cdot\sin 50^\circ}{\sin 110^\circ}=$ $=\frac{2\cdot\sin 50^\circ}{\sin(90^\circ+20^\circ)}=\frac{2\cdot\sin 50^\circ}{\cos 20^\circ}\approx\frac{2\cdot 0{,}766}{0{,}940}=\frac{1{,}532}{0{,}94}\approx 1{,}630$ (km). Bul joldi qag'iyda buziwshi aydawshi $\frac{1{,}630\ km}{140\ km/saat}\approx 0{,}0116\ saat=0{,}012\cdot 3600\ sekund\approx 42\ sekundta$ basip wo'tedi

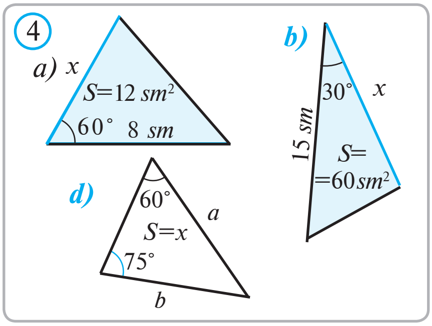
2. Yendi taslaq joldin' BC bo'legi uzi'nli'g'i'ni tabami'z: sinuslar teoremasi boyi'nsha, $\frac{BC}{\sin A}=\frac{AC}{\sin B}$. Bul ten'likten $BC=\frac{AC\cdot\sin A}{\sin B}=\frac{2\cdot\sin 20^\circ}{\sin 50^\circ}=\frac{2\cdot 0{,}342}{0{,}766}\approx 0{,}893$ (km).

Bul joldi MAI xizmetkeri $\frac{0{,}893\ km}{70\ km/saat}\approx 0{,}0128\ saat = 0{,}0128\cdot 3600\ sekund \approx$ $\approx 46\ sekundta$ basip wo'tedi. Demek, C kesispesine MAI xizmetkeri aydawshidan keshirek jetip keledi yeken. ***Juwabi':*** Yaq .



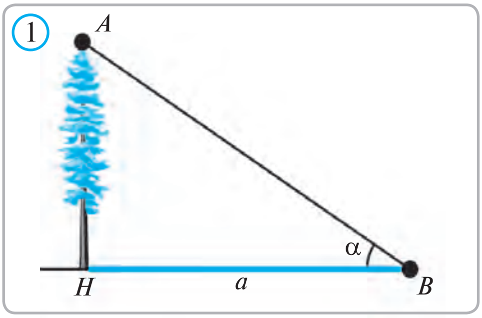
Soraw, ma'sele ha'm tapsi'rmalar

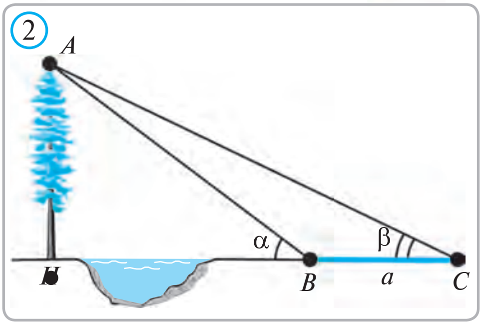
1. 4-su'wrettegi mag'liwmatlar boyi'nsha x tin' ma'nisin tabi'n'.
2. ABC u'shmu'yeshliginin' CD biyikligi 4 m. Yeger $\angle A=45^\circ$, $\angle B=30^\circ$ bolsa, u'shmu'yeshliktin' ta'replerin tabi'n'.
3. Bir noqatqa shamasi birdey bolg'an yeki ku'sh qoyi'lg'an (*5-su'wret*). Yeger bul ku'shlerdin' bag'itlari arasi'ndag'i mu'yesh 60°, bul ku'shlerdin' ten' ta'sir yetiwshisi 150 kg bolsa, bul ku'shlerdin' shamasin tabi'n'.
4. U'shmu'yeshliktin' yeki ta'repi 7 dm ha'm 11 dm, u'shinshi ta'repine tu'sirilgen medianasi bolsa 6 dm. U'shmu'yeshliktin' u'shinshi ta'repin tabi'n'.

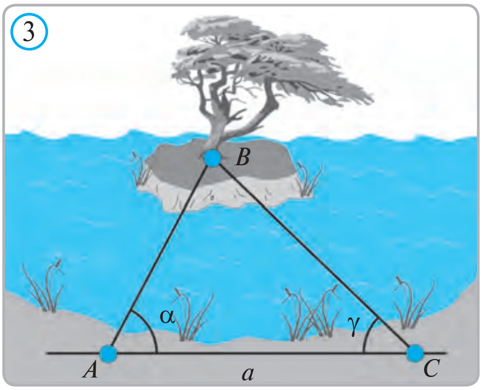


5. Ta'repleri 6 sm ha'm 8 sm bolg'an parallelogrammnin' bir diagonali *12 sm* bolsa, woni'n' yekinshi diagonali'n tabi'n'.
6. U'shmu'yeshliktin' 17 sm ge ten' ta'repi qarsisindag'i mu'yeshi 60° qa ten'. U'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusin tabi'n'.
7. Ten' qaptalli trapeciyanin' kishi ultani qaptal ta'repine ten', u'lken ultani bolsa 20 sm. Yeger trapeciyanin' bir mu'yeshi 120° bolsa, woni'n' perimetrin tabi'n'.

37 U'SHMU'YESHLIKLERDI SHESHIWDIN' A'MELIY ISTE QOLLANI'LI'WI'







1. Biyiklikti wo'lshew. Aytayiq, nenin'dur (misali', terektin') AH biyikligin wo'lshew za'ru'r bolsi'n (*1-su'wret*).

a) Bunin' ushi'n B noqati'n belgileymiz ha'm BH aralig'i' a ni' ha'm HBA mu'yeshi α ni wo'lsheymiz. Wonda, tuwri' mu'yeshli ABH u'shmu'yeshliginde

$$AH=BH\,\text{tg}\alpha=a\,\text{tg}\alpha.$$

b) Yeger biyikliktin' ultani' H noqati' bari'p bolmaytug'in noqat bolsa(*2-su'wret*), joqari'dag'i' usil menen AH biyikligin ani'qlay almaymiz. Wonda, to'mendegishe jol tutami'z:

1) H noqati' menen bir tuwri'da jatqan B ha'm C noqatlari'n belgileymiz;

2) BC arali'qti' wo'lshep a ni' tabami'z;

3) ABH ha'm ACH mu'yeshlerin wo'lshep $\angle ABH=\alpha$ ha'm $\angle ACH=\beta$ lardi' tabami'z;

4) ABC u'shmu'yeshligine sinuslar teoremasi'n qollan ($\angle BAC=\alpha-\beta$)

$$\frac{AB}{\sin\beta}=\frac{a}{\sin(\alpha-\beta)}\ \text{,yag'niy}\ AB=\frac{a\sin\beta}{\sin(\alpha-\beta)}.$$

5) Tuwri' mu'yeshli ABH u'shmu'yeshliginde AH biyikligin tabami'z:

$$AH=AB\sin\alpha=\frac{a\sin\alpha\cdot\sin\beta}{\sin(\alpha-\beta)}.$$

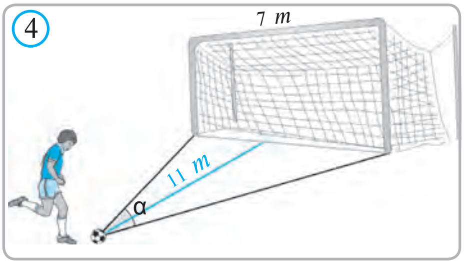
2. Bari'p bolmaytug'i'n noqatqa shekem bolg'an arali'qti' yesaplaw. Aytayi'q, A noqati'nan bari'p bolmaytug'in B noqati'na shekem bolg'an arali'qti' yesaplaw kerek (*3-su'wret*). Bul ma'seleni u'shmu'yeshliklerdin' uqsasli'q belgilerinen paydalani'p sheshkenimizdi yesletip wo'temiz. Yendi bul ma'seleni sinuslar teoremasinan paydalani'p sheshemiz.

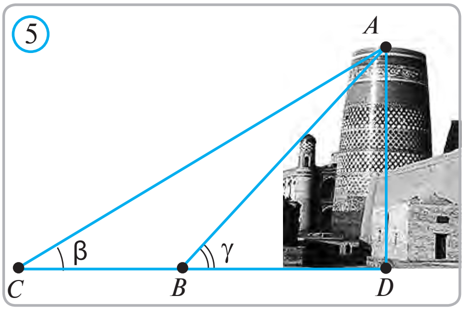
1) A ha'm B noqatlarinan ko'rinip turg'an tegis jerde C noqati'n belgileymiz. 2) AC aralig'in wo'lsheymiz: $AC=a$. 3) A'sbaplar ja'rdeminde ACB ha'm BAC mu'yeshlerin wo'lsheymiz: $\angle BAC=\alpha$, $\angle BAC=\gamma$.

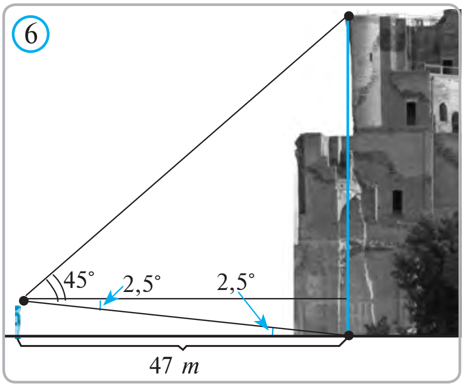
4) ABC u'shmu'yeshliginde $\angle B=180° - \alpha - \gamma$ bolg'ani' ushi'n, $\sin B = \sin(180° - \alpha - \gamma) = \sin(\alpha + \gamma)$.

Sinuslar teoreması boyi'nsha $\frac{AB}{\sin C}=\frac{AC}{\sin B}$ yaki $AB=\frac{a\sin\gamma}{\sin(\alpha+\gamma)}$.

Soraw, ma'sele ha'm tapsi'rmalar







1. 1-su'wrette $a=12$ m, $\alpha=42°$ bolsa, terektin' biyikligin yesaplan'.

2. 2-su'wrette $a=8$ m, $\alpha=43°$, $\beta=32°$ bolsa, terektin' biyikligin yesaplan'.

3. 3-su'wrette $a=60$ m, $\alpha=62°$, $\gamma=44°$ bolsa, AB aralig'in tabi'n'.

4. Futbol oyi'ni'nda 11 metrlik ja'riyma tobi'n da'rwazag'a bag'darlaw mu'yeshi α ni' tabi'n' (*4-su'wret*). Da'rwazani'n' ken'ligi 7 m.

5. 5-su'wrette Xiywa qalasindag'i' Kelteminara su'wretlengen. Yeger $\beta=45°$, $\gamma=24°$, $BC=50$ m bolsa, Kelteminara biyikligin tabi'n'.

6. Sayaxatshi Shaxrisabz qalasi'ndag'i' Aqsaraydi wonnan 47 m arali'qtan tamashalap tur (*6-su'wret*). Yeger wog'an Aqsaray ultani' gorizontqa sali'sti'rg'anda 2,5° qa ten' mu'yesh astinda, joqari' bo'limi bolsa 45° qa ten' mu'yesh asti'nda bolsa, Aqsaraydi'n' biyikligin tabi'n'.

7. U'sh jol ABC u'shmu'yeshligin quraydi'. Bul u'shmu'yeshlikte $\angle A=20°$, $\angle B=150°$. A noqati'nan jolg'a shi'qqan aydawshi' C noqati'na imkaniyati' bari'nsha tezirek jetip barmaqshi'. AC ha'm CB jollari' taslaq, AB asfalt jol boli'p, asfalt jolda taslaq jolg'a qarag'anda 2 barabar tezi'rek ha'reketleniw mu'mkin. Aydawshig'a qaysi' joldan ju'riwdi ma'sla'ha't beresiz?

Qi'zi'qli' ma'sele. **Pifagor teoremasi'ni'n' ja'ne bir "da'liyllemesi".**

Tuwri' mu'yeshli ABC u'shmu'yeshliginde $a=c\sin\alpha$, $b=c\cos\alpha$. Bul yeki ten'likti kvadratqa ko'terip ag'zama-ag'za qossaq ha'm $\sin^2\alpha+\cos^2\alpha=1$ yekenligin yesapqa alsaq, $a^2+b^2=c^2\sin^2\alpha+c^2\cos^2\alpha=c^2(\sin^2\alpha+\cos^2\alpha)=c^2$.
Demek, $a^2+b^2=c^2$. Bul "da'liyllew" logikaliq jaqtan natuwri' yekenligin da'liyllen'.

38–39 II BAPQA TIYISLI QOSI'MSHA MA'SELELER HA'M MAG'LIWMATLAR

I. Testler

1. Ta'repleri *a, b, c,* sa'ykes mu'yeshleri α, β, γ, maydani' *S* bolg'an u'shmu'yeshlik ushi'n qaysi' formula naduri's?

A. $a^2=b^2+c^2-2bc\cos\alpha$; B. $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$;
D. $S=\frac{1}{2}ab\sin\gamma$; E. $S=\frac{1}{2}ab\sin\alpha$.

2. Naduri's formulani tabi'n'.

A. $\sin^2\alpha+\cos^2\alpha=1$; B. $\sin(180°-\alpha)=\sin\alpha$;
D. $\cos(180°-\alpha)=\cos\alpha$; E. $\sin(90°-\alpha)=\cos\alpha$.

3. U'shmu'yeshliktin' u'sh ta'repi belgili bolsa, qaysi teoremadan paydalani'p, woni'n' mu'yeshlerin tabi'w mu'mkin?

A. Sinuslar teoremasi'; B. Kosinuslar teoremasi';
D. Fales teoremasi'; E. Geron formulasi';

4. U'shmu'yeshliktin' bir mu'yeshi 137° qa, yekinshi mu'yeshi 15° qa ten'. Yeger bul u'shmu'yeshliktin' u'lken ta'repi 22 ge ten' bolsa, wonin' kishi ta'repin tabi'n'.

A. 8,3; B. 9,3; D. 3,8; E. 6,5.

5. U'shmu'yeshliktin' 14 ha'm 19 g'a ten' bolg'an ta'repleri arasi'ndag'i' mu'yeshi 26°. Usi u'shmu'yeshliktin' u'shi'nshi ta'repin tabi'n'.

A. 1,2; B. 5,4; D. 6,9; E. 19,7.

6. Yeger yeki vektordin' uzi'nli'qlari' $|\bar{a}|=2$, $|\bar{b}|=5$ ha'm wolar arasi'ndag'i mu'yesh 45° bolsa, $\bar{a}$ ha'm $\bar{b}$ vektorlari'ni'n' skalyar ko'beymesin tabi'n'.

A. 52; B. 32 D. 102; E. 2.

7. $\bar{a}(4; -1)$ ha'm $\bar{b}(2; 3)$ vektorlari'ni'n' skalyar ko'beymesin tabi'n'.

A. 5; B. 3; D. 4; E. 9.

8. $\bar{a}(-\frac{1}{2}; \frac{\sqrt{3}}{2})$ ha'm $\bar{b}(\sqrt{3}; 1)$ vektorlar arasi'ndag'i mu'yeshti tabi'n'.

A. 30°; B. 60°; D. 90°; E. 45°.

9. U'shmuyeshlik mu'yeshlerinin' qatnasi 3:2:1 si'yaqli' bolsa, wonin' ta'repleri qatnasi'n tabi'n'.

A. 3:2:1; B. 1:2:3; D. $2:\sqrt{3}:1$; E. $\sqrt{3}:\sqrt{2}:1$.

10. Ta'repi 3 *sm* bolg'an duri's u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber radiusin tabi'n'.

A. $\sqrt{3}$; B. $\frac{\sqrt{3}}{3}$ D. $2\sqrt{3}$; E. $\frac{\sqrt{3}}{2}$

II. Ma'seleler

1. *ABC* u'shmu'yeshliginde $AB=6$ *sm*, $\angle A=60°$, $\angle B=75°$. *BC* ta'repin ha'm de *ABC* u'shmu'yeshligine si'rtlay si'zi'lg'an shen'berdin' radiusi'n tabi'n'.

2. Ta'repleri 5 *sm*, 6 *sm* ha'm 10 *sm* bolg'an u'shmu'yeshliktin' mu'yeshlerinin' kosinuslari'n tabi'n'.

3. ABC u'shmu'yeshliginde $\angle B=60°$, $AB=6$ *sm*, $BC=4$ *sm*. AC ta'repin ha'm de ABC u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi'n tabi'n'.

4. Ta'repleri 51 *sm*, 52 *sm* ha'm 53 *sm* bolg'an u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi'n tabi'n'.

5. U'shmu'yeshliktin' yeki ta'repi 14 *sm* ha'm 22 *sm*, u'shinshi ta'repine ju'rgizilgen medianasi' bolsa 12 *sm*. U'shmu'yeshliktin' u'shinshi ta'repin tabi'n'.

6. Parallelogarmmni'n' diagonallari' 4 *sm*, $4\sqrt{2}$ *sm* ha'm wolar arasi'ndag'i' mu'yesh 45°. Parallelogrammnin' a) maydani'n; b) perimetrin; d) biyikliklerin tabi'n'.

7. Ta'repleri 3 ha'm 5 bolg'an parallelogrammni'n' bir diagonali 4 ke ten''. Onin' yekinshi diagonali'n tabi'n'.

8. Ta'repleri a) 2, 2 ha'm 2,5; b) 24, 7 ha'm 25; d) 9, 5 ha'm 6 bolg'an u'shmu'yeshliktin' tu'rin ani'qlan'.

9. Parallelogrammnin' ta'repleri $7\sqrt{3}$ ha'm 6 *sm*. Yeger oni'n' dog'al mu'yeshi 120° bolsa, woni'n' maydani'n tabi'n'.

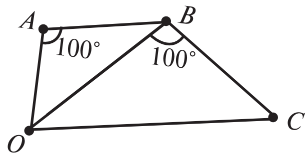
10. ABC u'shmu'yeshliginin' AB, BC ta'replerinde N, K noqatlari' ali'ng'an. Wonda $BN=2AN$, $3BK=2KC$. Yeger $AB=3$, $BC=5$, $CA=6$ bolsa, NK kesindisin tabi'n'.

11. ABC u'shmu'yeshliginde $\angle A=30°$, $BC=7$ *sm*. U'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi'n tabi'n'.

12. ABC u'shmu'yeshliginin' BE bissektrisasi' ju'rgizilgen. E noqati'nan BC ta'repke EF perpendikulyari tu'sirilgen. Yeger $EF=3$, $\angle A=30°$ bolsa, AE ni tabi'n'.

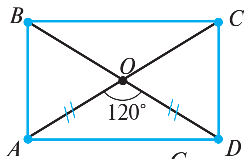
13. $ABCD$ tuwri'mu'yeshliginin' AD ta'repinin' wortasi' N noqati'nda. Yeger $AB=3$, $BC=6$ bolsa, $\overline{NB}\cdot\overline{NC}$ skalyar ko'beymesin tabi'n'.

14. $\bar{a}(2;x)$, $\bar{b}(-4;1)$ boli'p, $\bar{a}+\bar{b}$ ha'm $\bar{b}$ vektorlari perpendiculyar x ti' tabi'n'.



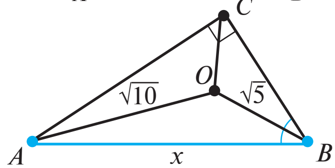
15. $\bar{m}(7;3)$ ha'm $\bar{n}(-2;-5)$ vektorlari' arasi'ndag'i mu'yeshti tabi'n'.

16. Su'wrette berilgenlerden paydalani'p, su'wrettegi yen' u'lken kesindini ani'qlan'.



17. $ABCD$ tuwri'mu'yeshliktin' diagonallari' O noqati'nda kesilisedi. Yeger $AO=12$ *sm*, $\angle AOD=120°$ bolsa, to'rtmu'yeshliktin' perimetrin tabi'n'.

18. Tuwri' mu'yeshli ABC u'shmu'yeshliginin' bissektrisalari' O noqatinda kesilisedi ($\angle C=90°$). Yeger $OA=\sqrt{10}$, $OB=\sqrt{5}$ bolsa, AB gipotenuzani' tabi'n'.



III. Wo'zin'izdi si'nap ko'rin' (u'lgi ushi'n tekseriw jumi'si')

1. Ta'repleri $a=45$, $b=70$, $c=95$ bolg'an u'shmu'yeshliktin' yen' u'lken mu'yeshin tabi'n'.
2. U'shmu'yeshlikte $b=5$, $\alpha=30°$, $\beta=50°$ bolsa, u'shmu'yeshlikti sheshin'.
3. PKH u'shmu'yeshliginde $PK=6$, $KH=5$, $\angle PKH=100°$. HF medianani'n' uzi'nli'g'i'n ha'm PFN u'shmu'yeshliginin' maydani'n tabi'n'.
4. *(Qosi'msha)*. U'shmu'yeshlikte $a=\sqrt{3}$, $b=1$, $\alpha=135°$ bolsa, β mu'yeshti tabi'n'.

Tariyix betlerinen. ***Sinus haqqinda***

Sinus haqqi'ndag'i' mag'li'wmat da'slep IV–V a'sirlerdegi hind astronomlari'ni'n' shig'armalari'nda ushi'raydi.

Worta Aziyali'q ali'mlar al-Xorazmiy, Beruniy, Ibn Sino, Abdurahmon al-Haziniy (XII a'sir) sinus ushi'n ***«al-jayb»*** atamasi'n isletken.

Ha'zirgi sinus belgisin Simpson, Eyler, D'alamber, Lagranj (XVII a'sir) ha'm basqalar qollag'an.

«Kosinus» atamasi' lati'nsha «komplimenti sinus» atamasi'ni'n' qi'sqarti'ri'lg'ani', wol «qosi'msha sinus», ani'g'i'rag'i' «qosi'msha dog'ani'n' sinusi'» dep ataladi'.

Kosinuslar teoremasi'n greklerde bilgen, wonin' da'liylli Yevklidtin "Negizler" shi'g'armasi'nda keltirilgen. Sinuslar teoremasi'ni'n wo'zine tan da'liylin Abu Rayhan Beruniy ayti'p ketken.

Tariyix betlerinen. Beruniy (toli'q ati' — Abu Rayxan Muxammad ibn Axmad) (973-1048)— worta a'sirdin' ulli' enciklopedist ali'mi'. Wol Xorezm u'lkesinin' Qiyat qalasi'nda tuwi'lg'an. Qi'yat A'miwda'ryani'n' won' jag'asi' — ha'zirgi Beruniy qalasi'ni'n' worni'nda bolg'an, wol jaqi'n ku'nlerge shekem Shabbaz dep atalg'an. Beruniydin' matematika ha'm pa'nnin' basqa tarawlari'na qosqan u'lesin jazi'p qaldi'rg'an 150 den aslam miynetinen de ko'riw mu'mkin. Wolardan yen' ko'lemlileri — "Hindstan", "Estelikler", "Masud ni'zamlari", "Geodeziya", "Mineralogiya" ha'm "Astronomiya".

Beruniy
(973 — 1048)

Beruniydin' ulli' miyneti "Masud ni'zamlari" tiykari'nan astronomiyag'a tiyisli bolsa da woni'n' matematikag'a tiyisli ko'plegen ashi'li'wlari' usi' miynetinde bayan yetilgen.

Bul miynetinde Beruniy yeki mu'yeshtin' qosi'ndi'si' ha'm ayirmasini'n' sinuslari', yeki eselengen ha'm yarim mu'yeshtin' sinuslari' haqqi'ndag'i' teoremalar menen ten' ku'shli bolg'an xordalar haqqi'nda teoremalari'n da'liyllegen, yeki gradusli' dog'ani'n' xordalari'n yesaplap tapqan, sinuslar ha'm tangensler kestelerin du'zgen, sinuslar teoremasi'n da'liyllegen.

III BAP

SHEN’BER UZI’NLI’G’I HA’M DO’N’GELEKTIN’ MAYDANI’

Bul bapti’ u’yreniw na’tiyjesinde siz to’mendegi bilim ha’m a’meliy ko’nlikpelerge iye bolasiz.

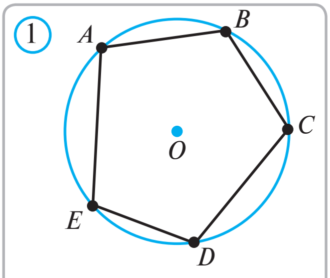
Bilimler:

√ *ko’pmu’yeshlikke si’rtlay ha’m ishley si’zi’lg’an shen’berlerdin’ qa’siyetlerin biliw.*
√ *duri’s ko’pmu’yeshliklerdin’ qa’siyetlerin biliw;*
√ *duri’s ko’pmu’yeshliktin’ maydani’n yesaplaw formulalari’n biliw;*
√ *shen’ber ha’m wonin’ dog’asi’ni’n’ uzi’nli’g’i’n yesaplaw formulalari’n biliw;*
√ *do’n’gelek ha’m wonin’ bo’leklerinin’ maydani’n tabi’w formulalarin biliw;*
√ *mu’yeshtin’ radian wo’lshemin biliw.*

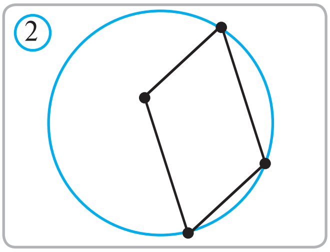
A’meliy ko’nlikpeler:

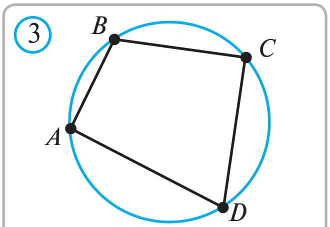
√ *duri’s ko’pmu’yeshliklerdi su’wretley aliw;*
√ *duri’s ko’pmu’yeshlikke si’rtlay ha’m ishley si’zi’lg’an shen’berlerdin’ radiuslari’n taba ali’w;*
√ *shen’ber ha’m dog’a uzi’nli’g’i’n yesaplay ali’w;*
√ *do’n’gelek ha’m woni’n’ bo’leklerinin’ maydani’n yesaplay ali’w.*

40 SHEN'BERGE ISHLEY SI'ZI'LG'AN KO'PMU'YESHLIK

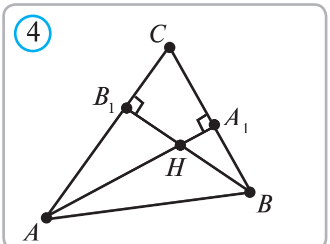


Shen'berge ishley si'zi'lg'an besmu'yeshlik. Besmu'yeshlikke si'rtlay si'zi'lg'an shen'ber.





$\angle A+\angle C=180°$
$\angle B+\angle D=180°$



Ani'qlama. Yeger ko'pmu'yeshliktin' mu'yeshleri shen'berde jatsa, onda bul ko'pmu'yeshlik shen'berge **ishley si'zi'lg'an**, shen'ber bolsa ko'pmu'yeshlikke **si'rtlay si'zi'lg'an** delinedi *(1-su'wret)*.

Qa'legen u'shmu'yeshlikke si'rtlay shen'ber si'zi'w mu'mkin yekenligi ha'm bul shen'berdin' worayi' u'shmu'yeshlik ta'replerinin' worta perpendiculyarlari' kesisken noqatta jatatug'i'ni'n 8-klasta u'yrengensiz.

Yeger ko'pmu'yeshliktin' mu'yeshlerinin' sani' u'shewden arti'q bolsa, ko'pmu'yeshlikke ha'r qashan da si'rtlay shen'ber si'zi'wg'a bola bermeydi. Mi'sali', tuwri' mu'yeshlikten wo'zgeshe parallelogramm ushi'n si'rtlay si'zi'lg'an shen'ber boli'wi' mu'mkin yemes (*2-su'wret*).

8-klastan ma'lim bolg'ani'nday, to'rtmu'yeshlikte qarama-qarsi' mu'yeshlerinin' qosi'ndi'si' 180° qa ten' bolg'anda ha'm tek usi'da jag'dayda g'ana si'rtlay shen'ber si'zi'w mu'mkin (*3-su'wret*).

1-ma'sele. Su'yir mu'yeshli ABC u'shmu'yeshliktin' AA_1 ha'm BB_1 biyiklikleri H noqati'nda kesilisedi. A_1HB_1C to'rtmu'yeshligi shen'berge ishley si'zi'lg'an yekenligin da'liyllen'.

Sheshiliwi. $AA_1 \perp B$ ha'm $BB_1 \perp AC$ bolg'ani' ushi'n (*4-su'wret*). $\angle HB_1C=\angle HA_1C=90°$. Wonda $\angle HB_1C+\angle HA_1C=180°$. to'rtmu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si' 360° bolg'ani' ushi'n: $\angle B_1CA_1+\angle B_1HC=180°$. Demek, A_1HB_1C to'rtmu'yeshlikke si'rtlay shen'ber siziw mu'mkin. Shen'berge ishley si'zi'lg'an ko'pmu'yeshliktin' to'beleri shen'ber worayi'nan ten'dey qashiqliqta jatqani ushi'n shen'berdin' worayi ko'pmu'yeshliktin' ta'replerinin' worta perpendikulyari'nda jatadi' *(5-su'wret)*. Demek, shen'berge ishley si'zi'lg'an ko'pmu'yeshliktin' ta'replerinin' worta perpendiculyarlari' bir noqatta kesilisiwi sha'rt.

2-ma'sele. Radiusi' 10 *sm* bolg'an shen'berge biyikligi 16 *sm* bolg'an ten' qaptalli su'yir mu'yeshli u'shmu'yeshlik ishley si'zi'lg'an. Woni'n' ta'replerin tabi'n'.

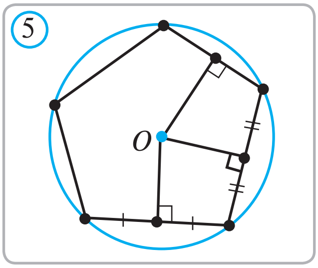
Sheshiliwi. *ABC* u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' worayi' *O* noqati' *AC* ta'repinin' worta perpendiculyari bolg'an *BD* biyiklikte jatadi' (*6-su'wret*). Wonda, $OD=BD-OB=16-10=6$ (*sm*) boladi ha'm Pifagor teoremasi boyi'nsha,

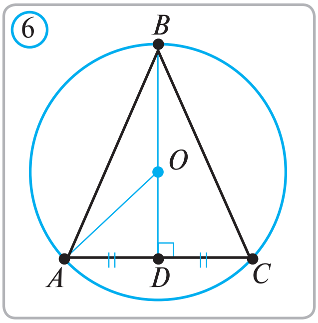
$$AD=\sqrt{OA^2-OD^2}=\sqrt{10^2-6^2}=8\ (sm),\ AC=2AD=16(sm).$$

Sonday-aq, tuwri' mu'yeshli ABD u'shmu'yeshliginde

$$AB=\sqrt{AD^2+BD^2}=\sqrt{8^2+16^2}=8\sqrt{5}\ (sm).$$

Juwabi': $8\sqrt{5}$ *sm*, $8\sqrt{5}$ *sm*, 16 *sm*.





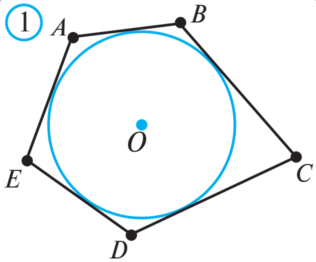
? *Soraw, ma'sele ha'm tapsi'rmalar*

1. Yeger ko'pmu'yeshlik shen'berge ishley si'zi'lg'an bolsa, wonin' ta'replerinin' worta perpendiculyarlari' bir noqatta kesilisetug'ini'n da'liyllen'.
2. Qanday u'shmu'yeshlik shen'berge ishley si'zi'lg'an boliwi mu'mkin? To'rtmu'yeshlik-she?
3. *ABCDE* besmu'yeshligi shen'berge ishley si'zi'lg'an bolsa, $\angle ACB=\angle AEB$ bolatug'i'ni'n da'liyllen'.
4. Katetleri 16 *sm* ha'm 12 *sm* bolg'an tuwri' mu'yeshli ushmu'yeshlikke si'rtlay si'zi'lg'an shen'ber radiusi'n tabi'n'.
5. Radiusi' 25 *sm* bolg'an shen'berge bir ta'repi 14 *sm* bolg'an tuwri'mu'yeshlik ishley si'zi'lg'an. Tuwri'mu'yeshliktin' maydani'n tabi'n'.
6. Radiusi' 10 *sm* bolg'an shen'berge ishley si'zi'lg'an a) ten' ta'repli u'shmu'yeshlik; b) kvadrat; c) ten' qaptalli' tuwri' mu'yeshli u'shmu'yeshliktin' ta'replerin tabi'n'.
7. Ta'repleri 16 *sm*, 10 *sm* ha'm 10 *sm* bolg'an u'shmu'yeshliklerge si'rtlay si'zi'lg'an shen'berdin' radiusi'n tabi'n'.
8. Shen'berge ishley si'zi'lg'an *ABCDEF* altimu'yeshlikte $\angle BAF+\angle AFB=90°$ bolsa, shen'ber worayi *AF* ta'repte jatatug'i'ni'n da'liyllen'.
9. Qa'legen ten' qaptalli' trapeciya shen'berge ishley si'zi'liwi mu'mkin yekenligin da'liyllen'.
10. Ten' qaptalli trapeciya sizi'n'. Wog'an si'rtlay si'zi'lg'an shen'ber jasan'.

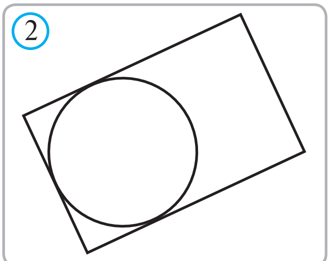
Qi'zi'qli' ma'sele. Won alti' jasli' Galua (E. Galua—francuz matematigi, 1811—1832) kolledjde woqi'p ju'rgen waqi'tlari'nda, wog'an oqi'ti'wshi'si bir saat ishinde u'sh ma'seleni sheship beriwdi sorag'an. Wol sheshimi an'sat bolmag'an bul ma'selelerdi 15 minutta sheship, ha'mmeni hayran qaldirg'an.

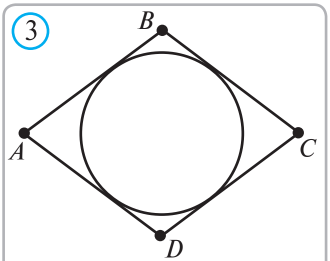
Ma'sele. Shen'berge ishley si'zi'lg'an to'rtmu'yeshliktin' to'rt ta'repi *a, b, c* ha'm *d* g'a ten'. Woni'n' diagonallarin tabi'n'.

41 SHEN'BERGE SI'RTLAY SI'ZI'LG'AN KO'PMU'YESHLIK

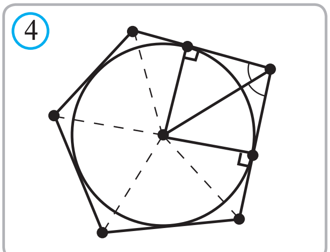


Shen'berge si'rtlay si'zi'lg'an ABCDE besmu'yeshlik. ABCDE besmu'yeshlikke ishley si'zi'lg'an shen'ber.





$AB + CD = AD + BC$



Ani'qlama. Yeger ko'pmu'yeshliktin' barli'q ta'repleri shen'berge uri'nba jasasa, wonda wol ko'p-mu'yeshlik shen'berge **si'rtlay si'zi'lg'an**, al shen'ber bolsa ko'pmu'yeshlikke **ishley si'zi'lg'an** delinedi *(1-su'wret).*

Qa'legen u'shmu'yeshlikke ishley shen'ber si'zi'w mu'mkin yekenligin ha'm bul shen'berdin' worayi' u'shmu'yeshliktin' bissektrisalari' kesisken noqati'nda yekenligi menen 8-klasta tani'sqansi'z.

Yeger ko'pmu'yeshliktin' mu'yeshlerinin' sani' u'shten arti'q bolsa, bul ko'pmu'yeshlikke ha'r qashan da ishley shen'ber si'zi'w mu'mkin bola bermeydi. Mi'sali', kvadrattan wo'zgeshe tuwri' mu'yeshlikke ishley shen'ber si'zi'wg'a bolmaydi' *(2-su'wret).*

Ja'ne 8-klastan belgili, to'rtmu'yeshlikke tek birg'ana qarama-qarsi' ta'replerinin' qosi'ndi'si' ten' bolg'anda ishley shen'ber si'zi'w mu'mkin *(3-su'wret).*

Shen'berge si'rtlay si'zi'lg'an ko'pmu'yeshliktin' ta'repleri shen'berge uri'ng'ani' ushi'n shen'ber worayi' usi' mu'yeshtin' bissektriasasi'nda jatadi' *(4-su'wret).* Demek, shen'berge si'rtlay si'zi'lg'an ko'pmu'yeshliktin' mu'yeshlerinin' bissektrisalari' bir noqatta kesilisedi.

Teorema. **Yeger r radiusli shen'berge si'rtlay si'zi'l-g'an ko'pmu'yeshliktin' maydani' S, yari'm perimetri p bolsa, $S=pr$ boladi'.**

Da'liyllew. Teoremani'n da'liylleniwin shen'berge si'rtlay si'zi'lgan $ABCDEF$ alti'mu'yeshlik ushi'n kel-tiremiz. Shen'ber worayi' O noqati'n ko'pmu'yeshliktin' to'beleri menen tutasti'ri'p, ko'pmu'yeshlikti u'sh-mu'yeshliklerge aji'ratamiz. Bul u'shmu'yeshlik-lerdin' biyiklikleri r ge ten' (*5-su'wret*). Wonda

$$S=S_{AOB}+S_{BOC}+\ldots+S_{FOA}=\frac{1}{2}AB\cdot r+\frac{1}{2}BC\cdot r+\ldots+$$

$$+\frac{1}{2}FA\cdot r=\frac{AB+BC+\ldots+FA}{2}\cdot r=pr.$$

Teorema da'liyllendi.

Ma'sele. Shen'berge si'rtlay si'zi'lg'an to'rtmu'yeshliktin' maydani' 21 sm^2 qa, perimetri bolsa7 *sm* ge ten'. Shen'berdin' radiusi'n tabi'n'.

Sheshiliwi: $S=pr$ formula boyi'nsha

$$r=\frac{S}{p}=\frac{21}{3,5}=6\ (sm).$$ ***Juwabi':*** 6 *sm.*

Soraw, ma'sele ha'm tapsi'rmalar

1. Ta'repi 6 *sm* bolg'an a) ten' ta'repli u'shmu'yeshlikke; b) kvadratqa si'rtlay si'zi'lg'an shen'ber radiusi'n tabi'n'.

2. Radiusi 5 *sm* bolg'an shen'berge si'rtlay si'zi'lg'an ko'pmu'yeshliktin' maydani' 18 sm^2. Ko'pmu'yeshliktin' perimetrin tabi'n'.

3. To'rtmu'yeshliklerdin' perimetrin tabi'n' (*6-su'wret*).

4. 7-su'wrettegi mag'li'wmatlar tiykari'nda soralg'an kesindini tabi'n'.

5. Shen'berge si'rtlay si'zi'lg'an parallelogamm romb bolatug'i'ni'n da'liyllen'.

6. Tuwri' mu'yeshli u'shmu'yeshlikke ishley si'zi'lg'an shen'berdin' radiusi katetler qosi'ndi'si' menen gipotenuza ayi'rmasinin' yari'mi'na ten' yekenligin da'liyllen'.

7. Shen'berge si'rtlay si'zi'lg'an ten' qaptalli trapeciyanin' wortasizig'i woni'n' qaptal ta'repine ten' yekenligin da'liyllen'.

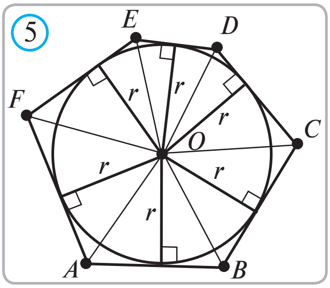
8. Ultanlari' 9 *sm* ha'm 16 *sm* bolg'an ten' qaptalli' trapeciya shen'berge si'rtlay si'zi'lg'an. Shen'berdin' radiusi'n tabi'n'.

9*. *ABCD* to'rtmu'yeshligi *O* worayi'na iye shen'berge si'rtlay si'zi'lg'an. *AOB* ha'm *COD* u'shmu'yeshliklerdin' maydanlari'ni'n' qosi'ndi'si' to'rtmu'yeshliktin' maydani'nin' yari'mi'na ten' yekenligin da'liyllen'.

10*. Shen'berge si'rtlay si'zi'lg'an trapeciyanin' ultanlari *a* ha'm *b* bolsa, woni'n' biyikligi $\sqrt{ab}$ g'a ten' yekenligin da'liyllen'.

11*. To'beleri *ABCD* to'rtmu'yeshliktin' bissektritsalari'ni'n' kesilisiwinen payda bolg'an noqatlardan ibarat. *EFPQ* to'rtmu'yeshlikke si'rtlay shen'ber si'zi'w mu'mkin yekenligin da'liyllen'.

5



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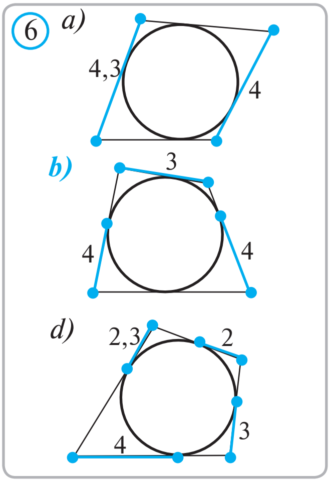

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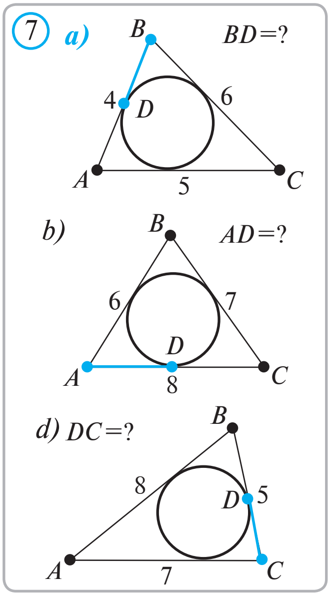


42 DURI’S KO’PMU’YESHLIKLER

Jedellestiriwshi shi’ni’g’i’w

1. Qanday figuralar ko’pmu’yeshlik delinedi?
2. Ko’pmu’yeshliktin’ mu’yeshleri, qon’si’las ta’repleri, diagonallari’ dep nege ayti’ladi’?
3. Do’n’es ko’pmu’yeshlik dep qanday ko’pmu’yeshlikke ayti’ladi’?
4. Do’n’es ko’pmu’yeshliktin’ ishki mu’yeshlerinin’ qosi’ndi’si’ haqqi’ndag’i teoremani’ ayti’n’.

Ani’qlama. Barli’q ta’repleri ten’ ha’m barli’q mu’yeshleri ten’ bolg’an do’n’es ko’pmu’yeshlik **duri’s ko’pmu’yeshlik** delinedi.

Ten’ ta’repli u’shmu’yeshlik, kvadrat duri’s ko’pmu’yeshlikke mi’sal boladi’. 1-su’wrette duri’s besmu’yeshlik, alti’mu’yeshlik ha’m segizmu’yeshlikler su’wretlengen.

Teorema. **Duri’s *n* mu’yeshtin’ ha’r bir mu’yeshi $\frac{n-2}{n}\cdot 180^\circ$ qa ten’.**

Da’liyllew. Duri’s *n* mu’yeshtin’ mu’yeshlerinin’ qosi’ndi’si’ $(n-2)\cdot 180^\circ$ qa ten’ (8-klass). Demek, woni’n’ ~~ha’r~~ bir mu’yeshi $\frac{n-2}{n}\cdot 180^\circ$qa ten’. ***Teorema da’liyllendi.***

Ma’sele. Duri’s $A_1A_2A_3A_4A_5$ besmu’yeshlikte A_1A_3 ha’m A_1A_4 diagonallari’ ten’ yekenligin ko’rsetin’ (*2-su’wret*).

$A_1A_2A_3A_4A_5$ — duri’s besmu’yeshlik ⇒ $A_1A_3 = A_1A_4$

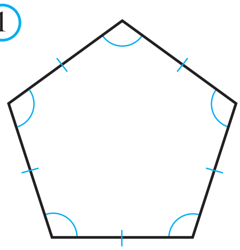
Sheshiliwi. U’shmu’yeshliklerdin’ ten’liginin’ *TMT* belgisi boyi’nsha, $A_1A_2A_3$ ha’m $A_1A_5A_4$ u’shmu’yeshlikleri wo’z ara ten’. Haqi’yqattan da, duri’s ko’pmu’yeshliktin’ ta’repleri ten’ ha’m mu’yeshleri ten’ bolg’ani’ ushi’n,

$A_1A_2 = A_1A_5$, $A_2A_3 = A_5A_4$ ha’m $\angle A_1A_2A_3 = \angle A_1A_5A_4$.

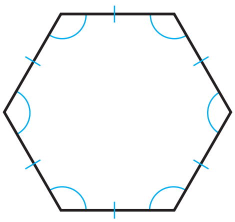
Demek, $\Delta A_1A_2A_3 = \Delta A_1A_5A_4$. Bunnan

$A_1A_3 = A_1A_4$ yekenligi kelip shig’adi’.

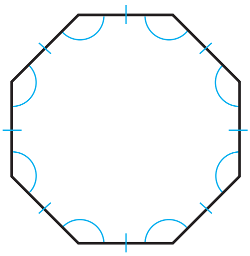
1



duri’s besmu’yeshlik



duri’s altimu’yeshlik



duri’s segizmu’yeshlik

Na'tiyje. Duri's besmu'yeshliktin' barli'q diagonallari' wo'z ara ten'.

Soraw, ma'sele ha'm tapsi'rmalar

1. Duri's bolmag'an ko'pmu'yeshliklerge mi'sallar ayti'n' ha'm ne ushi'n duri's yemesligin tu'sindirin'.
2. To'mendegi tasti'yi'qlawlardan duri'slari'n tabi'n':

   a) barli'q ta'repleri ten' bolg'an u'shmu'yeshlik duri's boladi';

   b) barli'q ta'repleri ten' to'rtmu'yeshlik duri's boladi';

   d) barli'q mu'yeshleri ten' to'rtmu'yeshlik duri's boladi';

   e) barli'q mu'yeshleri ten' romb duri's boladi';

   f) barli'q ta'repleri ten' tuwri' mu'yeshlik duri's boladi'.
3. Yeger a) $n=3$; b) $n=5$; d) $n=6$; e) $n=10$; f) $n=18$ bolsa, duri's n-mu'yeshliktin' mu'yeshlerin tabi'n'.
4. Duri's n mu'yeshliktin' si'rtqi' mu'yeshi nege ten' boladi? Yeger a) n=3; b) n=5; d) n=6; e) n=10; f) n=12 bolsa, duri's n mu'yeshliktin' si'rtqi' mu'yeshin tabi'n'.
5. Duri's n mu'yeshliktin' ha'r to'besinen birewden ali'ng'an si'rtqi' mu'yeshlerinin' qosi'ndi'si' 360° qa ten' yekenligin da'liyllen'.
6. Yeger duri's ko'pmu'yeshliktin' ha'r bir mu'yeshi a) 60°; b) 90°; d) 135°; e) 150° bolsa, bul ko'pmu'yeshliktin' ta'replerinin' sani'n tabi'n'.
7. Duri's $ABCDEF$ altimu'yeshligi berilgen.

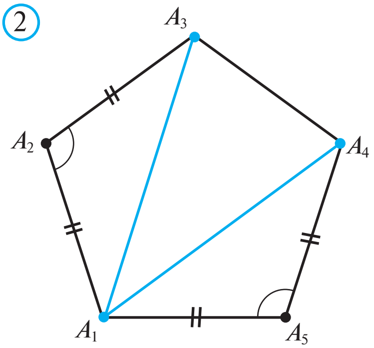
   a) AC ha'm BD diagonallari'ni'n' ten'ligin da'liyllen'.

   b) ACE — duri's u'shmu'yeshlik bolatug'i'ni'n da'liyllen'.

   d) AD, BE ha'm CF diagonallar wo'z ara ten'ligin da'liyllen'.
8. Ta'repi 10 *sm* bolg'an duri's a) besmu'yeshliktin'; b) altimu'yeshliktin'; d) segizmu'yeshliktin'; e) won eki mu'yeshliktin'; f) won segiz mu'yeshliktin' kishi diagonali'n yesaplan'.
9. Duri's to'rtmu'yeshliktin' kvadrat bolatug'i'ni'n da'liyllen'.

10*. Kvadratti'n' ta'repi a g'a ten'. Woni'n' ta'replerine ha'r bir to'besinen baslap diagonali'ni'n' yari'mi'na ten' kesindiler qoyi'ldi. Na'tiyjede 3-su'wrette su'wretlengen segizmu'yeshlik payda boldi'. Woni'n' tu'rin ani'qlan' ha'm maydani'n tabi'n'.

2



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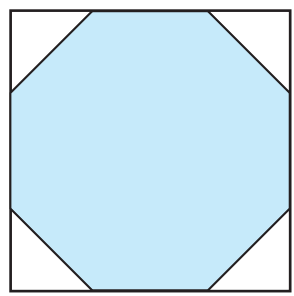
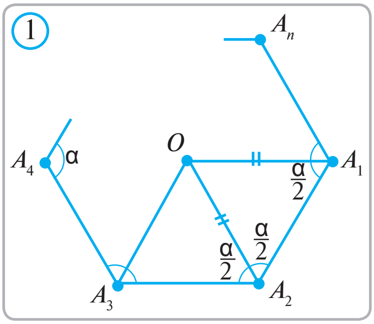
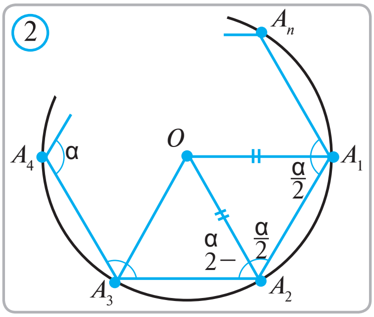
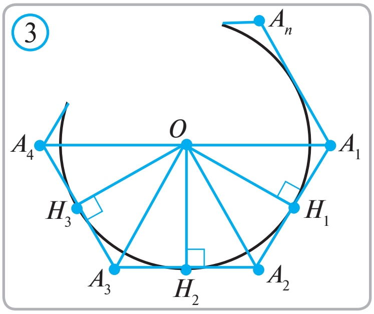


43 DURI'S KO'PMU'YESHLI'KKE I'SHLEY HA'M SI'RTLAY SI'ZI'LG'AN SHEN'BERLER







Jedellestiriwshi shinig'iw

1. Shen'berge ishley si'zi'lg'an ko'pmu'yeshlik dep, qanday ko'pmu'yeshlikke ayti'ladi'?
2. Shen'berge si'rtlay si'zi'lg'an ko'pmu'yeshlik dep, qanday ko'pmu'yeshlikke ayti'ladi'?
3. Qa'legen ko'pmu'yeshlik shen'berge ishley (si'rtlay) si'zi'lg'an boliwi mu'mkin be?

Teorema. **Ha'r qanday duri's ko'pmu'yeshlikke ishley shen'ber de, si'rtlay shen'ber de si'zi'w mu'mkin.**

Da'liyllew. Aytayi'q, $A_1A_2 \ldots A_n$ — duri's ko'pmu'yeshlik, O — A_1 ha'm A_2 mu'yeshleri bissektrisalari'ni'n' kesilisiw noqati' bolsi'n. Bul duri's ko'pmu'yeshliktin' mu'yeshin α menen belgileyik.

1. $OA_1 = OA_2 = \ldots = OA_n$ yekenligin da'liylleymiz (*1-su'wret*). Mu'yeshtin' bissektrisasi'ni'n' ta'riyplemesi boyi'nsha,

$$\angle OA_1A_2 = \angle OA_2A_1 = \frac{\alpha}{2}.$$

Demek, A_1OA_2 — ten' qaptalli' u'shmu'yeshlik. Bunnan, $OA_1 = OA_2$ kelip shig'adi'. ΔA_1A_2O ha'm ΔA_3A_2O u'shmu'yeshliklerdin' ten'liginin' TMT belgisi boyi'nsha ten', sebebi $A_1A_2 = A_3A_2$, A_2O — ta'repi uli'wma ha'm de

$$\angle OA_1A_2 = \angle OA_2A_1 = \frac{\alpha}{2}.$$

Soni'n' ushi'n $OA_3 = OA_1$. Da'l usinday jol tuti'p $OA_4 = OA_2$, $OA_5 = OA_3$ ha'm t. b. ten'likleri wori'nli bolatug'i'ni' ko'rsetiledi. Solay yeti'p, $OA_1 = OA_2 = \ldots OA_n$, yag'ni'y worayi O ha'm radiusi OA_1 bolg'an shen'ber ko'pmu'yeshlikke si'rtlay si'zi'lg'an shen'berden ibarat boladi' (*2-su'wret*).

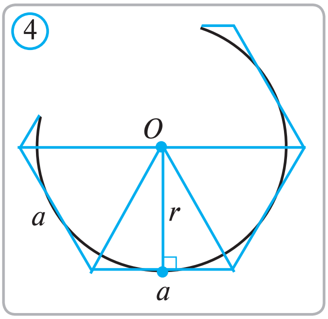
2. Joqari'da ayti'lg'anlar boyi'nsha ten' qaptalli A_1OA_2, A_2OA_3, ... A_nOA_1 u'shmu'yeshlikleri ten'. Sonin' ushi'n, bul u'shmu'yeshliklerdin' O to'besinen tu'sirilgen biyiklikleri de ten' boladi (*3-su'wret*).

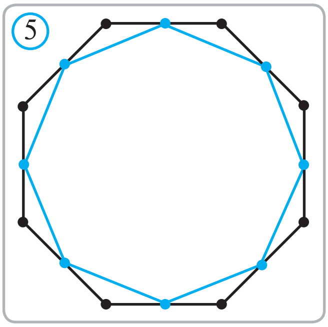
$$OH_1 = OH_2 = \ldots = OH_n.$$

Demek, O worayina iye ha'm radiusi' OH_1, kesindige ten' bolg'an shen'ber ko'pmu'yeshliktin' barli'q ta'replerine uri'nadi. Yag'ni'y, bul shen'ber ko'pmu'yeshlikke ishley si'zi'lg'an shen'ber boladi'. ***Teorema da'liyllendi.***

Na'tiyje. ***Duri's ko'pmu'yeshlikke ishley si'zi'lg'an ha'm si'rtlay si'zi'lg'an shen'berlerdin' oraylari bir noqatta boladi.***

Bul noqat duri's ko'pmu'yeshliktin' **worayi** delinedi. Ko'p-mu'yeshliktin' worayin woni'n' yeki qon'silas to'beleri menen tutastiriwshi nurlardan ibarat mu'yesh (1-su'wrettegi A_1OA_2, A_2OA_3 ... mu'yeshler) woni'n' **worayliq mu'yeshi** delinedi. Duri's ko'pmu'yeshliktin' worayinan ta'repine tu'sirilgen perpendikulyar (3-su'wrettegi OH_1, OH_2, ... kesindiler) woni'n' **apofemasi** delinedi.





Ma'sele. Yeger duri's *n* mu'yeshliktin' ta'repi *a*, wog'an ishley si'zi'lg'an shen'berdin' radiusi *r* bolsa, woni'n' *S* maydani' $S=\frac{1}{2}nar$ formulasi menen yesaplaw mu'mkin yekenligin da'liyllen'.(4-*su'wret*)

Sheshiliwi. Ko'pmu'yeshliktin' yarim perimetri $p=\frac{1}{2}na$ bolg'ani' ushi'n (shen'berge si'rtlay si'zi'lg'an, ko'pmuyeshliktin' maydani'n tabiw) $S=pr$ formulasi boyi'nsha $S=\frac{1}{2}nar$ boladi.

Soraw, ma'sele ha'm tapsi'rmalar

1. Maydani' 36 sm^2 bolg'an kvadratqa ishley ha'm si'rtlay si'zi'lg'an shen'berlerdin' radiuslarin tabi'n'.
2. Perimetri 18 *sm* bolg'an duri's u'shmu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an shen'berlerdin' radiuslarin yesaplan'.
3. Duri's altimu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi woni'n' ta'repine ten' bolatug'inin da'liyllen'.
4. Duri's ko'pmu'yeshliktin' ta'replerinin' wortalari ja'ne duri's ko'pmu'yeshlik payda yetetug'inin da'liyllen' (*5-su'wret*).
5. Duri's ko'pmu'yeshliktin' qa'legen yeki ta'repinin' worta perpendikulyarlari yaki bir noqatta kesilisiwi yaki bir tuwri'da jatatug'inin da'liyllen'.

6*. Duri's ko'pmu'yeshliktin' qa'legen yeki ta'repinin' worta perpendikulyarlari yaki bir noqatta kesilisiwi yaki bir tuwri'g'a jatatug'inin da'liyllen'.

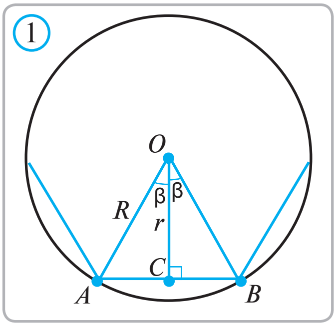
7. Shen'berge ishley si'zi'lg'an duri's ko'pmu'yeshliktin' bir ta'repi shen'berden a) 60°; b) 30°; d) 36°; e) 18°; f) 72° qa ten' dog'a ajiratadi.Ko'pmu'yeshliktin' neshe ta'repi bar?
8. Qag'azdan alti ten'dey duri's u'shmu'yeshlikqirqip alin'.Wolardan paydalani'p, duri's altimu'yeshlikjasan'.Ta'repleri ten' bolg'an duri's altimu'yeshliktin' ha'm u'shmu'yeshliktin' maydanlarinin' qatnasin tabi'n'.

44 DURI'S KO'PMU'YESHLIKTIN' TA'REPMENEN SI'RTLAY HA'M ISHLEY SI'ZI'LG'AN SHEN'BERLER RADIUSLARI ARASI'NDAG'I BAYLANIS

Jedellestiriwshi shinig'iw

Tuwri' mu'yeshli u'shmu'yeshliktin' su'yir mu'yeshinin' a) sinusi; b) kosinusi; c) tangensi dep nege ayti'ladi'?

Ta'repi a_n ge ten' bolg'an duri's n mu'yeshke si'rtlay si'zi'lg'an shen'berdin' R radiusi ha'm ishley si'zi'lg'an shen'berdin' r radiusin yesaplaw ushi'n tuwri' mu'yeshli ACO u'shmu'yeshliginen paydalanamiz. Bul jerde O — ko'pmu'yeshliktin' AB ta'repinin' wortasi (*1-su'wret*). Wonda,



$$\beta=\angle AOC=\frac{1}{2}\angle AOB=\frac{1}{2}\cdot\frac{360°}{n}=\frac{180°}{n};$$

$$R=OA=\frac{AC}{\sin\beta}=\frac{a_n}{2\sin\frac{180°}{n}};\quad r=OC=\frac{AC}{\operatorname{tg}\beta}=\frac{a_n}{2\operatorname{tg}\frac{180°}{n}};$$

$$r=OC=OA\cdot\cos\beta=R\cos\frac{180°}{n}.$$

Bul formulalardan paydalani'p, yari'm duri's ko'pmu'yeshliklerdin' ta'repi, ishley ha'm si'rtlay si'zi'lg'an shen'berlerdin' radiuslari arasi'ndag'i baylani'slardi tabami'z.

1. Duri's u'shmu'yeshlik ushi'n (n=3):

$$\beta=\frac{180°}{3}=60°;\quad R=\frac{a_3}{2\sin 60°}=\frac{a_3}{\sqrt{3}};\quad r=\frac{a_3}{2\operatorname{tg}60°}=\frac{a_3}{2\sqrt{3}};\quad R=2r.$$

2. Kvadrat ushi'n (n=4):

$$\beta=\frac{180°}{4}=45°;\quad R=\frac{a_4}{2\sin 45°}=\frac{a_4}{\sqrt{2}};\quad r=\frac{a_4}{2\operatorname{tg}45°}=\frac{a_4}{2};\quad R=r\sqrt{2}.$$

3. Duri's altimu'yeshlik ushi'n (n=6):

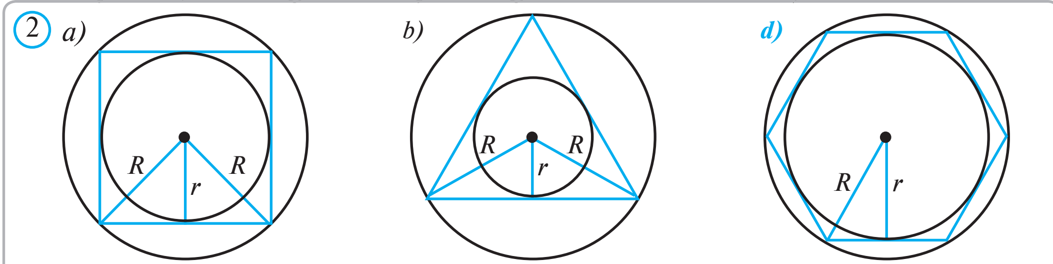
$$\beta=\frac{180°}{6}=60°;\quad R=\frac{a_6}{2\sin 30°}=a_6;\quad r=\frac{a_6}{2\operatorname{tg}30°}=\frac{a_6\sqrt{3}}{2};\quad R=\frac{2r}{\sqrt{3}}.$$

Ma'sele. Duri's n mu'yeshliktin' a_n ta'repin usi ko'pmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' R radiusi ha'm ishley si'zi'lg'an shen'berdin' r radiusi arqali an'latin'.

Sheshiliwi. $R=\frac{a_n}{2\sin\frac{180°}{n}}$ ha'm $r=\frac{a_n}{2\operatorname{tg}\frac{180°}{n}}$ formulalardan $a_n=2R\sin\frac{180°}{n}$ ha'm $a_n=2r\operatorname{tg}\frac{180°}{n}$ formulalarin payda yetemiz. Sonli'qtan, n=3 bolsa, $a_3=R\sqrt{3}=2r\sqrt{3}$.

Soraw, ma'sele ha'm tapsi'rmalar

1. Ta'repi 15 *sm* bolg'an a) duri's u'shmu'yeshlikke; b) duri's to'rtmu'yeshlikke; d) duri's altimu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an shen'berlerdin' radiuslarin yesaplan'.
2. 2-su'wrettin' won' ta'repinde *R* radiusli' shen'berge ishley si'zi'lg'an kvadrat, duri's u'shmu'yeshlik ha'm duri's altimu'yeshlik su'wretlengen. Da'pterin'izge berilgen kestelerdi ko'shirip, woni'n' bos keteklerin tolti'ri'n' (a_n — ko'pmu'yeshliktin' ta'repi, *P* — ko'pmu'yeshliktin' perimetri, *S* — woni'n' maydani', *r* — wog'an ishley si'zi'lg'an shen'ber radiusi).

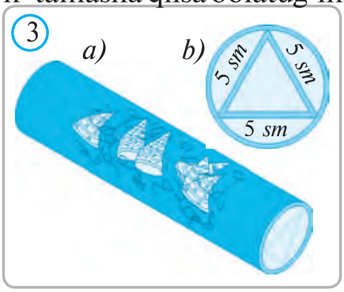


	R	r	a_4	P	S
1.			6		
2.		2			
3.	4				
4.				28	
5.					16

	R	r	a_3	P	S
1.	3				
2.					10
3.		2			
4.			5		
5.				6	

	R	r	a_6	P	S
1.	4				
2.		5			
3.			6		
4.				42	
5.					48

3. Radiusi 8 *sm* bolg'an shen'berge ishley si'zi'lg'an duri's won yeki mu'yeshliktin' bir to'besinen shi'qqan diagonallari'n tabi'n'.
4. Shen'berge ishley si'zi'lg'an duri's u'shmu'yeshliktin' perimetri 24 *sm*. Bul shen'berge ishley si'zi'lg'an kvadrattin' ta'repin tabin.
5. Csilindr formasindag'i' ag'ashtan ultani'ni'n' ta'repi 20 sm bolg'an: a) kvadrat; b) duri's altimu'yeshlik bolg'an prizma tu'rindegi bag'ana tayarlaw kerek. Ag'ashtin' kese-kesiminin' diametri keminde qansha boliwi za'ru'r?
6. 3. *a*-su'wrette su'wretlengen, tu'rlishe nag'i'slari'n tamasha qilsa bolatug'in "Kaleydoskop" dep atalg'an woyinshiq sizge tanis bolsa kerek. Woyinshiq truba ha'm 3 ayna bo'leklerinen ibarat. 3.*b*-su'wrette woni'n' kese-kesimi su'wretlengen ha'm wo'lshemleri berilgen. Kaleydoskoptin' kesekesiminin' radiusin tabi'n'.



45 BILIMIN'IZDI SINAP KO'RIN'

I. Testler

1. To'mendegi ko'pmu'yeshliklerdin' qaysi birinde ishley si'zi'lgan shen'ber joq?

A) U'shmu'yeshlikte; D) Kvadratdan wo'zgeshe rombida;

B) Kvadratta; E) Rombdan wo'zgeshe tuwri' mu'yeshli to'rtmu'yeshlikte;

2. To'mendegi ko'pmu'yeshliklerdin' qaysi birinde si'rtlay si'zi'lgan shen'ber joq?

A) U'shmu'yeshlikte; D) Kvadratdan wo'zgeshe rombida;

B) Kvadratta; E) Rombidan wo'zgeshe tuwri' mu'yeshler to'rtmu'yeshlikte

3. Shen'berge ishley si'zi'lg'an barliq *ABCD* to'rt mu'yeshlikler ushi'n naduri's ten'likti tabi'n'?

A) $\angle A+\angle B+\angle C+\angle D=360°$; D) $AB+CD=BC+AD$;

B) $\angle A+\angle C=180°$; E) $\angle B+\angle D=180°$.

4. Shen'berge si'rtlay si'zi'lg'an barliq *ABCD* to'rt mu'yeshlikler ushi'n naduri's ten'likti tabi'n'?

A) $\angle A+\angle B+\angle C+\angle D=360°$; D) $AB+CD=BC+AD$;

B) $\angle A+\angle C=180°$; E) $AB-BC=AD-CD$.

5. Ta'repleri 5 sm ha'm 12sm bolg'an tuwri' mu'yeshli to'rtmu'yeshlikke si'rtlay si'zi'lg'an shen'ber radiusin tabi'n'?

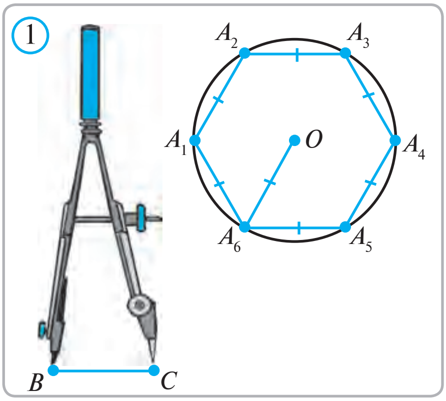
A) 6 *sm*; B) 6,5 *sm*; D) 7 *sm*; E) 7,5 *sm*.

6. Duri's 24 mu'yeshliktin' ishki mu'yeshlerin tabi'n'?

A) 120°; B) 135°; D) 150°; E) 165°.

7. Ha'r bir si'rtqi' muyeshi 60° bolg'an duri's ko'pmu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si'n tabi'n'?

A) 540°; B) 360°; D) 90°; E) 720°.



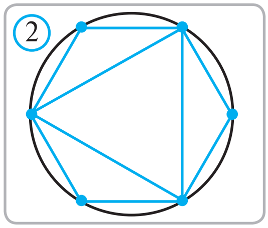
II. Siziwg'a tiyisli ma'seleler.

1. Ta'repi berilgen kesindige ten' duris altimu'yesh si'zi'n. Bunda turaqli altimu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi altimu'yeshliktin ta'repine ten' yekenliginen ha'm 1-su'wretten paydalani'n'.

2. 2-4-su'wretlerdegi mag'liwmatlardan paydalani'p, berilgen shen'berge ishley si'zi'lg'an a) duris u'shmu'yeshlik; b) kvadrat; d) duris segizmu'yeshlik si'zi'n'.

3. 5-su'wretten paydalani'p, berilgen shen'berge si'rtlay si'zi'lg'an duris altimu'yeshlik si'zi'n' (5-su'wrette su'wretlengen shen'berge si'rtlay si'zi'l'g'an altimu'yeshlik

ta'repleri sol shen'berge ishley si'zi'lg'an duri's altimu'yeshliktin' tobelerinen shen'berge ju'rgizilgen urinbalarda jatadi).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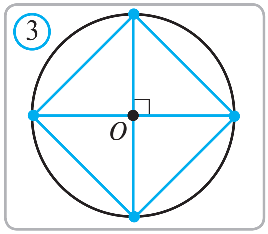
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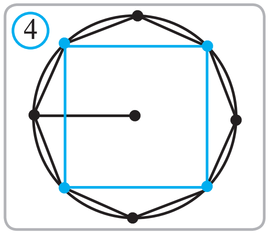
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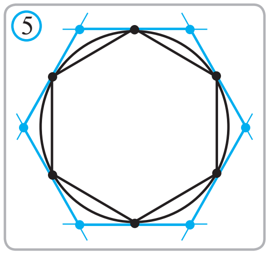
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III. Yesaplawg'a tiyisli ma'seleler

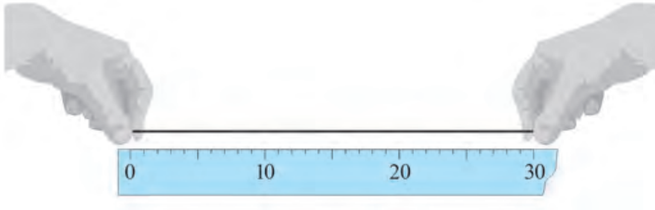
1. Duri's u'shmu'yeshlik, kvadrat ha'm duri's altimu'yeshliklerdin' ta'repleri bir-birine ten'. Wolardi'n' maydanlari'ni'n' qatnasi'n tabi'n'.
2. Bir shen'berge ishley si'zi'lg'an duri's altimu'yeshlik ha'm si'rtlay si'zi'lg'an altimu'yeshliktin' maydanlari'ni'n' qatnasi'n tabi'n'.
3. Duri's a) altimu'yeshlik; b) segizmu'yeshlik; c) won yeki mu'yeshliktin' parallel ta'repleri arasi'ndag'i arali'q 10 *sm* ge ten'. Ko'pmu'yeshliktin' ta'repin tabi'n'.
4. Radiusi' R bolg'an shen'berge $A_1A_2 \dots A_8$ duri's segizmu'yeshlik ishley si'zi'lg'an. $A_3A_4A_7A_8$ to'rtmu'yeshliginin' tuwri'mu'yeshlik yekenligin da'liyllen' ha'm woni'n' maydani'n tabi'n'.
5. Shen'berge si'rtlay si'zi'lg'an tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi' sol shen'berge uriniw noqatinda 4 *sm* ha'm 6 *sm* uzi'nli'qtag'i kesindilerge ajiraladi. U'shmu'yeshliktin maydani'n tabi'n'.
6. Duri's ushmu'yeshliktin' bir to'besinen shiqqan yen' u'lken ha'm yen' kishi diagonallari arasi'ndag'i mu'yeshti tabi'n'.

IV. Wo'zin'di sinap ko'rin' (baqlaw jumi'si' u'lgisi).

1. Katetleri 10 *sm* ha'm 24 *sm* bolg'an tuwri' mu'yeshli u'shmu'yeshlikke ishley si'zi'lg'an ha'm si'rtlay si'zi'lg'an shen'berdin' radiusin tabi'n'.
3. Ta'repleri 4 *sm* bolg'an duri's altimu'yeshliktin' bir ushi'nan shiqqan diagonallarin tabi'n'.
2. Radiusi 5 *sm* bolg'an shen'berge si'rtlay si'zi'lg'an rombnin' bir mu'yeshi 150° qa ten'.Rombi'nin': a) perimetrin b) diagonallarin; d) maydani'n tabi'n'.
4. *(Qosi'msha)*. Radiusi 3 *sm* bolg'an shen'berge ishley si'zi'lg'an duri's altimu'yeshlik ha'm duri's u'shmu'yeshlikler maydanlarinin' ayirmasin tabi'n'.

Tariyx betlerinen. Qalegen duri's ko'pmu'yeshlikti de cirkul ha'm sizg'ish ja'rdeminde jasawg'a bola bermeydi yeken. Buni 1801-jili nemis matematigi Karl Gauss (1777—1855) algebraliq usilda da'liyllegen. Wol yeger n saninin' $2^m p_1 p_2 \dots p_n$ jayilmasinda p_1, p_2, ... p_n tu'rli tu'bir sanlar $2^{2^k}+1$ ko'rinisinde bolsa g'ana duri's n-mu'yeshti tcirkul ha'm sizg'ish ja'rdeminde jasaw mu'mkin yekenligin da'liylledi. Bul jerde m ha'm k teris bolmag'an pu'tin sanlar.

46 SHEN'BERDIN' UZINLIG'I



Jedellestiriwshi shinig'iw

1. A'dette truba bo'leginin' kese-kesimi shen'berden ibarat boladi. Jin'ishke jipti bir ushi'nan baslap, trubag'abir ma'rte woran'. Bir ma'rte worawg'a ketken jip bo'legi trubanin' kese-kesimi, yag'ni'y shen'berdin' uzi'nlig'i boladi. Woni' su'wrette ko'rsetilgendey yetip si'zg'ish ja'rdeminde wo'lshen'.
2. Joqari'dag'i usil menen trubanin' kese-kesiminin' diametrin ani'qlan'.
3. Ani'qlang'an shen'ber uzi'nli'g'i'n woni'n' diametrine qatnasin yesaplan'.
4. Joqaridakeltirilgen wo'lshew ha'm yesaplaw ja'ne bir neshe tu'rli wo'lshemdegi trubabo'lekleri ushi'n daworinlap, shen'ber uzi'nli'g'i'nin' woni'n' diametrine qatnaslarin tabi'n'.
5. Ta'jiriybe na'tiyjesinde shen'ber uzi'nli'g'i'nin' woni'n' diametrine qatnasi haqqinda qanday juwmaq shi'g'ari'w mu'mkin?

Teorema. **Shen'ber uzi'nli'g'i'nin' shen'ber diametrine qatnasi shen'berdin' radiusina baylanisli yemes, yag'niy ha'r qanday shen'ber ushi'n bul qatnas bir qi'yli' san boladi.**

Da'liyllew. Qa'legen yeki shen'ber alamiz. Wolardi'n' radiuslari' R_1 ha'm R_2, uzinliqlari bolsa sa'ykes tu'rde C_1 ha'm C_2 bolsi'n. $\frac{C_1}{2R_1} = \frac{C_2}{2R_2}$ ten'ligin da'liyllewimiz kerek. Ha'r yeki shen'berge ishley duri's n-mu'yeshti sizamiz. Wolardi'n' perimetrl-erin sa'ykes tu'rde P_1 ha'm P_2 dep belgileyik. Wonda,

$P_1 = n \cdot 2R_1 \sin\frac{180°}{n}$, $P_2 = n \cdot 2R_2 \sin\frac{180°}{n}$ bolg'ani' ushi'n $\frac{P_1}{P_2} = \frac{2R_1}{2R_2}$ (*) boladi.

Bul ten'lik qa'legen n ushi'n duri's boladi. n sani u'lkeyip barsa, berilgen shen'berge ishley si'zi'lg'an n-mu'yeshliktin' perimetri P_1 usi shen'ber uzinlig'i C_1 ge jaqinlasip baradi. Sol siyaqli P_2 ha'm C_2 ge jaqinlasip baradi.

Sonin' ushi'n $\frac{P_1}{P_2}$ qatnasi $\frac{C_1}{C_2}$ qatnasina ten' boladi (bunin' toliq da'liyli matema-tikanin' joqari basqishlarinda u'yreniledi). Solay yetip, (*) ten'liginen $\frac{C_1}{C_2} = \frac{2R_1}{2R_2}$, bunnan bolsa $\frac{C_1}{2R_1} = \frac{C_2}{2R_2}$ ten'ligi kelip shig'adi.

Teorema da'liyllendi.

Shen'ber uzi'nli'g'i'n woni'n' diametrine qatnasi grek a'lipbesinin' π ha'ribi menen belgilew qabil yetilgen *("pi" dep oqiladi).* Shen'ber uzi'nli'g'i'n woni'n' diametrine qatnasin "π" ha'ribi menen belgilewdi ulli matematik Leonard Eyler (1707—1783) ilimge kiritken. Grekshede "shen'ber" so'zi usi ha'rip penen baslanadi. π irratcional san boli'p, a'meliyatta woni'n' 3,1416 g'a ten' bolg'an juwiq ma'nisinen paydalaniladi.

Solay yetip, $\frac{C}{2R}=\pi$. Bul ten'likten radiusi' R ge ten' shen'berdin' uzinlig'i ushi'n $C=2\pi R$ formulasin paydayetemiz.

Ma'sele. Ta'repi 6 *sm* bolg'an duri's u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' uzi'nli'g'i'n tabi'n'.

Sheshiliwi. Duri's u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusin tabiw formulasi $R=\frac{a_3}{\sqrt{3}}$ boyi'nsha $R=\frac{6}{\sqrt{3}}=2\sqrt{3}$ (*sm*). Yendi, shen'ber uzi'nli'g'i'n tabiw formulasinan $C=2\pi R=2\pi\cdot2\sqrt{3}=4\pi\sqrt{3}$ (*sm*). ***Juwabi':*** $4\pi\sqrt{3}$ *sm*.

? Soraw, ma'sele ha'm tapsi'rmalar

1. Qanday san π menen belgilenedi? Radiusi' R ge ten' shen'berdin' uzi'nli'g'i'n tabiw formulasinan paydalani'p kesteni toltirin' ($\pi\approx3,14$ dep yesaplan').

C			82	18π		6,28	
R	4	3			0,7		101,5

2. Yeger shen'ber radiusi' a) 3 ma'rte artsa; b) 3 *sm* ge artsa; d) 3 ma'rte kemise; e) 3 *sm* ge kemise, shen'ber uzinli'g'i' qanshag'a wo'zgeredi?

3. Yeger jer shari ekvatorinin' 40 millionnan bir bo'limi 1 *m* ge ten' bolsa, Jer sharinin' radiusin tabi'n'.

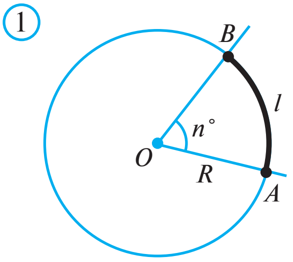
4. a) Ta'repi *a* g'a ten' bolg'an duri's u'shmu'yeshlikke; b) katetleri *a* ha'm *b* bolg'an tuwri' mu'yeshli u'shmu'yeshlikke; c) ultani' *a* ha'm qaptal ta'repi *b* bolg'an ten' qaptalli u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' uzi'nli'g'i'n tabi'n';

5. a) Ta'repi *a* g'a ten' kvadratqa; b) gipotenuzasi *c* g'a ten' bolg'an ten' qaptalli tuwri' mu'yeshli u'shmu'yeshlikke; c) gipotenuzasi *c*, su'yir mu'yeshi α bolg'an tuwri' mu'yeshli u'shmu'yeshlikke ishley si'zi'lg'an shen'berdin' uzi'nli'g'i'n tabi'n'.

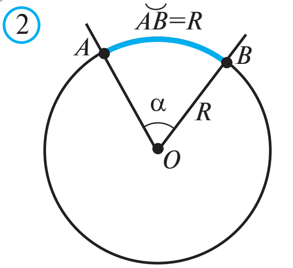
6. Teplovoz 1413 *m* jol ju'rdi. Bunda wonin do'n'gelegi 300 ma'rte aylandi. Teplovoz do'n'geleginin diametrin tabi'n'.

7. "Nexiya" avtomobili do'n'gelegi shen'berinin' radiusi 24 *sm* ge ten'. Avtomobil 100 km jol ju'rse, woni'n' do'n'gelegi neshe ma'rte aylanadi (*1-su'wret*)?

47 SHEN'BER DOG'ASININ' UZINLIG'I MU'YESHTIN' RADIAN O'LSHEMI



$l=\frac{\pi R}{180^\circ}\cdot n^\circ$



α=*1 radian* ≈ *57°17′45″*

1. n° li' worayliq mu'yeshke tirelgen dog'anin' uzinlig'i.

Aytayiq, radiusi R ge ten' bolg'an shen'berde n° li AOB worayliq mu'yesh berilgen bolsi'n (*1-su'wret*).

Bunda shen'berdin' AOB worayliq mu'yeshine tirelgen AB dog'asiinin' gradus wolshemin n° yaki n° li dog'a dep ju'ritiliwin yesletip wotemiz.

Radiusi R ge ten' bolg'an shen'ber, yag'ini'y 360° li dog'a uzinlig'i $2\pi R$ ge ten' bolg'ani' ushi'n,

1° li dog'auzinlig'i $\frac{2\pi R}{360^\circ}=\frac{\pi R}{180^\circ}$ boladi.

Wonda, n° li dog'auzinlig'i $l=\frac{\pi R}{180^\circ}\cdot n^\circ$ formula menen ani'qlanadi (*1-su'wret*).

2. Mu'yeshtin' radian wo'lshemi.

Mu'yeshtin' gradus wo'lshemi menen bir qatarda wonin' radian wo'lshemi de qollaniladi.

Shen'ber dog'asi uzi'nli'g'i'nin' radiusqa qatnasin joqaridag'i formulag'a tiykarlanip $\frac{l}{R}=\frac{\pi}{180^\circ}\cdot n^\circ$ g'a ten'. Demek, shen'ber dog'asi uzi'nli'g'i'nin' radiusina qatnasi tekusi dog'ag'a tirelgen worayliq mu'yeshtin' shamasina baylani'sli yeken. Bul qa'siyetten paydalani'p, mu'yeshtin' radian wo'lshemi sipatinda da'l usi qatnasti alamiz:

$$\alpha=\frac{l}{R}=\frac{\pi}{180^\circ}\cdot n^\circ.$$

A'dette, radian so'zi jazilmaydi. Ma'selen: 5 rad wornina 5 dep jaziladi.

Bir radian $\frac{180^\circ}{\pi}$ gradusqa ten': 1 radian$=\frac{180^\circ}{\pi}\approx 57^\circ17'45''$. Mu'yeshtin' gradus wo'lsheminen radian wo'lshemine wo'tiw ushi'n

$$\alpha=\frac{\pi}{180^\circ}\cdot n^\circ$$

formuladan paydalaniladi.

Solay yetip, n° li mu'yeshtin' radian wo'lshemin tabiw ushi'n woni'n' gradus wo'lshemin $\frac{\pi}{180^\circ}$ ge ko'beytiw jetkilikli yeken. Jeke halda, 180° mu'yeshtin' radian wo'lshemi π ge ten', 90^0 li, yag'iny tuwri' mu'yeshtin' radian wo'lshemi $\frac{\pi}{2}$ ge ten' boladi.

α radiang'a ten' worayliq mu'yeshke sa'ykes dog'asinin' uzinlig'i $l=\alpha R$ formulasimenen yesaplanadi.

Ma'sele. Mu'yeshleri 30° ha'm 45° bolg'an u'shmu'yeshliktin' mu'yeshlerinin' radian wo'lshemlerin tabi'n'.

Sheshiliwi. U'shmu'yeshliktin' 30° li mu'yeshinin' radian wo'lshemi $30°\cdot\frac{\pi}{180°}=\frac{\pi}{6}$, 45° li mu'yeshinin' radian wo'lshemi $45°\cdot\frac{\pi}{180°}=\frac{\pi}{4}$. U'shmu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si' 180° qa, yag'niy π ge ten' yekenligi haqqindag'i teoremag'a tiykarlanip, u'shmu'yeshliktin' u'shinshi mu'yeshinin' radian wo'lshemin tabami'z.

$$\pi-\frac{\pi}{6}-\frac{\pi}{4}=\frac{7\pi}{12}.$$

Juwabi': $\frac{\pi}{6}$; $\frac{\pi}{4}$ ha'm $\frac{7\pi}{12}$.

Soraw, ma'sele ha'm tapsi'rmalar

1. Radiusi 6 *sm* bolg'an shen'berdin' gradus wo'lshemi a) 30°; b) 45°; d) 90°; e) 120° bolg'an dog'a uzi'nli'g'i'n tabi'n'.

2. a) 40°; b) 60°; d) 75° qa ten' mu'yeshtin' radian wo'lshemin tabi'n'.

3. a) 1,2; b) $\frac{2\pi}{3}$; **d)** $\frac{5\pi}{6}$ radiang'a ten' mu'yeshtin' gradius wo'lshemin tabi'n'.

4. Yeger shen'berdin' radiusi 5 *sm* bolsa, woni'n' **a)** $\frac{\pi}{8}$; b) $\frac{2\pi}{5}$; d) $\frac{3\pi}{4}$ radiang'a ten' bolg'an dog'auzi'nli'g'i'n tabi'n'.

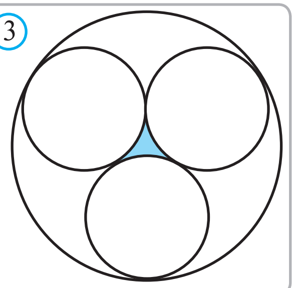
5. Radiusi 12 *sm* bolg'an shen'berge *ABC* u'shmu'yeshligi ishley si'zi'lg'an.Yeger a) $\angle A=30°$; b) $\angle A=120°$ bolsa, *A* noqatin wo'z ishine almag'an *BC* dog'a uzi'nli'g'i'n tabi'n'.

6. Shen'berdin' ten' xordalari shen'berden ten' dog'alar ajiratatug'inin da'liyllen'.

7*. Yeki shen'ber bir-birinin' worayinan wo'tedi. Bul shen'berlerdin' uliwma xordasi ha'r yeki shen'berden ajiratqan dog'alar uzinliqlarinin' qatnasin tabi'n'.

8*. Radiuslari ten' bolg'an u'sh shen'berler bir-birine si'rttan ha'm radiusi *R* ge ten' bolg'an shen'berge ishley urinadi (*3-su'wret*): a) shen'berlerdin' radiusin tabi'n'; b) boyalg'an figurani shegaralawshi dog'alardin' uzinliqlarinin' qosi'ndi'si'n tabi'n'.

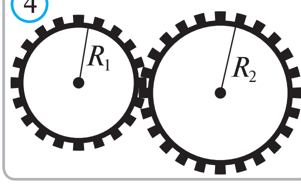
3



Qi'zi'qli' ma'sele

4-su'wrette su'wretlengen yeki tisli do'n'gelekler bir-birine "tisletilgen". Do'n'gelekler radiusi R_1 ha'm R_2. Birinshi do'n'gelek *n* ma'rte aylang'anda yekinshi do'n'gelek neshe ma'rte aylanadi?

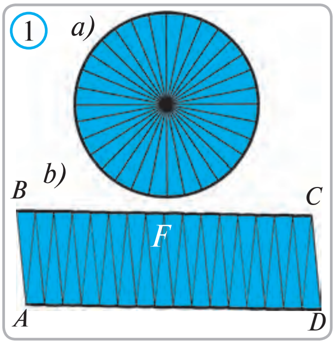
4

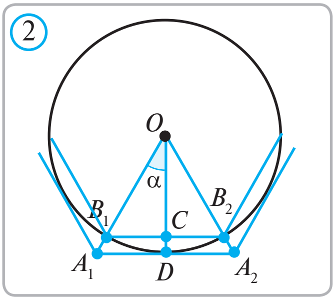


48 DO'N'GELEKTIN' MAYDANI'

Ani'qlama. Shen'ber ha'm tegisliktin' usi shen'ber menen shegarang'an (ishki) bo'limine ***do'n'gelek*** dep ataladi.

Worayi O noqat'inda ha'm radiusi R ge ten' bolg'an do'n'gelek tegisliktin' O noqatinan R den aspaytug'in arali'qta jatqan barliq noqatlardan quralg'an boladi.





Jedellestiriwshi shinig'iw

Bir bet qag'azg'a qali'n' si'zi'q penen shen'ber si'zi'n' ha'm 1.*a*-su'wrette ko'rsetilgendey, woni'n' bir neshe diametrlerin ju'rgizip, do'n'geleklerdi ten'dey bo'leklerge bo'lin'. Son' bul bo'leklerdi qiyip alin' ha'm 1.*b*-su'wrette ko'rsetilgendey yetip terip, F figurasi'n payda yetin'. Yeger do'n'gelek qa'legenshe ko'p ten'dey bo'leklerge bo'linip, bul bo'lekler su'wrette ko'rsetilgen ta'rtipte terilgende, na'tiyjede tuwri' mu'yeshlikke ju'da' jaqin F figura payda boladi.

a) F figurani tuwri' mu'yeshlik formasina ju'da' jaqin yekenligin yesapqa alip, woni'n' AB ta'repi shamamenen nege ten' bolatug'inin tabi'n'. (Ko'rsetpe: AB ta'repin do'n'gelek radiusi menen salistirin').

b) F figuranin' BC "ta'repi" shama menen nege ten' boladi? (Ko'rsetpe: BC ha'm AD ta'repleri qalin' si'zi'q penen si'zi'lg'anina, yag'niy shen'ber dog'alarinan ibarat yekenligine itibar berin').

d) F figuranin' $ABCD$ tuwri' mu'yeshlik formasina ju'da' jaqin yekenligin yesapqa alip, woni'n' maydani'n juwiq yesaplan'. F figurasinin' maydani' do'n'gelek maydani'na ju'da' jaqin yekenligin na'zerde tutip, do'n'gelek maydani' haqqinda juwmaq shig'arin'.

Teorema. **Radiusi' R ge ten' bolg'an do'n'gelektin' maydani' R^2 qa ten'.**

Da'liyllew. Radiusi R ha'm worayi O noqatta bolg'an shen'berdi qaraymiz.

Shen'berge si'rtlay si'zi'lg'an $A_1A_2 \ldots A_n$ ha'm ishley si'zi'lg'an $B_1B_2 \ldots B_n$ duri's n mu'yeshlerdin' maydanlari sa'ykes tu'rde S_n^I ha'm S_n^{II} bolsi'n (2-*su'wret*).

A_1OA_2 ha'm B_1OB_2 u'shmu'yeshliklerdin' maydanlarin tabami'z:

$$S_{A_1OA_2}=\frac{1}{2}A_1A_2\cdot OD=\frac{1}{2}A_1A_2\cdot R;\quad S_{B_1OB_2}=\frac{1}{2}B_1B_2\cdot OC=\frac{1}{2}B_1B_2\cdot OB_1\cos\alpha=\frac{1}{2}B_1B_2\cdot R\cos\alpha.$$

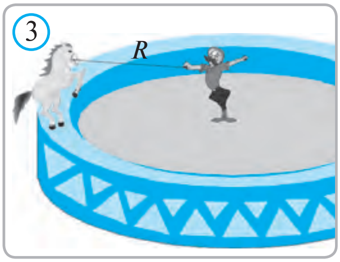
Wonda $S_n^I=n\cdot\frac{1}{2}AA_1\cdot R=\frac{1}{2}P_n^I R,\quad S_n^{II}=n\cdot\frac{1}{2}B_1B_2\cdot R\cos\alpha=\frac{1}{2}P_n^{II}R\cos\alpha$ (1)

Bul jerde P_n ha'm P_n sa'ykes tu'rde $A_1A_2...A_n$ ha'm $B_1B_2...B_n$ ko'pmu'yeshliktin' perimetrleri $\alpha=\frac{180°}{n}$ bolg'ani' ushi'n n nin' jeterlishe u'lken ma'nislerinde $\cos\alpha$ nin' ma'nisi birden, P_n ha'm P_n lerdin' ma'nisleri shen'ber uzinlig'i yag'niy $2\pi R$ den qa'legenshe kem parq qiladi. Wonda (1) ten'likler boyi'nsha, n nin' jeterlishe u'lken ma'nislerinde ko'pmu'yeshliklerdin' maydani' πR^2 qa jaqinlasip baradi. Bunnan, do'n'gelektin' maydani' ushi'n $S=\pi R^2$ formulakelip shig'adi. ***Teorema da'liyllendi***

Ma'sele. Cirk arenasi shen'berinin' uzinlig'i 41 *m*. Arena radiusin ha'm maydani'n tabi'n'.

Sheshiliwi 1) Shen'berdin' uzi'nli'g'i'n tabiw formulasinan radiusin tabamiz (*3-su'wret*):

$$R=\frac{C}{2R}\approx\frac{41}{2\cdot3,14}\approx6,53\ (m).$$



2) Do'n'gelektin' maydani'n yesaplaw formulasinan arenanin' maydani'n tabami'z:

$S=\pi R^2\approx3,14.6,53^2\approx133,84\ (m^2)$.

Juwabi': $R\approx6,53\ m$; $S=133,84\ m^2$.

Soraw, ma'sele ha'm tapsi'rmalar

1. Do'n'gelek maydani'n yesaplaw formulasin tiykarlan'.

2. Radiusi R ge ten' bolg'an do'n'gelektin' S maydani'n tabiw formulasinan paydalani'p, kesteni toltirin' ($\pi=3,14$ dep alin').

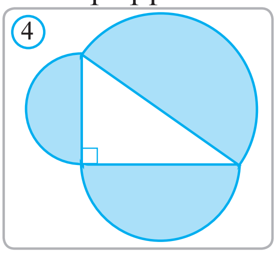
R	2	5		$\frac{2}{7}$		54,3		6,25
S			9		49π		$\sqrt{3}$	

3. Yeger do'n'gelek radiusi a) k ma'rte wo'sse; b) k ma'rte kemise, do'n'gelektin' maydani' qalay wo'zgeredi?

4. Ta'repi 5 *sm* bolg'an kvadratqa ishley si'zi'lg'an ha'm si'rtlay si'zi'lg'an do'n'geleklerdin' maydani'n tabi'n'.

5. Ta'repi $3\sqrt{3}$ *sm* bolg'an duri's u'shmu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an do'n'geleklerinin' maydani'n tabi'n'.

6. Radiusi' R bolg'an do'n'gelekten yen' u'lken kvadrat qirqip alindi. Do'n'gelektin' qalg'an bo'liminin' maydani'n tabi'n'.



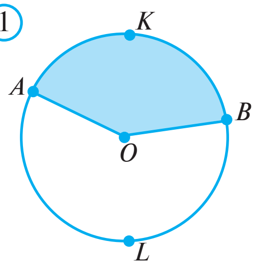
7. Ta'repleri 6 *sm* ha'm 7 *sm* bolg'an tuwri' mu'yeshlikke si'rtlay si'zi'lg'an do'n'gelektin' maydani'n tabi'n'.

8. Ta'repi 10 *sm* ha'm su'yir mu'yeshi 60° bolg'an rombig'a ishley si'zi'lg'an do'n'gelektin' maydani'n tabi'n'.

9*. Tuwri' mu'yeshli u'shmu'yeshliktin' ta'replerin diametr yetip yarim do'n'gelekler si'zi'lg'an. Gipotenuzag'a si'zi'lg'an yarim do'n'gelektin' maydani' katetlerge si'zi'lg'an yarim do'n'geleklerdin' maydanlarinin' qosi'ndi'si'na ten' bolatug'inin ko'rsetin' (*4-su'wret*).

49 DO'N'GELEKTIN' BO'LEKLERININ' MAYDANI'

1



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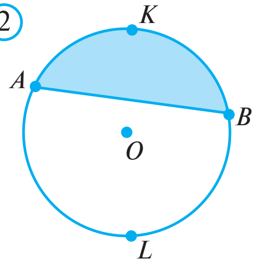

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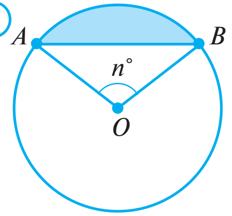
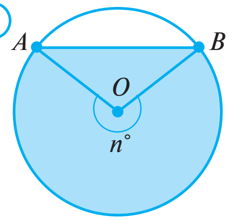


$S=\frac{\pi R^2}{360}\cdot n - S_{AOB}$

4



$S=\frac{\pi R^2}{360}\cdot n + S_{AOB}$

Ani'qlama. Do'n'gelektin' dog'asi ha'm bul dog'a aqirlarin do'n'gelek worayi menen tutastiriwshi yeki radiusi menen shegaralang'an bo'limi **sektor** delinedi. Sektordin' shegarasi bolg'an dog'a **sektor dog'asi** delinedi.

1-su'wrette AKB ha'm BLA dog'ali yeki sektor su'wretlengen (*wolardan birinshisi boyalg'an*).

Radiusi R ge ha'm dog'anin' gradus wo'lshemi $n°$ qa ten' bolg'an sektordin' S maydani'n tabiw ushi'n formula keltirip shig'aramiz. Dog'asi 1° qa ten' sektordin' maydani' do'n'gelek (yag'niy dog'asi 360° qa ten' sektor) maydani'nin' $\frac{1}{360}$ bo'limine ten' bolg'ani' ushi'n, dog'asi $n°$ bolg'an sektordin' maydani'

$$S=\frac{\pi R^2}{360}\cdot n \text{ yaki } S=\frac{1}{2}Rl$$

formula arqali tabiladi. Bul jerda l – $n°$ sektor jayinin' uzinligi.

Ani'qlama. Do'n'gelektin' dog'asi ha'm bul dog'a aqirlarin tutastiriwshi xordasi menen shegaralang'an bo'limi **segment** delinedi.

2-su'wrette AKB ha'm BLA dog'ali yeki segment su'wretlengen (*wolardan birinshisi boyalg'an*). Yarim do'n'gelekten parqli segmenttin' S maydani'

$$S=S_{sektor}\pm S_{\Delta}=\frac{\pi R^2}{360}\cdot n \pm S_{AOB}$$

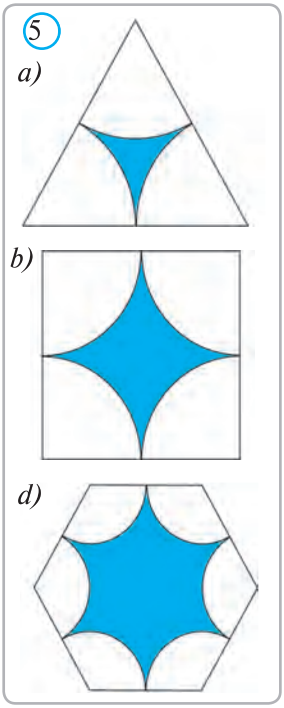
formula boyi'nsha yesaplanadi (*3-ha'm 4-su'wretlerge qaran'*).

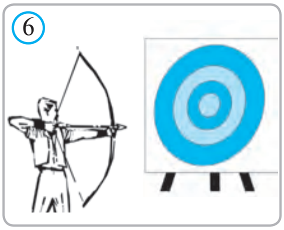
Ma'sele. Dog'anin' gradus wo'lshemi 72° bolg'an sektordin' maydani' 45π ge ten'. Sektor radiusin tabi'n'.

Sheshiliwi. Sektor maydani'n tabiw formulasi boyi'nsha, $\frac{\pi R^2}{360}\cdot 72=45\pi$. Bunnan, $R^2=\frac{45\pi\cdot 360}{72\pi}=225$, demek $R=15$. ***Juwabi':*** 15.

Soraw, ma'sele ha'm tapsi'rmalar

1. Sektor maydani'n tabiw formulasin keltirip shig'arin'.
2. Segment maydani'n tabiw formulasin keltirip shig'arin'.
3. Radiusi 7 *sm* bolg'an sektor ha'm segment maydanlarin tabi'n'. Bunda, woni'n' dog'asinin' gradius wo'lshemi a) 30°; b) 45°; d) 120°; e) 90°.
4. 5-su'wrette ta'repi *a* g'a ten' bolg'an duri's u'shmu'yeshlik, kvadrat ha'm duri's altimu'yeshlik su'wretlengen. Boyalg'an figuralardin' maydani'n tabi'n'. Bunda sektorlardin' radiuslari ko'p mu'yeshlik ta'repinin' yarimina ten'.
5. Nishanda radiuslari 1, 2, 3, 4 ke ten' bolg'an to'rt shen'ber bar. Yen' kishi do'n'gelektin' maydani'n ha'm ha'r bir jag'day da maydani'n tabi'n' (*6-su'wret*).
6. Radiusi 10 *sm* ge ten' bolg'an do'n'gelekke radiusqa ten' xorda ju'rgizilgen. Payda bolg'an segmentlerdin' maydani'n yesaplan'.
7. Radiuslari 15 *sm* bolg'an yeki do'n'gelektin' woraylari arasi'ndag'i arali'q 15 *sm*. Do'n'geleklerdin' uliwma bo'liminin' maydani'n tabi'n'.
8. Radiusi 10 *sm* bolg'an do'n'gelekke ishley ha'm si'rtlay si'zi'lg'an duri's won yeki mu'yeshliklerdin' maydani'n yesaplan'. Na'tiyjelerdi do'n'gelektin' maydani' menen salisti'ri'n'.



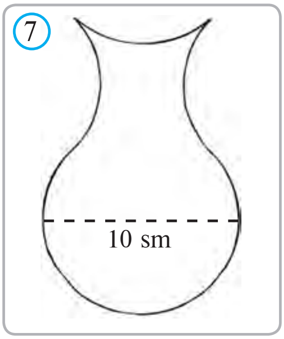


Qi'zi'qli' ma'sele

7-su'wrette su'wretlengen gu'ltu'bektin' su'wretin u'sh tuwri' si'zi'q penen:

a) sonday to'rt bo'lekke bo'lin', na'tiyjede wolardan tuwri' mu'yeshlik jiynaw mu'mkin bolsi'n;

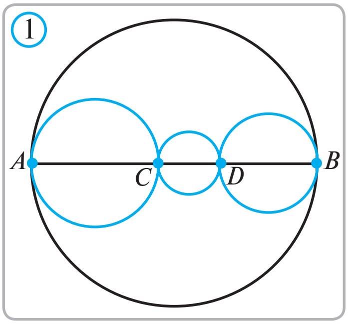
b) yeki tuwri' si'zi'q penen sonday u'sh bo'limge bo'lin', na'tiyjede wolardan kvadrat jasaw mu'mkin bolsi'n.



Tariyx betlerinen. Uzaq waqitlar dawaminda du'nyanin' ko'plep matematikleri "do'n'gelek kvadraturasi" dep at alg'an to'mendegi ma'seleni sheshiwge ha'reket yetken: tcirkul ha'm sizg'ish ja'rdeminde maydani' berilgen do'n'gelek maydani'na ten' bolg'an kvadrat jasaw. Tek XIX a'sirdin' aqirinda bul ma'sele sheshimge iye yemesligi da'liyllengen.

50 MA'SELELERDI SHESHIW

1-ma'sele. *C* ha'm *D* noqatlari shen'berdin' *AB* diametrin u'sh *AC*, *CD* ha'm *DB* kesindilerge ajiratadi. *AC*, *CD* ha'm *DB* diametrli shen'berlerdin' uzinliqlarinin' qosi'ndi'si' *AB* diametrli shen'ber uzi'nli'g'i'na ten' yekenligin da'liyllen' (*1-su'wret*).

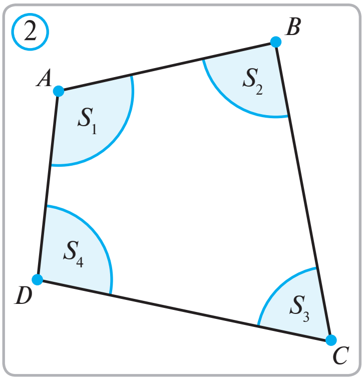


Sheshiliwi. Shen'ber uzi'nli'g'i'n tabiw formulasinan paydalani'p, *AC, CD* ha'm *DB* diametrli shen'berlerdin' C_1, C_2, C_3 uzinliqlarinin' qosi'ndi'si'n tabami'z:

$$C_1+C_2+C_3=AC\cdot\pi+CD\cdot\pi+DB\cdot\pi=\pi(AC+CD+DB).$$

$AC+CD+DB=AB$ ha'm *AB* diametrli shen'berdin' *C* uzunlig'i $AB\cdot\pi$ ge ten' bolg'ani' ushi'n $C_1+C_2+C_3=C$. Usi ten'likti da'liyllew talap yetilgen yedi.

2-ma'sele. *ABCD* to'rtmu'yeshliktin' to'belerin woray yetip birdey radiusli sektorlar jasalg'an (*2-su'wret*). Bul sektorlardan qa'legen yekewi uliwmanoqatqaiye yemes ha'm barlig'inin' radiusi 1 *sm*. Sektorlardin' maydanlarinin' qosi'ndi'si'n tabi'n'.



Sheshiliwi. 1) To'rtmu'yeshliktin' *A*, *B*, *C*, *D* mu'yeshleri sa'ykes tu'rde α_1, α_2, α_3, α_4 bolsi'n. Wonda, ko'p-mu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si' haqqindag'i teorema boyi'nsha,

$$\alpha_1+\alpha_2+\alpha_3+\alpha_4=360°.$$

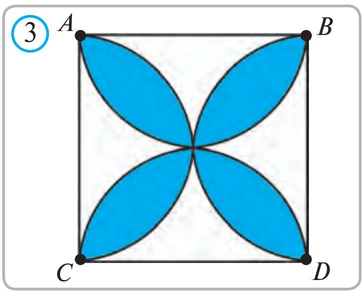
2) Sektor maydani'n tabiw formulasi ($R=1$ *sm*),

$$S_1=\frac{\pi}{360°}\cdot\alpha_1,\quad S_2=\frac{\pi}{360°}\cdot\alpha_2,\quad S_3=\frac{\pi}{360°}\cdot\alpha_3,\quad S_4=\frac{\pi}{360°}\cdot\alpha_4.\ (1)$$

3) (1) ten'liklerdin' sa'ykes bo'leklerin qosamiz. Wonda,

$$S_1+S_2+S_3+S_4=\frac{\pi}{360°}(\alpha_1+\alpha_2+\alpha_3+\alpha_4)=\frac{\pi}{360°}\cdot 360°=$$
$$=\pi\ (sm^2).$$

Juwabi': $\pi\ sm^2$.



Soraw, ma'sele ha'm tasirmalar

1. Perimetri 1 m bolg'an kvadrat ha'm uzinlig'i 1 *m* bolg'an shen'ber berilgen. Bul shen'ber menen shegaralang'an do'n'gelektin' maydani’ menen kvadrattin’ maydani’n salistirin’.

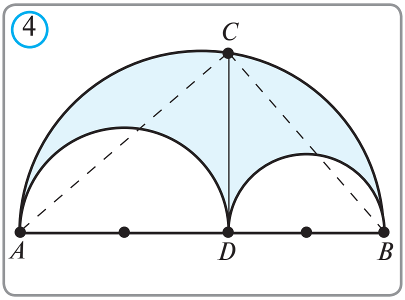
2. Radiusi 8 sm bolg’an do’n’gelekten 60° li sektor qirqip aling’an.Do’n’gelektin’ qalg’an bo’leginin’ maydani’n tabi’n’.

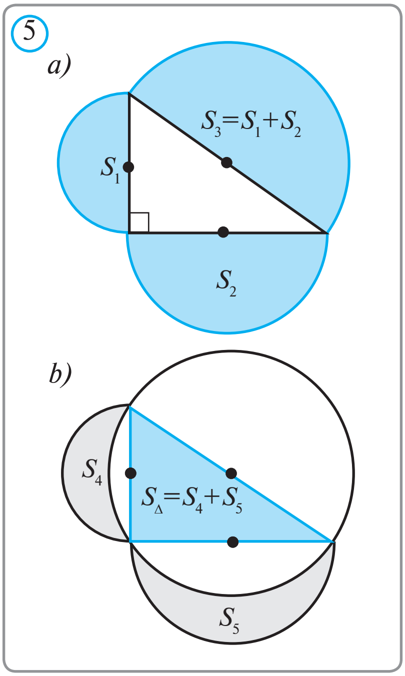
3. Diagonallari 6 *sm* ha’m 8 *sm* bolg’an rombig’a ishley si’zi’lg’an do’n’gelektin’ maydani’n yesaplan’.

4. 3-su’wrette boyalip ko’rsetilgen figura maydani’n tabi’n’. Wonda $ABCD$ — kvadrat, $AB=4$ *sm*.

5*. 4-su’wrette “Arximed pishag’i” dep ataliwshi figura boyalip ko’rsetilgen. Woni’n’ maydani’n $\frac{\pi \cdot CD^2}{4}$ formula menen yesplawdi da’liyllen’ (bunda $\angle ACB=90°$ ha’m $CD^2=AD \cdot DB$ yekenliginen paydalanin’).

6. Yeger $AD=6$ *sm*, $BD=4$ *sm* bolsa, 4-su’wrette boyalip ko’rsetilgen figuranin’ maydani’n ha’m perimetrin tabi’n’.





Tariyx betlerinen. **Gippokrat ayshalari.**

Gippokrat ayshalari — yeki shen’ber dog’alari menen shegaralang’an ha’m to’mendegi qa’siyetke iye bolg’an figuralar boladi. Bul shen’berler (ayshalar) radiuslari ha’m dog’alarinin’ xordalari boyi’nsha ayshalarg’a ten’dey kvadratlar jasaw mu’mkin.

Pifagor teoremasi’ qollani’lsa, 5-a su’wretlengen gipotenuzag’a jasalg’an yarim do’n’geleklerdin’ maydanlarinin’ qosi’ndi’si’na ten’ boladi’ (107-bet, 9*-ma’selege qaran’). Soni’n’ ushi’n 5.b-su’wrettegi ayshalardin’ maydanlarinin’ qosi’ndi’si’ u’shmu’yeshliktin’ maydani’na ten’ (baqlap ko’rin’!). Yeger su’wrettegi *u’shmu’yeshliktin’* wornina ten’ qaptalli tuwri’ mu’yeshli u’shmu’yeshlikti alsaq, payda bolg’an yeki ayshadan ha’r birinin’’ maydani’ u’shmu’yeshliktin’ maydani’nin’ yarimina ten’ boladi. Do’n’gelek kvadraturasi’ haqqindag’i ma’seleni sheshiwge uri’ni’p, grek matematigi Gippokrat (b.e.sh. V a’sir) ko’pmu’yeshlik penen ten’dey bir neshe tu’rli ayshalardi’ jasag’an.

Gippokrat ayshalarinin’ toliq kestesi tek XIX—XX a’sirlerde du’zilgen.

51 III BAPQA TIYISLI QOSIMSHA MA'SELELER HA'M MAG'LIWMATLAR

I. Testler

1. 45° gradiusli mu'yeshtin' radian wo'lshemi nege ten'?

A. 1 ge ten' B. $\frac{\pi}{2}$ ge ten' D. $\frac{\pi}{4}$ ge ten' E. $\sqrt{2}$ ge ten'.

2. Radiusi 3 *sm* bolg'an shen'berdin' gradus wo'lshemi 150° gradus bolg'an worayliq mu'yeshke tirelgen dog'a uzi'nli'g'i'n tabi'n'.

A. $\frac{5\pi}{2}$ *sm*; B. $\frac{5\pi}{3}$ *sm*; D. $\frac{10\pi}{3}$ *sm*; E. $\frac{5\pi}{4}$ *sm*.

3. Radiusi 6 sm bolg'an shen'berde $\frac{5\pi}{4}$ radiang'a ten' worayliq mu'yesh tirelgen dog'anin' uzi'nli'g'i'n tabi'n'.

A. $\frac{15\pi}{2}$ *sm*; B. $\frac{5\pi}{6}$ *sm*; D. $\frac{4\pi}{3}$ *sm*; E. $\frac{5\pi}{2}$ *sm*.

4. Ta'repi 5 *sm* ge ten' bolg'an kvadratqa si'rtlay si'zi'lg'an shen'berdin' maydani'n tabi'n'.

A. $5\sqrt{2}\pi$; B. $\sqrt{2}\pi$; D. $3\sqrt{2}\pi$; E. 5π.

5. Diametri 6 g'a ten' do'n'gelektin' maydani'n tabi'n'.

A. 9π; B. 6π; D. $3\sqrt{2}\pi$; E. 12π.

6. Dog'anin' gradius wo'lshemi 150°, radiusi 6 *sm* bolg'an do'gelek sektordin maydani'n tabi'n'.

A. 15π *sm*²; B. 6π *sm*²; D. $30\sqrt{2}\pi$ *sm*²; E. 24π *sm*².

7. Dog'anin' uzinlig'i 12 *sm* ha'm radiusi 6 *sm* bolg'an do'n'gelek sektordin' maydani'n tabi'n'.

A. 15π *sm*²; B. 6π *sm*²; D. $30\sqrt{2}\pi$ *sm*²; E. 24π *sm*².

8. Dog'anin' gradus o'lshemi 120°, radiusi 3 ke ten' bolg'an do'n'gelek segmenttin' maydani'n tabi'n'.

A. $6\pi - 4\sqrt{3}$; B. $6\pi + 4\sqrt{3}$; D. $3\pi - 4\sqrt{3}$; E. $3\pi + 4\sqrt{3}$.

II. Ma'seleler.

1. *ABCDEFKL* duri's segizmu'yeshliktin' ta'repi 6 *sm*. Woni'n' *AC* diagonali'n tabi'n'.

2. Kvadrat radiusi 4 *dm* bolg'an shen'berge ishley si'zi'lg'an. Kvadratti'n' qon'si'las ta'replerinin' wortalarinan wo'tiwshi xordanin' shen'berden aji'ratqan dog'alarinin' uzi'nli'g'i'n tabi'n'.

3. Shen'berdin' 90° li dog'asinin' uzinlig'i 15 *sm*. Shen'berdin' radiusin tabi'n'.

4. Radiusi' 20 g'a ten' shen'berden uzinlig'i 10 π ge ten' dog'a ajiratiladi. Bul dog'ag'a sa'ykes worayliq mu'yeshti tabi'n'.

5. Yeki dog'anin' uliwma xordasi bul do'n'geleklerdi shegaralawshi' shen'berlerden 60° li ha'm 120° li dog'alar ajiratadi. Do'n'geleklerdin' maydanlarinin' qatnasin tabi'n'.

6. Ta'repleri 3, 4, 5 bolg'an u'shmu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an do'n'geleklerdin' maydanlarin tabi'n'.

7. Do'n'gelek xordasi 60° li dog'ani kerip turadi. Bul xorda aji'ratqan segmentlerdin' maydanlarinin' qatnasin tabi'n'.

8. Duri's altimu'yeshliktin' maydani'nin' wog'an ishley si'zi'lg'an do'n'gelektin' maydani'na qatnasin tabi'n'.

9. Ta'repi a g'a ten' bolg'an $ABCDEF$ duri's altimu'yeshlik berilgen. Worayi A noqati ha'm radiusi a bolg'an shen'ber usi altimu'yeshlikti yeki bo'lekke ajiratadi. Ha'r bir bo'lektin' maydani'n tabi'n'.

10. Tuwri' mu'yeshli ABC u'shmu'yeshlikte $\angle A=72°$, $\angle C=90°$, $BC=15$ *sm*. BC diametrli shen'berdin' ABC u'shmu'yeshlik ishinde jatqan dog'asinin' uzi'nli'g'i'n tabi'n'.

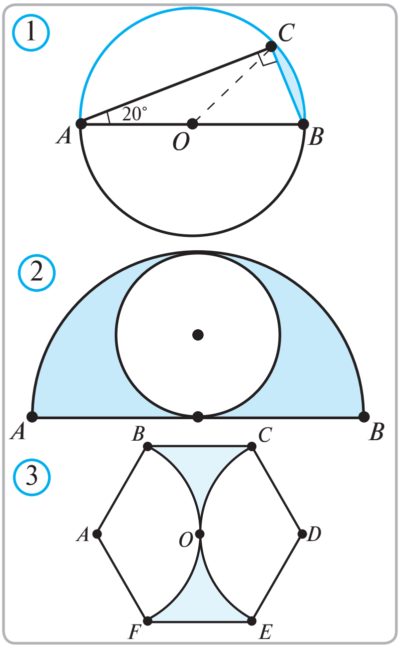
11. Do'n'gelekke ishley si'zi'lg'an duri's segizmu'yeshlik berilgen. Woni'n' yeki qon'silas to'belerine ju'rgizilgen radiuslar do'n'gelekti yeki sektorg'a ajiratadi. Bul sektorlardin' maydanlarinin' qatnasin tabi'n'.

12. Tuwri' mu'yeshli ABC u'shmu'yeshlikte $\angle A=20°$, $\angle C=90°$, $AB=18$ *sm*. BC kesindisi u'shmu'yeshlikke si'rtlay si'zi'lg'an do'n'gelekti yeki segmentke ajiratadi. Boyap ko'rsetilgen segment maydani'n tabi'n'.(*1-su'wret*).

13. Kishi shen'ber u'lken shen'berge ha'm de woni'n' AB diametrine urinadi'. Yeger diametrge uriniw noqati' shen'ber worayi' ha'm $AB=4$ bolsa, su'wrette boyalg'an figuranin' maydani'n tabi'n'. (*2-su'wret*).

14. Duri's $ABCDEF$ altimu'yeshliktin' ta'repi 6 g'a ten' ha'm worayi O noqati'nda. Woraylari A ha'm D noqatinda ha'm radiuslari ten' bolg'an shen'berler O noqatinda urinadi'. Boyalg'an oblast maydani'n tabi'n' (*3-su'wret*).

15. Tuwri' mu'yeshli ABC u'shmu'yeshlikte $\angle C=90°$, $AC=4$, $CB=2$. Worayi gipotenuzada bolg'an shen'ber u'shmu'yeshlik katetlerine urinadi'. Usi shen'berdin' uzi'nli'g'i'n tabi'n'.



III. Wo'zin'izdi sinap ko'rin' (u'lgi ushi'n baqlaw jumi'si')

1. Ta'repleri 6 *sm* bolg'an kvadratqa si'rtlay si'zi'lg'an shen'ber uzinlig'i' ha'm ishiley si'zi'lg'an do'n'gelektin' maydani'n tabi'n'.

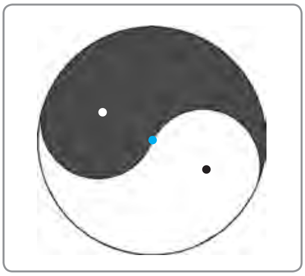
2. Ta'repi 24 *sm* bolg'an duri's ko'pmu'yeshlikke ishley si'zi'lg'an shen'ber radiusi $4\sqrt{3}$ *sm* ge ten' bolsa, wog'an si'rtlay si'zi'lg'an shen'ber radiusin tabi'n'.

3. 240° li shen'ber dog'asinin' uzinlig'i 24 *sm* bolsa,
a) shen'ber radiusin; b) dog'asi 240° bolg'an sektordin' maydani'n;
d) dog'asi 240° bolg'an segmenttin' maydani'n tabi'n'.

Qi'zi'qli' ma'sele

In ha'm Yan

Su'wrette ta'biyattag'i qarama-qarsiliqlardi sa'wlelendiriwshi "In ha'm Yan" dep atalg'an xitay belgisi (tamg'asi) su'wretlengen.



a) In ha'm Yan belgileri maydanlarinin ten'ligin ko'rsetin';

b) bir tuwri' si'zi'q penen bul belgilerdin' ha'r birin maydanlari ten' bo'lgan yeki bo'lekke ajiratin'.

d) In ha'm Yan nishanlari perimetrlerin (wolardi qorshap turg'an dog'alar uzinliqlarinin qosi'ndi'si'n) tabi'n'.

Tariyx betlerinen. Shen'ber uzi'nli'g'i'n yesaplaw ju'da' a'yyemnen ken' ko'lemdegi mashqala bolg'an. Shen'ber uzi'nli'g'i'n wog'an ishley si'zi'lg'an ko'pmu'yeshlik perimetrine almastiriw usi'li' ken' tarqalg'an.
Worta Aziyali matematikler de do'n'gelekke ishley si'zi'lg'an duri's ko'pmu'yeshliklerdi jasaw, wolardi'n' ta'replerin do'n'gelektin' radiusi arqali an'latiw ma'seleleri menen shug'illang'an. Abu Rayxan Beruniy "Qonuniy Masudiy" miynetinde do'n'gelekke ishley si'zi'lg'an ko'pmu'yeshliklerdin' ta'repin ani'qlaw menen shug'illanip, ishley si'zi'lg'an besmu'yeshlik, altimu'yeshlik, jetimu'yeshlik, ..., wonmu'yeshliktin' ta'replerin ani'qlaw usi'li'n ko'rsetedi. Bul yesaplaw na'tiyjesinde wol $\pi \approx 3{,}14$ ma'nisine iye boladi.

A'yyemgi Grek ha'm Misir qoljazbalarinda π di u'shke ten' dep alg'an. Bul sol da'wirdin' ani'qliq talabi ushi'n jeterli bolg'an. Keyinshelik rimliler π ushi'n 3,12 ni paydalang'an, π sani ushi'n Arximed bergen ma'nis 3,14 boli'p, bul a'meliy ma'selelerdi sheshiwde ju'da' qolayli.

Mirza Ulug'bektin' "Astronomiya mektebi"—talabalarinin' biri Jamshid Giyasiddin al-Koshiy 1424-jili jazg'an "Shen'ber uzinlig'i haqqinda kitap" atamasindag'i traktatinda shen'berge ishley ha'm si'rtlay si'zi'lg'an duri's ko'pmu'yeshlik ta'repleri sanin yeki yeseletiw joli menen $3 \cdot 2^{28} = 800\,335\,168$ ta'repli duri's ko'pmu'yeshliklerdin' perimetrlerin yesaplap, π ushi'n $\pi = 3{,}1\,415\,826\,535\,897\,932$ ma'nisin payda yetken. Bul 16 wonliq sang'a shekem ani'q boladi.

Biraq al-Koshiydin' miyneti uzaq waqitqa shekem Yevropada belgisiz bolg'an. Yevropalilardan Belgiyali Van Romen 1597-jili 2^{30} ta'repli duri's ko'pmu'yeshlikke Arximed usi'li'n qollanip, π ushi'n 17 wonliq sanlari ani'q bolg'an ma'nisti tapqan. Gollandiyali Rudolf Van Seylon (1540—1610) bul ani'qliqti 35 wonliq sanlarg'a shekem alip barg'an. Ha'zirgi da'wirde elektron yesaplaw mashinalari ja'rdeminde π ushi'n millionnan artiq wonliq sanlari ani'q bolg'an ma'nisleri tabilg'an. Ku'ndelik yesaplawlar ushi'n 3,14 ma'nisi, matematikaliq yesaplawlar ushi'n 3,1416 ma'nisi, ha'tte astronomiya ha'm kosmonavtika ushi'n 3,1415826 ma'nisi jetkilikli boladi'.

IV BAP

U’SHMU’YESHLIK HA’M SHEN’BERDEGI METRIKALIQ QATNASLAR

Usi bapti u’yreniw na’tiyjesinde siz to’mendegi bilim ha’m ko’nlikpelerge iye bolasiz:

Bilimler:

√ *proporcional kesindilerdin’ qa’siyetlerin biliw;*

√ *tuwri’ mu’yeshli u’shmu’yeshlikte gipotenuzag’a tu’sirilgen biyikliktin’ qa’siyetlerin biliw;*

√ *wo’z ara kesilisiwshi xordalar kesindiler haqqindag’i ha’m de shen’berdi kesip wo’tiwshi tuwri’nin’ kesindileri haqqindag’i qa’siyetlerdi biliw.*

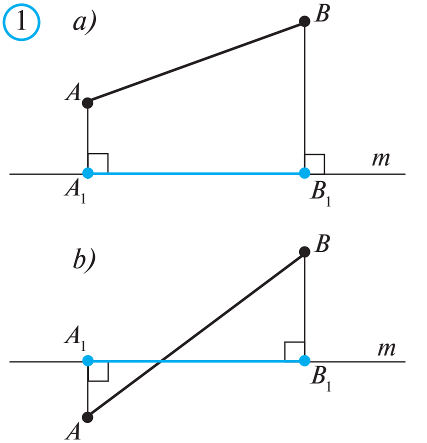
Ko’nlikpeler:

√ *kesindilerdin’ qatnasin ha’m proporcional kesindilerge tiyisli ma’selelerdi sheshe aliw;*

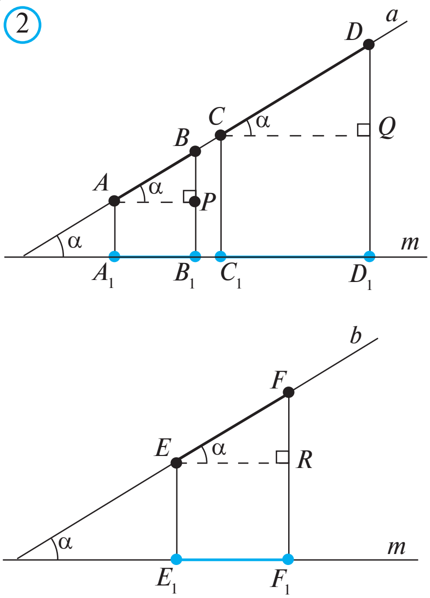
√ *tuwri’ mu’yeshli u’shmu’yeshlikte gipotenuzag’a tu’sirilgen biyikliktin’ qa’siyetlerinen paydalani’p ma’seleler sheshe aliw;*

√ *kesilisiwshi xordalar kesindilerinin’ ha’m kesiwshi tuwri’lar kesindilerinin’ qa’siyetlerinen paydalani’p ma’seleler sheshiw.*

52 KESINDILER PROEKTCIYASI HA'M PROPORTCIONALLIQ



A_1 — A noqatinin',
B_1 — B noqatinin',
A_1B_1 — AB kesindinin' *m ko'sherindegi proektsiyasi*



Jedellestiriwshi shinig'iw

1. Kesindilerdin' qatnasi neni an'latadi?
2. Qanday kesindiler proporcional delinedi?
3. Fales teoremasin aytin'.

Tegislikte m tuwri'si ha'm AB kesindisi berilgen bolsi'n. A ha'm B noqatlardan m tuwri'sina AA_1 ha'm BB_1 perpendikulyar tu'siremiz (1-su'wret). A_1B_1 kesindisi AB kesindisinin' m ko'sherindegi **proektciyasi (sayasi)** delinedi.

AB kesindisinin' m tuwri'dag'i A_1A_1 proekciyasin jasaw a'meli AB kesindisin m tuwri'sina **proekciyalaw** delinedi.

Teorema. **Bir tuwri'da yaki parallel tuwri'larda jatatug'in kesindiler berilgen bolsi'n. Wolardi'n' tap bir tuwri'g'a proekciyalari berilgen kesindilerge proporcional boladi.**

$a||b$,
A_1B_1 — AB nin',
C_1D_1 — CD nin',
E_1F_1 — EF nin'
m tuwri'si'zi'qqa proyek-tsiyalari (2-su'wret)

$\Rightarrow$

$$\frac{A_1B_1}{AB}=\frac{C_1D_1}{CD}=\frac{E_1F_1}{EF} \quad (1)$$

Da'liyllew. a) Yeger a ha'm b tuwri'lari m tuwri'sina parallel bolsa, $AB=A_1B_1$, $CD=C_1D_1$, $EF=E_1F$ boladi ha'm de (1) ten'lik wori'nli' yekenligi ani'q.

b) Yeger de a ha'm b tuwri'lari m tuwri'sina perpendikulyar bolsa, A_1 ha'm B_1, C_1 ha'm D_1, E_1 ha'm F_1 noqatlari u'stpe-u'st tu'sedi. Sonin' ushi'n A_1B_1, C_1D_1, E_1F_1 kesindilerinin' uzinlig'i nolge ten' boladi ha'm (1) ten'lik worinlanadi.

d) Yendi basqa jag'daylardi qaraymiz. 2-su'wrette su'wretlengenindey tuwri' mu'yeshli ABP, CDQ, EFR u'shmu'yeshliklerin jasaymiz.

Bunda $a||b$ bolg'ani ushi'n $\angle BAP=\angle DCQ=\angle FER$. Demek, *ABP, CDQ* ha'm *EFR* tuwri' mu'yeshli u'shmu'yeshlikleri uqsas. Bunnan

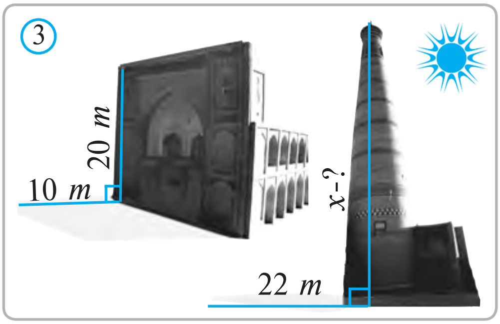
$\frac{A_1B_1}{AB}=\frac{C_1D_1}{CD}=\frac{E_1F_1}{EF}$ ten'ligin payda yetemiz. ***Teorema da'liyllendi.***

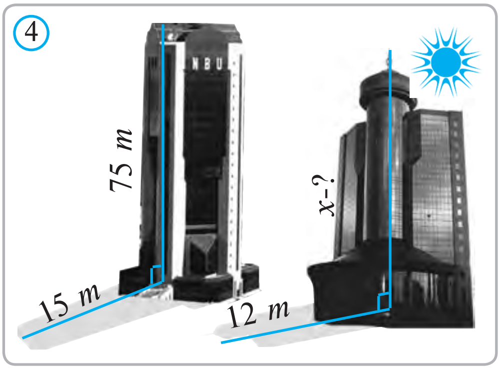
Ma'sele. *AB* ha'm *CD* kesindileri parallel tuwri'larda jatadi. Yeger $AB=12$ *sm*, $CD=15$ *sm* ha'm *AB* kesindinin' qanday da bir *m* tuwri'sindag'i proekciyasi 8 *sm* bolsa, *CD* kesindinin' usi *m* tuwri'sindag'i proektciyasin tabi'n'.

Sheshiliwi. *CD* kesindisinin' *m* tuwri'sindag'i proekciyasi *x* bolsi'n. Wonda, da'liyllengen teorema ha'm ma'sele sha'rtinen paydalani'p, proekciya du'zemiz:

$$\frac{x}{15}=\frac{8}{12}.$$

Bul ten'likten $x=10$ bolatug'inin tabami'z.
Juwabi': 10 *sm*.





Soraw, ma'sele ha'm tapsi'rmalar

1. Kesindinin' berilgen tuwri'dag'i proekciyasi degen ne?
2. Bir tuwri'da yaki parallel tuwri'larda jatqan kesindilerdin' da'l basqa bir tuwri'g'a proekciyalari berilgen kesindilerge proporcional yekenligin da'liyllen'.
3. *a* ha'm *b* tuwri'lari arasi'ndag'i mu'yesh 45° qa ten', *a* tuwri'sinda uzinlig'i 10 *sm* bolg'an *AB* kesindisi aling'an. *AB* kesindisinin' *b* tuwri'si'ndag'i' proektciyasin tabi'n'.
4. *AB* kesindisinin' ushlari' *l* tuwri'sinan 9 *sm* ha'm 14 *sm* qashiqliqta jatadi. Yeger *AB* kesindisi *l* tuwri'sin kesip wo'tpese ha'm *AB=13 sm* bolsa, *AB* kesindisinin' *l* tuwri'sindag'i proekciyasin tabi'n'.
5. *3*-ha'm *4*-su'wretlerdegi mag'liwmatlar tiykarinda imaratlardin' biyikliklerin tabi'n'.
6. Tuwri' ha'm wog'an parallel bolmag'an kesindi si'zi'n'. Kesindinin' tuwri'dag'i proekciyasi'n jasan'.
7. Koordinatalar tegisliginde *A*(2;3) ha'm *B*(3;-4) noqatlari belgilengen. *AB* kesindisinin' koordinata ko'sherlerindegi proekciyalari uzinliqlarin tabi'n'.
8. *a* ha'm *b* tuwri'lari' arasi'ndag'i mu'yesh α yekenligi belgili, *a* tuwri'sinda *AB* kesindisi aling'an. *AB* kesindisinin' *b* tuwri'sindag'i proekciyasin tabi'n'.

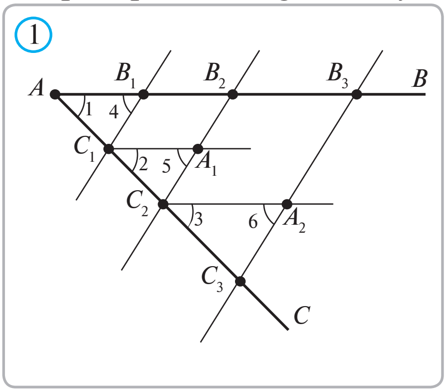
53 PROPORCIONAL KESINDILERDIN' QA'SIYETLERI

Fales teoremasinin' uluwmalastirilg'aninan ibarat a'hmiyetli qa'siyetin da'liylleymiz.

Teorema. **Mu'yeshtin' ha'r yeki ta'repin kesip wo'tken parallel tuwri'lar woni'n' ta'replerinen proportcional kesindiler ajiratadi.**

$\angle BAC,\ B_1C_1 || B_2C_2 || B_3C_3$ (*1-su'wret*) $\Rightarrow$ $\dfrac{AB_1}{AC_1}=\dfrac{B_1B_2}{C_1C_2}=\dfrac{B_2B_3}{C_2C_3}$

Da'liyllew. C_1 ha'm C_2 noqatlarinan AB g'a parallel C_1A_1 ha'm C_2A_2 tuwri'larin ju'rgizemiz. Bul jag'dayda, birinshiden, $\angle 1=\angle 2=\angle 3$ boladi, sebebi wolar wo'z ara parallel bolg'an AB, C_1A_1 ha'm C_2A_2 tuwri'larin AC kesip wo'tkende payda bolg'an sa'ykes mu'yeshler boladi'. Yekinshiden, $\angle 4=\angle 5=\angle 6$, sebebi wolar ta'repleri parallel bolg'an mu'yeshler boladi.



Demek, u'shmu'yeshliklerdin' uqsaslig'inin' MM belgisi boyi'nsha $\Delta AB_1C_1 \sim \Delta C_1A_1C_2 \sim \Delta C_2A_2C_3$ boladi.

Bul jag'dayda $\dfrac{AB_1}{AC_1}=\dfrac{C_1A_1}{C_1C_2}=\dfrac{C_2A_2}{C_2C_3}$ (1)

ten'liklerin payda yetemiz.

Bunnan tisqari, $B_1C_1A_1B_2$ ha'm $B_2C_2A_2B_3$ to'rtmu'yeshlikleri parallelogramm, sebebi

$B_1C_1 || B_2C_2 || B_3C_3$ — sha'rt boyi'nsha

$AB || C_1A_1 || C_2A_2$ — jasaw boyi'nsha.

Sonin' ushi'n, bul parallelogrammnin' qarama-qarsi ta'repleri wo'zara ten' boladi:

$$C_1A_1=B_1B_2 \quad \text{ha'm} \quad C_2A_2=B_2B_3. \qquad (2)$$

(1) ha'm(2) ten'liklerden $\dfrac{AB_1}{AC_1}=\dfrac{B_1B_2}{C_1C_2}=\dfrac{B_2B_3}{C_2C_3}$ bolatug'inlig'i kelip shig'adi.

Teorema da'liyllendi.

A'meliy shi'ni'g'iw. **Kesindini berilgen qatnasta bo'liw.**

Berilgen a kesindisin to'rt bo'lekke sonday yetip bo'lin', bo'leklerdin' wo'zara qatnasi $m:n:k:l$ siyaqli bolsi'n.

Bunin' ushi'n to'mendegilerdi qa'dembe-qa'dem worinlaymiz:

1-qa'dem. Qa'legen su'yir mu'yesh sizip, woni'n' bir ta'repine uzinliqlari $OA=m,\ AB=n,\ BC=l$ ha'm $CD=k$ g'a ten' bolg'an kesindilerdi 2-su'wrettegidey yetip, izbe-iz qoyip shig'amiz.

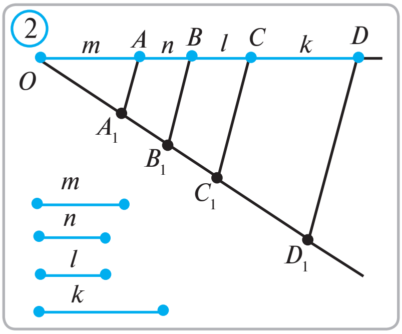
2-qa'dem. Mu'yeshtin' yekinshi ta'repindegi a kesindisine ten' OD_1 kesindisin qoyamiz.

3-qa'dem. D ha'm D_1 noqatlarin tutastiramiz.

4-qa'dem. A, B, C noqatlari arqali DD_1 ge parallel AA_1, BB_1 ha'm CC_1 kesindilerin ju'rgizemiz.

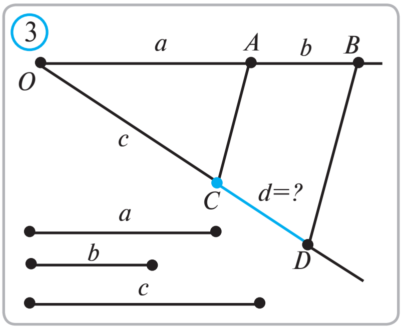
Joqaridag'i teorema boyi'nsha, berilgen $a=OD_1$ kesindisi A_1, B_1, C_1 ha'm D_1 noqatlari menen $m:n:l:k$ qatnasta bo'lingen boladi.

Tapsi'rma. Bul tastiyiqlawdi' wo'z betin'izshe tiykarlan'.



A'meliy tapsi'rma. **To'rtinshi proporcional kesindini jasaw.**

a, b ha'm *c* kesindileri berilgen. *a* ha'm *b* kesindilar *c* ha'm *d* kesindilerine proporcional, yag'niy $a:b=c:d$ yekenligin ma'lim. *d* kesindisin jasan' *(3-rasm)*.



1-qa'dem. Qa'legen su'yir mu'yesh sizip, woni'n' bir ta'repine $OA=a$ ha'm $AB=b$ kesindilerin 3-su'wrette ko'rsetilgendey yetip qoyamiz.

2-qa'dem. Yekinshi ta'repine bolsa $OC=c$ kesindisin qoyami'z.

3-qa'dem. A ha'm C noqatlarin tutastiramiz.

4-qa'dem. B noqatinan AC g'aparallel BD tuwri'sin ju'rgizemiz.

Tapsi'rma: CD izlenip atirg'an d kesindisi bolatug'inin tiykarlan'.

Soraw, ma'sele ha'm tapsi'rmalar

1. Uzinlig'i 42 *sm* bolg'an kesindi berilgen. Woni a) 5:2; **b)** 3:4:7; d) 1:5:1:7 qatnastag'i bo'lekshelerge bo'lin'.

2. Su'wrette ha'r bir bo'lek birlik kesindiden ibarat bolsa, AB ha'm CD, EF ha'm MN, AC ha'm DF, AN ha'm CE, EN ha'm BM kesindilerinin' qatnaslarin tabi'n'.

3. *m, n* kesindileri *l* ha'm *k* kesindilerine proporcional. Yeger **a)** $m=4$ *sm*, $n=3$ *sm* ha'm $l=8$ *sm*; b) $m=2$ *sm*, $n=3$ *sm* ha'm $l=7$ *sm* bolsa, *k* — to'rtinshi kesindini jasan' ha'm uzi'nli'g'i'n tabi'n'.

4. To'rtmu'yeshliktin' perimetri 54 *sm* ha'm ta'repleri 3:4:5:6 siyaqli qatnasta bolsa, woni'n' ha'r bir ta'repin ani'qlan'.

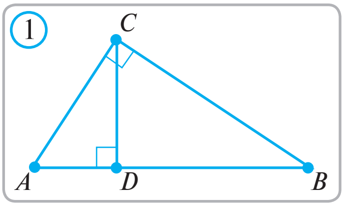
5. To'rtmu'yeshliktin' mu'yeshleri wo'zara 3:4:5:6 siyaqli qatnasta bolsa, woni'n' kishi mu'yeshi nege ten' yekenligin tabi'n'.

6. Uzinligi 4, 5 ha'm 6 bolg'an kesindiler berilgen. Uzinligi 4,8 ge ten' kesindi jasan'.

54 TUWRI' MU'YESHLI U'SHMU'YESHLIKTEGI PROPORTCIONAL KESINDILER

Qa'siyeti. Tuwri' mu'yeshli u'shmu'yeshliktin' tuwri' mu'yeshi to'besinen tu'sirilgen biyikligi woni wo'zine uqsas yeki u'shmu'yeshlikke ajiratadi.

ΔABC, $\angle C=90°$, *CD — biyiklik* (*1-su'wret*) ⇒ $\Delta ABC \backsim \Delta ACD$, $\Delta ABC \backsim \Delta CBD$



Da'lyillew. *ABC* ha'm *ACD* u'shmu'yeshlikleri tuwri' mu'yeshli boli'p, *A* mu'yeshi bolsa, wolar ushi'n ulawma. Demek, $\Delta ABC \backsim \Delta ACD$. Usi siyaqli, ΔABC ha'm ΔCBD da tuwri' mu'yeshli boli'p, wolar ushi'n $\angle B$ uliwma. Demek, $\Delta ABC \backsim \Delta CBD$.

1-su'wrettegi *AD* ha'm *DC* kesindilerge sa'ykes ra'wishte *AC* ha'm *BC* katetlerdin' gipotenuzadag'i proekciyasi dep ju'ritiledi.

Ani'qlama. Yeger *a*, *b* ha'm *c* kesindileri ushi'n $a:b=b:c$ bolsa, *b* kesindisi *a* ha'm *c* kesindileri arasi'ndag'i **worta proporcional kesindi** dep ataladi.

Worta proportcionalliq sha'rtin $b^2=ac$ yaki $b=\sqrt{ac}$ ko'rinisinde de jaziw mu'mkin.

Joqarida da'liyllengen qa'siyet ha'm 1-su'wretke tiykarlanatug'in bolsaq, worta proportcional kesindiler haqqindag'i to'mendegi teoremalar an'sat da'liyllenedi.

1-teorema. Tuwri' mu'yeshli u'shmu'yeshliktin' tuwri' mu'yeshi ushi'nan tu'sirilgen biyiklik katetlerdin' gipotenuzadag'i proekciyalari arasi'nda worta proporcional boladi.

Haqiyqattan da da'liyllengen qa'siyet boyi'nsha: $\Delta ACD \backsim \Delta CBD$. Bunnan,

$$\frac{AD}{CD}=\frac{CD}{BD} \Rightarrow CD^2=AD\cdot BD \Rightarrow CD=\sqrt{AD\cdot BD}.$$

2-teorema. Tuwri' mu'yeshli u'shmu'yeshliktin' kateti gipotenuza menen usi katettin' gipotenuzasindag'i proekciyasi arasi'nda worta proporcionali boladi (*1-su'wret*).

Haqiyqattan da, da'liyllengen qa'siyet boyi'nsha: $\Delta ABC \backsim \Delta ACD$. Bunnan,

$$\frac{AB}{AC}=\frac{AC}{AD} \Rightarrow AC^2=AB\cdot AD \Rightarrow AC=\sqrt{AB\cdot AD}.$$

Usig'an uqsas $BC=\sqrt{BD\cdot AB}$ yekenligin da'liyllew mu'mkin.

Ma'sele. Katetleri 15 sm ha'm 20 sm bolg'an tuwri' mu'yeshli u'shmu'yeshliktin' kishi katetinin' gipotenuzasindag'i proekciyasin tabi'n'.

ΔABC, $\angle C=90°$, *CD — biyiklik*, $AC=15$ *sm*, $BC=20$ *sm* (*1-su'wret*) ⇒ $AD=?$

Sheshiliwi. 1) Pifagor teoremasi'nan paydalani'p, u'shmu'yeshliktin' gipotenuzasin tabami'z: $AC^2 = AC^2 + BC^2 = 15^2 + 20^2 = 625$, yag'niy $AB = 25$ *sm*.

2) Yekinshi teoremadan paydalani'p AD ni tabami'z:

$$AC^2 = AB \cdot AD \Rightarrow AD = \frac{AC^2}{AB} = \frac{15^2}{25} = 9\ (sm).$$ ***Juwabi':*** 9 *sm*.

Bul yeki teoremadan na'tiyje sipatinda Pifagor teoremasinin' **Pifagordin' wo'zi jazip qaldirg'an da'liyli** kelip shig'adi (*1-su'wret*): 2-teorema boyi'nsha,

$$\left.\begin{aligned} AC^2 &= AD \cdot AB \\ BC^2 &= BD \cdot AB \end{aligned}\right\} \Rightarrow AC^2 + BC^2 = AD \cdot AB + BD \cdot AB = AB \cdot (\underbrace{AD + BD}_{AB}) = AB \cdot AB = AB^2.$$

Solay yetip, $AC^2 + BC^2 = AB^2$.

Soraw, ma'sele ha'm tapsi'rmalar

1. Da'liyllen' *(2-su'wret)*: a) $\Delta ACD \sim \Delta CBD \sim \Delta ABC$; b) $b^2 = b_c \cdot c$, $a^2 = a_c \cdot c$; **d)** $h_c^2 = a_c \cdot b_c$.

2. Tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasina tu'sirilgen biyikligi gipotenuzani 9 *sm* ha'm 16 *sm* ge ten' kesindilerge bo'ledi. U'shmu'yeshliktin' ta'replerin tabi'n'.

3. Tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi 15 *sm* ge, bir kateti bolsa 9 *sm* ge ten'. Yekinshi katetinin' gipotenuzasindag'i proekciyasin tabi'n'.

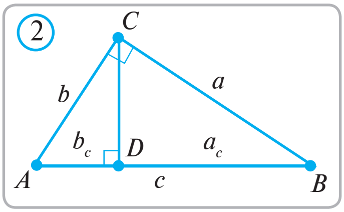
4. 3-su'wrettegi mag'liwmatlar tiykarinda ABC u'shmu'yeshliktin' ta'replerin tabi'n'.

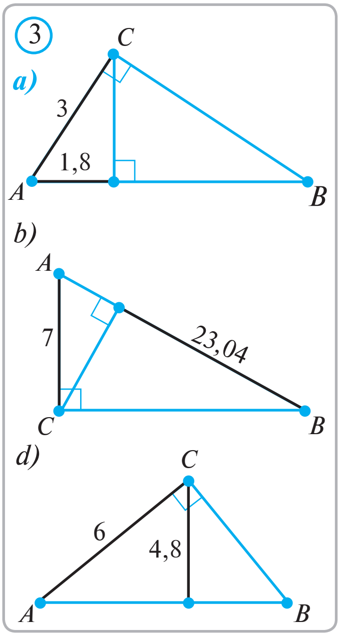
5*. Katetlerinin' qatnasi 4:5 siyaqli bolg'an tuwri' mu'yeshli u'shmu'yeshliktin' katetlerinin' gipotenuzasindag'i proekciyalarinin' qatnasin tabi'n'.

6*. Katetlerinin' qatnasi 3:2 siyaqli bolg'an tuwri' mu'yeshli u'shmu'yeshlik berilgen. Katetlerinin' gipotenuzasindag'i proekciyalarinan biri yekinshisinen 6 *sm* ge uzin. U'shmu'yeshliktin' maydani'n tabi'n'.

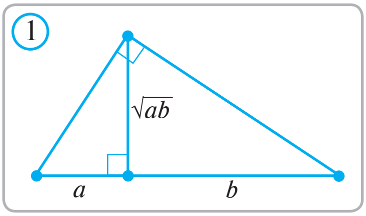
7. Katetlerinin' gipotenuzasindag'i proekciyalari 2 *sm* ha'm 18 *sm* bolg'an tuwri' mu'yeshli u'shmu'yeshliktin' maydani'n tabi'n'.

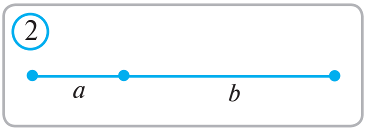
8*. ABC u'shmu'yeshliginde $\angle C = 90°$, CD — biyiklik, CE — bissektrissa ha'm $AE:EB = 2:3$. a) $AC:BC$; b) $S_{ACE}:S_{BCE}$; d) $AD:BD$ qatnaslari'n tabi'n'.

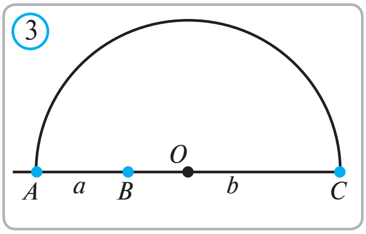


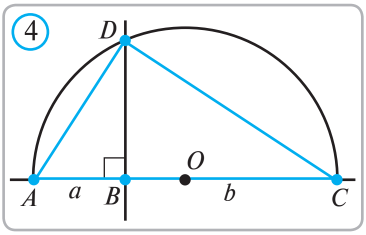


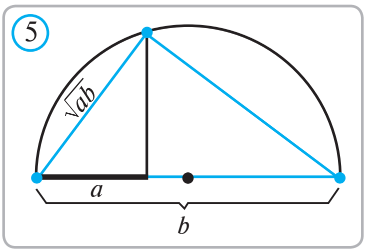
55 BERILGEN YEKI KESINDIGE WORTA PROPORTCIONAL KESINDINI JASAW











Tuwri' mu'yeshli u'shmu'yeshliktin' tuwri' mu'yeshinen tu'sirilgen biyikligi gipotenuzani a ha'm b kesindilerge bo'lse, biyiklik $\sqrt{ab}$ g'aten' bolatug'inin ko'rgen yedik (*1-su'wret*).

Demek, berilgen yeki kesindige worta proporcional kesindini jasaw ushi'n:

1) gipotenuzasinin' uzinlig'i $a+b$ g'aten' (*2-su'wret*);

2) tuwri' mu'yeshinen tu'sirilgen biyikligi usi gipotenuzani a ha'm b bo'leklerge bo'letug'in tuwri' mu'yeshli u'shmu'yeshlik jasaw jetkilikli.

Bunin' ushi'n tuwri' mu'yeshli u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'ber worayi gipotenuzanin' wortasinda jaylasqaninan paydalanamiz *(3-su'wret)*.

Jasaliwi:

1) tuwri' si'zi'q sizip, wog'an $AB=a$ ha'm $BC=b$ bolatug'inday yetip A, B ha'm C noqatlarin belgileymiz (*3-su'wret*).

2) AC kesindisinin' wortasi — O noqatin tabami'z. worayi O noqatinda bolg'an AC diametrli yarim shen'ber jasaymiz *(3-su'wret)*.

3) B noqatinan AC tuwri'sina perpendikulyar tuwri' ju'rgizemiz *(4-su'wret)*. Bul tuwri' yarim shen'berdi D noqatinda kesip wo'tken bolsi'n. Wonda ΔADC — tuwri' mu'yeshli u'shmu'yeshlik, $BD=\sqrt{ab}$ — biz jasawimiz kerek bolg'an kesindi boladi.

Jasaw wori'nlandi'.

Worta proportcional kesindini jasawda tuwri' mu'yeshli u'shmu'yeshliktin' kateti gipotenuza menen usi katettin' gipotenuzadag'i proekciyasi arasi'nda worta proportcional yekenligin paydalaniw da mu'mkin *(5-su'wret)*.

Soraw, ma'sele ha'm tapsi'rmalar

1. Uzinliqlari a ha'm b bolg'an kesindiler berilgen. Uzinlig'i $\sqrt{ab}$ bolg'an kesindini jasan'.

2. Uzinlig'i a ha'm b g'aten' kesindiler berilgen. Pifagor teoremasinan paydalani'p, uzinlig'i

a) $\sqrt{a^2+b^2}$; b) $\sqrt{a^2-b^2}$

bolg'an kesindilerdi jasan'.

3. Uzinlig'i 1 ge ten' bolg'an kesindi berilgen. Uzinlig'i a) $\sqrt{2}$; b) $\sqrt{3}$; d) $\sqrt{5}$; e) $\sqrt{6}$; f) $\sqrt{18}$; g) $\sqrt{30}$ bolg'an kesindilerdi jasan'.

4. 6-su'wrettegi mag'li'wmatlar tiykarinda ABC u'shmu'yeshliktin' maydani'n tabi'n'.

5. Shen'berdegi C noqatinan AB diametrge CD perpendikulyari' tu'sirilgen. Yeger $CD=12$ *sm*, $AD=24$ *sm* bolsa, do'n'gelektin' maydani'n tabi'n'.

6. Aldin'g'i ma'seledegi ABC u'shmu'yeshliktin' maydani'n tabi'n'.

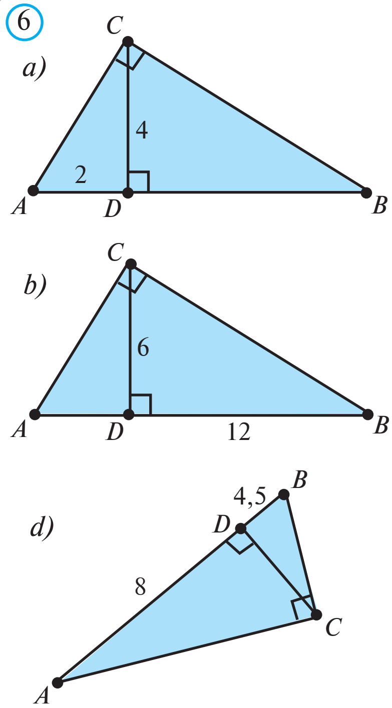
7. Tuwri' mu'yeshli u'shmu'yeshliktin' tuwri' mu'yeshinin' bissektrisasi' gipotenuzani $5:3$ siyaqli qatnasta bo'ledi. Tuwri' mu'yeshtin' to'besinen tu'sirilgen biyikliktin' gipotenuzadan ajiratqan kesindilerinin' qatnasin tabi'n'.

8. Radiusi 8 *sm* ge ten' do'n'gelekke bir mu'yeshi 30° bolg'an tuwri' mu'yeshli u'shmu'yeshliki shley si'zi'lg'an. Do'n'gelektin' u'shmu'yeshlikten si'rttag'i' bo'limi 3 segmentten ibarat. Usi segmentlerdin' maydanlarin tabi'n'.

9*. 7-su'wrette $AD=a$, $DB=b$, demek, $OC=\frac{a+b}{2}$ (O— shen'ber worayi). Su'wretten paydalani'p, $\frac{a+b}{2}\geq\sqrt{ab}$ ten'sizligin da'liyllen'.

Qi'zi'qli' ma'sele

Shen'berdin' AB diametri to'rt ten' bo'lekke bo'lindi ha'm 8-su'wrette ko'rsetilgendey yarim shen'berler jasaldi. Yeger $AB=d$ bolsa, su'wrette boyap ko'rsetilgen ha'r bir figuranin' maydani'n yesaplan'.

6


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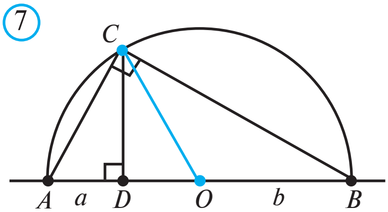

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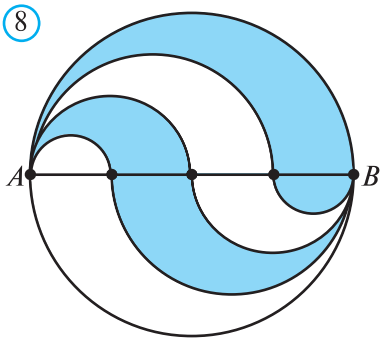
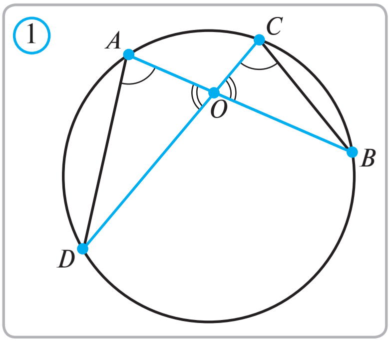


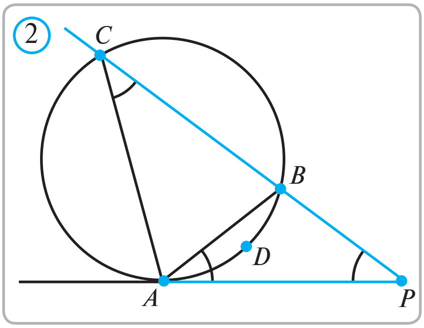
56 SHEN'BERDEGI PROPORTCIONAL KESINDILER

1-teorema. **Shen'berdin' AB ha'm CD xordalari' O noqatinda kesilisse, $AO \cdot OB = CO \cdot OD$ ten'ligi wori'nli' boladi.**



Da'liyllew. AB ha'm CD xordalari *(1-su'wret)* ko'rsetilgen ta'rtipte jaylasqan bolsi'n. To'belerin AD ha'm BC xoradalari menen tutastiramiz. Sonda BAD ha'm BCD mu'yeshleri bir dog'ag'atireledi, demek, $\angle BAD = \angle BCD$. Ja'ne, $\angle AOD = \angle BOC$ yekenligi belgili. Bul yeki ten'likten MM belgisi boyi'nsha AOD ha'm COB u'shmu'yeshliklerdin' uqsaslig'i kelip shig'adi. Uqsas u'shmu'yeshliklerdin' sa'ykes ta'repleri bolsa proporcional: $\frac{OD}{OB} = \frac{AO}{CO}$ yaki $AO \cdot OB = CO \cdot OD$. ***Teorema da'liyllendi.***

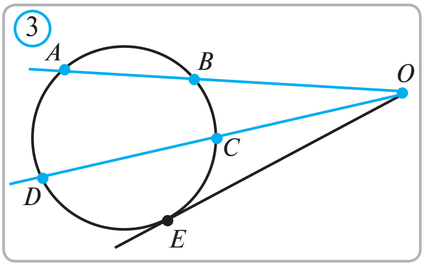
2-teorema. **Shen'berdin' si'rtindag'i P noqatinan shen'berge PA urinba (A – uriniw noqati) ha'm shen'berdi B ha'm C noqatlarinda kesip wo'tiwshi tuwri' ju'rgizilgen bolsa, $PA^2 = PB \cdot PC$ boladi.**



Da'liyllew. ABP ha'm CPA u'shmu'yeshliklerin qaraymiz *(2-su'wret)*. Wonda,

$\angle C = \frac{\overset{\frown}{ADB}}{2} = \angle BAP$ ha'mde $\angle P$ — bul u'shmu'yeshlikler ushi'n uliwma mu'yesh. Demek, ABP ha'm CPA u'shmu'yeshlikleri yeki mu'yeshi boyi'nshauqsas. Bunnan, $\frac{PA}{PC} = \frac{PB}{PA}$ yaki $PA^2 = PB \cdot PC$. ***Teorema da'liyllendi.***

Ma'sele. A, B, C ha'm D noqatlari shen'berdi AB, BC, CD ha'm AD dog'alarg'a ajiratadi. Yeger AB ha'm DC nurlari O noqatinda kesilisse, wonda $OA \cdot OB = OC \cdot OD$ ten'ligi wori'nli' bolatug'inin da'liyllen'.

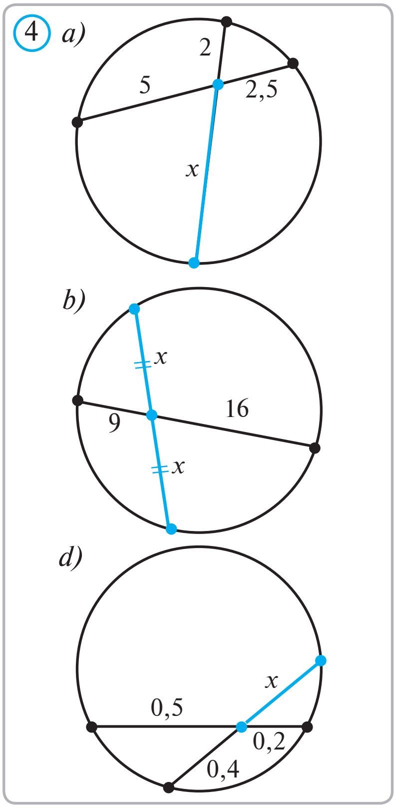


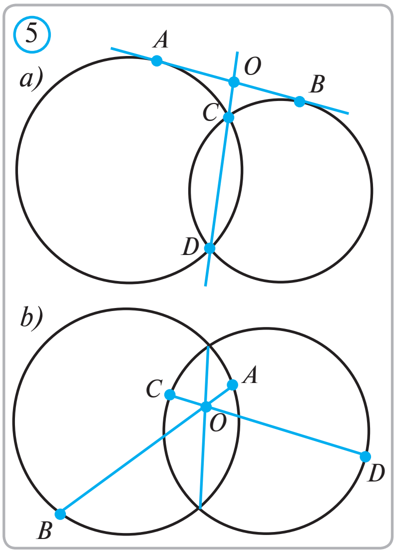
Sheshiliwi. Ma'sele sha'rtine sa'ykes si'zi'lma sizamiz *(3-su'wret)* ha'm O noqatinan OE urinba ju'rgizemiz. Wonda, 2-teoremag'a tiykarlanip,

$$\left.\begin{array}{l} OB \cdot OA = OE^2 \\ OC \cdot OD = OE^2 \end{array}\right\} \Rightarrow OA \cdot OB = OC \cdot OD.$$

Soraw, ma'sele ha'm tapsi'rmalar

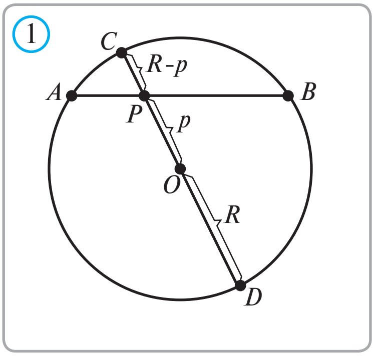
1. 4-su'wrette x penen belgilengen belgisiz kesindini tabi'n'.
2. A noqatinan shen'berge AB urinba (B — uriniw noqati) ha'm shen'berdi C ha'm D noqatlarinda kesip wo'tetug'in kesiwshi ju'rgizilgen. Yeger
   a) $AB=4$ *sm*, $AC=2$ *sm* bolsa, AD kesindisin;
   b) $AB=5$*sm*, $AD=10$ *sm* bolsa, AC kesindisin;
   d) $AC=3$ *sm*, $AD=2{,}7$ *sm* bolsa, AB kesindisin tabi'n'.
3. Shen'berge $ABCD$ to'rtmu'yeshligi ishley si'zi'lg'an. AB ha'm DC nurlari' O noqatinda kesilisedi. Yeger a) $AO=10$ *dm*, $BO=6$ *dm*, $DO=15$ *dm* bolsa, OC kesindisin;
   b) $CD=10$ *dm*, $OD=8$ *dm*, $AB=4$ *dm* bolsa, OB kesindisin tabi'n'.
4. Shen'berdin' AB diametri ha'm bul diametrge perpendikulyar CD xordasi E noqatinda kesilisedi. Yeger $AE=2$ *sm*, $EB=8$ *sm* bolsa, CD xordani tabi'n'.
5. AB ha'm CD kesindileri O noqatindakesilisedi. Yeger $AO\cdot OB=BO\cdot OD$ bolsa, A, B, C ha'm D noqatlarinin' bir shen'berde jatatug'inin da'liyllen'.
6. Radiusi 13 *dm* bolg'an shen'ber worayinan 5 *dm* qashiqliqta P noqati aling'an. P noqatinan uzinlig'i 25 *dm* bolg'an AB xorda ju'rgizilgen. AP ha'm PB kesindilerin tabi'n'.
7. 3-su'wrette AOD ha'm BOC u'shmu'yeshliklerdin' uqsas yekenliginen paydalani'p, $AO\cdot OB=CO\cdot OD$ ten'ligin da'liyllen'.
8*. 5-su'wretlerdegi mag'liwmatlar tiykarinda $AO\cdot OB=CO\cdot OD$ ten'ligin da'liyllen'.
9*. Yeki shen'ber C noqatindaurinadi'. AB tuwri' si'zi'q birinshi shen'berge B noqatindaurinadi. $\angle ACB=90^\circ$ yekenligin da'liyllen'.





57 MA'SELELER SHESHIW

Aldi'n'g'i' sabaqta shen'ber kesilisiwshileri ha'm xordalarinin' qa'siyetlerin da'liyllegen yedik. Yendi bul qa'siyetlerdin' ayi'rim jag'daylari' menen tanisamiz.



1-ma'sele. P noqati R radiusli' shen'berdin' ishki oblastindag'i wonin' worayinan p arali'qta jaylasqan bolsi'n. Wonda P noqatinan wo'tiwshi qa'legen AB xorda ushi'n

$$AP \cdot PB = R^2 - p^2 \qquad (1)$$

ten'ligi worinli' boladi.

Da'liyllew. P noqati arqali shen'berdin' CD diametrin ju'rgizemiz. Wonda, $PC = R - p$, $PD = R + p$ (*1-su'wret*). Kesip o'tiwshi xordalar haqqindag'i teorema boyi'nsha,

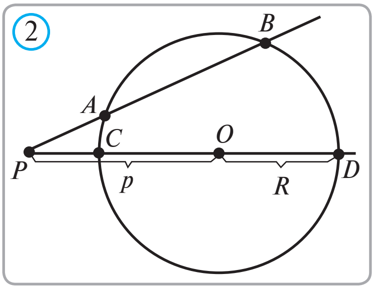
$$AP \cdot PB = CP \cdot PD = (R - p)(R + p) = R^2 - p^2.$$

(1) ten'lik da'ylillendi.

2-ma'sele. Radiusi 6 *sm* bolg'an shen'berdin' O worayinan 4 *sm* qashi'qli'qta P noqati alindi. P noqati' arqali' AB xorda ju'rgiziledi. Yeger $AP = 2$ *sm* bolsa, PB kesindisin tabi'n'.

Sheshiliwi. Ma'sele sha'rti boyi'nsha $R = 6$ *sm*, $d = 4$ *sm*, $AP = 2$ *sm*. Wonda (1) ten'lik boyi'nsha, $2 \cdot PB = 6^2 - 4^2 = 36 - 16 = 20$. Bunnan, $PB = 10$ *sm*.

Juwabi': $PB = 10$ *sm*.



3-ma'sele. P noqati R radiusli shen'berdin' si'rtqi' oblastinda woni'n' worayinan p aralig'inda jaylasqan bolsi'n. Wonda P noqat arqali' wo'tiwshi ha'm shen'berdi A ha'm B noqatlardakesip wo'tiwshi qa'legen tuwri' si'zi'q ushi'n ,

$$PA \cdot PB = p^2 - R^2 \qquad (2)$$

ten'ligi wori'nli' boli'wi'n da'liyllen.

Da'liyllew. Shen'berdin' O worayi arqali wo'tiwshi PO tuwri'si shen'ber menen C ha'm D noqatlarda kesilissin (*2-su'wret*). Onda, sha'rt boyi'nsha, $PC = p - R$, $PD = p + R$. Shen'berdin' si'rtqi' oblasti'ndag'i' noqattan ju'rgizilgen kesiwshiler haqqindag'i Teorema boyi'nsha,

$$PA \cdot PB = PC \cdot PD = (p - R)(p + R) = p^2 - R^2.$$

Solay yetip (2) ten'lik da'liyllendi.

4-ma'sele. Radiusi' 7 *sm* bolg'an shen'berdin' worayinan 13 *sm* qashiqliqtag'i *P* noqatinan wo'tiwshi tuwri' shen'berdi *A* ha'm *B* noqatlarda kesip wo'tedi. Yeger $PA=10$ *sm* bolsa, *AB* xordani tabi'n'.

Sheshiliwi. Sha'rt boyi'nsha, $R=7$ *sm*, $p=13$ sm. Wonda, (2) formulag'a ko're,

$$PA \cdot PB = p^2 - R^2 = 13^2 - 7^2 = 169 - 49 = 120.$$

Bunnan, $PB = \dfrac{120}{PA} = \dfrac{120}{10} = 12$ (*sm*). Demek,

$AB = PB - PA = 12 - 10 = 2$ (*sm*). ***Juwabi':*** 2 *sm*.

Soraw, ma'sele ha'm tapsi'rmalar

1. Radiusi 5 *sm* bolg'an shen'ber worayinan 3 *sm* qashiqliqta *P* noqati aling'an. *AB* xorda *P* noqati arqali wo'tedi. Yeger $PA=2$ *sm* bolsa, *AB* xorda uzi'nli'g'i'n tabi'n'.

2. Radiusi 5 *m* bolg'an shen'ber worayinan 7 *m* qashiqliqta *P* noqati aling'an. *P* noqati arqali wo'tiwshi tuwri' shen'berdi *A* ha'm *B* noqatinda kesip wo'tedi. Yeger $PA=4$ *m* bolsa, *AB* xorda uzi'nli'g'i'n tabi'n'.

3. 3-su'wrettegi mag'liwmatlar tiykarinda *x* penen belgilengen kesindini tabi'n' (*O*—shen'ber worayi).

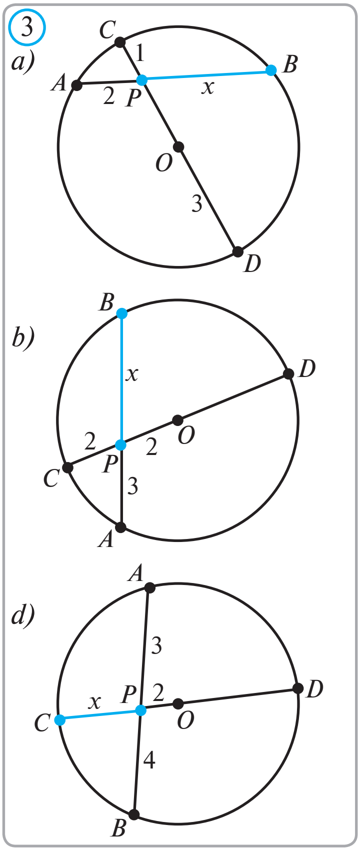
4. 4-su'wretten paydalani'p, ma'seleni sheshin'. Wonda
a) $PC=5$ *dm*, $OD=7$ *dm*, $AB=2$ *dm*, $PA=?$
b) $PA=5$ *dm*, $AB=4$ *dm*, $PC=3$ *dm*, $OD=?$

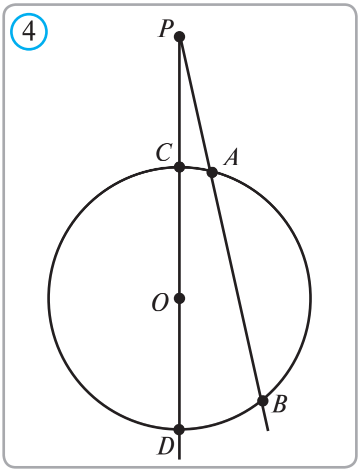
5. Shen'berdin' $AB=7$ *sm* ha'm $CD=5$ *sm* xordalari *P* noqatinda kesilisedi. Yeger $CP:PD=2:3$ bolsa, *P* noqati *AB* xordasin qanday qatnasta bo'ledi?

6. Shen'berdin' *C* noqatinan *AB* diametrge *CD* perpendikulyari tu'sirilgen. Yeger $AD=2$ *sm*, $DB=18$ *sm* bolsa, *CD* kesindisin tabi'n'.

7*. Shen'berge ishley si'zi'lg'an *ABCD* to'rt-mu'yeshliktin' diagonallari *K* noqatinda kesilisedi. Yeger $AB=2$, $BC=1$, $CD=3$ ha'm $CK:KA=1:2$ bolsa, *AD* kesindisin tabi'n'.

8*. Shen'berge ishley si'zi'lg'an *ABCD* to'rt-mu'yeshlikte $AB:DC=1:2$ ha'm $BD:AC=2:3$ bolsa, $DA:BC$ qatnasi'n tabi'n'.





58 IV BAPQA TIYISLI QOSI'MSHA MA'SELELER HA'M MAG'LIW-MATLAR

I. Testler

1. **Tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasina tu'sirilgen biyikligi haqqinda naduri's tastiyiqlawdi ko'rsetin':**
A. Katetlerinen kishi;
B. U'shmu'yeshlikti yeki uqsas u'shmu'yeshliklerge ajiratadi;
D. Katetlerinin' gipotenuzadag'i proekciyalari arasi'nda worta proporcional;
E. Gipotenuzanin' yarimina ten'.
2. ***AB* ha'm *CD* xordalar *O* noqatinda kesilisedi. Naduri's tastiyiqlawdi tabi'n':**
A. $\angle DAB = \angle DCB$; B. *AOD* ha'm *COB* u'shmu'yeshlikler uqsas;
D. $AO \cdot OB = CO \cdot OD$; E. $AO = CO$.
3. **Duri's tastiyiqlawdi tabi'n':**
A. Ten' kesindilerdin' proektciyalari da ten' boladi;
B. U'lken kesindinin' proekciyasi u'lken boladi;
D. Bir tuwri' si'zi'qtag'i ten' kesindilerdin' proekciyalari ten' boladi;
E. Proekciya uzinlig'i proektciyalaniwshi kesindi uzi'nli'g'i'na ten' boladi;
4. **Tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasina tu'sirilgen biyiklik woni yeki u'shmu'yeshlikke bo'ledi. Bul u'shmu'yeshlikler:**
A. Ten'; B. Ten'les; D. Uqsas; E. Ten' qaptalli.
5. **Uzinlig'i *a* ha'm *b* bolg'an kesindilerdin' worta proporcionali nege ten'?**
A. $a+b$; B. $\sqrt{ab}$; D. $\frac{a+b}{2}$; E. $a:b$.
6. ***ABCD* to'rtmu'yeshligi *O* worayli shen'berge ishley si'zi'lg'an. Naduri's tastiyiqlawdi ko'rsetin':**
A. $\Delta AOB \sim \Delta COD$; B. $\angle A + \angle C = \angle B + \angle D$;
D. $AO \cdot OB = CO \cdot OD$; E. $AB \cdot CD = BC \cdot AD$.

II. Ma'seleler

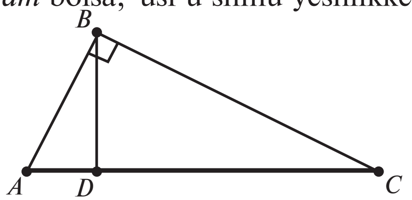
1. Tuwri' mu'yeshli u'shmu'yeshlik katetlerinin' qatnasi 3:4 ke ten'. Bul u'shmu'yeshliktin' gipotenuzasi 50 *sm*. U'shmu'yeshliktin' tuwri' mu'yeshi to'besinen tu'sirilgen biyikligi gipotenuzadan qanday uzinliqtag'i kesindiler ajiratadi?
2. Shen'berdin' *AB* ha'm *CD* xordalarita E noqatinda kesilisedi.Yeger $AE=5$ *sm*, $BE=2$ *sm* ha'm $EC=2{,}5$ *sm* bolsa, *ED* ni tabi'n'.
3. Radiusi 6 *m* bolg'an shen'berdin' worayinan 10 *m* qashiqliqta *K* noqati alindi ha'm *K* noqatinan shen'berge urinba ju'rgizildi. Urinbanin' uriniw noqati *P* menen *K* noqati arasi'ndag'i arali'qti tabi'n'.
4. *ABC* u'shmu'yeshliginde $\angle C = 90°$ ha'm *CD* biyikligi 4,8 *dm*. Yeger $AD=3{,}6$

dm bolsa, *AB* ta'repin tabi'n'.

5. Shen'berdin' *AB* ha'm *CD* xordalari *O* noqatta kesilisedi. Yeger $AO=6$, $OB=4$ ha'm $CO=3$ bolsa, *OD* kesindisin tabi'n'.
6. Shen'berde *A*, *B*, *C*, *D* noqatlari belgilengen, *BA* ha'm *CD* nurlari *O* noqatinda kesilisedi. Yeger $OA=5$, $AB=4$, $OD=6$ bolsa, *DC* xordasin tabi'n'.
7. Shen'berge *B* noqatinda uriniwshi tuwri'sinin' u'stinen *A* noqati belgilendi. Yeger $AB=12$ ha'm *A* noqatinan shen'berge shekem yen' qisqaarali'q 8 bolsa, shen'ber radiusin tabi'n'.
8. Yarim shen'berdegi *C* noqatinan *AB* diametrge tu'sirilgen *CD* perpendikulyar *AB* kesindide 4 ha'm 9 g'aten' kesindilerdi ajiratadi. *CD* kesindisin tabi'n'.
9. Tuwri' mu'yeshli u'shmu'yeshliktin' biyikligi gipotenuzani 3 *dm* ha'm 12 *dm* ge ten' kesindilerge bo'ledi. U'shmu'yeshliktin' maydani'n tabi'n'.
10. Radiusi 5 *sm* bolg'an *O* worayina iye shen'berdin' *AB* xordasinda *D* noqati aling'an. Yeger $AD=2$ *sm*, $DB=4{,}5$ *sm* bolsa, *OD* kesindisin tabi'n'.
11. Radiusi 5 *sm* bolg'an *O* worayina iye shen'berdi *A* ha'm *B* noqatlarda kesip wo'tiwshi tuwri'da *P* noqati alindi. Yeger $PA=5$ *m*, $AB=2{,}8$ *m* bolsa, *OP* aralig'in tabi'n'.
12. To'rt parallel tuwri' berilgen. Wolar mu'yeshtin' ta'replerin *A* ha'm A_1, *B* ha'm B_1, *C* ha'm C_1 ha'm de *D* ha'm D_1 noqatlarindakesip wo'tedi. Yeger $AB=8$, $CD=12$ ha'm $C_1D_1=9$ bolsa, A_1B_1 kesindisin tabi'n'.
13. Shen'ber mu'yeshke ishley si'zi'lg'an. Yeger mu'yesh ushi'nan shen'berge shekem bolg'an arali'q radiusqa ten' bolsa, mu'yeshtin' shamasin tabi'n'.
14. Shen'berge *AB* diametrinin' *B* ushi'nan *BC* urinba ha'm *AC* kesiwshi ju'rgizilgen. *AC* kesiwshisi shen'ber menen *D* noqatindakesilisedi. Yeger $AD=DC$ bolsa, *CBD* mu'yeshin tabin.
15. Tuwri' mu'yeshli u'shmu'yeshliktin' katetlerinin' qatnasi 2:3 tu'rinde. U'shmu'yeshliktin' gipotenuzag'a tu'sirilgen biyiklik woni yeki ushmu'yeshlikke bo'ledi. Wolardi'n' maydanlarinin' qatnasin tabi'n'.

III. Wo'zin'izdi sinap ko'rin' (u'lgi ushi'n baqlaw jumi'si')

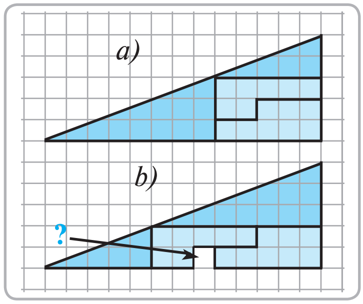
1. Shen'berdin sirti'ndag'i' noqattan wog'an shekem bolg'an yen' qisqa arali'q 2 *sm* ge, uriniw noqatinashekem bolg'an arali'q bolsa 6 *sm* ge ten'. Shen'berdin' radiusin tabi'n'.
2. ΔABC tuwri' mu'yeshli, $AD=9$ *dm*, $BD=16$ *dm* bolsa, usi u'shmu'yeshlikke ishley si'zi'lg'an shen'ber radiusi'n yesaplan'.



3. Noqattan tuwri'g'a yeki qiya ju'rgizilgen. Yeger qiyalar 1:2 qatnasta boli'p, wolardin' proekcsiyalari 1 *m* ha'm 7 *m* bolsa, qiyalardi'n' uzi'nliqlari'n tabin'.

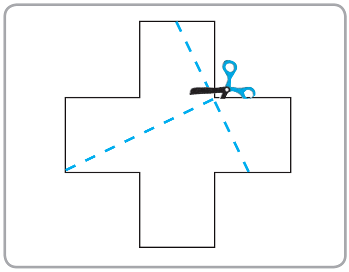
4.* *(Qosi'msha ma'sele).* PQ ha'm wonnan uzin ET kesindileri berilgen. Sonday $ABCD$ to'rtmu'yeshligin jasan', na'tiyjede $AB=BC=PQ$; $BD=ET$ boli'p, diagonallari kesilisetug'in O noqati ushi'n $AO\cdot OC=BO\cdot OD$ ten'ligi wori'nli' bolsi'n.

Qi'zi'qli' ma'seleler



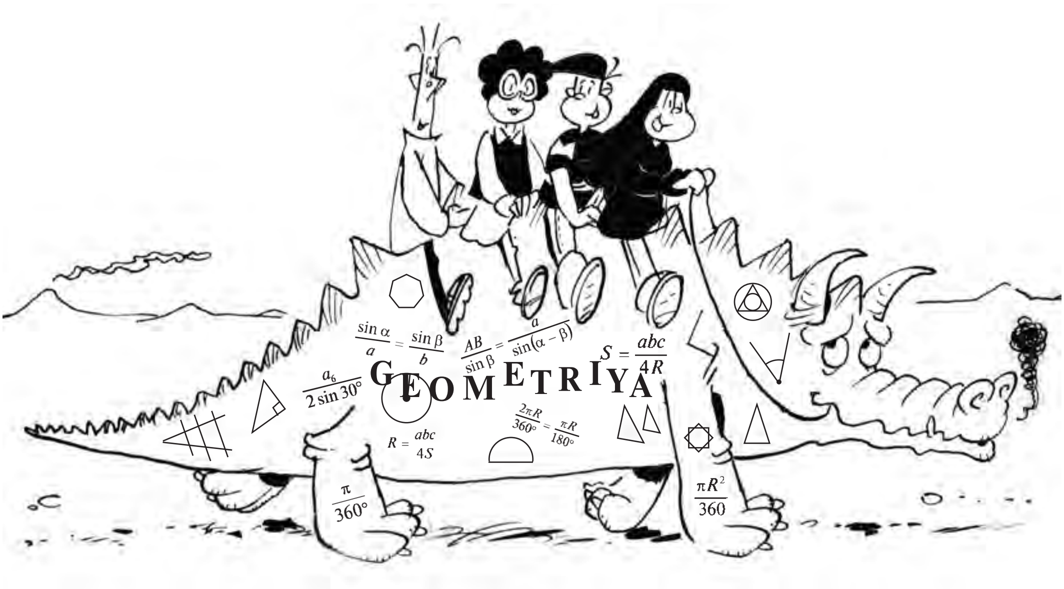
U'shmu'yeshlik 4-a su'wrette ko'rsetilgenindey yetip, to'rt bo'lekke bo'lingen ha'm 4-b su'wrette ko'rsetilgendey etip qayta jiynalg'an. Artiq kvadrat qalay payda bolg'ani'n aytip berin'.

(Yunon) Grek



Eramizdan a'ldi'n'gi 500-jillarda payda bolg'an bul forma wo'mirdin' belgisi sipatinda nan u'stinde si'zi'lg'an. *(4 su'wret)*

Bul formani qalin' qag'azg'a sizip alip woni su'wrette ko'rsetilgen si'zi'qlar boylap qirqin'. Payda bolg'an bo'leklerden kvadrat jasaw mu'mkinshiligine isenim payda yetin'.



V BAP

PLANIMETRIYA KURSI BOYI’NSHA TA’KIRARLAW

Usi bapti’ u’yreniw na’tiyjesinde siz to’mendegi bilim ha’m ko’nlikpe-lerge iye bolasiz:

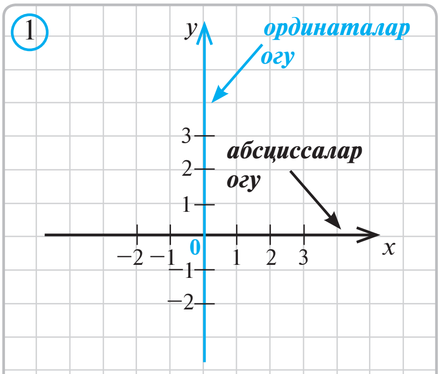
√ *geometriyani’n’ planimetriya bo’limi boyi’nsha wo’tilgen temalardi yeske aliw;*

√ *planimetriya kursi’ boyi’nsha wo’zlestirilgen bilim, ta’jiriybe ha’m ko’nlikpelerdi bekkemlew;*

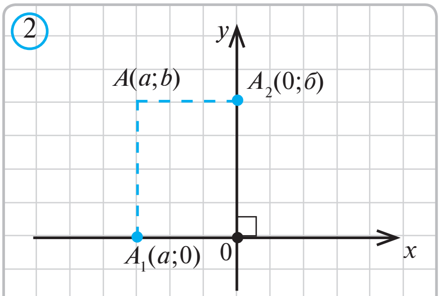
√ *juwmaqlawshi baqlaw jumi’si’na tayarliq ko’riw.*

59 KOORDINATALAR USI'LI'

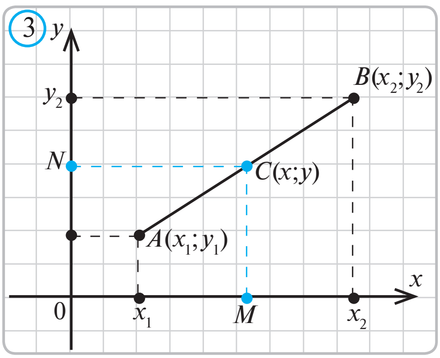
Tegisliktegi tuwri' mu'yeshli koordinatalar sistemasi menen 7-klass algebra kursinda tanisqansiz (*1—2-su'wret*). To'mende usi temag'a tiyisli geometriyaliq ma'selelerdi qaraymiz.



0 — координаталардын башталышы



(a, b)— A чекитинин координаталары: a — анын абсциссасы; b — анын ординатасы.



1-ma'sele. To'beleri koordinatalar tegisliginin' birinshi shereginde bolg'an AB kesindisi berilgen bolsi'n: $A(x_1;y_1)$ ha'm $B(x_2;y_2)$, $x_1>0$, $y_1>0$, $x_2>0$, $y_2>0$ (*3-su'wret*). AB kesindisinin' wortasi bolg'an $C(x;y)$ noqatinin' koordinatalarin tabi'n'.

Sheshiliwi. Bul jag'dayda CN kesindi ultanlarinin' uzinliqlari x_1 ha'm y_1 bolg'an trapeciyanin' worta sizig'i, CM kesindisi bolsa, ultanlarinin' uzinliqlari x_2 ha'm y_2 bolg'an trapeciyanin' worta sizig'i boladi.

Trapeciyanin' worta sizig'inin' qa'siyeti boyi'nsha,

$$x = \frac{x_1 + x_2}{2}; \quad y = \frac{y_1 + y_2}{2} \qquad (1)$$

boladi.

Bul formulalardin' duri's yekenligin AB kesindisinin' basqa jag'daylari ushi'n da usig'an uqsas ko'rsetiw mu'mkin.

2-ma'sele. To'beleri $A(-1;-2)$, $B(2;-5)$, $C(1;-2)$, $D(-2,1)$ noqatlarinda bolg'an $ABCD$ to'rtmu'yeshliginin' parallelogramm yekenligin da'liyllen'.

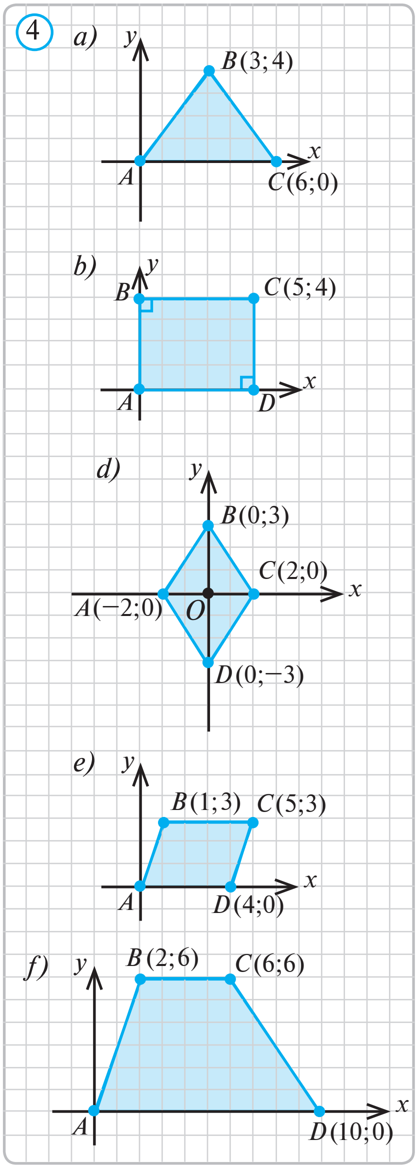
Sheshiliwi. (1) formuladan paydalani'p, to'rtmu'yeshliktin' AC ha'm BD diagonallari wortasinin' koordinatalarin tabami'z:

$$AC: \; x = \frac{-1+1}{2} = 0, \quad y = \frac{-2+(-2)}{2} = -2;$$

$$BD: \; x = \frac{2+(-2)}{2} = 0, \quad y = \frac{-5+1}{2} = -2$$

Demek, $ABCD$ to'rtmu'yeshliginin' ha'r yeki diagonali'nin' wortasi da bir $(0; -2)$ noqati boladi yeken. Basqasha aytqanda $ABCD$

to'rtmu'yeshliginin' diagonallari (0; −2) noqatinda kesilisedi ha'm usi noqatta ten' yekige bo'linedi. Bul *ABCD* to'rtmu'yeshliginin' parallelogramm boliwi belgilerinin' biri boli'p yesaplanadi.



Soraw, ma'sele ha'm tapsi'rmalar

1. Ko'pmu'yeshliklerdin' maydanlarin yesaplan' (*4-su'wret*).

2. Shen'berdin' 8 *sm* ge ten' xordasi shen'berden 90° qa ten' dog'ani ajiratadi. Shen'ber worayinan xordag'a shekemgi bolg'an qashiqliqti tabi'n'.

3. Ta'repleri a) 5,5 ha'm 6; b) 17, 65, 80 bolg'an u'shmu'yeshliktin' maydani'n tabi'n'.

4. Ta'repleri a) 13, 13, 12; b) 35, 29, 8 bolg'an u'shmu'yeshlikke ishley si'zi'lg'an shen'berdin' radiusin tabi'n'.

5. Ushlari to'mendegishe bolg'an kesindilerdin' wortasinin' koordinatalarin tabi'n':

a) $A(1;-2)$, $B(5;6)$;
b) $A(4;-3)$, $B(1;2)$;
d) $A(-4;5)$, $B(2;3)$;
e) $A(-0,7;2)$, $B(-0,3;4,2)$.

6*. Yeger $A(1;\ 0)$, $B(2;\ 3)$, $C(3;\ 2)$ bolsa, *ABCD* parallelogrammnin' *D* to'besinin' koordinatalarin tabi'n'.

7*. Parallelogramm mu'yeshlerinin' bissektrisalari' kesilisken noqatlar tuwri'mu'yeshliktin' to'beleri bolatug'inin da'liyllen'.

8. Katetleri 40 *sm* ha'm 30 *sm* bolg'an tuwri'mu'yeshli u'shmu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an shen'berlerdin' radiuslarin tabi'n'.

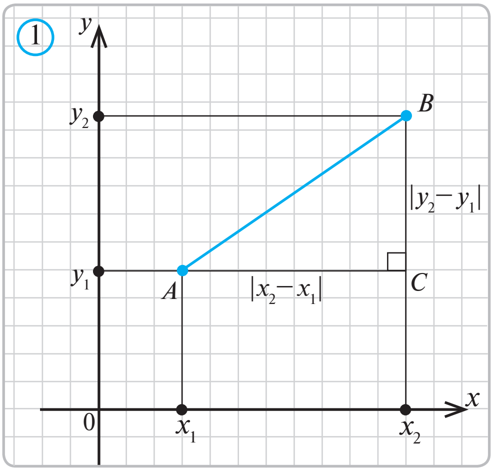
9. Shen'berge ishley si'zi'lg'an to'rtmu'yeshliktin' u'sh mu'yeshi 2:3:4 siyaqli qatnas payda yetedi. Woni'n' mu'yeshlerin tabi'n'.

10. Radiusi 6 *sm* bolg'an shen'berdin' 60° qaten' dog'asin kerip turg'an xordasin tabi'n'.

11. Radiuslari 6 *sm* bolg'an shen'berlerdin' woraylari arasi'ndag'i arali'q $6\sqrt{2}$ *sm* ge ten'. Shen'berlerdin' uliwma xordasinin' uzi'nli'g'i'n tabi'n'.

60 KOORDINATALAR USI’LI’ HA’M VEKTORLAR

1-ma’sele. Koordinatalar tegisliginde berilgen $A(x_1;y_1)$ ha’m $B(x_2;y_2)$ noqatlari ara-sindag’i arali’q $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ formula menen yesaplanatug’inin ko’rsetin’.



Sheshiliwi. Aytayiq, A ha’m B noqatlari 1-su’wrettegidey jaylasqan bolsi’n ($x_1 \neq x_2$, $y_1 \neq y_2$). A ha’m B noqatlardan koordinata ko’sherlerine parallel tuwri’lar ju’rgizemiz ha’m wolardi’n’ kesilisiw noqatin C menen belgileymiz.

Wonda, $AC=|x_2-x_1|$ ha’m de $BC=|y_2-y_1|$. ABC tuwri’ mu’yeshli u’shmu’yeshlikke Pifagor teoremasin qollansaq,

$$AB^2=AC^2+BC^2=(x_2-x_1)^2+(y_2-y_1)^2$$

boladi. Bunnan,

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

formulani payday ytemiz. Bul formulanin’ $x_1=x_2$ yaki $y_1=y_2$ bolg’andada duri’r bolatug’inin tekserip ko’rin’.

2-ma’sele. Yeger $A(-3;-1)$, $B(1;-1)$, $C(1;-3)$, $D(-3;-3)$ bolsa, $ABCD$ tuwri’mu’yeshlik yekenligin da’liyllen’.

Sheshiliwi. 1) AC diagonali’ wortasinin’ x, y koordinatalarin tabami’z:

$$x=\frac{-3+1}{2}=-1; \qquad y=\frac{-1-3}{2}=-2.$$

BD diagonali wortasinin’ x, y koordinatalarin tabami’z:

$$x=\frac{1-3}{2}=-1; \qquad y=\frac{-1-3}{2}=-2.$$

Demek, $ABCD$ to’rtmu’yeshligin diagonallari bir (1; 2) noqatinda kesilisedi ha’m usi noqatta ten’ yekige bo’linedi yeken. Bul, $ABCD$ parallelogramm yekenligin ko’rsetedi.

2) $ABCD$ parallelogramm diagonallari’ni’n’ uzi’nli’g’i’n tabami’z:

$$AC=\sqrt{(1-(-3))^2+(-3-(-1))^2}=\sqrt{4^2+(-2)^2}=\sqrt{20};$$

$$BD=\sqrt{(1-(-3))^2+(-1-(-3))^2}=\sqrt{4^2+2^2}=\sqrt{20}.$$

Demek, $ABCD$ parallelogrammnin’ diagonallari wo’z ara ten’. Bul (tuwri’-mu’yeshliktin’ belgisi boyi’nsha), $ABCD$ — tuwri’mu’yeshlik yekenligin bildiredi.

Soraw, ma'sele ha'm tapsi'rmalar

1. Yeger a) $A(2;7)$, $B(-2;7)$; b) $A(-5;-1)$, $B(5;-7)$; d) $A(-3;0)$, $B(0;4)$; e) $A(0;3)$, $B(-4;0)$ bolsa, AB kesindisinin' uzi'nli'g'i'n yesaplan'.

2. Yeger $M(4;0)$, $N(12;-12)$, $P(5;-9)$ bolsa, MNP u'shmu'yeshliginin' perimetrin tabi'n'.

3. Kollinear $\bar{x}$ ha'm $\bar{y}$ vektorlarin si'zi'n' ha'm $2\bar{x}+3\bar{y}$ vektorin jasan'.

4. Yeger A, B, C ha'm D noqatlari' bir tuwri'da jatpasa ha'm $\overline{AB}=0,7\overline{DC}$ bolsa, $ABCD$ to'rtmu'yeshliginin' tu'rin ani'qlan'.

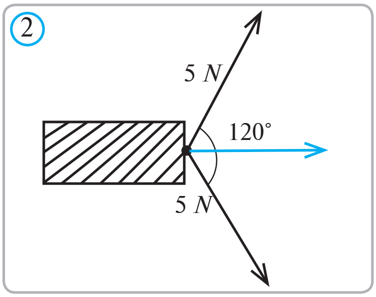
5. Kollinear yemes $\bar{a}$ ha'm $\bar{b}$ vektorlari' berilgen. Yeger $3\bar{a}-x\bar{b}=y\bar{a}+4\bar{b}$ bolsa, x ha'm y sanlarin tabi'n'.

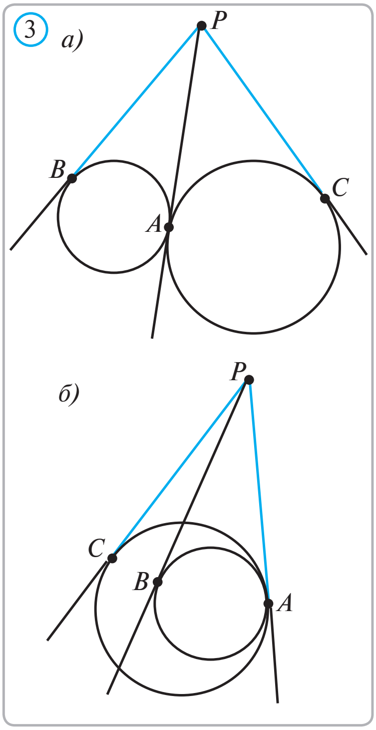
6. Yeger AA_1, BB_1 ha'm CC_1 kesindileri ABC u'shmu'yeshliklerinin' medianalari ha'm O — qa'legen noqat bolsa, $\overline{OA}+\overline{OB}+\overline{OC}=\overline{OA_1}+\overline{OB_1}+\overline{OC_1}$ ten'ligin da'liyllen'.

7. ABC u'shmu'yeshliginin' medianalari O noqatinda kesilisedi. $\overline{AB}$, $\overline{BC}$ ha'm $\overline{CA}$ vektorlarin $\bar{a}=\overline{OA}$ ha'm $\bar{b}=\overline{OB}$ vektorlari arqali an'latin'.

8. Denege ha'r biri $5N$ bolg'an yeki ku'sh ta'sir ko'rsetip atir (*2-su'wret*). Yeger bul ku'shlerdin' bag'itlari arasi'ndag'i mu'yesh 120° bolsa, wolardi'n' ten' ta'sir yetiwshisinin' shamasin tabi'n'.

9. Ten' ta'repli u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusi 6 sm. U'shmu'yeshliktin' perimetrin ha'm maydani'n tabi'n'.





10. Shen'berge A noqatinan ju'rgizilgen urinbada B noqati belgilendi. B noqatinan shen'berdin' yen' jaqin noqatina shekem bolg'an arali'q 4 *sm* ge, yen' uzaq noqatina shekem bolg'an arali'q 8 *sm* ge ten'. AB kesindisin tabi'n'.

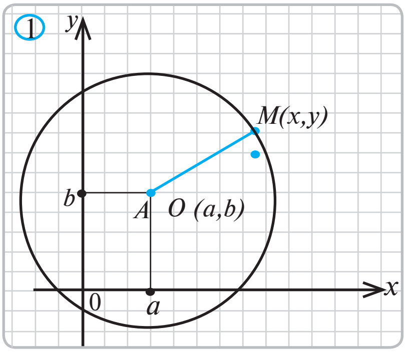
11*. Radiuslari ha'r tu'rli bolg'an yeki shen'ber A noqatinda PA tuwri'sinaurinadi. Bul shen'berlerge sa'ykes tu'rde PA dan wo'zgeshe PB ha'm PC urinbalar ju'rgizilgen. Yeger B ha'm C bul urinbalardin' shen'berge uriniw noqatlari bolsa, $PC=PB$ ten'ligin da'liyllen' (*3-su'wret*).

61 SHEN'BER HA'M DO'N'GELEK

1-ma'sele. Koordinatalar tegisliginde worayi $O(a; b)$ noqatta ha'm radiusi R bolg'an shen'berdegi qa'legen $M(x; y)$ noqattin' x ha'm y koordinatalari

$$(x - a)^2 + (y - b)^2 = R^2 \qquad (1)$$

ten'likti qanaatlandiriwin da'liyllen'.



Sheshiliwi. $O(a; b)$ — berilgen shen'berdin' worayi, $M(x; y)$ — ysi shen'berdin' qa'legen noqati bolsa, wonda OM=R boladi. Koordinatalar tegisliginde berilgen yeki noqat arasi'ndag'i arali'qti tabiw formulasina tiykarlanip (134-bettegi 1- maselege qaran')

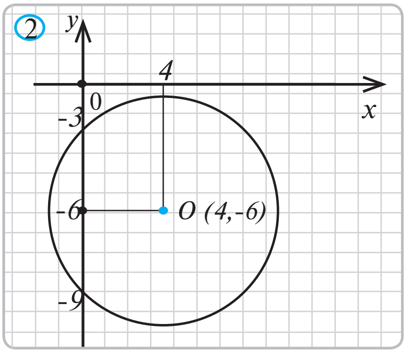
$$OM = \sqrt{(x - a)^2 + (y - b)^2}.$$

Solay yetip,

$$\sqrt{(x - a)^2 + (y - b)^2} = R.$$

Son'g'i ten'liktin' eki jag'inda kvadratqa ko'terip, (1) ten'likti payda yetemiz.

Yesletpe. (1) ten'leme worayi *(a,b)* noqatta bolg'an *R* radiusli **shen'ber ten'lemesi** delinedi.



2-ma'sele. Koordinatalar tegisliginde usi

$$(x - 4)^2 + (y + 6)^2 = 25$$

ten'leme menen ani'qlang'an shen'berdi wordinatalar ko'sherinen ajiratqan kesindinin' ushlarini'n koordinatlari'n tabi'n'

Sheshiliwi. Berilgen shen'ber menen ordinatalar ko'sheri kesilisken noqatlardin' abscisalari nolge ten' boladi. $x=0$ bolg'anda, berilgen ten'lemeden paydalani'p, bul noqatlardin' ordinatasin tabamiz: $(0 - 4)^2 + (y + 6)^2 = 25$, $(y + 6)^2 = 9$, $y = -9$ yoki $y = -3$. Demek , shen'ber ha'm ordinatalar ko'sheri (0; — 9) ha'm (0; — 3) noqatta kesilisedi. Bul noqatlar arasi'ndag'i arali'q 6 birlikke ten'. ***Juwabi':*** 6.

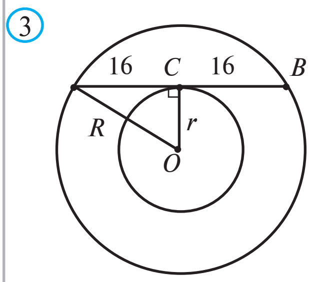
3-masele. Worayi *O* noqatta jaylasqan eki don' gelek jag'dayinda duziledi. U'lken don'gelektin' 32 *sm* ge ten' *AB* xordasi kishi don'gelekke *C* noqatta urinadi.(*3-suwret*) Yeger sagi'ynanin' ken'ligi 8 *sm* bolsa, wol jag'dayda halqanin' sagi'ynani'n tabi'n'.

Sheshiliwi. U'lken don'gelektin' radiusi' *R* menen, kishkenesin bolsa *r* menen belgileymiz. Maselenin' sha'rtine ko're $OA=R=r+8$(sm) ha'm $OC=r$. Bunnan basqa, C noqatta AB xordanin' wortasi yag'niy $AC=16sm$, OCA u'shmu'yeshlik bolsa tuwri' mu'yeshli boladi.

Pifagor teoremasinan paydalani'p, $OC^2+CA^2=OA^2$ bolg'ani' ushi'n,

$$r^2 + 16^2=(r + 8)^2$$

ten'lemesin payda yetemiz. Bul ten'lemeni sheship, $r=12$ *sm* yelkenligin tabami'z. Wonda $R=r+8=20$ (*sm*) boladi. Ulken don'gelektin' maydani'nan kishkenesin ayi'ri'p, berilgen jag'dayda maydani' S ti tabami'z:



$S=\pi R^2 - \pi r^2=20^2\pi - 12^2\pi=400\pi - 144\pi = 256\pi$ (sm^2). ***Juwabi':*** 256π sm^2.

Soraw, ma'sele ha'm tapsi'rmalar

1. To'mendegi ten'lemeler menen berilgen shen'berler worayinin' koordinatalarin ha'm radiusin tabi'n'. Usi shen'berlerdi jasan'.

a) $(x - 1)^2 + (y + 2)^2 = 4$; b) $(x - 4)^2 + (y - 3)^2 = 16$;
d) $x^2 + y^2 = 25$; e) $x^2 + (y - 2)^2 = 9$.

2. Shen'berge ishiley si'zi'lg'an $ABCD$ to'rtmu'yeshliktin' A, B ha'm C ushlarindag'i mu'yeshlerinin' qatnaslari 1:2:3 tu'rinde to'rtmu'yeshliktin' ishki mu'yeshlerin tabi'n'.

3. Shen'berdin' 1:8 bo'limine sa'ykes orayliq mu'yeshti tabi'n'.

4. Worayi A noqatda bolg'an shen'berge B noqat aling'an. Worayi B noqattan bolg'an basqa shen'ber A noqattan wo'tedi. Bul yeki shen'ber C noqatta kesilisedi. ACB mu'yeshti tabi'n'.

5. Shen'berdin' AB ha'm CD xordalari O noqatta kesilisedi. Yeger $AO=4$ *sm*, $BO=6$ *sm* ha'm $CD=11$ *sm* bolsa, OC ha'm OD kesindilerdi tabi'n'.

6. Shen'berge ishley si'zi'lg'an tuwri' to'rtmu'yeshliktin' diogonali bir ta'repinen yeki ma'rte u'lken. Bul to'rtmu'yeshlik ushlarinin' shen'berden ajiratqan bo'leginin' gradi'us wo'lshemlerin tabi'n'.

7. Shen'berge si'rtlay si'zi'lg'an trapeciyanin' worta sizig'i 7 *sm*. Trapeciyanin' perimetrin tabi'n'.

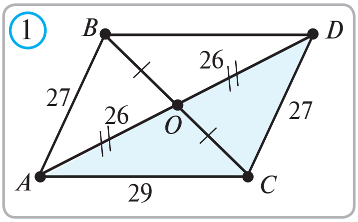
8*. Radiusi 15 *sm* bolg'an do'n'gelektin' worayinan 7 *sm* uzaqli'qtag'i K noqattan 27 *sm* uzaqliqtag'i AB xorda wo'tkerilgen AK ha'm BK kesindilerdi tabi'n'.

9. Duri's segiz mu'yeshliktin' bir ushi'nan shiqqan yen' u'lken ha'm yen' kishi diagonallar arasi'ndag'i mu'yeshti tabi'n'.

10. To'beleri koordinatalar tegisligindegi $A(-3; 4)$, $B(3; 4)$, $C(3; -8)$ noqatlarda bolg'an u'shmu'yeshlik berilgen. a) $\angle ABC=90^\circ$ yekenligin ko'rsetin'; b) ABC u'shmu'yeshlikke si'rtlay si'zi'lg'an do'n'gelektin' worayin, radiusi ha'm maydani'n tabi'n'.

62 TA'KIRARLAW

Ma'sele. ABC u'shmu'yeshliginde AO mediana, $AO=26$, $AB=27$ ha'm $AC=29$. U'shmu'yeshliktin' maydani'n tabi'n'.



Sheshiliwi. AO nurinda A noqatinan $AD=2AO=52$ bolatug'inday yetip D noqatin tan'laymiz (*1-su'wret*). Bunda $BO=OC$, $AO=OD$ bolg'anliqtan $ABCD$ — parallelogramm boladi.

ABC ha'm ADC u'shmu'yeshliginin' maydanlari ten'. Geron formulasinan paydalani'p, ADC u'shmu'yeshliginin' maydani'n yesaplaymiz:

$P=\dfrac{29+52+27}{2}=54;$ $S=\sqrt{54\cdot(54-29)(54-52)(54-27)}=270.$ ***Juwabi':*** 270.

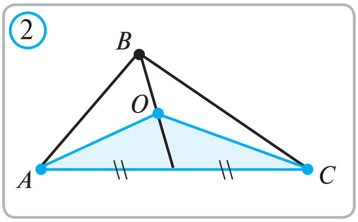
Soraw, ma'sele ha'm tapsi'rmalar

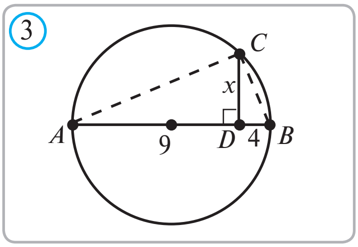
1. ABC ha'm EFK u'shmu'yeshlikleri uqsas: AB ha'm EF, BC ha'm FK wolardi'n' sa'ykes ta'repleri. Yeger $AB=4$ *sm*, $BC=5$ *sm*, $CA=7$ *sm* ha'm $EF:AB=2{,}1$ bolsa, EFK u'shmu'yeshliginin' ta'replerin tabi'n'.

2. ABC ha'm $A_1B_1C_1$ u'shmu'yeshlikleri uqsas ha'm wolardi'n' sa'ykes ta'replerinin' qatnasi $6:5$ ke ten'. ABC u'shmu'yeshliginin' maydani' $A_1B_1C_1$ u'shmu'yeshlik maydani'nan 77 dm^2 ge artiq. U'shmu'yeshliklerdin' maydani'n tabi'n'.

3. ABC u'shmu'yeshliginin' medianalari kesilisken noqat O bolsi'n. Yeger AOC u'shmu'yeshliginin' maydani' 4 sm^2 bolsa, ABC u'shmu'yeshliginin' maydani'n tabi'n' (*2-su'wret*).

4. Shen'berdin' C noqatinan AB diametrine CD perpendikulyari ju'rgizilgen. Yeger $AD=9$, $DB=4$ bolsa, CD kesindisin tabi'n' (*3-su'wret*).





5. Ta'repi 6 m, bul ta'repine irgeles jatqan mu'yeshleri 30° ha'm 45° bolg'an u'shmu'yeshliktin' maydani'n tabi'n'.

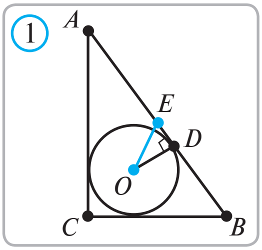
6. Ultanlari 28 *dm* ha'm 16 *dm*, qaptal ta'repleri bolsa 25 dm ha'm 17 dm bolg'an trapeciyanin' biyikligin tabi'n'.

7. Radiusi 2 *sm* bolg'an shen'berge maydani' 20 sm^2 bolg'an ten' qaptalli trapeciya si'rtlay si'zi'lg'an. Trapeciyata'replerinin' uzinliqlarin tabi'n'.

8. Tuwri' mu'yeshli u'shmu'yeshlikke ishley si'zi'lg'an shen'berdi gipotenuzag'auriniw noqati, gipotenuzani 2 *sm* ha'm 3 *sm* bolg'an kesindilerge ajiratadi. U'shmu'yeshliktin' katetlerin tabi'n'.

63 TA'KIRARLAW

Ma'sele. Katetleri 3 ha'm 4 bolg'an tuwri' mu'yeshli u'shmu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an shen'berlerdin' woraylari arasi'ndag'i arali'qti tabi'n' (*1-su'wret*).



Sheshiliwi. 1) ABC u'shmu'yeshliginde $\angle C=90°$, $AC=4$ ha'm $BC=3$ bolsi'n. Bunda, Pifagor teoremasi' boyi'nsha,

$$AB=\sqrt{3^2+4^2}=5.$$

Tuwri' mu'yeshli u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' E worayi gipotenuzanin' ortasinda boladi:

$$BE=\frac{AB}{2}=\frac{5}{2}.$$

3) U'shmu'yeshlikke ishley si'zi'lg'an shen'berdin' radiusi OD ni tabami'z (D—ishley si'zi'lg'an shen'berdin' gipotenuzag'a uriniw noqati):

$$OD=\frac{AC+BC-AB}{2}=\frac{4+3-5}{2}=1.$$

4) BD ha'm DE kesindilerin tabami'z:

$$BD=\frac{AB+BC-AC}{2}=\frac{5+3-4}{2}=2; \qquad ED=BE-DE=\frac{5}{2}-2=\frac{1}{2}.$$

5) Tuwri' mu'yeshli ODE u'shmu'yeshliginen OE kesindilerin tabami'z:

$$OE=\sqrt{OD^2+ED^2}=\sqrt{1+\frac{1}{4}}=\frac{\sqrt{5}}{2}. \qquad \textbf{Juwabi':}\ \frac{\sqrt{5}}{2}.$$

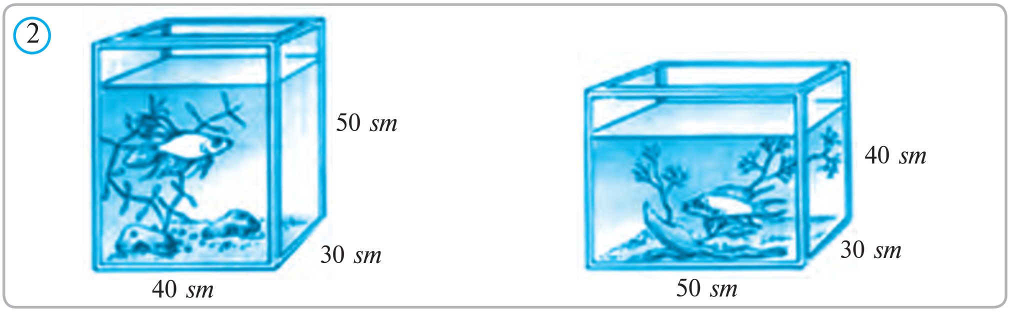
? Soraw, ma'sele ha'm tapsi'rmalar

1. Ten' qaptalli ABC u'shmu'yeshliginde $AB=AC=4$ *sm* ha'm $\angle A=30°$ bolsa, onin' BE biyikligin tabi'n'.
2. Trapeciyanin' ultanlari 5 *dm* ha'm 8 *dm*, qaptal ta'repleri bolsa 3,6 *dm* ha'm 3,9 *dm*. Trapeciyanin' qaptal ta'replerinin' dawami O noqatinda kesilisedi. O noqatinan trapeciyanin' ushlarina shekem bolg'an arali'qlardi tabi'n'.
3. A mu'yeshinin' bir ta'repine $AB=5$ *sm* ha'm $AC=16$ *sm* kesindiler, yekinshi ta'repine bolsa $AD=8$ *sm* ha'm $AF=10$ *sm* kesindiley qoyi'lg'an. ACD ha'm AFB u'shmu'yeshlikleri uqsaspa? Juwabi': n'izdi da'liyllen'.
4. Tuwri'mu'yeshliktin' maydani' 9 dm^2, diagonallari payda yetken mu'yeshlerden biri bolsa $120°$ qaten'. Tuwri'mu'yeshliktin' ta'replerin tabi'n'.
5. Yeger ten' qaptalli u'shmu'yeshliktin' ultani 24 *sm* ha'm qaptal ta'repi 13 *sm* bolsa, wonday jag'dayda u'shmu'yeshlikke si'rtlay si'zi'lg'an shen'berdin' radiusin tabi'n'.

6. Rombi'ni'n' biyikligi 12 *sm* boli'p, diagonallarinan biri 15 *sm*. Rombi'ni'n' maydani'n tabi'n'.

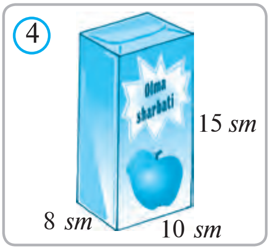
7. Yeger *ABCD* parallelogrammda $A(1;-3)$, $B(-2;4)$ ha'm $C(-3;1)$ bolsa, woni'n' *D* to'besinin' koordinatalarin tabi'n'.

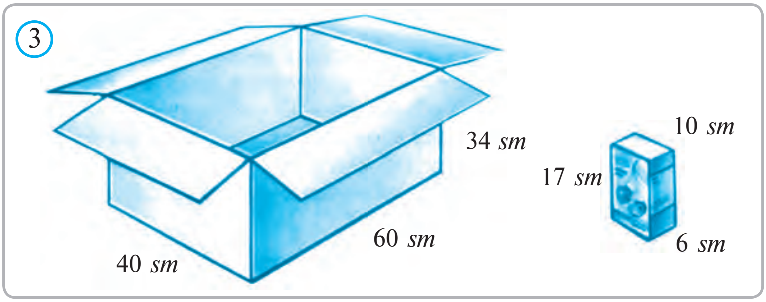
8. Yeki akvariumg'a joqarg'i shetinen 10 sm to'menlew suw quyildi (*2-su'wret*). Qaysi akvariumda suw ko'p?



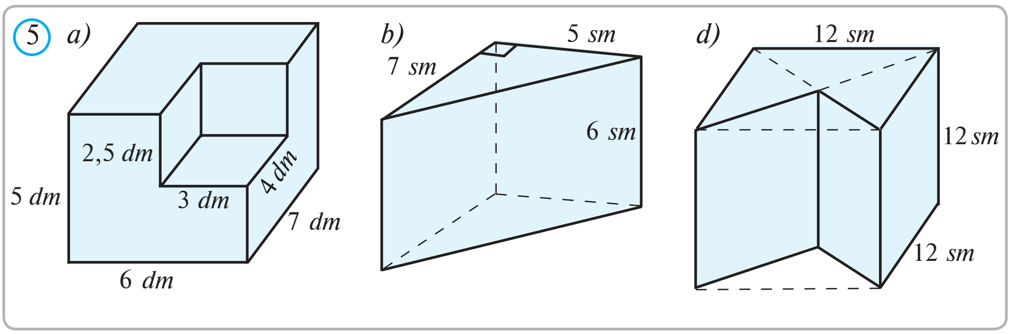
9. Qutig'a neshe paket miywe sherbeti siyadi' (*3-su'wret*)?

10. 1 litrli miywe sherbeti paketi tuwri' mu'yeshli parallelepiped formasinda (*4-su'wret*). Bir qadaq ushi'n qansha material kerek boladi?





11*. 5-su'wrette ko'rsetilgen ag'ash bo'leklerinin' ko'lemin yesaplan'.



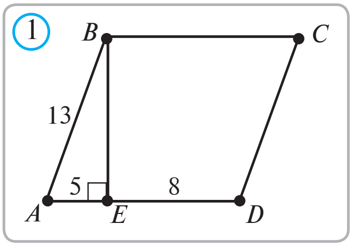
64 TA'KIRARLAW

Ma'sele. Rombi'ni'n' dog'al mu'yeshinin' to'besinen ju'rgizilgen biyiklik romb ta'replerinen birin su'yir mu'yeshinin' to'besinen baslap yesaplag'anda 5 *sm* ha'm 8 *sm* bolg'an kesindilerge ajiratadi. Rombi'nin' maydani'n yesaplan'.

Sheshiliwi. $ABCD$ romb, $\angle B > 90°$, BE — biyiklik, $AE = 5$ *sm*, $ED = 8$ *sm* bolsi'n (*1-su'wret*).

1) Rombi'ni'n' ta'replerin tabami'z:

$$AD = AE + ED = 5 + 8 = 13\ (sm).$$



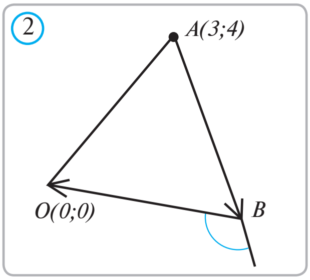
2) Tuwri' mu'yeshli ABE u'shmu'yeshlikke Pifagor teoremasin qollanip, BE biyikligin tabami'z:

$$BE = \sqrt{AB^2 - AE^2} = \sqrt{13^2 - 5^2} = 12\ (sm).$$

3) Rombi'ni'n' maydani'n tabami'z: $S = AD \cdot BE = 13 \cdot 12 = 156\ (sm^2)$.

Juwabi': 156 sm^2.

Soraw, ma'sele ha'm tapsi'rmalar



1. Yeger ten' ta'repli AOB u'shmu'yeshliginde $O(0;0)$ ha'm $A(3:4)$ yekenligi belgili bolsa, $\overline{AB} \cdot \overline{BO}$ skalyar ko'beymeni tabi'n' (*2-su'wret*).
2. Ultanlari AB ha'm CD bolg'an $ABCD$ trapeciyanin' diagonallari' O noqatindakesilisedi.Yeger $OB=8$ *sm*, $OD=20$ *sm* ha'm $OC=50$ *sm* bolsa, AO kesindisin tabi'n'.
3. Yeger $AB = 1{,}7$ *sm*, $BC = 3$ *sm*, $CA = 4{,}2$ *sm*, $A_1B_1 = 34$ *dm*, $B_1C_1 = 60$ *dm* ha'm $C_1A_1 = 84$ *dm* bolsa, ABC ha'm $A_1B_1C_1$ u'shmu'yeshlikleri uqsaspa?
4. Perimetri 36 *sm* bolg'an parallelogrammnin' diagonallari kesilisiwden payda bolg'an yeki u'shmu'yeshliktin' birin perimetri yekinshisinikinen 8 *sm* artiq bolsa, parallelogrammnin' ta'replerin tabi'n'.
5. 60° qa ten' mu'yeshke bir-birine si'rttan uriniwshi yeki shen'ber ishley si'zi'lg'an. Kishi shen'berdin' radiusi 1 *sm* bolsa, u'lken shen'berdin' radiusin tabi'n'.
6. U'lken ultani AD bolg'an $ABCD$ trapeciyanin' AC diagonali CD ta'repine perpendikular ha'm $\angle BAC = \angle CAD$. Yeger trapeciyanin' perimetri 20 *sm* ha'm $\angle D = 60°$ bolsa, AD ta'repinin' uzi'nli'g'i'n tabi'n'.
7. Shen'ber diametrinin' ushlari shen'berdin' qanday da bir urinbasinan 18 *sm* ha'm 12 *sm* uzaqliqta yekenligi belgili bolsa, shen'berdin' uzi'nli'g'i'n tabi'n'.
8. Ultanlarinin' uzinliqlari ha'm maydani' sa'ykes tu'rde 8 *sm*, 14 *sm* ha'm 44 sm^2 bolg'an ten' qaptalli trapeciyanin' qaptal ta'repin tabi'n'.

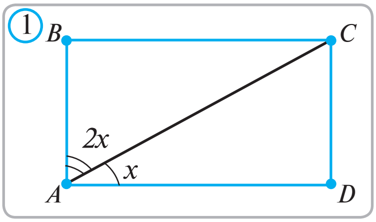
65 TA'KIRARLAW

Ma'sele. Tuwri'mu'yeshliktin' diagonallari 12 *sm* ge ten' ha'm to'rtmu'yeshliktin' mu'yeshi 2:1 qatnasta bo'ledi. Tuwri'mu'yeshliktin' perimetrin tabi'n'.

ABCD — tuwri'mu'yeshlik,
$AC=12$ *sm*, $\angle BAC:\angle CAD=2:1$

$P=?$

Sheshiliwi. 1) Yeger $\angle CAD=x$ desek, $\angle BAC=2x$ ha'm $\angle CAD+\angle BAC==x+2x=90°$ boladi. Bunnan $x=30°$.



2) Tuwri' mu'yeshli *ADC* u'shmu'yeshliktin' katetlerin tabami'z:

$$CD=AC\sin CAD=12\cdot\sin30°=12\cdot\frac{1}{2}=6\ (sm),$$

$$AD=AC\cdot\cos CAD=12\cdot\cos30°=12\cdot\frac{\sqrt{3}}{2}=6\sqrt{3}\ (sm).$$

3) To'rtmu'yeshliktin' perimetrin tabami'z:

$$P=2(AD+CD)=2(6+6\sqrt{3})=12(1+\sqrt{3})\ (sm).$$

Juwabi': $12(1+\sqrt{3})$ *sm*.

Soraw, ma'sele ha'm tapsi'rmalar

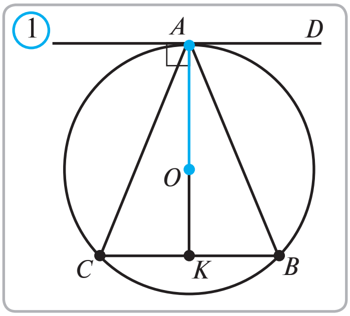
1. ABC ha'm $A_1B_1C_1$ u'shmu'yeshlikler uqsas, $AB=6$ *sm*, $BC=9$ *sm*, $CA=10$ *sm*. $A_1B_1C_1$ u'shmu'yeshliktin' u'lken ta'repi 7,5 *sm*. Wonin' qalg'an ta'replerin tabi'n'.
2. *ABC* u'shmu'yeshliginin' *AB* ta'repine parallel tuwri' *AC* ta'repin *A* to'besinen baslap yesaplag'anda 2:7 siyaqli qatnasta bo'ledi. Yeger $AB=10$ *sm*, $BC=18$ *sm* ha'm $CA=21{,}6$ *sm* bolsa, tuwri' *ABC* u'shmu'yeshliginen ajiratqan u'shmu'yeshliktin' ta'replerin tabi'n'.
3. Ten' qaptalli trapeciyanin' qaptal ta'repi worta sizig'ina ten' ha'm perimetri 48 *sm*. Trapeciyanin' qaptal ta'repinin' uzi'nli'g'i'n tabi'n'.
4. Ultanlari 6 *sm* ha'm 3 *sm* bolg'an tuwri' mu'yeshli trapetsiyag'a ishley si'zi'lg'an shen'berdin' radiusin tabi'n'.
5. Yeger $A_1A_4=2{,}24$ bolsa, onda $A_1A_2A_3A_4A_5A_6$ duri's alt'imu'yeshliktin' perimetrin tabi'n'.
6. $N(7;3)$ ha'm $M(-3;5)$ bolsa, *NM* diametrli shen'berdin' uzi'nli'g'i'n tabi'n'.
7. To'besindegi mu'yeshi 120° boli'p, radiusi 10 *sm* bolg'an shen'berge ishley si'zi'lg'an ten' qaptalli u'shmu'yeshliktin' maydani'n tabi'n'.
8. Yeger *ABCD* to'rtmu'yeshliginde $AB=5$ *sm*, $BC=13$ *sm*, $CD=9$ *sm*, $DA=15$ *sm* ha'm $AC=12$ *sm* bolsa, *ABCD* to'rtmu'yeshliginin' maydani'n tabi'n'.

9*. 90° li worayliq mu'yeshke sa'ykes dog'anin' uzinlig'i 15π ge ten'. Shen'berge si'rtlay si'zi'lg'an duri's u'shmu'yeshliktin' maydani'n tabi'n'.

66 TA'KIRARLAW

Ma'sele. Shen'berdin' AB xordasi 10 sm. Xordanin' A ushi'nda AD urinba, B ushi'nan bolsa usi urinbag'a parallel BC xorda ju'rgizildi. Yeger $BC=12$ *sm* bolsa, shen'berdin' radiusin tabi'n'.

Sheshiliwi: 1) A noqati ha'm shen'ber worayi — O noqati arqali wo'tiwshi tuwri' BC xordasin K noqatindakesip o'tsin. AD urinba bolg'anliqtan $AK \perp AD$, $AD \parallel BC$ bolg'ani' ushi'n $AK \perp BC$.



2) $AK \perp BC$, yag'niy $OK \perp BC$ bolg'ani' ushi'n $CK=KB$. AK kesindisi ABC u'shmu'yeshliginin' medianasi ha'm biyikligi yeken. Demek, $AC=AB=10$ *sm*.

3) Geron formulasinan paydalani'p ABC u'shmu'yeshliginin' maydani'n tabami'z:

$$p=\frac{a+b+c}{2}=\frac{10+10+12}{2}=16\ (sm),$$

$$S=\sqrt{p\cdot(p-a)(p-b)(p-c)}=\sqrt{16\cdot(16-10)(16-10)(16-12)}=48\ (sm^2).$$

4) ABC u'shmu'yeshligine si'rtlay si'zi'lg'an shen'berdin' radiusin tabami'z:

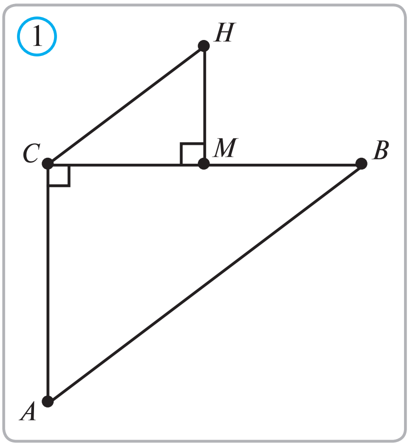
$$R=\frac{abc}{4S}=\frac{12\cdot10\cdot10}{4\cdot48}=6{,}25\ (sm).$$

Juwabi': 6,25 *sm*.

Soraw, ma'sele ha'm tapsi'rmalar

1. U'shmu'yeshliktin' ta'repleri sa'ykes tu'rde 13 *sm*, 14 *sm* ha'm 15 *sm* ge ten'. U'shmu'yeshlikke ishley ha'm si'rtlay si'zi'lg'an do'n'gelektin' maydanlarinin' qatnasin tabi'n'.
2. Yeger $\angle BDC=40°$ ha'm $\angle CBD=60°$ bolsa, shen'berge ishley si'zi'lg'an $ABCD$ to'rtmu'yeshliginin' A ha'm C mu'yeshlerin tabi'n'.
3. Shen'berge ishley si'zi'lg'an ten' qaptalli trapeciyanin' ultanlari 4 sm ha'm 16 sm bolsa, shen'berdin' radiusin tabi'n'.
4. Tuwri' mu'yeshli u'shmu'yeshliktin' katetlerine tu'sirilgen medianalari $\sqrt{52}$ *sm* ha'm $\sqrt{73}$ *sm* ge ten'. U'shmu'yeshliktin' maydani'n tabi'n'.
5. Katetleri 6 *m* ha'm 8 *m* bolg'an tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasina tu'sirilgen biyikligin tabi'n'.
6. Tuwri' mu'yeshli u'shmu'yeshliktin' bir kateti 5 *sm* ha'm gipotenuzasi 13 *sm* bolsa, wonin' maydani'n tabi'n'.
7. x tin' qanday ma'nislerinde $\bar{a}(x; 7)$ ha'm $\bar{b}(5; 2-x)$ vektorlari perpendikulyar boladi?
8. U'shmu'yeshliktin' yeki ta'repi 10 *sm* ha'm 12 *sm*, wolardi'n' arasi'ndag'i su'yir mu'yeshtin' sinusi 0,8 ge ten'. U'shmu'yeshliktin' u'shinshi ta'repin tabi'n'.

67–68 JUWMAQLAWSHI BAQLAW JUMI'SI'



I. U'lgi ushi'n baqlaw jumi'si'

1. $ABCD$ parallelogrammda $\angle A = 45°$, $AD = 4$. parallelogrammnin' AB ta'repinin' dawamina $\angle PDA = 90°$ qa ten' bolatug'in BP kesindi qoyi'ldi. BC ha'm PD kesindiler T noqatinda kesilisedi, bunda $PT : TD = 3:1$.

a) $\Delta BPT \backsim \Delta CDT$ yekenligin da'liyllen', bul u'shmu'-yeshliklerdin' maydanlarinin' qatnasin tabi'n'.

b) $ABCD$ parallelogrammnin' maydani'n tabi'n'.

d) AB ha'm TD kesindilerinin' wortalarin tutastiriwshi kesindinin' uzi'nli'g'i'n tabi'n'.

e) $\overline{AB}$ vektorin $\overline{CA}$ ha'm $\overline{TB}$ vektorlari arqali an'latin'.

f) CAD mu'yeshinin' sinusin tabi'n'.

2. (*Qosi'msha*) 1-su'wrette $BC \perp AC$, $MH \perp BC$, $2MC = BC$, $MH = 0,5AC$ bolsa, $AB || CH$ yekenligin da'liyllen'.

II. Baqlaw jumi'si' ushi'n ko'rsetpeli testler.

1. Yeger tuwri' mu'yeshli u'shmu'yeshliktin' biyikligi gipotenuzasin 6 *sm* ha'm 54 *sm* kesindilerge ajiratsa, bul u'shmu'yeshliktin' maydani'n tabi'n':

A) 648 sm^2; B) 324 sm^2; D) 1080 sm^2; E) 540 sm^2.

2. *C* noqatinan wo'tkerilgen bir kesiwshi shen'berdi *A* ha'm *B*, yekinshisi bolsa *D* ha'm *E* noqatlarinda kesedi. Yeger $CD = 18$ *sm*, $CB = 8$ *sm*, $CD = 8$ *sm* bolsa, *DE* kesindisinin' uzi'nli'g'i'n tabi'n':

A) 17 *sm*; B) 1 *sm*; D) 9 *sm*; E) duri's juwap ko'rsetilmegen.

3. Yeger $A(-5;2\sqrt{3})$, $B(-4;2)$, $C(-2;\sqrt{3})$, $D(0;2)$ bolsa, *ABCD* to'rtmu'yeshliginin' diagonallari arasi'ndag'i mu'yeshti tabi'n':

A) 30°; B) 60°; D) 90°; E) duri's juwap ko'rsetilmegen.

4. Yeger parallelogrammnin' diagonallari 10 *sm* ha'm $8\sqrt{2}$ *sm* ge ten' ha'm wolar arasi'ndag'i mu'yesh 45° bolsa, parallelogrammnin' ta'replerin tabi'n':

A) $\sqrt{17}$ *sm* ha'm $\sqrt{97}$ *sm*; B) 5 *sm* ha'm 6 *sm*;
D) $\sqrt{34}$ *sm* ha'm $\sqrt{63}$ *sm*; E) duri's juwap ko'rsetilmegen.

5. Radiusi 8 sm bolg'an shen'berge ishley si'zi'lg'an duri's altimu'yeshtin' maydani'n tabi'n'.

A) $48\sqrt{3}$ sm^2; B) $192\sqrt{3}$ sm^2; D) $96\sqrt{2}$; E) duri's juwap ko'rsetilmegen.

6. Worayliq mu'yeshi 140^o, maydani' 31,5π *sm*^{2} bolg'an sektordin' radiusin ani'qlan':

A) 9 *sm*; B) 18 *sm*; D) 9π *sm*; E) duri's juwap ko'rsetilmegen.

7. Ultaninin' uzinlig'i 15 *sm* bolg'an u'shmu'yeshliktin' ultanina parallel kesindi ju'rgizilgen. Yeger payda bolg'an trapeciyani'n' maydani' u'shmu'yeshliktin' maydani'nin' $\frac{3}{4}$ bo'legin quraytug'inlig'i belgili bolsa, kesindinin' uzi'nli'g'i'n tabi'n':

A) 6,5; B) 7; D) 7,5; E) 5.

8. Qaptal ta'repi $2\sqrt{39}$ *sm* bolg'an ten' qaptalli u'shmu'yeshliktin' biyikliginin' ultanina qatnasi' 3 : 4 ke ten' bolsa, u'shmu'yeshliktin' maydani'n tabi'n':

A) 260; B) 245; D) 310; E) 72.

9. $a(4;4\sqrt{3})$ ha'm $b(8\sqrt{3};8)$ vektorlari arasi'ndag'i mu'yeshti tabi'n':

A) 45°; B) 90°; D) 30°; E) 60°.

10. Ten' qaptalli trapeciyanin' ultanlari 10 *sm* ha'm 16 *sm*, qaptal ta'repi bolsa 5 *sm*. Trapeciyanin' maydani'n tabi'n':

A) 45; B) 50; D) 48; E) 52.

11. Tuwri' mu'yeshli u'shmu'yeshliktin' gipotenuzasi 13 *sm* boli'p, katetlerinen biri yekinshisinen 7 sm u'lken. U'shmu'yeshliktin' maydani'n tabi'n':

A) 30 *sm*^{2}; B) 25 *sm*^{2}; D) 45 *sm*^{2}; E) 40 *sm*^{2}.

12. Ta'repi 5 *sm* bolg'an rombi'ni'n' bir diagonali 6 sm ge ten'. Rombi'ni'n' maydani'n tabi'n':

A) 24 *sm*^{2}; B) 30 *sm*^{2}; D) 29 *sm*^{2}; E) 40 *sm*^{2}.

13. Diagonali $6\sqrt{2}$ bolg'an kvadratqa ishley si'zi'lg'an shen'berdin' uzi'nli'g'i'n tabi'n':

A) 10π; B) 8π; D) 9π; E) 6π.

14. Ta'repi $6\sqrt{2}$ *sm* bolg'an kvadratqa si'rtlay si'zi'lg'an do'n'gelektin' maydani'n tabi'n':

A) 9π; B) 12π; D) 15π; E) 18π.

15. Biyiklikleri 4 *sm* ha'm 6 *sm* bolg'an parallelogrammnin' maydani' 36 *sm*^{2} ge ten'. Woni'n' perimetrin tabi'n':

A) 26 *sm*; B) 30 *sm*; D) 29 *sm*; E) 36 *sm*.

16. Perimetri 30 *sm* bolg'an parallelogrammnin' ta'repleri 2 : 3 qatnasta. Yeger parallelogrammnin' su'yir mu'yeshi 30^o bolsa, woni'n' maydani'n tabi'n':

A) 26 *sm*^{2}; B) 27 *sm*^{2}; D) 29 *sm*^{2}; E) 30 *sm*^{2}.

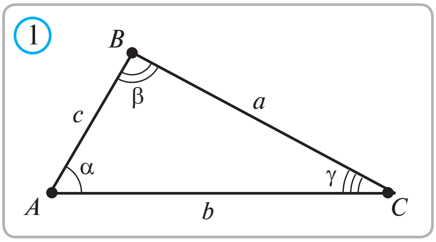
17. Yeger *ABC* u'shmu'yeshliginde $AB=6\sqrt{3}$ *sm*, $BC=12$ *sm* ha'm $\angle C=60°$ bolsa, u'shmu'yeshliktin' *A* mu'yeshin tabi'n':

A) 45°; B) 90°; D) 30°; E) 60°.

PLANIMETRIYAG'A TIYISLI TIYKARG'I' TU'SINIK HA'M MAG'LIWMATLAR

U'SHMU'YESHLIKLER

1°. Tiykarg'i' tu'sinikler



Tegislikte u'sh noqat berilgen boli'p, u'shewi bir tuwri'da jatpasin. Usi noqatlardin' ha'r yekewin kesindiler menen tutastiramiz. Payda bolg'an forma *u'shmu'yeshlik* dep ataladi. Noqatlar u'shmu'yeshliktin' to'beleri *kesindiler ta'repleri* dep ataladi. Belgileniwi: *A, B, C* — to'beleri, *a, b, c* — ta'repleri *(1-su'wret).*

U'shmu'yeshliku'sh mu'yeshke iye: $\angle BAC$, $\angle CBA$, $\angle ACB$. Belgileniwi: α , β, γ.

Mediana — U'shmu'yeshliktin' to'besi menen qarsi ta'repi ortasin tutastiriwshi kesindi. U'shmu'yeshlikte 3 medianaboli'p, olar m_a, m_b, m_c tu'rinde belgilenedi.

Bissektrisa — u'shumu'yeshlik to'besin woni'n qarsi'ndag'i ta'rep penen tutashti'ri'wshi' ha'm usi' to'bedegi mu'yesh bissektrisasi'nda jati'wshi' kesindi. Ushu'yeshlikte u'sh bessektrisiw boli'p, wolar l_a, l_b, l_c ko'rinisinde belgilenedi.

Biyiklik — U'shmu'yeshliktin' to'besinen wonin' qarsisindag'i ta'repte jatqan tuwri'g'a tu'sirilgen perpendikulyar.

U'shmu'yeshlikte u'sh biyiklik boli'p, wolar h_a, h_b, h_c tu'rinde belgilenedi.

Worta si'zi'q — yeki ta'rep wortalarin tutastiriwshi kesindi.

Wortasi'zi'qlardin' sani 3.

Perimetr — u'sh ta'reptin' uzinliqlarinin' qosi'ndi'si'. Belgileniwi *P.*

U'shmu'yeshlikler ta'replerine qarap u'sh tu'rge bo'linedi:

a) ten' ta'repli ($a=b=c$); b) ten' qaptalli (a, b, c lardin' qanday dabir ekewi ten'); d) tu'rli ta'repli (a, b, c lardin' hesh qanday yekewi ten' emes).

U'shmu'yeshliktin' u'sh ta'repine de urinip wo'tiwshi shen'ber wog'an ishley si'zi'lg'an shen'ber delinedi (bunday shen'ber boladi ha'm jalg'iz). Ishley si'zi'lg'an shen'berdin' radiusi r menen belgilenedi.

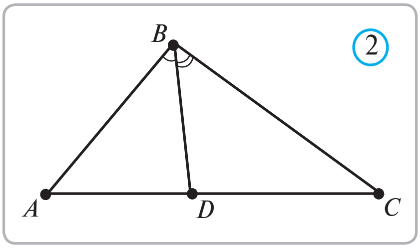
U'shmu'yeshliktin' u'sh to'besinen, de wo'tiwshi shen'ber wog'an si'rtlay si'zi'lg'an shen'ber delinedi ha'm woni'n' radiusi R menen belgilenedi (bunday shen'ber bar ha'm jalg'iz).

2°. Tiykarg'i' tu'sinikler

1) $\alpha+\beta+\gamma=180°$. U'shmu'yeshliktin' ishki mu'yeshlerinin' qosi'ndi'si' 180° qaten'. 2) U'sh medianabir noqattakesilisedi. Bul noqat medianani' 2:1 qatnasta bo'ledi. Medianau'shmu'yeshlikti yeki maydani' ten' u'shmu'yeshliklerge bo'ledi. Medianalardin' uzinliqlari

$$m_a=\frac{1}{2}\sqrt{2b^2+2c^2-a^2}; \quad m_b=\frac{1}{2}\sqrt{2a^2+2c^2-b^2}; \quad m_c=\frac{1}{2}\sqrt{2a^2+2b^2-c^2}$$

formulalarinan tabiladi.



3) U'sh bissektrisa bir noqatta kesilisedi. Bul noqat ishley si'zi'lg'an shen'ber worayi boladi. Bissektrisa wo'zi tu'sirilgen ta'repti qalg'an ta'replerge proporcional bolg'an bo'leklerge bo'ledi *(2-su'wret)*.

BD bissektrisa bolsa, $\frac{AB}{AD}=\frac{BC}{DC}$.

Bissektrisa uzinliqlari:

$$l_a=\frac{2\sqrt{bc}}{b+c}\sqrt{p(p-a)};\quad l_b=\frac{2\sqrt{ac}}{a+c}\sqrt{p(p-b)};$$

$l_c=\frac{2\sqrt{ab}}{a+b}\sqrt{p(p-c)},\quad p=\frac{1}{2}(a+b+c)$ formulalarinan tabiladi.

4) U'shmu'yeshliktin' biyiklikleri yaki wolardin' dawamlari bir noqatta kesilisedi. Biyikliktin' uzinliqlari

$$h_a=\frac{2S}{a};\qquad h_b=\frac{2S}{b};\qquad h_c=\frac{2S}{c}$$

formulalarinan tabiladi. Bul jerde S - u'shmu'yeshliktin' maydani'.

5) U'shmu'yeshliktin' ta'replerinin' worta perpendikulyarlari bir noqatta kesilisedi. Bul noqat u'shmu'yeshlikke *si'rtlay si'zi'lg'an shen'ber worayi* boladi.

6) U'shmu'yeshliktin' *worta sizig'i* u'shinshi ta'repke parallel ha'm yariminaten'.

7) Sinuslar teoreması:

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}=2R.$$

8) Kosinuslar teoreması:

$$a^2=b^2+c^2-2bc\cos\alpha,\quad b^2=a^2+c^2-2ac\cos\beta,\quad c^2=a^2+b^2-2ab\cos\gamma$$

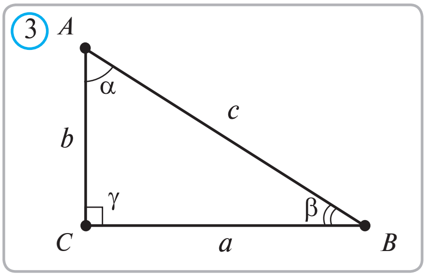
9) U'shmu'yeshliktin' maydani'n yesaplaw formulalari:

$$S=\frac{1}{2}ah_a=\frac{1}{2}bh_b=\frac{1}{2}ch_c;\qquad S=\frac{1}{2}ab\sin\gamma=\frac{1}{2}bc\sin\alpha=\frac{1}{2}ac\sin\beta;$$

10) Geron formulasi:

$$S=\sqrt{p(p-a)(p-b)(p-c)},\qquad p=\frac{a+b+c}{2};\qquad S=\frac{abc}{4R},\qquad S=pr.$$

3°. Za'ru'r jeke jag'daylar



a) Tuwri' mu'yeshli u'shmu'yeshlik (*3-su'wret*).

$\angle\gamma=90°$, $\alpha+\beta=90°$, AC ha'm BC — katetler, AB— gipotenuza. Pifagor teoremasi: $a^2+b^2=c^2$.

$$S=\frac{1}{2}ab;\qquad R=\frac{c}{2};\qquad r=\frac{a+b-c}{2};$$

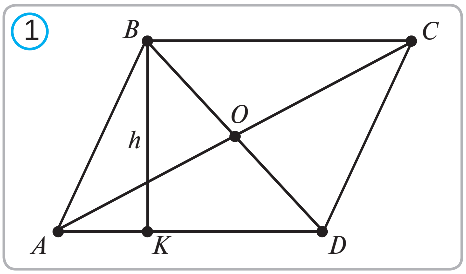
$$\frac{a}{c}=\sin\alpha;\qquad \frac{a}{c}=\cos\beta;\qquad \frac{b}{c}=\sin\beta;\qquad \frac{b}{c}=\cos\alpha.$$

$$\frac{a}{b}=\text{tg}\alpha; \quad \frac{a}{b}=\text{ctg}\beta; \quad \frac{b}{a}=\text{ctg}\alpha; \quad \frac{b}{a}=\text{tg}\beta.$$

b) Ten' ta'repli u'shmu'yeshlik

$$\alpha=\beta=\gamma=60^\circ, \quad S=\frac{a^2\sqrt{3}}{4}, \quad r=\frac{a\sqrt{3}}{6}, \quad R=\frac{a\sqrt{3}}{3}.$$

TO'RTMU'YESHLIKLER



1°. Parallelogramm

Qarama-qarsi ta'repleri parallel bolg'an to'rt-mu'yeshlik *parallelogramm* delinedi *(1-su'wret).*

Qon'silas bolmag'an to'belerin tutastiriwshi kesindi *diagonal* delinedi.

AB ha'm *CD; AD* ha'm *BC* parallel ta'repler; *BD* ha'm *AC* diagonallar.

Tiykarg'i' qa'siyetler ha'm qatnasiqlar:

1) Diagonallardin' kesilisiw noqati parallelogrammnin' simmetriyaworayi boladi.

2) Qarama-qarsi ta'replerdin' uzinliqlari wo'z-ara ten':

$$AB=CD \text{ va } AD=BC.$$

3) Parallelogrammnin' qarama-qarsi mu'yeshleri wo'z-ara ten':

$$\angle BAD=\angle BCD \text{ va } \angle ABC=\angle ADC.$$

4) Qon'silas mu'yeshlerdin' qosi'ndi'si' 180^0 qaten':

5) Diagonallar kesilisiw noqatinda ten' yekige bo'linedi: $BO=OD$ ha'm $AO=OC$

6) Ta'replerinin' kvadratlarinin' qosi'ndi'si', diagonallari'ni'n' kvadratlarinin' qosi'ndi'si'naten':

$$AB^2+BC^2+CD^2+DA^2=AC^2+BD^2 \quad \text{yamasa} \quad 2(AB^2+BC^2)=AC^2+BD^2.$$

7) Parallelogramm maydani':

a) $S=ah_a$, bul jerde $a=AD$ ta'rep, $h_a=BK$ — biyiklik;

b) $S=ab\sin\alpha$, bul jerde $b=AB$ — ta'rep, $\alpha=\angle BAD$ — *AB* ha'm *AD* ta'repleri arasi'ndag'i mu'yesh.

2°. Romb

Barliq ta'repleri wo'z-ara ten' bolg'an parallelogramm *romb* dep ataladi.

Parallelogramm ushi'n orinli bolg'an barliq qa'siyetler romb ushi'n da worinli.

Rombinin' qosi'msha qa'siyetleri.

1) Rombi'ni'n' diagonallari wo'z-ara perpendikulyar.

2) Rombi'ni'n' diagonallari ishki mu'yeshlerdin' bissektrisalari boladi.

3) Rombi'ni'n' maydani' $S=\frac{1}{2}d_1d_2$, bul jerde d_1, d_2 — rombi'ni'n' diagonallari.

3°. Tuwri' mu'yeshlik

Barliq mu'yeshleri 90° qaten' bolg'an parallelogramm *tuwri'mu'yeshlik* dep ataladi.

1) Tuwri’mu’yeshliktin’ diagonallari wo’z-ara ten’.

2) Tuwri’mu’yeshliktin’ maydani’ $S=ab$, bul jerde a ha’m b — tuwri’mu’yeshliktin’ qon’silas ta’repleri.

4°. Kvadrat

Barliq ta’repleri wo’z-ara ten’ bolg’an tuwri’mu’yeshlik *kvadrat* delinedi.

Romb ha’m tuwri’mu’yeshlikler ushi’n wori’nli’ bolg’an barliq qa’siyetler kvadrat ushi’n da wori’nli’.

Yeger a — kvadrattin’ ta’repi, d diagonali bolsa: $S=a^2$; $S=\frac{d^2}{2}$; $d=a\sqrt{2}$.

5°. Trapeciya

Ultanlar dep atalatug’in yeki ta’repi wo’z-ara parallel ha’m qaptal ta’repler dep atalatug’i’n, qalg’an yeki ta’repi bolsa parallel bolmag’an to’rtmu’yeshlik *trapetsiya* dep ataladi.

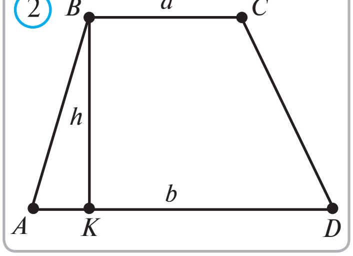
Qaptal ta’replerinin’ wortalarin tutastiriwshi kesindi trapeciyanin’ *worta sizig’i* dep ataladi.

Tiykarg’i’ qa’siyetler

1) Trapeciyanin’ wortasizig’i ultanlarg’a parallel boladi ha’m ultanlardin’ qosi’ndi’si’nin’ yariminaten’.

2) Trapeciyanin’ maydani’ $S=\frac{a+b}{2}h$, bul jerde a ha’m b — ultanlar, h bolsabiyiklik (*2-su’wret*).

2



SHEN’BER, DO’N’GELEK

1°. Tegislikte R on’ sani ha’m O noqati berilgen bolsi’n. O noqatinan R aralig’inda jaylasqan noqatlardan quralg’an figura *shen’ber* dep ataladi. O noqati *shen’berdin’ worayi*, woray menen shen’berdegi noqatti tutastiriwshi kesindi *radius*, R sani bolsa *radius uzinlig’i* dep ataladi. Shen’berdegi yeki noqatti tutastiriwshi kesindi *xorda*, woraydan wo’tiwshi xorda bolsa *diametr* dep ataladi.

Tegisliktin’ shen’ber menen shegaralang’an shekli bo’limi — *do’n’gelek* dep ataladi.

Tiykarg’i’ qatnaslar

1) $D=2R$, bul jerde: D — diametrdin’ uzinlig’i.

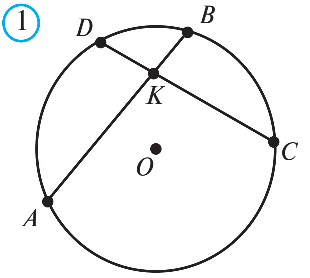
2) $L=2\pi R$ — shen’berdin’ uzinlig’i.

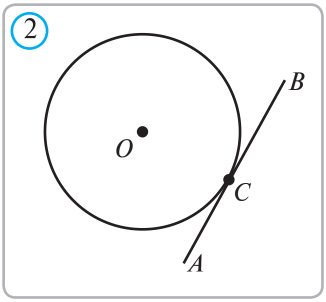
3) $S=\pi R^2$ — do’n’gelektin’ maydani’.

4) AB ha’m CD xordalar K noqatinda kesilisse (*1-su’wret*), $AK\cdot KB=CK\cdot KD$ qatnaslar worinlanadi.

5) Xordani ten’ yekige bo’liwshi diametr usi xordag’aperpendikulyar boladi.

1





6)Ten' xordalar woraydan ten' arali'qta jaylasqan ha'm kerisinshe woraydan ten' arali'qta jaylasqan xordalar wo'z-ara ten'.

2°. Urinba

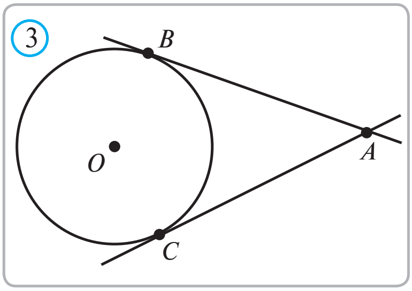
Shen'ber (yamasa do'n'gelek) menen birden-bir uliwma noqatqa iye bolg'an tuwri' *urinba* dep ataladi. Noqat bolsa *uriniw noqati* dep ataladi *(2-su'wret)*.

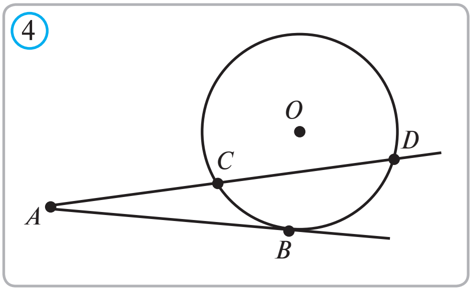
Shen'ber menen 2 uliwma noqatqa iye bolg'an tuwri' *kesiwshi* dep ataladi.

Urinbanin' qa'siyetleri

1) Uriniw noqatina wo'tkerilgen radius urinbag'aper pendikulyar boladi.

2) Do'n'gelek si'rtindag'i noqattan usi do'n'gelekke yeki urinba wo'tkeriw mu'mkin. Bul urinbalardin' kesindileri wo'z-ara ten' (*3-su'wret*): $AB=AC$.



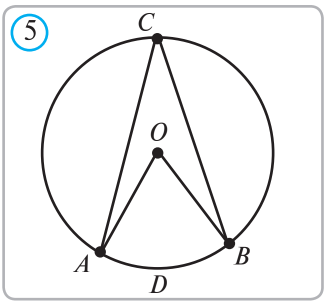


3) Yeger AC kesiwshi boli'p, shen'berdi C ha'm D noqatlarda kesip wo'tse, AB urinba bolsa, $AB^2=AD \cdot AC$ ten'ligi worinli boladi (*4-su'wret*).

3°. Orayliq ha'm ishley si'zi'lg'an mu'yeshler.

Shen'berdegi yeki noqat ja'rdeminde shen'ber yeki bo'lekke bo'linedi. Bul bo'lekke *dog'alar* dep ataladi. Belgileniwi: $\overset{\frown}{ADB}$; $\overset{\frown}{ACB}$.

AOB mu'yeshi ADB dog'ag'a tirelgen *worayliq mu'yesh*, (*5-su'wret*), ACB mu'yeshi bolsa ADB dog'ag'a tirelgen ha'm shen'berge *ishley si'zi'lg'an mu'yesh* delinedi. Bul mu'yeshler arasi'nda,



$$\angle ACB = \frac{1}{2}\angle AOB$$

qatnasi' wori'nli'.

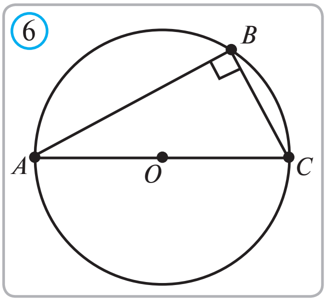
Demek, yari'm shen'berge (diametrge) tirelgen ishki mu'yesh tuwri' mu'yesh boladi *(6-su'wret)*. Bir dog'ag'a tirelgen shen'berge ishley si'zi'lg'an mu'yeshler ten' boladi.

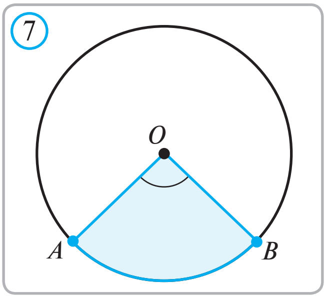
4°. Sektor ha'm segment

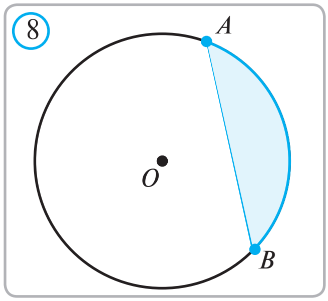
Do'n'gelektin' yeki radius penen shegaralang'an bo'legi *sektor* delinedi (*7-su'wret*).Sektor dog'asinin' uzinlig'i: $l=\frac{\pi R\alpha}{180}$, bul jerde, α — worayliq mu'yeshtin' gradius wo'lshemi.

Sektor maydani': $S=\frac{\pi R^2\alpha}{360}$; $S=\frac{1}{2}Rl$.

Segment — do'n'gelektin' xordasi ha'm usi xorda tirelgen dog'a menen shegaralang'an bo'legi (*8-su'wret*).







Segment maydani': $S=S_{sektor}\pm S_{\Delta}=\frac{\pi R^2}{360}\cdot\alpha\pm\frac{1}{2}R^2\sin\alpha$

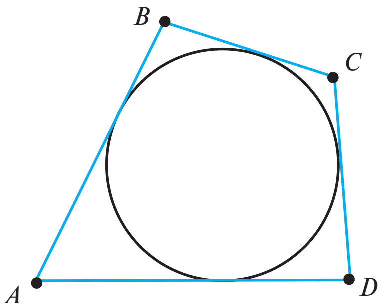
DURI'S KO'PMU'YESHLIKLER

Duri's n mu'yeshtin' ta'repi a_n, perimetri P_n, maydani' S_n, ishley si'zi'lg'an shen'ber radiusi r_n, si'rtlay si'zi'lg'an shen'ber radiusi R_n, ishki mu'yeshi $_n$ bolsa,

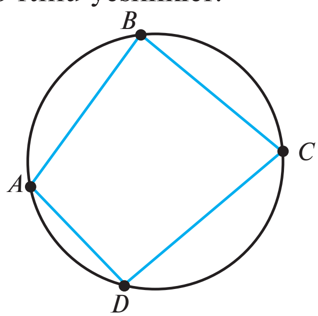
$$P_n=na_n,\qquad S_n=\frac{1}{2}P_n r_n=\frac{1}{2}na_n r_n,\qquad \alpha_n=\frac{(n-2)\cdot180^\circ}{n}$$

$$R_n=\frac{a_n}{2\sin\frac{180^\circ}{n}},\qquad r_n=\frac{a_n}{2\mathrm{tg}\frac{180^\circ}{n}}$$

Shen'berge ishley ha'm si'rtlay si'zi'lg'an to'rtmu'yeshlikler.



$BC+AD=AB+CD$



$\angle A+\angle C=180^\circ$

$\angle B+\angle D=180^\circ$

10 nan 99 g'a shekem bolg'an natural sanlardin' kvadratlarinin' kestesi

onliq / *birlik*	1	2	3	4	5	6	7	8	9
0	100	400	900	1600	2500	3600	4900	6400	8100
1	121	441	961	1681	2601	3721	5041	6561	8281
2	144	484	1024	1764	2704	3844	5184	6724	8464
3	169	529	1089	1849	2809	3969	5329	6889	8649
4	196	576	1156	1936	2916	4036	5476	7056	8836
5	225	625	1225	2025	3025	4225	5625	7225	9025
6	256	676	1296	2116	3136	4356	5776	7396	9216
7	289	729	1369	2209	3249	4489	5929	7569	9409
8	324	784	1444	2304	3364	4624	6084	7744	9604
9	361	841	1521	2401	3481	4761	6241	7921	9801

Ayirim shamalardin' kestesi

$\pi \cong 3,1416$	$\sqrt{8} \cong 2,8284$
$\sqrt{2} \cong 1,4142$	$\sqrt{10} \cong 3,1623$
$\sqrt{3} \cong 1,7320$	$\frac{1}{\sqrt{2}} \cong 0,7071$
$\sqrt{5} \cong 2,2360$	$\frac{1}{\sqrt{3}} \cong 0,5774$
$\sqrt{6} \cong 2,4495$	$\frac{1}{\sqrt{\pi}} \cong 0,3183$
$\sqrt{7} \cong 2,6457$	

Trigonometriyaliq funktciyalardi'n' ma'nislerinin' kestesi

α°	sinα	cosα	tgα	ctgα	α°	sinα	cosα	tgα	ctgα
1	0,0175	1,000	0,0175	57,3	**46**	0,719	0,695	1,036	0,966
2	0,0349	0,999	0,0349	28,6	**47**	0,731	0,682	1,072	0,933
3	0,0523	0,999	0,0524	19,1	**48**	0,743	0,669	1,111	0,900
4	0,0698	0,998	0,0699	14,3	**49**	0,755	0,656	1,150	0,869
5	0,0872	0,996	0,0875	11,4	**50**	0,766	0,643	1,192	0,839
6	0,1045	0,995	0,1051	9,51	**51**	0,777	0,629	1,235	0,810
7	0,1219	0,993	0,1228	8,14	**52**	0,788	0,616	1,280	0,781
8	0,139	0,990	0,141	7,11	**53**	0,799	0,602	1,327	0,754
9	0,156	0,988	0,158	6,31	**54**	0,809	0,588	1,376	0,727
10	0,174	0,985	0,176	5,67	**55**	0,819	0,574	1,428	0,700
11	0,191	0,982	0,194	5,145	**56**	0,829	0,559	1,483	0,675
12	0,208	0,978	0,213	4,507	**57**	0,839	0,545	1,540	0,649
13	0,225	0,974	0,231	4,331	**58**	0,848	0,530	1,600	0,625
14	0,242	0,970	0,249	4,011	**59**	0,857	0,515	1,664	0,601
15	0,259	0,966	0,268	3,732	**60**	0,866	0,500	1,732	0,577
16	0,276	0,961	0,287	3,487	**61**	0,875	0,485	1,804	0,554
17	0,292	0,956	0,306	3,271	**62**	0,883	0,469	1,881	0,532
18	0,309	0,951	0,325	3,078	**63**	0,891	0,454	1,963	0,510
19	0326	0,946	0,344	2,904	**64**	0,899	0,438	2,050	0,488
20	0,342	0,940	0,364	2,747	**65**	0,906	0,423	2,145	0,466
21	0,358	0,934	0,384	2,605	**66**	0,914	0,405	2,246	0,445
22	0,375	0,927	0,404	2,475	**67**	0,921	0,391	2,356	0,424
23	0,391	0,921	0,424	2,356	**68**	0,927	0,375	2,475	0,404
24	0,405	0,914	0,445	2,246	**69**	0,934	0,358	2,605	0,384
25	0,423	0,906	0,466	2,145	**70**	0,940	0,342	2,747	0,364
26	0,438	0,899	0,488	2,050	**71**	0,946	0,326	2,904	0,344
27	0,454	0,891	0,510	1,963	**72**	0,951	0,309	3,078	0,325
28	0,469	0,883	0,532	1,881	**73**	0,956	0,292	3,271	0,306
29	0,485	0,875	0,554	1,804	**74**	0,961	0,276	3,487	0,287
30	0,500	0,866	0,577	1,732	**75**	0,966	0,259	3,732	0,268
31	0,515	0,857	0,601	1,664	**76**	0,970	0,242	4,011	0,249
32	0,530	0,848	0,625	1,600	**77**	0,974	0,225	4,331	0,231
33	0,545	0,839	0,649	1,540	**78**	0,978	0,208	4,507	0,213
34	0,559	0,829	0,675	1,483	**79**	0,982	0,191	5,145	0,194
35	0,574	0,819	0,700	1,428	**80**	0,985	0,174	5,67	0,176
36	0,588	0,809	0,727	1,376	**81**	0,988	0,156	6,31	0,158
37	0,602	0,799	0,754	1,327	**82**	0,990	0,139	7,11	0,141
38	0,616	0,788	0,781	1,280	**83**	0,993	0,1219	8,14	0,1228
39	0,629	0,777	0,810	1,235	**84**	0,995	0,1045	9,51	0,1051
40	0,643	0,766	0,839	1,192	**85**	0,996	0,0872	11,4	0,0875
41	0,656	0,755	0,869	1,150	**86**	0,998	0,0698	14,3	0,0699
42	0,669	0,743	0,900	1,111	**87**	0,999	0,0523	19,1	0,0524
43	0,682	0,731	0,933	1,072	**88**	0,999	0,0349	28,6	0,0349
44	0,695	0,719	0,966	1,036	**89**	1,000	0,0175	57,3	0,0175
45	0,707	0,707	1,000	1,000	**90**	1,000	0,0000	-	0,0000

JUWAPLAR HA'M KO'RSETPELER

1-sabaq. **1.** 50°; 130°; 133°; 97°. **2.** 12 *sm*. **3.** 65°; 70°; 45°. **4.** 105°; 130°; 125°. **5.** 35°; 35°; 110°. **6.** 94°; 56°; 30°. **7.** 110°; 130°; 120°. **8.** *Ko'rsetpe:* to'rtu'shmu'yeshliktin' ha'r birinin' ta'repleri da'slepki u'shmu'yeshliktin' sa'ykes ta'replerinin' yarmina ten'. **9.** *Ko'rsetpe: DF* kesindi *ABH* u'shmu'yeshliktin'de, *CEB* u'shmu'yeshliktin' de worta sizig'i boladi. **10.** *Ko'rsetpe: ANC* ha'm *CKA* u'shmu'yeshliklerdin' ha'mde ishki almasiwshi mu'yeshlerdin' ten'liginen paydalanin'.

2-sabaq. **1.** $6\sqrt{6}$. **2.** 36. **3.** 30°. **4.** a) 80°; 80°; 20°; b) 70°; 70°; 40°. **5.** 6,72 *sm*. **6.** 54. **7.** 5 *sm*; $25\sqrt{3}$ sm^2; 120°; 30°; 30°. **8.** 55°; 60°; 65°. **9.** 90°. **10.** 140°.

3-sabaq. **2.** 78°; 102°; 78°; 102°. **3.** 53°; 37°. **4.** 110°; 70°; 110°; 70°. **5.** 45°; 135°; 45°; 135°. **6.** 20 *sm* yaki 28 *sm*. **7.** *Ko'rsetpe:* Da'slep ta'repleri $AB=2$ *sm*, $BC=6$ *sm* bolg'an *ABCD* tuwri'mu'yeshlik jasan'. Keyin woraylari *B* ha'm *C* noqatlarda radiusi 3 *sm* bolg'an shen'berler jasan'.

4-sabaq. **1.** 30 *sm*. **2.** 13 *sm*. **3.** *Ko'rsetpe:* 3-sabaqtan 7-ma'selege qaran'. **4.** $880\sqrt{41}$ sm^2. **5.** a) 4 *sm*, 8 *sm*; b) 45°, 90°; d) $16+8\sqrt{2}$ *sm*, 32 sm^2. **6.** $18\sqrt{3}$ sm^2. **7.** 30 sm^2. **8.** 28 *sm*; $28\sqrt{2}$ *sm*.

5-sabaq. **3.** U'shmu'yeshlikler uqsas. **5.** 5; 8; $\frac{1}{2}$. **6.** 72; 162; 90.

6-sabaq. **3.** 12 *m*. **4.** 7,5 *sm*; 12,5 *sm*; 15 *sm*. **5.** 73,5 m^2; 37,5 m^2. **6.** U'shmu'yeshlikler uqsas.

7-sabaq. **3.** a) 4,5; b) 10,5; d) 4,5. **4.** a) 10; b) 6; d) 4,5. **5.** a) 5 *sm*, 3,5 *sm*; b) $5\frac{5}{7}$ *sm*, $2\frac{2}{7}$ *sm*. **6.** a) 8; b) 3,5; d) 12,5.

8-sabaq. **4.** a) awa; b) awa; d) yaq. **5.** $2\frac{1}{3}$ *sm*, 9. **6.** a) 15 *sm*; 20 *sm*; b) 24 *sm*; 18 *sm*; d) 144 sm^2; 256 sm^2. **8.** 19,2 *m*.

9-sabaq. **2.** awa; **3.** a) ha'm d); e) ha'm f). **4.** 108 sm^2. **5.** 4 *sm*; 6 *sm*. **7.** 4,8 *sm*. **9.** 12.

10-sabaq. **2.** a) ha'm d); b) ha'm e); g) ha'm f). **3.** 36 *m* yaki 20,25 *m*. **4.** 12 *sm*; 14 *sm*. **6.** $5\frac{5}{11}$ *sm*.

11-sabaq. **3.** a) 15; b) $3\frac{2}{11}$; d) $3\frac{5}{17}$. **4.** 18 *sm*; 6 *sm*. **5.** 29 dm^2. **6.** 6 *dm*. **7.** $m:n$.

12-sabaq. **1.** $3\frac{3}{17}$ *m*; 13,6 *sm*. **7.** $n:m$. **8.** a) $S:4$; b) $S:2$; d) $S:4$.

13-sabaq. **II.** **1.** 12 sm^2. **2.** 8,4. **3.** 2,4. **4.** 24. **5.** 8. **6.** 1,6. **III.** **1.** 6 *sm*. **2.** 65 *dm*; 52 *dm*. **3.** $AB||EF$.

14-sabaq. **5.** 1 *km* 750 *m*. **8.** 7,2 *sm*. **9.** $k=\frac{1}{2}$ yaki $k=2$.

15-sabaq. **4.** $k=2$. **5.** 6 sm^2; 24 sm^2. **6.** 104 sm^2. **7.** Ha'r yeki jag'dayda $k=1$. **8.** 1,2 m^2. **9.** 16 *sm*, 32 *sm*.

16-sabaq. **4.** $\frac{2}{3}$; $\frac{4}{9}$. **5.** X^*X ha'm Y^*Y nurlarinin' kesilisiw noqati gomotetiyaorayi boladi. **6.** $OX_1=2\cdot OX$. **7.** *Ko'rsetpeler:* Temadag'i ma'selelerdin' sheshiminen paydalanin'.

8. a) $OA_1=\frac{2}{3}OA$; b) $OA_1=20A$; d) $OA_1=30A$; e) $OA_1=OA$. **9.** *Ko'rsetpe:* Temadag'i 3-su'wretten paydalanin'.

17-sabaq. **4.** a) $P_2=42$; $k=\frac{1}{2}$; b) $S_1=12$, $k=2$; d) $P_1=150\sqrt{2}$, $k=\sqrt{2}$; e) $P_1=10$, $S_2=216$.

18-sabaq. **1.** ≈6,97 *m*. **2.** 300 *m*.**3.** ≈72 *m*. **4.** 6,6 *m*.

19-sabaq. **1.** 9. **2.** 12 *dm*. **3.** 8 *m*. **4.** 24 *dm*². **6.** *Ko'rsetpe: ABC* u'shmu'yeshlik si'zi'n', ko'pmu'yeshliklerdi jasaw temasindag'i 1-ma'seleden paydalani'p si'zi'lg'an u'shmu'yeshlik ta'replerinen u'sh ma'rte kishi u'sh mu'yeshlik jasan'.

20-sabaq. **1.** 72°; 72°; 36°. **3.** 12 *sm*². **4.** 15 000 000 *km*. **5.** a) Awa; b) Awa; **7.** 6 *sm*, 12 *sm*, 18 *sm*. **8.** 63 *m*.

21-sabaq. **II. 1.** 8 *sm*. **2.** $4\frac{4}{9}$ *sm*. **3.** 48 *m*. **4.** 4 *sm*; 0,5 *sm*². **5.** $5\frac{1}{3}$ *m*. **6.** 867 *km*. **III. 1.** 7,5 *m*. **2.** 6 *sm*. **3.** a) 7,5 *sm*; b) 6 *sm*; d) 16,2 *sm*. *Qi'zi'qli' ma'seleler:* **1.** Wo'zgermeydi. **2.** a) Awa; b) Yaq. **3.** *Ko'rsetpe:* Sizg'ish penen ha'r bir quwirshaqtin' boyin wo'lshen' ha'm olardin' qatnasin tabi'n'.

22-sabaq. **4.** $\sin A=\frac{5}{13}$; $\cos A=\frac{12}{13}$; $\mathrm{tg}A=\frac{5}{12}$; $\mathrm{ctg}A=\frac{12}{5}$. **5.** a) $\sin A=\frac{7}{25}$; $\cos A=\frac{24}{25}$; $\mathrm{tg}A=\frac{7}{25}$; $\mathrm{ctg}A=\frac{25}{7}$. $\sin B=\frac{24}{25}$; $\cos B=\frac{7}{25}$; $\mathrm{tg}B=\frac{25}{7}$; $\mathrm{ctg}B=\frac{7}{25}$. **6.** $BC=\frac{11}{20}$; $AB=\frac{61}{20}$. **7.** $AB=34$; $AC=30$.

23-sabaq. **4.** a) 15 *sm*; b) 8 *sm*; d) 36,125 *sm*; e) 31,875 *sm*. **5.** $\frac{15\sqrt{55}}{8}$ *sm*. **7.** 42*sm*². **8.** 21 *sm*². **9.** 32 *sm*². **10.** 180 *sm*².

24-sabaq. **3.** $2\sqrt{3}$ *dm*, $4\sqrt{3}$ *dm*. **4.** a) $12+4\sqrt{3}$; b) $6+6\sqrt{3}$; d) $16+8\sqrt{2}$. **5.** a) $\angle A=45°$, $\angle B=45°$; b) $\angle A=60°$; $\angle B=30°$; d) $\angle A=30°$, $\angle B=60°$. **6.** $\frac{25\sqrt{3}}{3}$ *sm*². **7.** 7 *sm*; 24 *sm*, $\cos A=\frac{24}{25}$, $\mathrm{tg}A=\frac{7}{24}$; $\mathrm{ctg}A=\frac{24}{7}$. **8.** 120°; 120°; 60°; 60°.

25-sabaq. **1.** 36 *sm*². **2.** 24 *sm*. **3.** a) $6\sqrt{3}$; b) 30; d) $\frac{105\sqrt{3}}{4}$. **4.** $(24+4\sqrt{3})$ *sm*; $(24+8\sqrt{3})$ *sm*². **5.** $10\sqrt{3}$ *sm*. **6.** a) $\frac{\sqrt{3}}{6}$; b) $\frac{1}{2}$; d) $\frac{\sqrt{3}}{2}$. **7.** ≈807 *m*². **8.** ≈88 *m*.

26-sabaq. **2.** tangens 90° ta, kotangens 0° ha'm 180° ta.**3.** $\sin\alpha>0$, $\cos\alpha<0$, $\mathrm{tg}\alpha<0$, $\mathrm{ctg}\alpha<0$. **6.** sin45°=sin135°; cos45°>cos135°.

27-sabaq. **2.** 1) $\sin^2\alpha$; 2) $\cos^2\alpha$; 3) 1; 4) $\cos^2\alpha$; 5) $\cos^2\alpha$; 6) $\sin^2\alpha$. **3.** a) $-\frac{3}{5}$; b) $\frac{\sqrt{5}}{3}$; d) 0. **4.** $6\sqrt{3}$ *sm*². **5.** $0,8\sqrt{3}$ *sm*, $1,6\sqrt{3}$ *sm*. **6.** a) $\frac{\sqrt{3}}{2}$; b) $\frac{\sqrt{5}}{3}$; e) 0. **9.** a) $A\left(\frac{3\sqrt{2}}{2};\frac{3\sqrt{2}}{2}\right)$; e) $A(-2;0)$; f) $A\left(\frac{4\sqrt{3}}{3};2\right)$.

28-sabaq. **4.** a) 150°; b) 135°; d) 135°; e) 150°. **5.** a) 0; b) 1; d) 0; e) −3,5; **6.** a) 1; b) 1; d) 1. **7.** 3,5 *sm*. **8.** $36\sqrt{3}$ *sm*². **9.** a) $\frac{1}{2}$; $-\frac{1}{2}$; b) $\pm\frac{\sqrt{15}}{4}$; d) 0. **10*.** a) 30°; b) 135°; d) 150°.

29-sabaq. **III. 2.** 1000, 37°. **3.** 2°. **4.** 34°. **5.** $2\sqrt{3}$; $4\sqrt{3}$. **6.** $3\sqrt{3}$ *sm*. **7.** 5 *sm*. **8.** 12, $24\sqrt{3}$. **9.** 20 *sm*, 100 sm². **10.** 4, $16\sqrt{3}$. **11.** 30°; 60°. **13.** 12 *sm*; $4\sqrt{3}$ *sm*; $8\sqrt{3}$ *sm*. **14.** 32 *sm*². **15.** $-\frac{15}{17}$; $-\frac{8}{15}$; $-\frac{15}{8}$. **16.** $\frac{4\sqrt{3}}{3}$. **17.** $12(\sqrt{3}+1)$, $72(\sqrt{3}+1)$. **IV. 1.** $\frac{15}{17}$; $-\frac{8}{15}$; . **2.** $2\sqrt{77}$; 13°; 77°. **4.** *Ko'rsetpe:* U'shmu'yeshliktin' ten'ligi haqqindag'i teoremadan paydalanin'.

30-sabaq. **2.** a) 6 sm^2; b) 73,5 sm^2; d) 6 sm^2. **3.** 36 sm^2. **4.** $49\sqrt{2}$ sm^2. **5.** $54\sqrt{3}$ sm^2. **6.** $2\frac{2}{3}$ *sm*; $4{,}5\sqrt{2}$ *sm*. **7.** $\frac{h_a h_b}{2\sin\alpha}$. **8.** $4{,}8\sqrt{3}$ *sm*.

31-sabaq. **2.** a) $BC=6$; b) $AB=8\sqrt{2}$; d) $AC=7\sqrt{2}$. **3.** a) $\sin C=\frac{1}{3}$; b) $\sin A=\frac{21}{40}$; d) $\sin B=\frac{16}{21}$. **4.** 4,8 *dm*. **5.** 30° yamasa 150°. **6.** Mu'mkin. **7.** $AB\approx 21{,}1$ *m*; $\angle B\approx 37°$, $\angle C\approx 76°$. **8.** 76°; 26,1 *sm*; 23,8 *sm*.

32-sabaq. **2.** a) $\sqrt{13}$ *sm*; b) 4 *m*; d) $\sqrt{283}$ *dm*. **3.** $\frac{1}{5}$; $\frac{19}{35}$; $\frac{5}{7}$. **4.** $2\sqrt{13}$ *sm* yaki $2\sqrt{109}$ *sm*. **5.** $\sqrt{31}$ *sm*, $\sqrt{91}$ *sm*. **6.** $\sqrt{109}$ *sm*, $\sqrt{39}$ *sm*.

7. *Ko'rsetpe:* *ADC* ha'm *BDC* u'shmu'yeshliklerge kosinuslar teoremesinan paydalanip, a^2 ha'm c^2 ti tabin, keyin bul ten'liklerdi ag'zama-ag'za qosin'. **8.** $\frac{\sqrt{106}}{2}$ *sm*; $\frac{\sqrt{151}}{2}$ *sm*; $\frac{\sqrt{190}}{2}$ *sm*.

33-sabaq. **1.** $\angle B$ ha'm $\angle C$. **2.** *AB* ha'm *BC*. **3.** a) su'yir muyeshli; b) tuwri' mu'yeshli; d) dog'al mu'yeshli. **4.** a) $8\frac{1}{8}$; b) $8\frac{1}{8}$; d) $24\frac{1}{6}$; e) $\frac{35\sqrt{6}}{24}$. **6.** *Ko'rsetpe:* Sinuslar teoremesinan paydalanin'. **7.** *Ko'rsetpe:* 6-ma'selege uqsas sheshiledi.

8. *Ko'rsetpe:* Sinuslar teoremesinan paydalanin'.

34-sabaq. **1.** a) $10\sqrt{3}$; b) $28\sqrt{2}$; d) 12; e) ≈0,1532. **2.** a) –2,5; b) 0; d) 2. **3.** a) 8; b) 24; d) 8; e) 0. **5.** a) –7,5; d) 0. **6.** $a\perp b$, $c\perp d$.

35-sabaq. **1.** a) $\alpha=90°$, $a=b=5$, $c=5\sqrt{2}$. b) $\gamma\approx 45°$; $b\approx 17{,}9$, $c\approx 14{,}6$; d) $\alpha=20°$; $b\approx 65.8$; $c\approx 88{,}6$; e) $\gamma=119°$; $a\approx 16{,}7$; $b\approx 11{,}2$. **2.** a) $c\approx 5{,}29$; $\alpha\approx 79°6'$; $\beta\approx 138°21'$; b) $c\approx 53{,}09$; $\alpha\approx 11°39'$; $\beta\approx 38°21'$; d) $a\approx 19{,}9$; $\beta\approx 58°19'$; $\gamma\approx 936°41'$; e) $a\approx 22{,}9$; $\beta\approx 21°$; $\gamma\approx 15°$. **3.** a) $\alpha\approx 29°$; $\beta\approx 47°$; $\gamma\approx 104°$; b) $\alpha\approx 54°$; $\beta\approx 13°$; $\gamma\approx 113°$; d) $\alpha\approx 34°$; $\beta\approx 44°$; $\gamma\approx 102°$; e) $\alpha\approx 39°$; $\beta\approx 93°$; $\gamma\approx 48°$.

36-sabaq. **1.** a) $2\sqrt{3}$ *sm*; b) 16 *sm*; d) $\frac{ab\sqrt{2}}{4}$. **2.** $4\sqrt{2}$ *m*; 8 *m* va $4+4\sqrt{3}$ *m*. **3.** $50\sqrt{3}$ *kg*. **4.** 14 *sm*. **5.** $2\sqrt{14}$ *sm*. **6.** $6\sqrt{3}$ *sm*. **7.** 50 *sm*.

37-sabaq. **1.** ≈10,8 *m*. **2.** ≈15 *m*. **3.** ≈43,4 *m*. **4.** ≈35°. **5.** ≈73,2 *m*. **6.** ≈49 *m*. **7.** Asfalt jayilg'an.

38-39-sabaq. **II.** **1.** $3\sqrt{6}$, $3\sqrt{2}$. **2.** $\frac{111}{120}$; 0,89; –0,65. **3.** $2\sqrt{7}$ *sm*; $\frac{2\sqrt{21}}{3}$ *sm*. **4.** $30\frac{1}{30}$ *sm*. **5.** 28 *sm*. **6.** 8 sm^2; $(4+4\sqrt{5})$ *sm*; $h_a=4$ *sm*, $h_b=0{,}8\sqrt{5}$ *sm*. **7.** $2\sqrt{13}$. **8.** a) su'yir mu'yeshli; b) tuwri' mu'yeshli, d) dog'al mu'yeshli. **9.** 63 sm^2. **10.** ≈3,7 *sm*. **11.** 7 *sm*. **12.** 6. **13.** 0. **14.** –9. **15.** 135°. **16.** OC≈9,6. **17.** $(24+24\sqrt{3})$ *sm*. **18.** 5. **III.** **1.** ≈109°. **2.** $\gamma=100°$, $a\approx 3{,}25$; $c\approx 6{,}43$. **3.** 6,25; 14,76.

40-sabaq. **2.** a) Ha'r qanday ush'mu'yeshlik shen'berge ishley si'zi'liwi mu'mkin. b) Qarama-qarsi mu'yeshlirinin' qosi'ndi'si' 180° bolg'an to'rtmu'yeshlikler. **3.** Bir dog'ag'a uring'an mu'yeshleri ten'. **4.** 10 *sm*. **5.** 672 sm^2. **6.** a) $10\sqrt{3}$ *sm*; b) $10\sqrt{2}$ *sm*; d) $10\sqrt{2}$ *sm*; $10\sqrt{2}$ *sm*; 20 *sm*. **7.** $8\frac{1}{3}$ *sm*. **8.** ΔABF da, $\angle BAF+\angle AFB=90°$, $\angle ABF=90°$. Demek, *AF* – diametr. **9.** Qarama-qarsi mu'yeshlerinin' qosi'ndi'si' 180°, yag'niy shen'berge ishley siziw mu'mkin. **10.** *Ko'rsetpe:* bir u'ltan ha'm bir qaptal ta'reptin' worta perpendekulyari' kesisken noqat shen'ber worayi' boladi'.

41-sabaq. **2.** 7,2 *sm*. **3.** a) 16,6; b) 22; d) 22,6. **4.** a) 2,5; b) 3,5; d) 2. **8.** 6 *sm*.

42-sabaq. **3.** a) 60°; b) 108°; d) 120°; e) 144°; f) 160°. **4.** a) 120°; b) 72°; d) 120°; e) 36°; f) 30°. **5.** a) 3; b) 4; d) 8; e) 12.

43-sabaq. **1.** 3 *sm* ha'm $3\sqrt{2}$ *sm*. **2.** $\sqrt{3}$ ha'm $2\sqrt{3}$. **7.** a) 6; b) 12; d) 10; e) 20; f) 5.

44-sabaq. **3.** 8 *sm*; $8\sqrt{2}$ *sm*; $8\sqrt{3}$ *sm*; $8\sqrt{2}+3$ *sm*; 16 *sm*.
4. $\frac{8\sqrt{6}}{3}$ *sm*; **5.** a) $20\sqrt{2}$ *sm*; b) 40 *sm*. **6.** $\frac{5\sqrt{3}}{3}$ *sm*.

45-sabaq. **I. 1.** E; **2.** D; **3.** D; **4.** B; **5.** B; **6.** E; **7.** E. **III. 1.** $\sqrt{3}$:4: $6\sqrt{3}$. **2.** 3:4. **3.** a) ≈5,780 *sm*; b) ≈4,142 *sm*; d) ≈2,679 *sm*. **4.** $S=\sqrt{2}R^2$. **5.** 24 sm^2. **IV. 1.** 4 sm; 13 sm. **2.** $4\sqrt{3}$ *sm*; 8 *sm*. **3.** a) 80 *sm*; b) $20\sqrt{2-\sqrt{3}}$ *sm*; $40\sqrt{2-\sqrt{3}}$ *sm*; d) 200 sm^2. 4. $\frac{27\sqrt{3}}{4}$ sm^2.

46-sabaq. **2.** a) 3 ma'rte artadi; b) 6π *sm* ge artadi; d) 3 ma'rte kemeyedi; e) 6π *sm* ge kemeyedi. **3.** 6369 *km*. **4.** a) $\frac{2\pi\sqrt{3}a}{3}$; b) $\pi\sqrt{a^2+b^2}$; d) $\frac{2\pi b^2}{\sqrt{4b^2-a^2}}$. **5.** a) πa; b) $\pi c(\sqrt{2}-1)$; d) $\pi c(\sin\alpha+\cos\alpha-1)$. **6.** 1,5 *m*. **7.** 66348 ma'rte.

47-sabaq. **1.** a) π *sm*; b) 1,5π *sm*; d) 3π *sm*; e) 4π *sm*. **2.** a) $\frac{2\pi}{9}$; b) $\frac{\pi}{3}$; d) $\frac{5\pi}{12}$. **3.** a) ≈69°; b) 120°; d) 150°. **4.** a) $\frac{5\pi}{8}$ *sm*; b) 2π *sm;* d) $\frac{15\pi}{4}$ *sm*; **5.** a) 4π; b) 16π. **7.** 2.

48-sabaq. **3.** k^2 ma'rte artti; b) k^2 ma'rte kemeyedi. **4.** 6,25π sm^2; 12,5π sm^2. **5.** 2,25π sm^2; 9π sm^2. **6.** $(\pi-2)R^2$. **7.** 21,25 π sm^2. **8.** 7,5 sm^2.

49-sabaq. **3.** a) $\frac{49}{12}\pi$ sm^2; $\frac{49(\pi-3)}{12}$ sm^2; b) 6,125π sm^2; $\frac{49(\pi-2\sqrt{2})}{8}$ sm^2; d) $\frac{49\pi}{3}$ sm^2; $\frac{49(4\pi-3\sqrt{3})}{12}$ sm^2; e) $\frac{49\pi}{4}$ sm^2; $\frac{49(\pi-2)}{4}$ sm^2. **4.** a) $a^2\left(\frac{\sqrt{3}}{4}-\frac{\pi}{8}\right)$; b) $a^2\left(1-\frac{\pi}{4}\right)$ d) $\frac{3\sqrt{3}-\pi}{2}a^2$; **5.** π sm^2; 3π sm^2; 5π sm^2; 7π sm^2. **6.** $\frac{25(2\pi-3\sqrt{3})}{3}$ sm^2; $\frac{25(10\pi+3\sqrt{3})}{3}$ sm^2; **7.** $\frac{75\cdot(4\pi-3\sqrt{3})}{2}$ sm^2. **8.** $S_1<S<S_2$; 300 sm^2 < 314sm^2 < 321,48 sm^2.

50-sabaq. **1.** Do'n'gelektin' maydani' u'lken. **2.** $\frac{160}{3}\pi$ sm^2. **3.** 5,76π sm^2. **4.** 8.(π—2) sm^2. **6.** 6π sm^2; 10π *sm*.

51-sabaq. **II. 1.** $6\sqrt{2+\sqrt{2}}$. **2.** $\frac{8\pi}{3}$ *dm*. **3.** 30 *sm*. **4.** 90°. **5.** 3. **6.** π va 6,25π. **7.** $\frac{10\pi+3\sqrt{3}}{2\pi-3\sqrt{3}}$. **8.** $\frac{2\sqrt{3}}{\pi}$. **9.** $\frac{9\sqrt{3}-2\pi}{6}.a^2$. **10.** 1,5π. **11.** 7. **12.** ≈9π—26,04. **13.** π. **14.** $54\sqrt{3}$—24π. **15.** $\frac{3\pi}{8}$. **III. 2.** $8\sqrt{3}$ *sm*. **3.** a) $\frac{18}{\pi}$ *sm*; b) $\frac{216}{\pi}$ sm^2; d) $\frac{216\pi+81\sqrt{3}}{\pi^2}$ sm^2.

52-sabaq. **3.** $5\sqrt{2}$ *sm*. **4.** 12 *sm*. **5.** 44 *m*, 60 *m*. **7.** 1:7. **8.** *AB*cosα.

53-sabaq. **1.** a) 30 *sm*, 12 *sm*; b) 9 *sm*, 12 *sm*, 21 *sm*; d) 3 *sm*, 15 *sm*, 3 *sm*, 21 *sm*.

3. 6 *sm*; 10,5 *sm*. **4.** 9 *sm*, 12 *sm*, 15 *sm*, 18 *sm*. **5.** 60°. **6.** 21 *sm*.

54-sabaq **1.** *Ko'rsetpe:* $\Delta ACD \backsim \Delta CBD \backsim \Delta ABC$. **2.** 25 *sm*, 15 *sm*, 20 *sm*. **3.** $9\frac{3}{5}$ *sm*. **4.** a) 5, 4; b) 24, 25; d) 8,10. **5.** 16:25. **6.** 56,16 *sm*^{2}. **7.** 60 *sm*^{2}. **8.** $\frac{2}{3}$; $\frac{4}{9}$; $\frac{2}{3}$.

55-sabaq. **2.** *Ko'rsetpe:* a) katetleri a ha'm b bolg'an tuwri' mu'yeshli u'shmu'yeshlik jasan'; b) gipotenuzasi a, bir kateti b bolg'an tuwri' mu'yeshli u'shmu'yeshlik jasan'. **3.** *Ko'rsetpe:* Katetleri $AB=BC=1$ bolg'an ΔABC jasan'. Keyin kateti $CC_1=1$ ha'm $\angle C_1=90°$ bolg'an ΔBCC_1 jasan' ha'm t.b. **4.** a) 20; b) 45; d) 37,5. **5.** 225 *sm*^{2}. **6.** 180 *sm*^{2}. **7.** 25:9. **9.** $OC \geq OD$ bolg'ani' ushi'n ten'sizlik ha'r qashan duri's.

56-sabaq. **1.** a) 6,25; b) 12; d) 0,25. **2.** a) 8 *sm*; b) 2,5 *sm;* d) 0,9 *sm*. **3.** a) 4 *dm*; b) 4 *dm*. **4.** 4 *sm*. **6.** 9 *dm*; 16 *dm*.

57-sabaq. **1.** 10 *sm*. **2.** 2 *sm*. **3.** a) 2,5; b) 4; d) 2. **4.** a) $4\sqrt{6}-1$ *sm*; b) 6 *sm*. **5.** 1:6. **6.** 6 *sm*. **7.** 3. **8.** 1:4.

58-sabaq. **II.** **1.** 18 *sm*; 32 *sm*. **2.** 4 *sm*; **3.** 8 *sm*; **4.** 6,4 *dm*. **5.** 8 *sm*. **6.** 1,5. **7.** 5. **8.** 6. **9.** 45 *dm*^{2}. **10.** 4 *sm*. **11.** 8 *sm*. **12.** 6. **13.** 60°. **14.** 45°. **15.** 4:9. **III.** **1.** 8 *sm*. **2.** 5 *dm*. **3.** 4 *sm*; 8 *sm*.

59-sabaq. **1.** a) 12 (kv.b.); b) 20 (kv.b.); d) 12 (kv.b.); e) 12 (kv.b.); f) 42 (kv.b.). **2.** 4 *sm*. **3.** a) 12; b) 288. **4.** a) $\frac{6\sqrt{133}}{19}$; b) $2\frac{1}{3}$. **5.** a) (3;2); b) (2,5;—0,5); d) (—1;4); e) (—0,5;3,1). **6.** D(2;—1). **8.** 10 *sm*; 25 *sm*. **9.** 60°; 90°; 120°; 90°. **10.** 6 *sm*. **11.** $6\sqrt{2}$ *sm*.

60-sabaq. **1.** a) 4; b) 6; d) 5; e) 5. **2.** $4\sqrt{13}+\sqrt{82}+\sqrt{58}$. **4.** Trapeciya. **5.** $x=4$, $y=3$. **7.** $b-a$; $-a-2b$; $2a+b$. **8.** 5*N*. **9.** $18\sqrt{3}$; $27\sqrt{3}$. **10.** $4\sqrt{2}$ *sm*. **11.** $PA=PB$ ha'm $PA=PC$ bolg'ani' ushi'n $PB=PC$.

61-sabaq. **2.** 45°; 90°; 135°; 90°. **3.** 45°. **4.** 60°. **5.** 3 sm; 8 sm. **7.** 28 sm. **9.** 45°.

62-sabaq. **1.** 8,4 *sm*, 10,5 *sm*, 14,7 *sm*. **2.** 175 *dm*^{2}; 252 *dm*^{2}. **3.** 12 *sm*^{2}. **4.** 6. **5.** $9(3-\sqrt{3})$ *sm*^{2}. **6.** 8 *sm*. **7.** 5 *sm*; 2 *sm*; 5 *sm*; 8 *sm*. **8.** 3 *sm*, 4 *sm*.

63-sabaq. **1.** 2 *sm*. **2.** 6 *dm*; 9,6 *dm*; 6,5 *dm*; 10,4 *dm*. **3.** Awa. **4.** $\sqrt[4]{27}$; $3\sqrt[4]{3}$. **5.** 16,9 *sm*. **6.** 150 *sm*^{2}. **7.** (0;—6). **8.** Birinshisinde. **9.** 80 ta. **10.** 7 *dm*^{2}. **11.** a) 180 *dm*^{3}; b) 105 *sm*^{3}; d) 1296 *sm*^{3}.

64-sabaq. **1.** —12,5. **2.** 20 *sm*. **3.** Awa. **4.** 5 *sm*, 13 *sm*. **5.** 3 *sm*. **6.** 8 *sm*. **7.** 30π *sm*. **8.** 5 *sm*. **9.** 25 *sm* yaki 20 $\sqrt{2}$ *sm*.

65-sabaq. **1.** 4,5 *sm*; 6,75 *sm*. **2.** $\frac{20}{9}$ *sm*, 4 *sm*; 4,8 *sm*. **3.** 12 *sm*. **4.** 2 *sm*. **5.** 6,72. **6.** $2\sqrt{26}\pi$ **7.** $25\sqrt{3}$ *sm*^{2}. **8.** 84 *sm*^{2}. **9.** $675\sqrt{3}$ *sm*^{2}.

66-sabaq. **1.** $4\frac{129}{1024}$. **2.** 100°; 80°. **3.** 4 *sm*. **4.** 24 *sm*^{2}. **5.** 4,8 *m*. **6.** 30 *sm*^{2}. **7.** 7. **8.** 10 *sm* yaki $2\sqrt{97}$ *sm*.

67–68-sabaq. **1.** a) 9; b) 4 *sm*^{2}; d) 3,5 *sm*; e) $\frac{4}{3}$ $TB-CA$; f) 0,2. **2.** $\Delta CMH \backsim \Delta BCA$.

X 30 Haydarov Bahodir Qayumovich
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Ijarag'a berilgen sabaqliq jag'dayin ko'rsetiwshi keste

T/r	Woiwshinin' ati ha'm familiyasi	Woqiw jili	Sabaqliqtin' aling'andag'i jag'dayi	Klass basshi-sinin' qoli	Sabaqliqtin' tapsiril-g'andag'i jag'dayi	Klass basshi-sinin' qoli
1						
2						
3						
4						
5						
6						

Sabaqliq ijarag'a berilip, woqi'w ji'li' aqi'ri'nda qaytari'p ali'ng'anda joqari'dag'i' keste klass basshi'si' ta'repinen to'mendegi bahalaw wo'lshemlerine tiykarlanip toltiriladi.

Taza	Sabaqliqti birinshi ret paydalaniwg'aberilgendegi jag'dayi.
Jaqsi	Muqabapu'tin, sabaqliqtin' tiykarg'i' bo'liminen ajiralmag'an. Barliq betleri bar. Jirtilmag'an, betleri almastirilmag'an, betlerinde jaziw ha'm si'zi'qlar joq.
Qanaatlandirarli	Muqabajelingen, biraz si'zi'lip, shetleri qayrilg'an, sabaqliqtin' tiykarg'i' bo'liminen alinip qaliw jag'dayi bar, paydalaniwshi ta'repinen qanaatlanarli qa'lpine keltirilgen.Aling'an betler qayta islengen, ayrim betleri si'zi'lg'an.
Qanaatlandirarsiz	Muqabag'asi'zi'lg'an, jirtilg'an, tiykarg'i' bo'liminen ajiralg'an yamasa pu'tinley joq, qanaatlandirarsiz islengen.Betleri jirtilg'an, betleri toliq emes, sizip, boyap taslang'an. Sabaqliqti qayta tiklewge bolmaydi.