MATEMATIKA 11

ALGEBRA VA ANALIZ ASOSLARI
GEOMETRIYA
I QISM

Umumiy o'rta ta'lim maktablarining 11-sinflari va o'rta maxsus, kasb-hunar ta'limi muassasalari uchun darslik

O'zbekiston Respublikasi Xalq ta'limi vazirligi tomonidan tasdiqlangan

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Darslikning “Algebra va analiz asoslari” bo‘limida ishlatilgan belgilar va ularning talqini:

- △ – masalani yechish (isbotlash) boshlandi
- ▲ – masalani yechish (isbotlash) tugadi
- (!) – nazorat ishlari va test (sinov) mashqlari
- (?) – savol va topshiriqlar
- ■ – asosiy ma’lumot
- * – murakkabroq mashqlar

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Algebra va analiz asoslari

I BOB. HOSILA VA UNING TATBIQLARI

1–2 O‘ZGARUVCHI MIQDORLAR ORTTIRMALARINING NISBATI VA UNING MA’NOSI. URINMA TA’RIFI. FUNKSIYA ORTTIRMASI

O‘zgaruvchi miqdorlar orttirmalarining nisbati

Turli o‘lchov birliklariga ega bo‘lgan ikkita o‘zgaruvchi miqdor nisbatini hisoblash inson hayotida tez-tez uchrab turadi.

Masalan, avtomashinaning ***tezligi*** uning yurgan yo‘lining vaqtga nisbati km/soat yoki m/s larda o‘lchanadi, yoqilg‘i sarflashi esa km/litr yoki 100 km/litr larda o‘lchanadi.

Xuddi shunday, basketbolchining mahorati bir o‘yinda to‘plagan ochkolar soni bilan belgilanadi.

Misol. O‘quv ishlab chiqarish majmuasida 11-sinf o‘quvchilari orasida matn terishning sifati va tezligi bo‘yicha sinov o‘tkazilmoqda.

Karim 3 minut mobaynida 213 ta so‘zni terib, 6 ta imloviy xatoga, Nargiza esa 4 minut mobaynida 260 ta so‘zni terib, 7 ta imloviy xatoga yo‘l qo‘ygani ma’lum bo‘ldi. Ularning natijalarini solishtiring.

△ Har bir o‘quvchi uchun tegishli nisbatlarni tuzamiz:

Karim:

matn terishning tezligi $\frac{213 \text{ ta so‘z}}{3 \text{ min}} = 71 \ \frac{\text{so‘z}}{\text{min}}$;

matn terishning sifati $\frac{6 \text{ ta xato}}{213 \text{ ta so‘z}} \approx 0{,}0282 \frac{\text{xato}}{\text{so‘z}}$.

Nargiza:

matn terishning tezligi $\frac{260 \text{ ta so‘z}}{4 \text{ min}} = 65 \ \frac{\text{so‘z}}{\text{min}}$;

matn terishning sifati $\frac{7 \text{ ta xato}}{260 \text{ ta so‘z}} \approx 0{,}0269 \frac{\text{xato}}{\text{so‘z}}$.

Demak, Karim matnni Nargizaga nisbatan tezroq tergan bo‘lsa-da, Nargiza bu ishni sifatliroq bajargan. ▲

Mashqlar

1. Puls chastotasini tekshirish uchun barmoqlar uchi arteriya tomiri o‘tadigan joyga qo‘yiladi va zarbalarni his qilish uchun shu joy bosiladi.

Madina pulsni o‘lchaganda bir minutda 67 ta zarbani his qildi.

a) Puls chastotasining ma’nosini tushuntiring. U qanday kattalik (belgi)?

b) Har soatda Madinaning yuragi necha marta uradi?

2. Karim uyida 14 bet matn terib, 8 ta imloviy xatoga yo‘l qo‘ydi. Agar 1 betda o‘rtacha 380 ta so‘z bo‘lsa:

a) Karimning matn terish sifatini aniqlang va yuqoridagi misolda olinяgan natija bilan solishtiring. Karimning matn terish sifati yaxshilandimi?

b) Karim 100 ta so‘z terganda o‘rtacha qancha xato qiladi?

3. Ma’ruf 12 soat ishlab 148 m 20 cm, Murod esa 13 soat ishlab 157 m 95 cm ariq tozaladi. Ularning mehnat unumdorligini solishtiring.

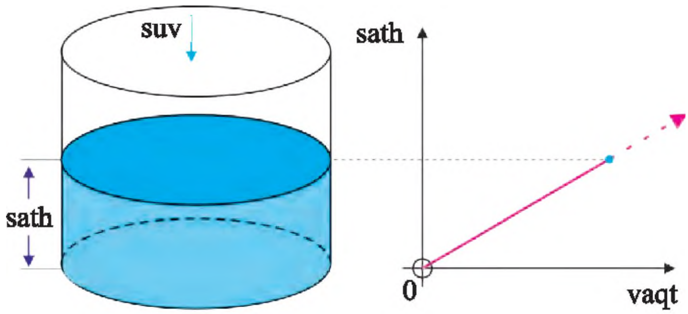
4. Avtomashina yangi shina protektorining chuqurligi 8 mm ni tashkil qiladi. 32178 km yurilganidan so‘ng yemirilish natijasida shina protektorining chuqurligi 2,3 mm bo‘lgani ma’lum bo‘ldi.

a) 1 km masofa yurilganda shina protektori chuqurligi qanday o‘zgaradi?

b) 10000 km masofa yurilganda-chi?

5. Madina Qarshi shahridan soat 11:43 da chiqib, soat 15:49 da Guliston shahriga yetib keldi. Agar u 350 km masofa yurgan bo‘lsa, uning o‘rtacha tezligi necha $\frac{\text{km}}{\text{soat}}$ bo‘ldi?

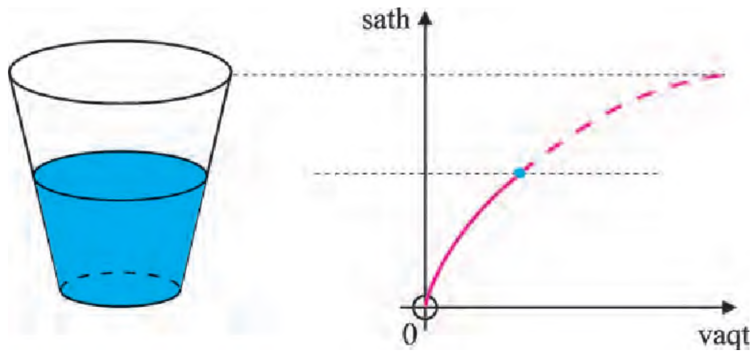
Misol. Silindr shaklidagi idish suv bilan bir xil tezlikda to‘ldirilmoqda. Bunda silindrik idish ichiga vaqtga proportsional bo‘lgan suv (hajmi) quyilayotgani bois suv sathining (balandligining) vaqtga nisbatan bog‘lanishi chiziqli funksiya ko‘rinishida bo‘ladi (1-rasmga qarang).



1-rasm.

Bu holda idishdagi suv sathining vaqtga bo'lgan nisbati (ya'ni sathning *o'zgarish tezligi*) o'zgarmas son bo'lib qolaveradi.

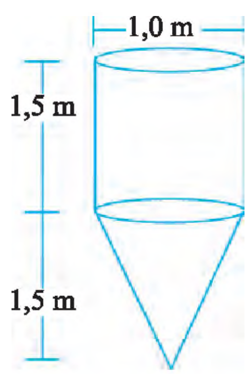
Endi boshqa shakldagi idishni qaraymiz (2-rasm):



2-rasm.

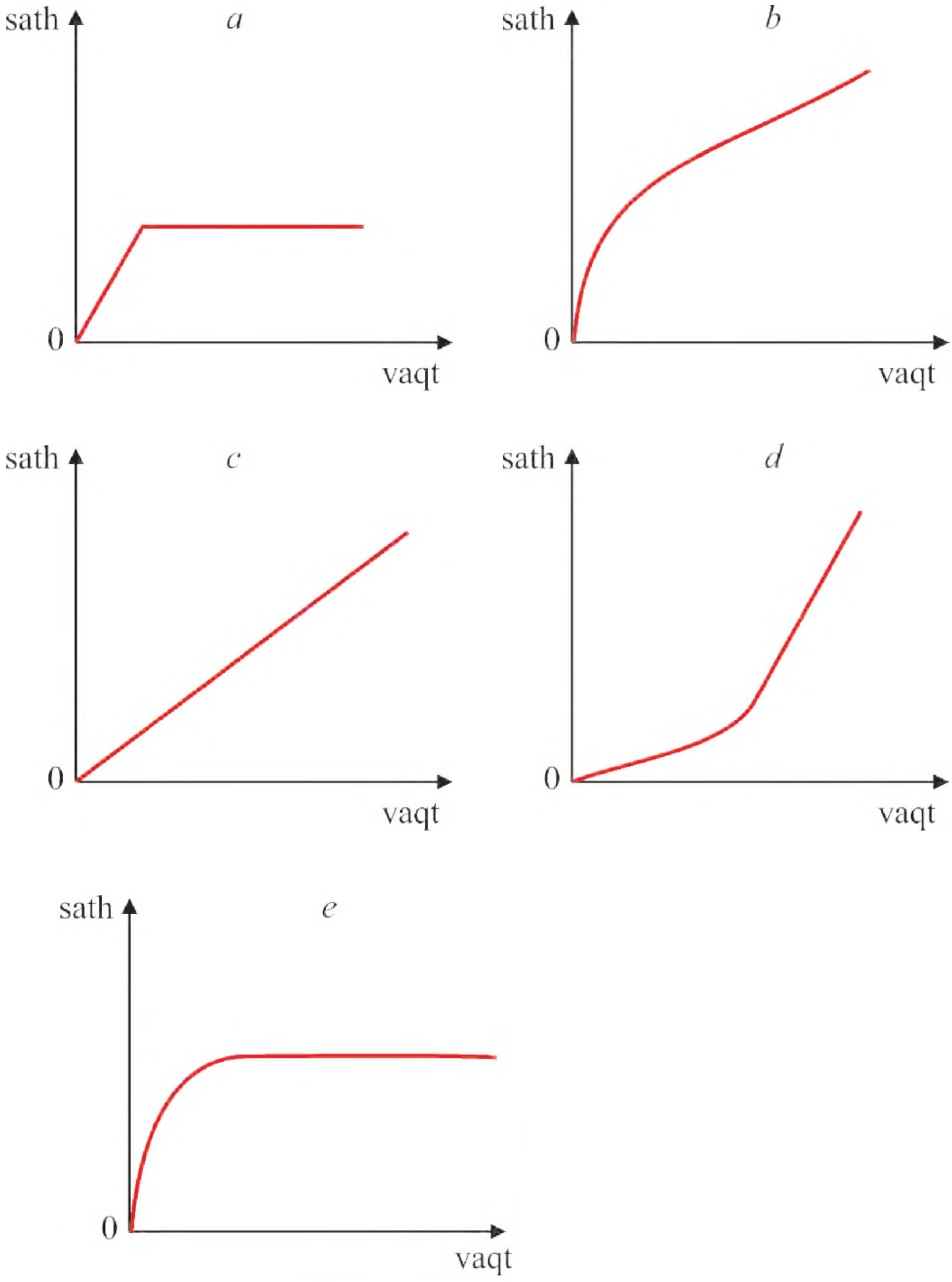
2- rasmda suv sathining o'zgarish tezligining vaqtga nisbatan bog'lanishi aks ettirilgan.

1-savol. 3-rasmda suv quyishga mo'ljallangan idish tasvirlangan.



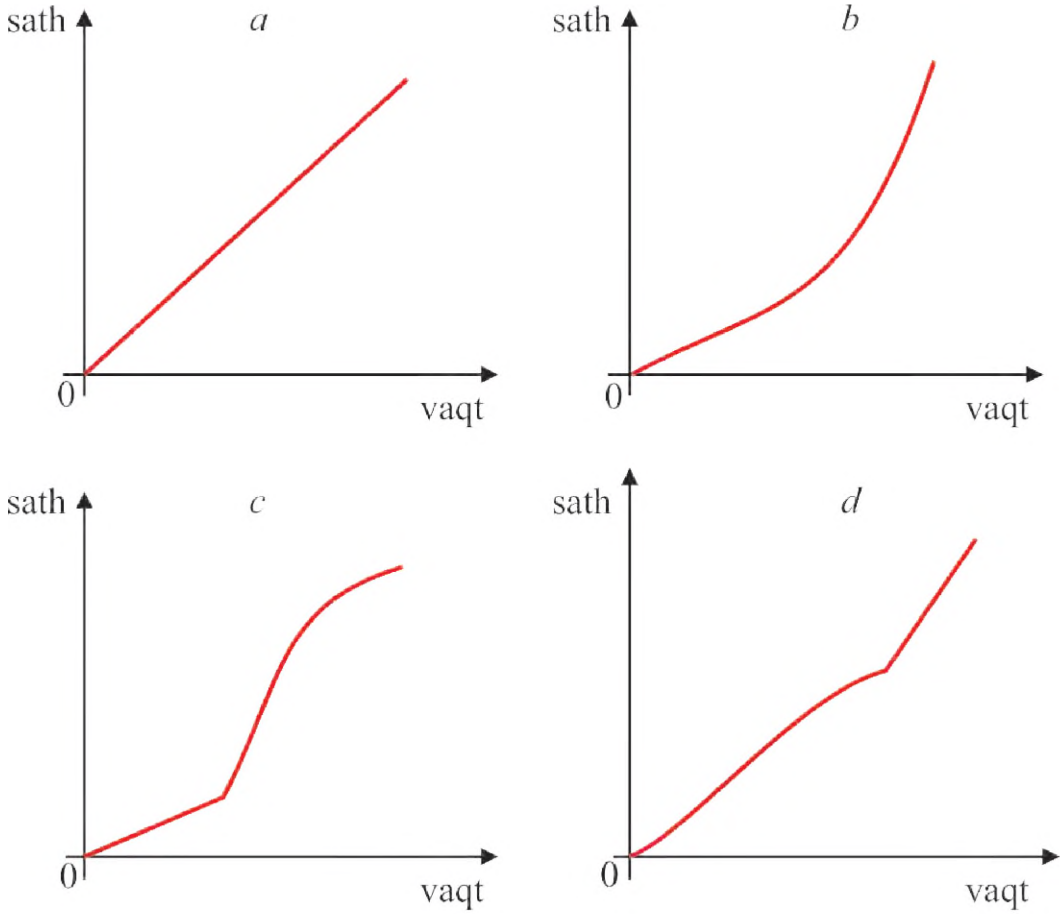
3-rasm.

Boshida unda suv yo‘q edi. Keyin u “bir sekundda bir litr” tezlikda to‘ldirila boshlandi. Suv sathining vaqtga nisbatan o‘zgarishi 4-rasmdagi qaysi grafikda to‘g‘ri ko‘rsatilgan?



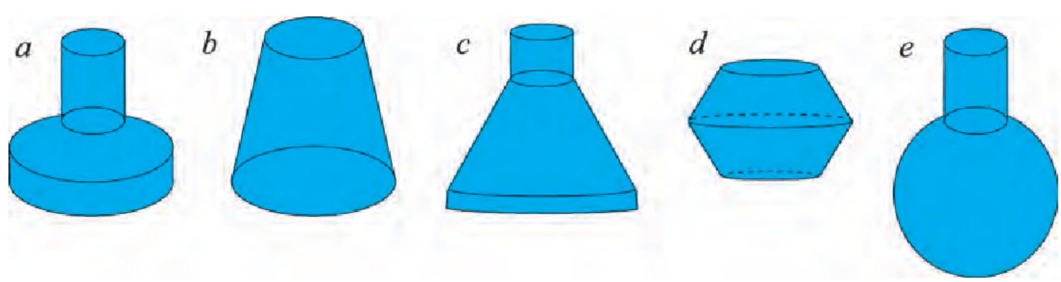
4-rasm.

2-savol. Suv sathining vaqtga nisbatan o‘zgarishi 5-rasmdagi grafiklarda berilgan:



5-rasm.

Ular 6-rasmdagi qaysi idishlarga mos keladi?



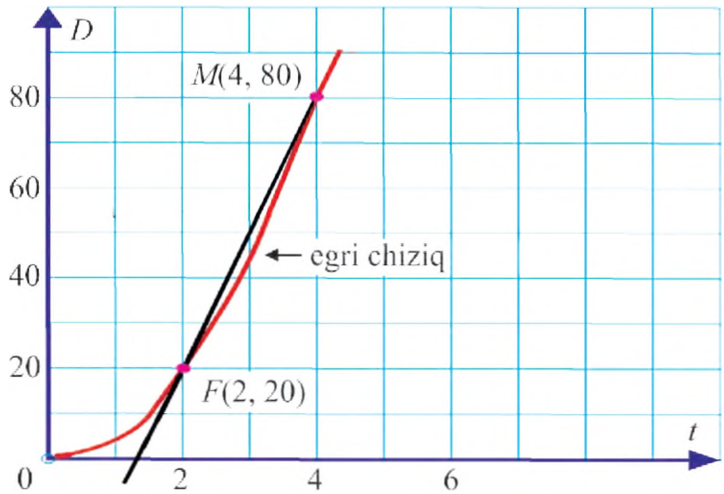
6-rasm.

O‘zgarishning o‘rtacha tezligi

Ikkita o‘zgaruvchi miqdorning bir-biriga bog‘lanishi chiziqli funksiya ko‘rinishida bo‘lsa, bu miqdorlar orttirmalarining nisbati o‘zgarmas son bo‘ladi.

Ikkita o‘zgaruvchi miqdorning bir-biriga bog‘lanishi chiziqli funksiya ko‘rinishida bo‘lmasa, biz bu o‘zgaruvchi miqdorlarning berilgan oraliqdagi o‘rtacha nisbatini topa olamiz. Agar oraliqlar turlicha olinsa, hisoblangan o‘rtacha nisbatlar ham turlicha bo‘ladi.

1-misol. Moddiy nuqtaning vaqtga nisbatan to‘g‘ri chiziq bo‘ylab harakat qonuni grafikda tasvirlangan (7- rasm). *FM* kesuvchining burchak koeffitsiyentini toping.



7-rasm.

△ Grafikda t=2 sekundga mos bo‘lgan F nuqtani va undan farqli (masalan, t=4 sekundga mos bo‘lgan) M nuqtani belgilaylik. $2 \le t \le 4$ vaqt oralig‘ida o‘rtacha tezlik $\frac{(80-20)\,m}{(4-2)\,s} = 30\frac{\text{m}}{\text{s}}$ ga teng ekanligini topamiz.

Ko‘rinib turibdiki, *FM* kesuvchining burchak koeffitsiyenti 30 ga teng ekan. ▲

Savol. F nuqtani qo‘zg‘almas hisoblab, t ning quyida berilgan qiymatlariga mos bo‘lgan M nuqtalar uchun *FM* kesuvchilarning burchak koeffitsiyentlarini hisoblab, jadvallarni to‘ldiring:

t	burchak koeffitsiyenti
0	
1,5	
1,9	
1,99	

t	burchak koeffitsiyenti
3	
2,5	
2,1	
2,01	

Qanday xulosaga keldingiz?

2-misol. Populatsiyadagi sichqonlar soni haftalar kechishi bilan quyidagicha o'zgaradi (8-rasm):

8-rasm.

3- va 6- hafta oralig'ida sichqonlar soni o'rtacha qanday o'zgargan? 7 haftalik vaqt oralig'da-chi?

△ Sichqonlar populatsiyasining o'sish tezligi

$\frac{(240-110)\text{ta sichqon}}{(6-3)\text{ta hafta}} \approx 43\frac{\text{sichqon}}{\text{hafta}}$, ya'ni 3- va 6- hafta oralig'ida sichqonlar soni haftasiga o'rtacha 43 taga ko'paygan.

Xuddi shunday 7 haftada $\frac{(315-50)\text{ta sichqon}}{(7-0)\text{ta hafta}} \approx 38\frac{\text{sichqon}}{\text{hafta}}$.

7 hafta oralig'ida sichqonlar soni haftasiga o'rtacha 38 taga ko'paygan. ▲

Umumiy holda: x miqdor a dan b gacha o'zgarganda $y=f(x)$ miqdor o'zgarishining o'rtacha tezligi

$$\frac{f(b)-f(a)}{b-a}$$

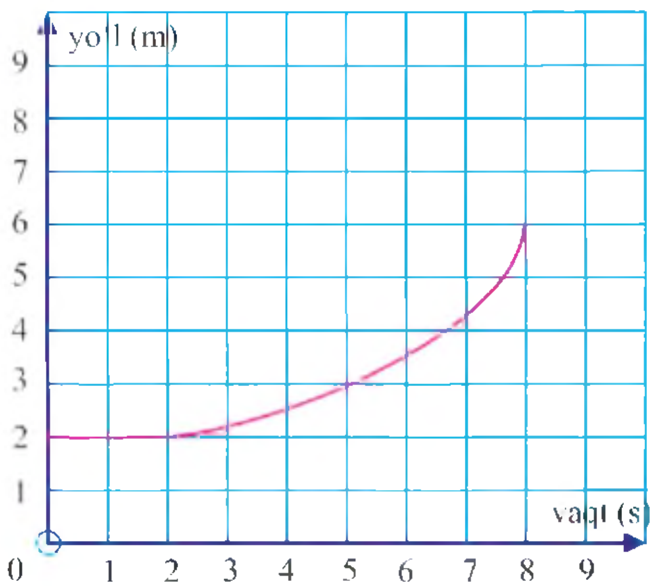
orttirmalar nisbatiga teng, bu yerda $f(b)-f(a)$ – funksiya orttirmasi, $b-a$ esa argument orttirmasi.

$h=b-a$ deb belgilasak, o'rtacha tezlik $\frac{f(a+h)-f(a)}{h}$ ko'rinishni oladi.

$\frac{f(a+h)-f(a)}{h}$ kasr suratini $y=f(x)$ *funksiyaning* argumenti x ning h *orttirmasiga* mos keluvchi *orttirmasi* deb atash qabul qilingan. Kasrning o'zi esa *ayirmali nisbat* deb atashadi.

Mashqlar

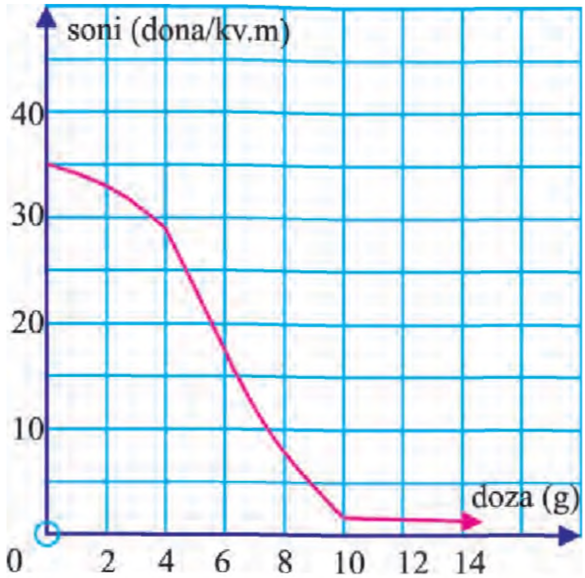
6. Nuqtaning to'g'ri chiziq bo'ylab yurgan yo'li vaqtga qanday bog'langanligi 9-rasmdagi grafikda tasvirlangan.



9-rasm.

Nuqtaning
a) dastlabki 4 sekund;
b) so'nggi 4 sekund;
c) 8 sekund mobaynidagi o'rtacha tezligini toping.

7. 1) Dalaga turli miqdordagi (dozadagi) dori bilan ishlov berilganda 1 m^2 da mavjud bo'lgan zararli hasharotlar sonining o'zgarishi 10-rasmdagi grafikda ko'rsatilgan.



10-rasm.

a) 1) doza 0 grammdan 10 grammgacha oshirilsa; 2) 4 grammdan 7 grammgacha oshirilsa, 1 m^2 da mavjud bo'lgan zararli hasharotlar sonining o'zgarishini toping.

b) doza 10 grammdan 14 grammgacha oshirilsa, qanday hodisa ro'y beradi?

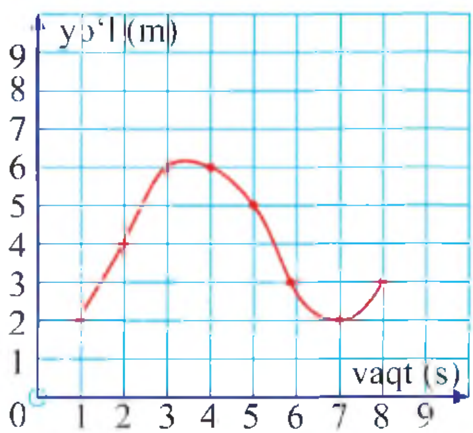
2) Moddiy nuqtaning to'g'ri chiziq bo'yicha harakat qonuni $s(t)$ ning grafigi rasmda berilgan.

a) s(2), s(3), s(5), s(7) sonlar nechaga teng?

b) Qaysi oraliqlarda funksiya o'suvchi?

c) Qaysi oraliqda funksiya kamayuvchi?

d) s(3)–s(1), s(5)–s(4), s(7)–s(6), s(8)–s(6) orttirmalarni hisoblang.



3–4 LIMIT HAQIDA TUSHUNCHA

x ning qiymatlari 2 dan kichik bo'lib, 2 ga yaqinlasha borganda $f(x)=x^2$ funksiyaning qiymatlari jadvalini qaraylik:

x	1	1,9	1,99	1,999	1,9999
$f(x)$	1	3,61	3,9601	≈ 3,996 00	≈ 3,999 60

Jadvaldan ko'rinib turibdiki, x ning qiymatlari 2 ga yaqin bo'laversa (*yaqinlashsa*), $f(x)$ funksiyaning mos qiymatlari ham 4 soniga yaqinlashaveradi.

Bunday holatda x argument (o'zgaruvchi) 2 ga *chapdan yaqinlashganda* $f(x)$ ning qiymatlari 4 soniga *yaqinlashadi* deymiz.

Endi x ning qiymatlari 2 dan katta bo'lib, 2 ga yaqinlasha borganida $f(x)=x^2$ funksiyaning qiymatlari jadvalini qaraylik:

x	3	2,1	2,01	2,001	2,0001
$f(x)$	9	4,41	4,0401	≈ 4,004 00	≈ 4,000 40

Bunday holatda x argument 2 ga *o'ngdan yaqinlashganda,* $f(x)$ funksiya qiymatlari 4 soniga *yaqinlashadi* deymiz.

Yuqoridagi ikki holatni umumlashtirib, x argument 2 ga *yaqinlashganda,* $f(x)$ ning qiymatlari 4 soniga *yaqinlashadi* deymiz va buni quyidagicha yozamiz:

$$\lim_{x\to 2} x^2 = 4.$$

Bu yozuv shunday o'qiladi: x argument 2 ga yaqinlashganda, $f(x) = x^2$ funksiyaning *limiti* 4 ga teng.

Umumiy holda *funksiya limiti* tushunchasiga quyidagicha yondashiladi:

$x \neq a$ bo'lib, uning qiymatlari a soniga yaqinlashsa, $f(x)$ ning mos qiymatlari A soniga *yaqinlashsin.* Bu holda A sonni x a ga *yaqinlashganda* $f(x)$ funksiyaning *limiti* deyiladi va bunday belgilanadi:

$$\lim_{x\to a} f(x) = A.$$

Ayrim hollarda mazkur holatni x ning qiymatlari a ga *intilganda* $f(x)$ funksiya A ga *intiladi,* deymiz.

$\lim\limits_{x\to a} f(x) = A$ yozuv o‘rniga $x\to a$ da $f(x)\to A$ yozuv ham qo‘llaniladi.

Eslatma. x ning qiymati a ga *intilganda* $x \neq a$ sharti bajarilishining muhimligini aytib o‘tish joiz.

Misol. $x\to 0$ bo‘lganda $f(x)=\dfrac{5x+x^2}{x}$ funksiyaning limitini toping.

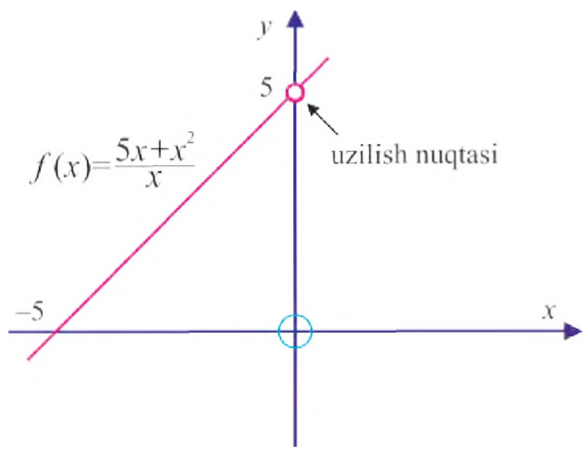
△ $x \neq 0$ sharti bajarilmasin, ya’ni $x=0$ bo‘lsin. $x=0$ qiymatni $f(x)$ ga bevosita qo‘yib ko‘rsak, $\dfrac{0}{0}$ ko‘rinishdagi *aniqmaslikka* ega bo‘lamiz.

Boshqa tomondan, $f(x)=\dfrac{x(5+x)}{x}$ bo‘lgani uchun bu funksiya ushbu

$$f(x)=\begin{cases} 5+x, & \text{agar } x\neq 0 \text{ bo‘lsa,} \\ \text{aniqlanmagan}, & \text{agar } x=0 \text{ bo‘lsa,} \end{cases}$$

ko‘rinishni oladi.

$y=f(x)$ funksiyaning grafigi (0; 5) koordinatali nuqtasi “olib tashlangan” $y=x+5$ to‘g‘ri chiziq ko‘rinishida bo‘ladi (11-rasm):



11-rasm.

(0; 5) koordinatali nuqta $y=f(x)$ funksiyaning *uzilish nuqtasi* deyiladi.

Ko‘rinib turibdiki, bu nuqtadan farqli bo‘lgan nuqtalarda x ning qiymatlari 0 ga *yaqinlashganda* $f(x)$ funksiyaning mos qiymatlari 5 ga yaqinlashadi, ya’ni uning *limiti* mavjud:

$$\lim_{x\to 0}\frac{5x+x^2}{x}=5.$$ ▲

Amalda, funksiya limitini topish uchun, lozim bo'lsa, tegishli soddalashtirishlarni bajarish maqsadga muvofiq.

1-misol. Limitlarni hisoblang:

a) $\lim\limits_{x\to 2} x^2$; b) $\lim\limits_{x\to 0} \frac{x^2+3x}{x}$; c) $\lim\limits_{x\to 3} \frac{x^2-9}{x-3}$.

△ a) x ning qiymatlari 2 ga *yaqinlashganda* x^2 ning qiymatlari 4 ga yaqinlashadi, ya'ni $\lim\limits_{x\to 2} x^2 = 4$.

b) $x \neq 0$ bo'lgani uchun

$$\lim_{x\to 0} \frac{x^2+3x}{x} = \lim_{x\to 0} \frac{x(x+3)}{x} = \lim_{x\to 0} (x+3) = 3.$$

c) $x \neq 3$ bo'lgani uchun

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x+3)(x-3)}{x-3} = \lim_{x\to 3} (x+3) = 6. \ ▲$$

Mashqlar

Limitni hisoblang (**8–11**):

8. a) $\lim\limits_{x\to 3}(x+4)$; b) $\lim\limits_{x\to -1}(5-2x)$; c) $\lim\limits_{x\to 4}(3x-1)$

d) $\lim\limits_{x\to 2}(5x^2-3x+2)$; e) $\lim\limits_{h\to 0} h^2(1-h)$; f) $\lim\limits_{x\to 0}(x^2+5)$.

9. a) $\lim\limits_{x\to 5} 5$; b) $\lim\limits_{h\to 2} 7$; c) $\lim\limits_{x\to 0} c$, c – o'zgarmas son.

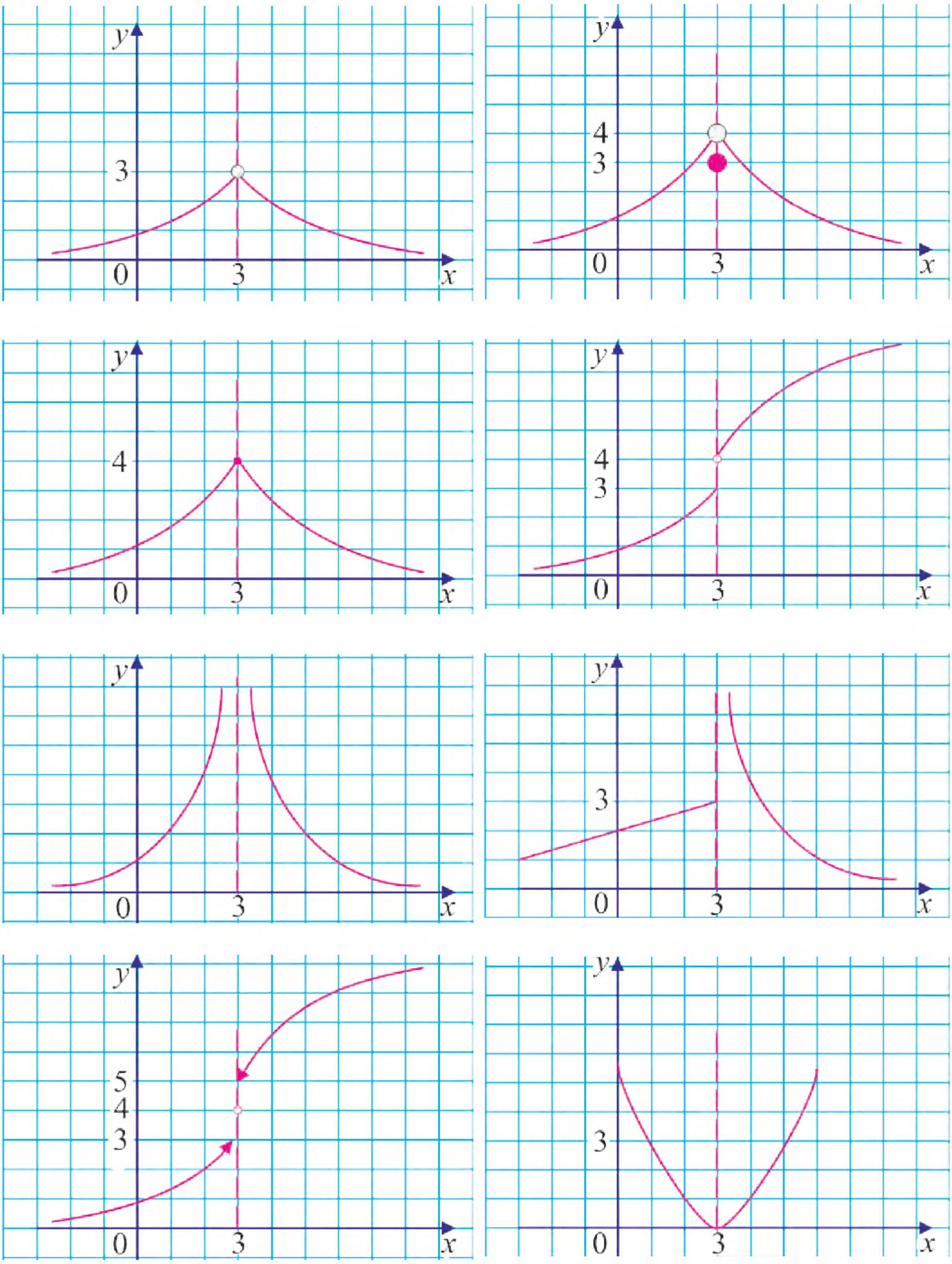
10. a) $\lim\limits_{x\to 1} \frac{x^2-3x}{x}$; b) $\lim\limits_{h\to 2} \frac{h^2+5h}{h}$; c) $\lim\limits_{x\to 0} \frac{x-1}{x+1}$; d) $\lim\limits_{x\to 0} \frac{x}{x}$.

11. a) $\lim\limits_{x\to 0} \frac{x^2-3x}{x}$; b) $\lim\limits_{x\to 0} \frac{x^2-5x}{x}$; c) $\lim\limits_{x\to 0} \frac{2x^2-x}{x}$.

d) $\lim\limits_{h\to 0} \frac{2h^2+6h}{h}$; e) $\lim\limits_{h\to 0} \frac{3h^2-4h}{h}$; f) $\lim\limits_{h\to 0} \frac{h^3-8h}{h}$;

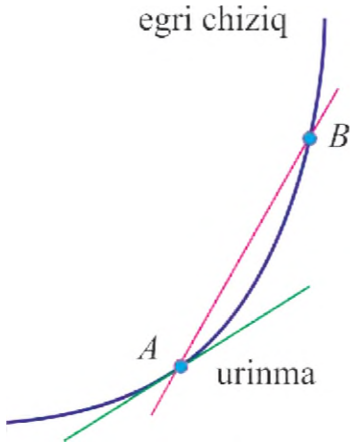
g) $\lim\limits_{x\to 1} \frac{x^2-x}{x-1}$; h) $\lim\limits_{x\to 2} \frac{x^2-2x}{x-2}$; i) $\lim\limits_{x\to 3} \frac{x^2-x-6}{x-3}$.

12. Quyidagi funksiyalardan qaysi biri $x \to 3$ da limitga ega? Shu limitni toping.

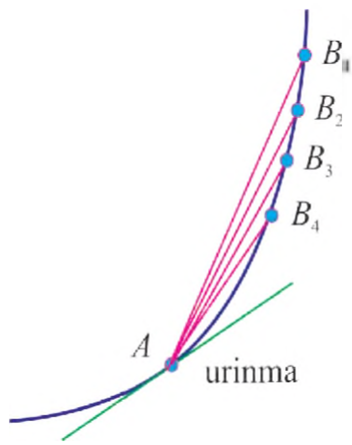


HOSILA, UNING GEOMETRIK VA FIZIK MA'NOSI

12-rasmda egri chiziq, kesuvchi va urinma tasvirlangan.



12-rasm.

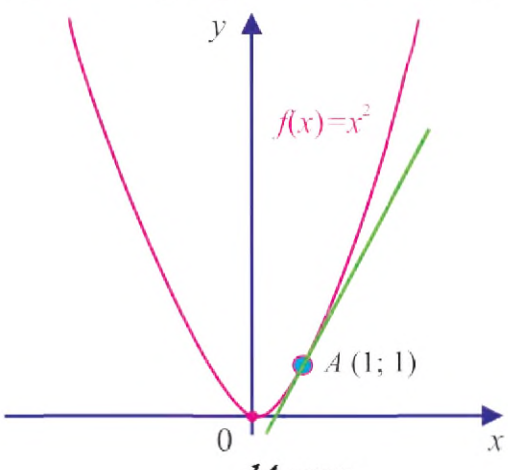


13-rasm.

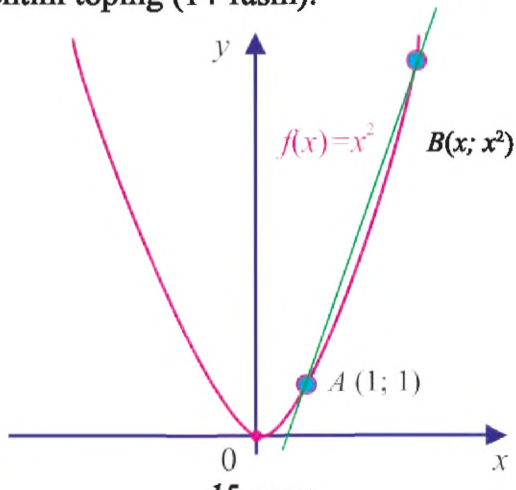
B nuqta B_1, B_2, ... holatlarni ketma-ket qabul qilib, A nuqtaga *egri chiziq bo'ylab* yaqinlashsa (13-rasm), mos kesuvchilarning egri chiziqqa A nuqtada o'tkazilgan urinma holatini olishga intilishini *intuitiv tarzda* qabul qilamiz.

Bu holda, ravshanki, AB to'g'ri chiziqning burchak koeffitsiyenti urinmaning burchak koeffitsiyentiga yaqinlashadi.

1-misol. $f(x) = x^2$ funksiyaning grafigiga $A(1;\ 1)$ nuqtada urinadigan to'g'ri chiziqning burchak koeffitsiyentini toping (14-rasm).



14-rasm.



15-rasm.

△ $f(x) = x^2$ funksiyaning grafigiga tegishli ixtiyoriy $B(x, x^2)$ nuqtani qaraylik (15-rasm).

AB to'g'ri chiziqning burchak koeffitsiyenti

$\frac{f(x)-f(1)}{x-1}$ yoki $\frac{x^2-1}{x-1}$ ga teng.

B nuqta A nuqtaga egri chiziq bo'ylab yaqinlashganda, x ning qiymati 1 ga yaqinlashadi, bunda $x \neq 1$.

Demak, AB to'g'ri chiziqning burchak koeffitsiyenti urinmaning burchak koeffitsiyenti k ga yaqinlashadi, ya'ni:

$$k = \lim_{x\to 1}\frac{x^2-1}{x-1} = \lim_{x\to 1}\frac{(x+1)(x-1)}{x-1} = \lim_{x\to 1}(x+1) = 2\cdot$$

Shunday qilib, $k = 2$. ▲

$y = f(x)$ funksiya berilgan bo'lsin. Uning grafigiga tegishli bo'lgan $A(x; f(x))$ va $B(x+h; f(x+h))$ nuqtalarni qaraylik (16-rasm).

AB to'g'ri chiziqning burchak koeffitsiyenti

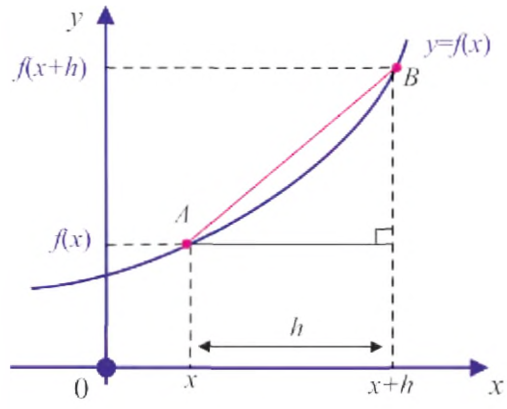
$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

ayirmali nisbatga teng.

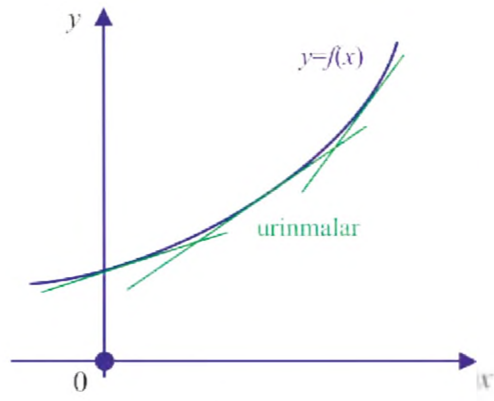
B nuqta A nuqtaga egri chiziq bo'ylab yaqinlashganda $h\to 0$, ya'ni h orttirma nolga intiladi, AB kesuvchi esa funksiya grafigiga A nuqtada o'tkazilgan urinmaga intiladi.

Shu bilan birga, AB to'g'ri chiziqning burchak koeffitsiyenti urinmaning burchak koeffitsiyentiga yaqinlashadi.

Boshqacha aytganda, h ning qiymati 0 ga intilganda ixtiyoriy $(x; f(x))$ nuqtada o'tkazilgan urinmaning burchak koeffitsiyenti $\frac{f(x+h)-f(x)}{h}$ ayirmali nisbatning limit qiymatiga, ya'ni $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ qiymatga teng bo'ladi.



16-rasm.



17-rasm.

x ning mazkur limit mavjud bo‘lgan ixtiyoriy qiymatiga funksiya grafigiga $(x, f(x))$ nuqtada o‘tkazilgan urinmaning burchak koeffitsiyentining yagona qiymatini mos qo‘yish mumkin (17-rasm).

Demak, $\lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ formula yangi funksiyani ifodalaydi.

Mana shu funksiya $y=f(x)$ funksiyaning **hosilaviy funksiyasi**, yoki sodda qilib **hosilasi** deb ataladi.

Ta’rif. $y=f(x)$ funksiyaning **hosilasi deb quyidagi limitga** (agar **u mavjud bo‘lsa) aytiladi:**

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}. \qquad (1)$$

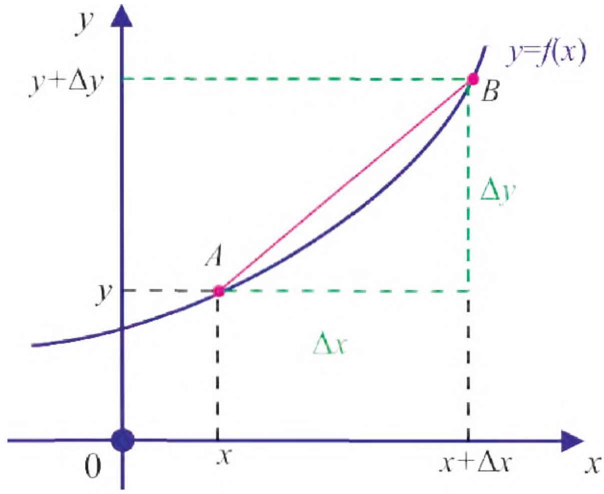
Odatda $y=f(x)$ **funksiyaning hosilasi** $f'(x)$ **kabi belgilanadi.**
Hosilani topish amali ***differensiallash*** **deyiladi.**

$f'(x)$ **belgilash o‘rniga** $\frac{dy}{dx}$ **kabi belgilash** ham **qabul qilingan.**

Bu **belgilashning “kasr” ko‘rinishda** ekanligini **quyidagicha tushuntirish mumkin.**

Agar **orttirmalarni** $h = \Delta x$, $f(x+\Delta x) - f(x) = \Delta y$ **deb belgilasak,**

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ dan quyidagiga ega bo‘lamiz (18-rasm): $f'(x) = \lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$.



18-rasm.

Yuqoridagi mulohazalardan shunday xulosaga kelamiz: $y = f(x)$ funksiya hosilasining x_0 nuqtadagi qiymati funksiya grafigiga shu nuqtada o'tkazilgan urinmaning burchak koeffitsiyentiga teng. Hosilaning ***geometrik ma'nosi*** shundan iboratdir.

2-misol. Moddiy nuqta $s=s(t)$ (s – metrlarda, t – sekundlarda o'lchanadi) qonunga muvofiq to'g'ri chiziq bo'ylab harakat qilmoqda. Shu moddiy nuqtaning vaqtning t momentidagi (paytidagi) tezligi $v(t)$ ni toping.

△ Ma'lumki, oniy tezlik nuqtaning kichik vaqt oralig'i Δt dagi o'rtacha tezligi $v(t) = \dfrac{s(t+\Delta t)-s(t)}{\Delta t}$ ga taqriban teng. Δt nolga intilganda oniy tezlik va o'rtacha tezlik orasidagi farq ham nolga intiladi. Demak, moddiy nuqtaning t momentdagi oniy tezligi

$$v(t) = \lim_{\Delta t \to 0} \frac{s(t+\Delta t)-s(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = s'(t). \ ▲$$

Shunday qilib, t momentdagi oniy tezlik nuqtaning harakat qonuni $s(t)$ funksiyadan olingan hosilaga teng ekan.

Hosilaning *fizik ma'nosi* ana shundan iborat. Umuman aytganda, *hosila funksiyaning o'zgarish tezligidir.*

Misollar

Hosila ta'rifidan foydalanib, funksiyalarning hosilasini toping:

1. $f(x)=x^2$; **2.** $f(x)=5$; **3.** $f(x)=x^3-7x+5$;

4. $f(x)=x^4$; **5.** $f(x)=\frac{1}{x}$; **6.** $f(x)=\sqrt{x}$; 7. $f(x)=\sqrt[3]{x}$.

△ **1.** $h\neq 0$ bo'lgani uchun

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2+x^2}{h}=$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}=\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}}=\lim_{h\to 0}(2x+h)=2x.$$

2. $h\neq 0$ bo'lgani uchun $f(x+h)=5$, $f(x+h)-f(x)=5-5=0$,

$$\frac{f(x+h)-f(x)}{h}=\frac{0}{h}=0 \quad \text{Demak, } f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=0.$$

3. $h\neq 0$ bo'lgani uchun

$f(x+h)=(x+h)^3-7(x+h)+5=x^3+3x^2h+3xh^2+h^3-7x-7h+5;$

$f(x+h)-f(x)=x^3+3x^2h+3xh^2+h^3-7x-7h+5-x^3+7x-5=$

$=3x^2h+3xh^2+h^3-7h.$

$$\frac{f(x+h)-f(x)}{h}=\frac{3x^2h+3xh^2+h^3-7h}{h}=3x^2+3xh+h^2-7.$$

$h\to 0$ da $3xh+h^2\to 0$ bo'lgani uchun

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=3x^2-7.$$

4. Qisqa ko'paytirish formulalariga ko'ra $a^4-b^4=(a-b)(a+b)(a^2+b^2)$.

Demak, $(x+h)^4-x^4=(x+h-x)(x+h+x)((x+h)^2+x^2)=$

$=h(2x+h)(2x^2+2xh+h^2)=2hx(2x+h)(x+h)+h^3(2x+h)=$

$=2hx(2x^2+h(3x+h))+h^3(2x+h)$; $h\to 0$ bo'lsa,

$2h^2x(3x+h)\to 0$ va $h^3(2x+h)\to 0$ bo'lgani uchun

$$\lim_{h\to 0}\frac{(x+h)^4-x^4}{h}=\lim_{h\to 0}(4x^3+2hx(3x+h))+h^2(2x+h))=4x^3.$$

Demak, $f'(x)=(x^4)'=4x^3$.

5. $f(x)=\frac{1}{x}$, $x\neq 0$ bo'lsin,

$$f(x+h)-f(x)=\frac{1}{x+h}-\frac{1}{x}=\frac{x-(x+h)}{(x+h)x}=-\frac{h}{(x+h)x},$$

$$\frac{f(x+h)-f(x)}{h}=\frac{-1}{(x+h)x}.$$

$h\to 0$ da $x+h\to x$ bo‘lgani uchun $f'(x)=-\frac{1}{x^2}$ bo‘ladi.

6. $f(x)=\sqrt{x},\ x>0,\ x+h>0$ bo‘lsin, $\frac{f(x+h)-f(x)}{h}=\frac{\sqrt{x+h}-\sqrt{x}}{h}$ ayirmali nisbatni tuzamiz va uni soddalashtiramiz:

$$\frac{f(x+h)-f(x)}{h}=\frac{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=$$

$$=\frac{(x+h)-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\frac{1}{\sqrt{x+h}+\sqrt{x}}.$$

$h\to 0$ da $\sqrt{x+h}\to\sqrt{x}$ bo‘lgani uchun $f'(x)=\frac{1}{2\sqrt{x}}$ bo‘ladi.

7. Ayirmali nisbatni tuzamiz:

$$\frac{f(x+h)-f(x)}{h}=\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}=\frac{\left(\sqrt[3]{x+h}-\sqrt[3]{x}\right)\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}\right)}{h\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}\right)}=$$

$$=\frac{x+h-x}{h\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}\right)}=\frac{h}{h\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}\right)}=$$

$$=\frac{1}{\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}}.$$

$h\to 0$ da $\frac{1}{\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}}\to\frac{1}{3\sqrt[3]{x^2}}$. Demak, $\left(\sqrt[3]{x}\right)'=\frac{1}{3\sqrt[3]{x^2}}$.

Javob: **1.** $2x$. **2.** 0. **3.** $3x^2-7$. **4.** $4x^3$. **5.** $-\frac{1}{x^2}$. **6.** $\frac{1}{2\sqrt{x}}$. **7.** $\frac{1}{3\sqrt[3]{x^2}}$.▲

Eslatish joizki, x miqdor x dan $x+h$ gacha o'zgarganda $y=f(x)$ miqdor o'zgarishining **o'rtacha tezligi**

$$\frac{f(x+h)-f(x)}{h}$$

ayirmali nisbatga teng.

Bundan $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ifoda $y=f(x)$ miqdor o'zgarishining **oniy tezligini** bildiradi.

Mashqlar

13. Quyidagi funksiyaning hosilasi nimaga teng?

a) $f(x)=x^3$; b) $f(x)=x^{-1}$; c) $f(x)=x^{\frac{1}{2}}$; d) $f(x)=c$.

14. Jadvalni daftaringizga ko'chiring va to'ldiring:

a)

$f(x)$	$f'(x)$
x^1	
x^2	
x^3	
x^{-1}	
$x^{\frac{1}{2}}$	

b) Fikringizcha, $y=x^n$ funksiya hosilasi nimaga teng (bu yerda n – ratsional son) ?

15. Ta'rifdan foydalanib, funksiya hosilasini toping:

a) $f(x)=2x+3$; b) $f(x)=3x^2+5x+1$; c) $f(x)=2x^3+4x^2+6x-1$.

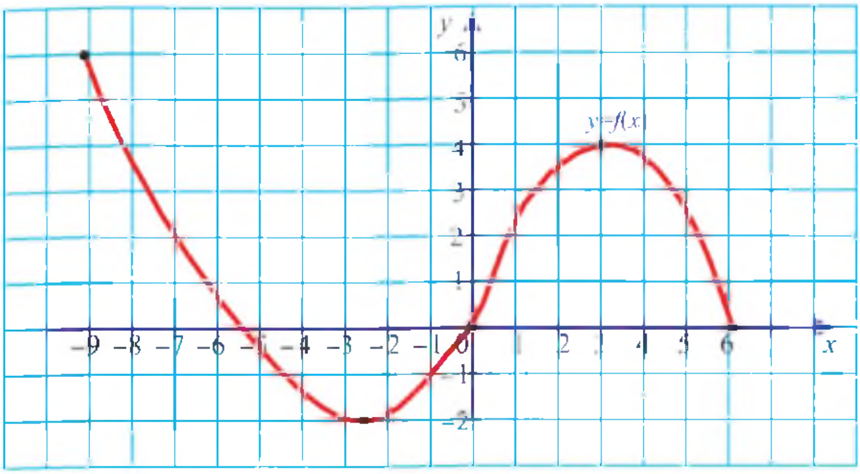
16*. Daftaringizga ko'chiring va to'ldiring:

a) $f(x)=ax+b$ uchun $f'(x)=...$;
b) $f(x)=ax^2+bx+c$ uchun $f'(x)=...$;
c) $f(x)=ax^3+bx^2+cx+d$ uchun $f'(x)=...$

17*. Quyidagi tasdiqlarni isbotlang:

a) $f(x)=cg(x)$ bo'lsa, u holda $f'(x)=cg'(x)$;
b) $f(x)=g(x)+h(x)$ bo'lsa, u holda $f'(x)=g'(x)+h'(x)$.

18*. Funksiya grafigiga qarab hosilalar qiymatlarini solishtiring:



a) $f'(-7)$ va $f'(-2)$;
c) $f'(-9)$ va $f'(0)$;
b) $f'(-4)$ va $f'(2)$;
d) $f'(-1)$ va $f'(5)$.

19. 1) Yuqoridagi funksiya grafigiga qarab ushbu shartlarni qanoatlantiradigan x_1, x_2 nuqtalarni toping (x_1, x_2 – Ox o'qidagi nuqtalar: –9, –8, ..., 5, 6):

a) $f'(x_1) > 0$, $f'(x_2) > 0$;
b) $f'(x_1) < 0$, $f'(x_2) > 0$;
c) $f'(x_1) < 0$, $f'(x_2) < 0$;
d) $f'(x_1) > 0$, $f'(x_2) < 0$.

2) Grafikka qarab ushbu savollarga javob bering:

a) funksiya qaysi oraliqda o'suvchi? qaysi oraliqda kamayuvchi?

b) funksiyaning [0; 3], [3; 6], [–9; –6] oraliqlaridagi orttirmalarini hisoblang.

3) Funksiya qaysi nuqtada eng katta, qaysi nuqtada eng kichik qiymatni qabul qiladi?

4) Funksiya qaysi nuqtalarda nolga aylanyapti?

5) Qaysi oraliqda funksiya musbat qiymatlarni qabul qilyapti?

6) Qaysi oraliqda funksiya manfiy qiymatlarni qabul qilyapti?

7–9 HOSILANI HISOBLASH QOIDALARI

Agar $f(x)$ va $g(x)$ funksiyalarning har biri hosilaga ega bo‘lsa, u holda quyidagi differensiallash qoidalari o‘rinlidir:

1. Yig‘indining hosilasi hosilalar yig‘indisiga teng:

$$(f(x)+g(x))'=f'(x)+g'(x). \quad (1)$$

2. Ayirmaning hosilasi hosilalar ayirmasiga teng:

$$(f(x)-g(x))'=f'(x)-g'(x). \quad (2)$$

1-misol. Funksiyaning hosilasini toping:

1) $f(x)=x^3+x^2-x+10$; 2) $f(x)=\sqrt{x}-\dfrac{1}{\sqrt{x}}$.

△ Hosilani topishda 1, 2-qoidalaridan va hosilalar jadvalining 1, 3-bandlaridan foydalanamiz, ya’ni:

1) $f'(x)=(x^3)'+(x^2)'-(x)'+10=3x^2+2x-1$;

2) $f'(x)=\left(x^{\frac{1}{2}}\right)'-\left(x^{-\frac{1}{2}}\right)'=\dfrac{1}{2}x^{-\frac{1}{2}}+\dfrac{1}{2}x^{-\frac{3}{2}}$.

Javob: 1) $3x^2+2x-1$; 2) $\dfrac{1}{2}x^{-\frac{1}{2}}+\dfrac{1}{2}x^{-\frac{3}{2}}$. ▲

3. O‘zgarmas ko‘paytuvchini hosila belgisidan tashqariga chiqarish mumkin:

$$(cf(x))'=c\cdot f'(x), \quad c - \text{o‘zgarmas son}. \quad (3)$$

2-misol. Funksiyaning hosilasini toping:

1) $f(x)=7x^3-5x^2+4$; 2) $f(x)=3\sqrt{x}+\dfrac{5}{x}-x^3$.

△ Hosilani topishda 1, 2, 3-qoidalaridan va hosilalar jadvalining 1, 3-bandlaridan foydalanamiz, ya’ni:

1) $f'(x)=(7x^3-5x^2+4)'=(7x^3)'-(5x^2)'+(4)'=21x^2-10x$;

2) $f'(x)=\left(3\sqrt{x}+\dfrac{5}{x}-x^3\right)'=3\left(\sqrt{x}\right)'+5\cdot\left(\dfrac{1}{x}\right)'-(x^3)'=\dfrac{3}{2\sqrt{x}}-\dfrac{5}{x^2}-3x^2$.

Javob: 1) $21x^2-10x$;; 2) $\dfrac{3}{2\sqrt{x}}-\dfrac{5}{x^2}-3x^2$. ▲

4. Ko‘paytmaning hosilasi:

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x). \qquad (4)$$

3- misol. Funksiyaning hosilasini toping:

1) $f(x)=(2x+4)(3x+1)$; 2) $f(x)=(3x^2+4x+1)(2x+6)$; 3) $f(x)=\sqrt[3]{x}\cdot(x^2-5x)$.

△ Hosilani topishda 1, 3, 4-qoidalardan va hosilalar jadvalining 1-, 3- bandlaridan foydalanamiz, ya'ni:

1) $f'(x)=((2x+4)(3x+1))'=(2x+4)'(3x+1)+(2x+4)(3x+1)'=$
$=2(3x+1)+3(2x+4)=6x+2+6x+12=12x+14;$

2) $f'(x)=((3x^2+4x+1)(2x+6))'=(3x^2+4x+1)'(2x+6)+$
$+(3x^2+4x+1)(2x+6)'=(6x+4)(2x+6)+2(3x^2+4x+1)=18x^2+52x+26;$

3) $f'(x)=\left(\sqrt[3]{x}\cdot\left(x^2-5x\right)\right)'=\left(\sqrt[3]{x}\right)'\left(x^2-5x\right)+\sqrt[3]{x}\left(x^2-5x\right)'=$

$$=\frac{1}{3\sqrt[3]{x^2}}\left(x^2-5x\right)+\sqrt[3]{x}\,(2x-5)=\frac{x^2-5x}{3\sqrt[3]{x^2}}+(2x-5)\sqrt[3]{x}=\frac{x^2-5x+3(2x-5)\sqrt[3]{x^3}}{3\sqrt[3]{x^2}}=$$

$$=\frac{x^2-5x+6x^2-15x}{3\sqrt[3]{x^2}}=\frac{7x^2-20x}{3\sqrt[3]{x^2}}=\frac{x(7x-20)}{3\sqrt[3]{x^2}}=\frac{\sqrt[3]{x}}{3}(7x-20).$$

Javob: 1) $12x+14;$; 2) $18x^2+52x+26$; 3) $\frac{\sqrt[3]{x}}{3}(7x-20)$. ▲

5. Bo‘linmaning hosilasi:

$$\left(\frac{f(x)}{g(x)}\right)'=\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}, \quad \text{bunda} \quad g(x)\neq 0. \qquad (5)$$

4- misol. Funksiyaning hosilasini toping:

1) $f(x)=\frac{x+1}{x-2}$; 2) $f(x)=\frac{3x+7}{x-5}$; 3) $f(x)=\frac{\sqrt{x}}{5x-7}$.

△ Hosilani topishda 1, 3, 5-qoidalardan va hosilalar jadvalining 1, 3- bandlaridan foydalanamiz, ya'ni:

1) $f'(x)=\left(\frac{x+1}{x-2}\right)'=\frac{(x+1)'(x-2)-(x+1)(x-2)'}{(x-2)^2}=\frac{x-2-(x+1)}{(x-2)^2}=-\frac{3}{(x-2)^2}$;

2) $f'(x)=\left(\frac{3x+7}{x-5}\right)'=\frac{(3x+7)'(x-5)-(3x+7)(x-5)'}{(x-5)^2}=$

$$=\frac{3(x-5)-(3x+7)\cdot 1}{(x-5)^2}=\frac{3x-15-3x-7}{(x-5)^2}=-\frac{22}{(x-5)^2};$$

3) $f'(x)=\left(\frac{\sqrt{x}}{5x-7}\right)'=\frac{(\sqrt{x})'\cdot(5x-7)-\sqrt{x}\cdot(5x-7)'}{(5x-7)^2}=$

$$=\frac{\frac{1}{2\sqrt{x}}(5x-7)-\sqrt{x}\cdot 5}{(5x-7)^2}=\frac{5x-7-10x}{2\sqrt{x}(5x-7)^2}=-\frac{7+5x}{2\sqrt{x}(5x-7)^2}.$$

Javob: 1) $-\frac{3}{(x-2)^2}$; 2) $-\frac{22}{(x-5)^2}$; 3) $-\frac{7+5x}{2\sqrt{x}(5x-7)^2}$. ▲

5- misol. Funksiyalarning hosilasini toping:

1) $f(x) = \sin x$; 2) $f(x) = \cos x$; 3) $f(x) = \operatorname{tg} x$.

△ 1) Ayirmali nisbatni topishda sinuslar ayirmasini ko‘paytmaga keltirish formulasidan foydalanamiz:

$$\frac{\sin(x+h)-\sin x}{h}=\frac{2\sin\frac{h}{2}\cos\frac{2x+h}{2}}{h}=\frac{\sin\frac{h}{2}}{\frac{h}{2}}\cos\frac{2x+h}{2}.$$

$h\to 0$ da $\frac{\sin\frac{h}{2}}{\frac{h}{2}}\to 1,\ \ \cos\frac{2x+h}{2}\to\cos x$ ekanini isbotlash mumkin.

Demak, $(\sin x)'=\cos x$.

2) Ayirmali nisbatni topishda kosinuslar ayirmasini ko‘paytmaga keltirish formulasidam foydalanamiz:

$$\frac{\cos(x+h)-\cos x}{h}=-\frac{2\sin\frac{h}{2}\sin\frac{2x+h}{2}}{h}=-\frac{\sin\frac{h}{2}}{\frac{h}{2}}\sin\frac{2x+h}{2}=-\frac{\sin\frac{h}{2}}{\frac{h}{2}}\cdot\sin(x+\frac{h}{2}).$$

$h\to 0$ da; $\sin(x+\frac{h}{2})\to\sin x$ ekanini isbotlash mumkin.

Demak, $(\cos x)' = -\sin x$.

3) Hosilani topishning 5-qoidasi hamda shu misolning 1-, 2-qism javoblaridan foydalanib, $f(x)=\operatorname{tg}x=\frac{\sin x}{\cos x}$ funksiyaning hosilasini topamiz:

$$f'(x)=(\operatorname{tg}x)'=\left(\frac{\sin x}{\cos x}\right)'=\frac{(\sin x)'\cos x-\sin x(\cos x)'}{\cos^2 x}=$$

$$=\frac{\cos x\cos x-\sin x(-\sin x)}{\cos^2 x}=\frac{\cos^2 x+\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}.$$

Javob: 1) $(\sin x)'=\cos x$; 2) $(\cos x)'=-\sin x$; 3) $(\mathrm{tg}x)'=\frac{1}{\cos^2 x}$. ▲

Hosilani hisoblashda differensiallash qoidalari va quyidagi jadvaldan foydalanish maqsadga muvofiqdir.

Hosilalar jadvali

№	*Funksiyalar*	*Hosilalar*
1	c – o‘zgarmas	0
2	$kx+b$, k, b – o‘zgarmaslar	k
3	x^p, p – o‘zgarmas	px^{p-1}
4	$\sin x$	$\cos x$
5	$\cos x$	$-\sin x$
6	$\mathrm{tg}x$	$\frac{1}{\cos^2 x}$
7	$\mathrm{ctg}x$	$-\frac{1}{\sin^2 x}$
8	a^x, $a>0$	$a^x \ln a$
9	e^x	e^x
10	$\ln x$	$1/x$
11	$\lg x$	$\frac{1}{x\cdot\ln 10}$
12	$\log_a x$, $a>0$, $a=1$	$\frac{1}{x\cdot\ln a}$

? Savol va topshiriqlar

1. Hosilani hisoblash qoidalarini ayting. Har bir qoidaga misol keltiring.
2. Hosilalar jadvalining 4-, 5- bandlarini isbotlang.
3. Funksiyaning $x=x_0$ nuqtadagi hosilasi nima-yu, hosilaviy funksiya nima? Ularning qanday farqi bor? Misollarda tushuntiring.

Mashqlar

Hosilani toping (**20–22**):

20. 1) $y = x^4$; 2) $y = \frac{1}{x^2}$; 3) $y = \frac{1}{x^3}$.

21. 1) $y = x^4 - x^2 + x$; 2) $y = \frac{1}{x} + x$; 3) $y = x^3 + \sqrt[3]{x}$;

4) $y = x^4 + x^3 + x^2 - x - \frac{1}{x} - \frac{1}{x^2}$.

22. 1) $y = (x-1)(x^2-5)$; 2) $y = \frac{x^2-4}{x-2}$;

3) $y = (x^4 - \sqrt{x})(x^2 + x)$; 4) $y = \frac{\sqrt{x}+1}{x-1}$.

23. Moddiy nuqtaning berilgan t_0 vaqtdagi tezligini hisoblang:

1) $s(t) = t^3 - 2t^2 + t$; $t_0 = 5$; 2) $s(t) = 5t + t^3 + \sqrt{t}$, $t_0 = 4$.

24. Funksiyaning abssissasi berilgan nuqtadagi hosilasini hisoblang:

1) $f(x) = x^2 + 5x - 3$, $x_0 = 1$; 3) $f(x) = 2\sqrt{x} + x^3 + \frac{1}{2}$, $x_0 = 4$;

2) $f(x) = 4 - 3x$, $x_0 = -2$; 4) $f(x) = x^2 + \lg 2$, $x_0 = 1$.

Hosilani toping (**25–29**):

25. 1) $y = 2x^3 - 4x^2 + 5$; 3) $y = \frac{4}{x} + \frac{x}{4}$;

2) $y = 7x^2 - 2x + \sqrt{7}$; 4) $y = x^2 + \frac{1}{x^2}$.

26. 1) $y = (x-2)(x+2)$; 3) $y = \frac{x^2-9}{x-3}$;

2) $y = (x+2)^3$; 4) $y = x^2 + \lg 7 + \sin\frac{\pi}{9}$.

27. 1) $y = x^8 + 7x^2 + 5x$; 2) $y = 2x^8 + x^6$;

3) $y = \frac{x^4}{x^6-1}$; 4) $y = \frac{x^2+x+1}{x^3-1}$;

5) $y = x^{-2} + \frac{1}{x}$; 6) $y = x^4 - 4x$;

7) $y = \sqrt[5]{x^4} + \sqrt[3]{x^2}$; 8) $y = (x^5 + x^{-5})(x^2 + x^{-2})$.

28. 1) $f(x)=x^5+5x^4+4x^3+3x^2+2x+1$; | 2) $f(x)=\sin^2 x+\cos^2 x$;

3) $f(x)=\dfrac{x}{\cos x}$; 4) $f(x)=\text{tg}x$; 5) $y=8^x$;

6) $y=\log_2 x+\log_2 3$; 7) $y=2^x x$; 8) $y=x\ln x$;

9) $y=e^x\cos x$; 10) $y=2e^x-\ln x+\dfrac{1}{x}$.

29. 1) $y=2^x\sin x$; 2) $y=e^x(\cos x+\sin x)$; 3) $y=x\text{tg}x$;

4) $y=\dfrac{\ln x}{x}$; 5) $y=3\sin^2 x$; 6) $y=5x+\sqrt{x}+\sqrt[3]{x}$;

7) $y=(x+1)(\ln x+1)$; | 8) $y=(2+x)^3$; 9) $y=(3x+5)^6+2019$.

30. Moddiy nuqtaning berilgan t_0 vaqtdagi tezligini toping:

1) $s(t)=t^2+5t+1$, $t_0=1$; 2) $s(t)=4t^3+\dfrac{1}{t}+1$, $t_0=1$.

31. Funksiyaning berilgan nuqtadagi hosilasini toping:

1) $f(x)=(x+1)^3$, $x_0=-1$; 2) $f(x)=\sin x$, $x_0=\dfrac{\pi}{2}$.

32. Hosilani toping:

1) $y=2\sin x$;	2) $y=\sqrt{3}-\text{tg}x$;	3) $y=-3\cos x$;	4) $y=\text{tg}x-\text{ctg}x$;
5) $y=4x-\cos x$;	6) $y=x^2\sin x$;	7) $y=\dfrac{x}{\sin x}$;	8) $y=x\sin x+\cos x$.

33. Funksiyaning x_0 nuqtadagi hosilasini hisoblang:

1) $f(x)=\dfrac{2x+1}{3x-5}$, $x_0=2$; 2) $f(x)=\text{tg}x-x+2$, $x_0=\dfrac{\pi}{4}$;

3) $f(x)=x(\lg x-1)$, $x_0=10$; 4) $f(x)=\text{tg}x-\dfrac{1}{2}\ln x$, $x_0=\dfrac{\pi}{4}$.

34. Hosilani nolga aylantiradigan nuqtani toping:

1) $f(x)=x^4-4x$; 2) $f(x)=\text{tg}x-x$;

3) $f(x)=x^8-2x^4+3$; 4) $f(x)=\log_2 x-\dfrac{x}{\ln 2}$.

10–12 MURAKKAB FUNKSIYANING HOSILASI

Murakkab funksiya. $y = (x^2 + 3x)^4$ funksiyani qaraylik. Agar biz $g(x) = x^2 + 3x$, $f(x) = x^4$ belgilashlarni kiritsak, $y = (x^2 + 3x)^4$ funksiya $y = f(g(x))$ ko'rinishini oladi. Biz $y = f(g(x))$ funksiyani *murakkab funksiya* deymiz.

1-misol. Agar $f(x) = x^2$ va $g(x) = \dfrac{x-2}{x+3}$ bo'lsa, quyidagilarni toping:

1) $f(g(2))$; 2) $f(g(-4))$; 3) $g(f(1))$;

4) $f((-4))$; 5) $f(f(1))$ 6) $g(g(-1))$.

△ Berilgan funksiyalardan foydalanib, hisoblashlarni bajaramiz:

1) $f(g(x)) = f\left(\dfrac{x-2}{x-3}\right)$, bundan $f(g(2)) = f\left(\dfrac{2-2}{2+3}\right) = f(0) = 0^2 = 0$;

2) $f(g(-4)) = f\left(\dfrac{-4-2}{-4+3}\right) = f(6) = 6^2 = 36$;

3) $g(f(1)) = g(1^2) = g(1) = \dfrac{1-2}{1+3} = -\dfrac{1}{4}$;

4) $g(f(-4)) = g((-4)^2) = g(16) = \dfrac{16-2}{16+3} = \dfrac{14}{19}$;

5) $f(f(1)) = f(1^2) = f(1) = 1^2 = 1$;

6) $g(g(-1)) = g\left(\dfrac{-1-2}{-1+3}\right) = g\left(-\dfrac{3}{2}\right) = \dfrac{-\frac{3}{2}-2}{-\frac{3}{2}+3} = \dfrac{-3{,}5}{1{,}5} = -\dfrac{7}{3}$.

Javob: 1) 0; | 2) 36; | 3) $-\dfrac{1}{4}$; | 4) $\dfrac{14}{19}$; | 5) 1; | 6) $-\dfrac{7}{3}$. ▲

Murakkab funksiyaning hosilasi uchun ushbu formula o'rinli:

$$(f(g(x)))' = f'(g(x)) \cdot g'(x) \qquad (1)$$

2-misol. Funksiyaning hosilasini toping (k, b – o‘zgarmas sonlar):

1) $f(x) = (kx + b)^n$; 2) $f(x) = \sin(kx + b)$;
3) $f(x) = \cos(kx + b)$; 4) $f(x) = \mathrm{tg}(kx + b)$.

△ 1) $f(t) = t^n$ va $t(x) = kx + b$ funksiyalarga (1) formulani qo‘llaymiz:

$$((kx+b)^n)' = (t^n)' \cdot (kx+b)' = nt^{n-1} \cdot k = n \cdot k \cdot (kx + b)^{n-1}.$$

2) $f(t)=\sin t$ va $t(x) = kx+b$ funksiyalarga (1) formulani qo‘llaymiz:

$$(\sin(kx+b))' = (\sin t)' \cdot (kx+b)' = k \cdot \cos t = k \cdot \cos(kx + b).$$

3) $f(t) = \cos t$ va $t(x) = kx + b$ funksiyalarga (1) formulani qo‘llaymiz:

$$(\cos(kx + b))' = (\cos t)' \cdot (kx+b)' = -k \cdot \sin t = -k \cdot \sin(kx + b).$$

4) $f(t) = \mathrm{tg}\, t$ va $t(x) = kx+b$ funksiyalarga (1) formulani qo‘llaymiz:

$$(\mathrm{tg}(kx + b))' = (\mathrm{tg}\, t)' \cdot (kx + b)' = \frac{1}{\cos^2 t} \cdot k = \frac{k}{\cos^2(kx+b)}.$$

Javob: 1) $((kx + b)^n)' = n \cdot k \cdot (kx + b)^{n-1}$; 2) $(\sin(kx + b))' = k \cdot \cos(kx+b)$;
3) $(\cos(kx+b)' = -k \cdot \sin(kx+b)$; 4) $(\mathrm{tg}(kx+b)' = \frac{k}{\cos^2(kx+b)}$. ▲

3-misol. $f(x) = \sin 8x \cdot e^{(3x+2)}$ funksiya hosilasini toping.

△ Hosilani topishning 4-qoidasi hamda (1) formulani qo‘llab hosilani topamiz:

$$f'(x) = (\sin 8x e^{(3x+2)})' = (\sin 8x)' e^{3x+2} + \sin 8x \cdot (e^{3x+2})' = \cos 8x e^{3x+2} \cdot (8x)' +$$
$$+ \sin 8x e^{3x+2} \cdot (3x+2)' = e^{3x+2} \cdot (8\cos 8x + 3\sin 8x).$$

Javob: $e^{3x+2} \cdot (8\cos 8x + 3\sin 8x)$. ▲

4-misol. $h(x) = (x^3 + 1)^5$ funksiyaning $x_0 = 1$ nuqtadagi hosilasini toping.

△ (1) formuladan foydalanib hosilani hisoblaymiz:

$$h'(x) = 5(x^3+1)^4(x^3+1)' = 5(x^3+1)^4 3x^2 = 15x^2(x^3+1)^4.$$

Demak, $h'(1) = 15(1^3+1)^4 \cdot 1^2 = 15 \cdot 16 = 240$. *Javob*: 240. ▲

5-misol. $f(x) = 2^{\cos x}$ funksiyaning hosilasini toping.

△ (1) formuladan foydalanib hosilani hisoblaymiz:

$f'(x) = 2^{\cos x} \ln 2 (\cos x)' = -\sin x\, 2^{\cos x} \ln 2$. *Javob*: $-\sin x\, 2^{\cos x} \ln 2$. ▲

6-misol. $f(x) = \mathrm{tg}^5 x$ funksiyaning hosilasini toping.

△ (1) formuladan foydalanib hosilani hisoblaymiz:

$$f'(x) = 5\,\mathrm{tg}^4 x(\mathrm{tg}\,x)' = 5\mathrm{tg}^4 x\frac{1}{\cos^2 x}.$$

Javob: $\dfrac{5\mathrm{tg}^4 x}{\cos^2 x}$. ▲

7-misol. $h(x)=3^{\cos x}\cdot\log_7(x^3+2x)$ funksiyaning hosilasini toping.

△ $f(x)=3^{\cos x}$ va $g(x)=\log_7(x^3+2x)$ belgilashlarni kiritib, (1) formulani – murakkab funksiya hosilasini topish formulasini qo'llaymiz:

$$f'(x)=(3^{\cos x})'=3^{\cos x}\ln 3\cdot(\cos x)'=-3^{\cos x}\ln 3\cdot\sin x,$$

$$g'(x)=(\log_7(x^3+2x))'=\frac{1}{(x^3+2x)\ln 7}\cdot(x^3+2x)'=\frac{3x^2+2}{(x^3+2x)\ln 7}$$

hamda $h(x)$ funksiyani 2 ta funksiyaning ko'paytmasi deb qaraymiz:

$$h'(x) = (3^{\cos x}\cdot\log_7(x^3+2x))' = (3^{\cos x})'\cdot\log_7(x^3+2x) +$$

$$+ 3^{\cos x}\cdot(\log_7(x^3+2x))' = -3^{\cos x}\cdot\ln 3\cdot\sin x\cdot\log_7(x^3+2x)+\frac{3^{\cos x}(3x^2+2)}{(x^3+2x)\ln 7}.$$

Javob: $-3^{\cos x}\cdot\ln 3\cdot\sin x\cdot\log_7(x^3+2x)+\dfrac{3^{\cos x}(3x^2+2)}{(x^3+2x)\ln 7}$. ▲

? Savol va topshiriqlar

1. Murakkab funksiya deb nimaga aytiladi? Misol keltiring.
2. Murakkab funksiyaning aniqlanish sohasi qanday topiladi?
3. Murakkab funksiya hosilasini topish formulasini yoza olasizmi?
4. Murakkab funksiya hosilasini topishni 1–2 ta misolda ko'rsating.

Mashqlar

35. Agar $f(x)=x^2-1$ bo‘lsa, ko‘rsatilgan funksiyalarni toping:

1) $f(\frac{1}{x})$; 2) $f(2x)$; 3) $f(x^2-1)$; 4) $f(x+1)-f(x-1)$.

36. Agar $f(x)=\frac{x+1}{x-1}$ bo‘lsa, ko‘rsatilgan funksiyalarni toping:

1) $f(\frac{1}{x})$; 2) $f(\frac{1}{x^2})$; 3) $f(x-1)$; 4) $f(x+1)$.

37. Agar $f(x)=x^2$, $g(x)=x-1$ bo‘lsa, quyidagilarni toping:

1) $f(g(x))$; 2) $f(f(x))$; 3) $g(g(x))$; 4) $g(f(x))$.

38. Agar $f(x)=x^3$, $g(x)=x^2+1$ bo‘lsa, quyidagilarni toping:

1) $\frac{f(x^2)}{g(x)-1}$; 2) $f(x)+3g(x)+3x-2$;

3) $f(g(x))$; 4) $g(f(x))$.

Tenglikdan foydalanib, $f(x)$ ni toping (**39–42**):

39. $f(x+1)=x^2-1$. **40*.** $f(x)+3\cdot f(\frac{1}{x})=\frac{1}{x}$.

41. $f(x+3)=x^2-4$. **42*.** $2f(x)+f(\frac{1}{x})=x$.

Hosilani toping (**43–44**):

43. 1) $f(x)=(3x-2)^5$; 2) $f(x)=e^{\sin x}$; 3) $f(x)=(4-3x)^7$;

4) $f(x)=\sin^2 x$; 5) $f(x)=\frac{1}{(2x+9)^3}$; 6) $f(x)=\ln(4x-1)$;

7) $f(x)=\sqrt{4x-5}$; 8) $f(x)=(2x-1)^{10}$; 9) $f(x)=\cos^8 x$.

44*. 1) $e^{\sin x}\cdot \mathrm{tg}\frac{1}{x}$; 2) $3^{\mathrm{ctg}x}\cdot \log_a \cos x$; 3) $\ln\cos x$;

4) $(x^2-5x+4)^3\cdot 10^{\mathrm{tg}x}$; 5) $7^{\log 3x}\cdot(x^3-2x+1)^3$; 6) $3^{\cos x}\cdot(x^2-8x+4)^2$;

7) $\mathrm{ctg}x\cdot\ln(x^2+x)$; 8) $x^2\cos^{30}x+4$; 9) $5\ln x\cdot \mathrm{ctg}x$.

13–14 FUNKSIYA GRAFIGIGA O'TKAZILGAN URINMA VA NORMAL TENGLAMALARI

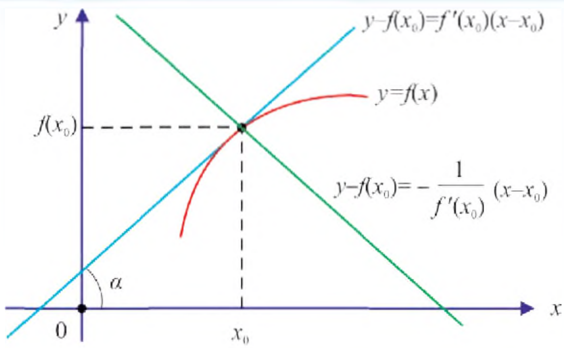
Urinma tenglamasi. $y=f(x)$ funksiyaga grafigining $(x_0;f(x_0))$ nuqtasidan o'tuvchi urinma tenglamasini topamiz (19-rasm). Urinma to'g'ri chiziq bo'lgani uchun uning umumiy ko'rinishi $y=kx+b$ bo'ladi. Hosilaning geometrik ma'nosiga ko'ra $k=\mathrm{tg}\alpha=f'(x_0)$, ya'ni urinma tenglamasi $y=f'(x_0)x+b$ ko'rinishini oladi. Bu urinma $(x_0;\ f(x_0))$ nuqtadan o'tgani uchun $f(x_0)=f'(x_0)x_0+b$ bo'ladi, bundan $b=f(x_0)-f'(x_0)x_0$. Topilgan b ni urinma tenglamasiga qo'yib,

$$y=f'(x_0)x+f(x_0)-f'(x_0)x_0 \quad \text{yoki}$$

$$y-f(x_0)=f'(x_0)(x-x_0) \qquad (1)$$

tenglamani hosil qilamiz.

$y-f(x_0)=f'(x_0)(x-x_0)$ tenglama $(x_0;f(x_0))$ nuqtada $y=f(x)$ funksiyaga o'tkazilgan urinma tenglamasi bo'ladi.



19-rasm.

1-misol. $f(x)=x^2-5x$ funksiya grafigiga $x_0=2$ abssissali nuqtada o'tkazilgan urinma tenglamasini yozing.

△ Avval funksiyaning va funksiyadan olingan hosilaning $x_0=2$ nuqtadagi qiymatini topamiz:

$$f(x_0)=f(2)=2^2-5\cdot2=-6,\quad f'(x)=2x-5,\quad f'(2)=2\cdot2-5=-1.$$

Topilganlarni (1) tenglamaga qo'yib, urinma tenglamasini hosil qilamiz:

$y-(-6)=-1\cdot(x-2)$ yoki $y=-x-4$. *Javob:* $y=-x-4$. ▲

2-misol. $f(x)=x^3-2x^2$ funksiya grafigiga $x_0=1$ abssissali nuqtada o'tkazilgan urinma tenglamasini yozing.

△ Avval funksiyaning va funksiyadan olingan hosilaning $x_0=1$ nuqtadagi qiymatini topamiz:

$$f(x_0)=f(1)=1^3-2\cdot1^2=-1, \qquad f'(x)=3x^2-4x, \qquad f'(1)=3\cdot1^2-4\cdot1=-1.$$

Topilganlarni (1) tenglamaga qo'yib, urinma tenglamasini hosil qilamiz:

$y-(-1)=-1(x-1)$ yoki $y=-x$. *Javob:* $y=-x$. ▲

Agar $y=f(x)$ funksiya grafigining x_0 abssissali nuqtasida o'tkazilgan urinma $y=kx+b$ to'g'ri chiziqqa parallel bo'lsa, $f'(x_0)=k$ bo'ladi. Bu shart orqali funksiyaning berilgan to'g'ri chiziqqa parallel bo'lgan urinmasi topiladi.

3-misol. $f(x)=x^2-3x+4$ funksiya uchun $y=2x-1$ to'g'ri chiziqqa parallel bo'lgan urinma tenglamasini yozing.

△ Urinmaning berilgan to'g'ri chiziqqa parallellik shartiga ko'ra, $f'(x_0)=2$ yoki $2x_0-3=2$ tenglamani hosil qilamiz. Bu tenglamada $x_0=2,5$ bo'lgani uchun urinma abssissasi $x_0=2,5$ bo'lgan nuqtadan o'tadi. Hisoblashlarni bajaramiz:

$$f(x_0)=f(2,5)=2,5^2-3\cdot2,5+4=6,25-7,5+4=2,75$$

$$f'(x_0)=f(2,5)=2.$$

Endi urinma tenglamasini topamiz:

$$y-2,75=2(x-2,5) \text{ yoki } y=2x-2,25.$$

Javob: $y=2x-2,25$. ▲

4-misol. $f(x)=x^3-2x^2+3x-2$ funksiya grafigiga $x_0=4$ abssissali nuqtada o'tkazilgan urinma tenglamasini tuzing va urinma bilan Ox o'qining musbat yo'nalishi tashkil qilgan burchakning sinusini toping.

△ Avval funksiyaning va funksiyadan olingan hosilaning $x_0=4$ nuqtadagi qiymatini topamiz:

$$f(x_0)=f(4)=3\cdot4^3-2\cdot4^2+3\cdot4-2=170, \quad f'(x)=3x^2-4x+3,$$

$$f'(4)=3\cdot4^2-4\cdot4+3=35.$$

Topilganlarni (1) tenglamaga qo'yib, urinma tenglamasini hosil qilamiz:

$y-170=35(x-4)$ yoki $y=35x+30$.

Hosilaning geometrik ma'nosiga ko'ra $\operatorname{tg}\alpha=35$, bundan

$$\sin\alpha = \frac{1}{\sqrt{1+\text{ctg}^2\alpha}} = \frac{1}{\sqrt{1+\frac{1}{\text{tg}^2\alpha}}} = \frac{\text{tg}\alpha}{\sqrt{1+\text{tg}^2\alpha}} = \frac{35}{\sqrt{1+35^2}} = \frac{35}{\sqrt{1226}}.$$

Javob: $y=35x+30$; $\sin\alpha = \frac{35}{\sqrt{1226}}$. ▲

5*-misol. $f(x)=x^2$ parabolaga abssissasi x_0 bo‘lgan A nuqtada o‘tkazilgan urinma Ox o‘qini $\frac{1}{2}x_0$ nuqtada kesib o‘tadi. Shu da’voni isbotlang.

△ $f'(x)=2x$, $f(x_0)=x_0^2$, $f'(x_0)=2x_0$.

Urinma tenglamasi (1) ga ko‘ra $y=2x_0\cdot x-x_0^2$ bo‘ladi. Uning Ox o‘qi bilan kesisish nuqtasi $\left(\frac{x_0}{2};0\right)$ ekani ravshan. Bundan $y=x^2$ parabolaga abssissasi x_0 bo‘lgan A nuqtada o‘tkazilgan urinmani yasash usuli kelib chiqadi: A nuqta va $\left(\frac{x_0}{2};0\right)$ nuqta orqali o‘tuvchi to‘g‘ri chiziq $y=x^2$ parabolaga A nuqtada urinadi.

Normal tenglamasi. $y=f(x)$ funksiya grafigiga $x=x_0$ abssissali nuqtada o‘tkazilgan urinmaga $x=x_0$ nuqtada perpendikular bo‘lgan

$$y-f(x_0)=-\frac{1}{f'(x_0)}(x-x_0) \quad (2)$$

to‘g‘ri chiziqqa $y=f(x)$ funksiya grafigining x_0 abssissali nuqtasida o‘tkazilgan normal deyiladi (19- rasm).

6-misol. $f(x)=x^5$ funksiya grafigiga $x_0=1$ abssissali nuqtada o‘tkazilgan normal tenglamasini tuzing.

△ Hosila formulasiga ko‘ra $f'(x)=5x^4$ bo‘ladi. Funksiya va uning hosilasining $x_0=1$ nuqtadagi qiymatlarini hisoblaymiz: $f(1)=1^5=1$ va $f'(1)=5\cdot1^4=5$. Bu qiymatlarni normalning tenglamasiga qo‘yamiz va $y-1=-\frac{1}{5}(x-1)$ yoki $y=-\frac{1}{5}x+\frac{6}{5}$ tenglamani hosil qilamiz.

Javob: $y=-\frac{1}{5}x+\frac{6}{5}$. ▲

Eslatma: $f(x)=x^5$ funksiya grafigiga $x_0=1$ abssissali nuqtada o'tkazilgan urinma tenglamasi $y=5x-4$ bo'ladi (isbotlang!). Urinma va normalning burchak koeffitsiyenti ko'paytmasi $5\cdot(-\frac{1}{5})=-1$ ekaniga e'tibor bering.

? Savol va topshiriqlar

1. $y=f(x)$ funksiya grafigiga x_0 abssissali nuqtada o'tkazilgan urinma tenglamasini yozing.
2. $y=f(x)$ funksiya grafigiga x_0 abssissali nuqtada o'tkazilgan normal tenglamasini yozing.
3. Berilgan funksiyaning biror to'g'ri chiziqqa parallel bo'lgan urinmasi qanday topiladi? Misolda tushuntiring.

Mashqlar

45. Funksiya grafigiga abssissasi $x_0=1$; $x_0=-2$; $x_0=0$ bo'lgan nuqtada o'tkazilgan urinma tenglamasini yozing:

1) $f(x)=2x^2-5x+1$;	2) $f(x)=3x-4$;	3) $f(x)=6$;
4) $f(x)=x^3-4x$;	5) $f(x)=e^x$;	6) $f(x)=2^x$;
7) $f(x)=2^x+\ln 2$;	8) $f(x)=\sin x$;	9) $f(x)=\cos x$;
10) $f(x)=\cos x-\sin x$;	11) $f(x)=e^x x$;	12) $f(x)=x\cdot\sin x$.

46. Funksiya uchun $y=7x-1$ to'g'ri chiziqqa parallel bo'lgan urinma tenglamasini yozing:

1) $f(x)=x^3-2x^2+6$; 2) $f(x)=4x^2-5x+3$; 3) $f(x)=8x-4$.

47. Berilgan $f(x)$ va $g(x)$ funksiyalarning urinmalari parallel bo'ladigan nuqtalarni toping:

1) $f(x)=3x^2-5x+4$, $g(x)=4x-5$;

2) $f(x)=8x+9$, $g(x)=-5x+8$;

3) $f(x)=7x+11$, $g(x)=7x-9$;

4) $f(x)=x^3-8$, $g(x)=x^2+5$;

5) $f(x)=x^3+x^2$, $g(x)=5x-7$;

6) $f(x)=x^4+11$, $g(x)=x^3+10$.

48. Funksiya grafigiga abssissasi a) $x_0 = 1$; b) $x_0 = -2$; d) $x_0 = 0$ bo‘lgan nuqtada o‘tkazilgan normal tenglamasini toping:

1) $f(x)=3x^2-5x+1$;	2) $f(x)=3x-40$;	3) $f(x)=7$;
4) $f(x)=x^3-10x$;	5) $f(x)=e^x$;	6) $f(x)=12^x$;
7) $f(x)=\sin x$;	8) $f(x)=\cos x$;	9) $f(x)=\cos x-\sin x$;
10) $f(x)=e^{\pi x}$;	11) $f(x)=x\cdot\cos x$;	12) $f(x)=x\cdot\sin x$.

Nazorat ishi namunasi

I variant

1. $f(x)=x^3+2x^2-5x+3$ funksiya uchun $x_0=2$ va $\Delta x=0,1$ bo‘lganda funksiya orttirmasining argument orttirmasiga nisbatini toping.

2. $f(x)=-8x^2+4x+1$ funksiyaning $x_0=-3$ nuqtadagi hosilasini hisoblang.

3. $f(x)=x^3-7x^2+8x-5$ funksiya grafigiga $x_0=-4$ abssissali nuqtada o‘tkazilgan urinma tenglamasini yozing.

4. Moddiy nuqta $s(t)=8t^2-5t+6$ qonuniyat bilan harakatlanmoqda. Agar t – sekund, s – metrlarda o‘lchanadigan bo‘lsa, nuqtaning $t_0=8$ sekunddagi oniy tezligini toping.

5. Ko‘paytmaning hosilasini toping: $(3x^2-5x+4)\cdot e^x$.

II variant

1. Bo‘linmaning hosilasini toping: $\dfrac{x^2-5x+6}{x+1}$.

2. Murakkab funksiyaning hosilasini toping: $\mathrm{ctg}^{15}x$.

3. $f(x)=\sqrt{x\sqrt{x}}$ funksiyaning $x_0=\dfrac{1}{16}$ nuqtadagi hosilasini hisoblang.

4. $f(x)=\ln(x+1)$ funksiya grafigiga $x=0$ nuqtada o‘tkasilgan urinma tenglamasini yozing.

5. $s(t)=0,5t^2-6t+1$ qonuniyati bilan harakatlanayotgan moddiy nuqtaning $t=16$ sekunddagi oniy tezligini toping. (t – sekundda, s – metrlarda o‘lchanadi).

MASALALAR YECHISH

49. $y=f(x)$ funksiya uchun x_0 va x nuqtalarga mos h va Δy ni hisoblang:

1) $f(x)=4x^2-3x+2$, $x_0=1$, $x=1,01$; 2) $f(x)=(x+1)^3$, $x_0=0$, $x=0,1$.

50. Agar $x_0=3$ va $\Delta x=0,03$ bo'lsa, berilgan funksiyalar uchun: a) funksiya orttirmasini; b) funksiya orttirmasining argument orttirmasiga nisbatini toping:

1) $f(x)=7x-5$; 2) $f(x)=2x^2-3x$; 3) $f(x)=x^3+2$; 4) $f(x)=x^3+4x$.

51. Agar $x_0=2$ va $\Delta x=0,01$ bo'lsa, berilgan funksiyalar uchun: a) funksiya orttirmasini; b) funksiya orttirmasining argument orttirmasiga nisbatini toping:

1) $f(x)=-4x+3$; 2) $f(x)=-8$; 3) $f(x)=x^2+10x$; 4) $f(x)=x^3-10$.

52. $x\to 0$ bo'lsa, funksiya qaysi songa intiladi:

1) $f(x)=x^3-2x^2+3x+4$; 2) $f(x)=x^5-6x^4+8x-7$;

3) $f(x)=(x^2-5x+1)(x^3-7x^2-11x+6)$;

4) $f(x)=\dfrac{x^2-x-19}{x^2+7x-28}$; 5) $f(x)=\dfrac{x^3-8x}{x^3+x^2+x+1}$?

53. Funksiyaning hosilasini toping:

1) $y=17x$; 2) $y=29x-3$; 3) $y=-15$; 4) $y=16x^2-3x$;

5) $y=-5x+40$; 6) $y=18x-x^2$; 7) $y=x^2+15x$;

8) $y=16x^3+5x^2-2x+14$; 9) $y=3x^3+2x^2+x$.

54. Funksiyaning hosilasini: a) $x=-3$; b) $x=1,1$; c) $x=0,4$; d) $x=-0,2$ nuqtalarda hisoblang:

1) $y=15x$; 2) $y=9x+3$; 3) $y=-20$; 4) $y=5x^2+x$;

5) $y=-8x+4$; 6) $y=8x-x^2$; 7) $y=x^2+25x$; 8) $y=x^3+5x^2-2x+4$.

55. $y=f(x)$ funksiya hosilasini ta'rifga ko'ra toping:

1) $f(x)=2x^2+3x+5$; 3*) $f(x)=\dfrac{x+1}{x}$;

2) $f(x)=(x+2)^3$; 4*) $f(x)=\dfrac{x^2+1}{x}$.

56. $y=f(x)$ funksiyaning x_0 nuqtadagi hosilasini toping:

1) $f(x)=4x^3+3x^2+2x+1$, $x_0=1$;
2) $f(x)=\frac{1}{3}x^3+\sin 22°$, $x_0=-1$;
3) $f(x)=(2x+1)(\sqrt{x}-1)$, $x_0=4$;
4) $f(x)=\frac{x^3-1}{x^2+1}$, $x_0=-3$.

57. Moddiy nuqta $s(t)=\frac{4}{3}t^3-t+5$ qonuniyat bilan harakatlanmoqda (s metrda, t – sekundda). Moddiy nuqtaning 2-sekunddagi tezligini toping.

58. Funksiyaning hosilasini toping:

1) $y=\frac{1}{\sqrt{x}}+2\sqrt{x}$;
2) $y=\sqrt[3]{x}+2x^3$;
3*) $y=\sqrt[5]{x}+x\cdot \mathrm{tg}x-\log_3 x$;
4) $y=(2x+3)^3$;
5*) $y=x\cdot \ln x\cdot(x+1)$;
6) $y=(x+\sqrt{x})(\sqrt{x}-2)$;
7) $y=\frac{x+2}{\sin x}$;
8) $y=10^x+\log_2 5+\cos 15°$;
9) $y=3^{-x}\cdot\sin x$;
10*) $y=\mathrm{tg}x\cdot\cos x+7^x\cdot x^7$;
11) $f(x)=\frac{1}{4}x^4-8x^2+3$;
12) $f(x)=\frac{\sqrt{2}}{2}x-\sin x+5$;
13) $f(x)=x^{10}-80x$;
14) $f(x)=8x-\frac{2^x}{\ln 2}$.

59. Funksiya hosilasining x_0 nuqtadagi qiymatini hisoblang:

1) $f(x)=\frac{1}{\cos x}$, $x_0=0$;
2) $f(x)=(x^2+3x)\ln x$, $x_0=1$;
3) $f(x)=\frac{\mathrm{arctg}x}{1+x^2}$, $x_0=1$;
4) $f(x)=e^x(x-\ln 2)$, $x_0=\ln 2$.

60*. $f'(x)>0$ tengsizlikni yeching:

1) $f(x)=x\cdot\ln 27-3^x$;
2) $f(x)=\sin x-2x$;

61. Moddiy nuqta $s(t)=\frac{1}{3}t^3-\frac{3}{2}t^2+2t$ qonuniyat bilan harakatlanmoqda.

Moddiy nuqtaning tezligi qachon nolga teng boʻladi? Buning ma’nosi nima?

62. Hosilani toping: 1) $y=x^5-x^4+x$; | 2) $y=\frac{1}{x^2}-x$; | 3) $y=x^4+\sqrt[5]{x}$.

63. Moddiy nuqtaning t_0 vaqtdagi tezligini toping:

1) $x(t)=t^4-2t^3+t$, $t_0=-5$; | 2) $x(t)=-5t+t^2-\sqrt{t}$, $t_0=4$.

Hosilani toping (**64–66**):

64. 1) $y=(x+2)(x^2-5x)$; | 2) $y=\frac{x^2-3x}{x+8}$; | 3) $y=(x^4+\sqrt{x})(x^3-5x)$;

4) $y=2x^3+4x^2+5x$; | 5) $y=\frac{14}{x}-\frac{x}{14}$; | 6) $y=7x^2+12x+\sqrt{2019}$.

65*. 1) $y=\frac{x^8}{x^{10}-1}$; 2) $y=\frac{x^3+x+1}{x^5+7}$; 3) $y=(x^{10}+x^{-10})(x^8+x^{-8})$.

66*. 1) $y=\frac{3^x\cdot\sin x}{\cos x}$; 2) $y=e^{5x}(\cos x-\sin x)$;

3) $y=x\text{ctg}x$; 4) $y=\frac{\ln x}{x^2}$.

67*. Hosilani x_0 nuqtada hisoblang:

1) $f(x)=\frac{5x+1}{13x-5}$, $x_0=-2$; 2) $f(x)=\text{ctg}x-2x+2$, $x_0=\frac{-\pi}{4}$;

3) $f(x)=x^2(\lg x-1)$, $x_0=1$; 4) $f(x)=\text{ctg}x-\frac{1}{20}\ln x$, $x_0=1$.

68*. Murakkab funksiyaning hosilasini toping:

1) $x^2\cdot\sin x$; 2) $\log_{15}\cos x$; 3) $\ln\text{ctg}x$;

4) $\text{tg}^{35}x$; 5) $e^{\text{ctg}x}$; 6) $23^{\cos x}$;

7) $35^{\sin x}$; 8) $(x^2-10x+7)\ln\cos x$;

9) $\frac{x^5-6x+4}{e^x}$; 10) $e^{-3x}(x^4-3x^2+2)$; 11) $\ln\text{tg}x$;

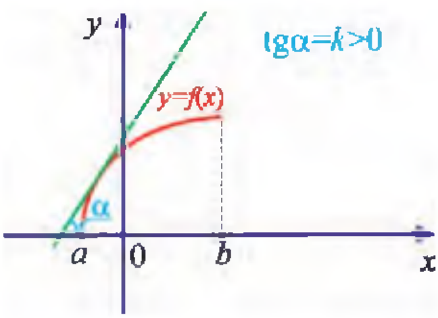
12) $\frac{x^3+7x+1}{e^{2x}}$; 13) $e^{5x}(x^5+8x+11)$; 14) $\ln\cos 2x$.

18–21 HOSILA YORDAMIDA FUNKSIYANI TEKSHIRISH VA GRAFIKLARNI YASASH

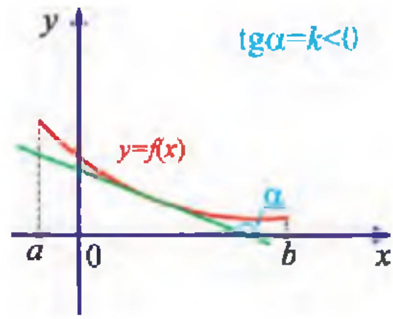
Funksiyaning o'sishi va kamayishi. O'suvchi va kamayuvchi funksiyalar bilan tanishsiz. Endi funksiyaning o'sish va kamayish oraliqlarini aniqlash uchun hosila tushunchasidan foydalanamiz.

1-teorema. $y = f(x)$ funksiya $(a; b)$ oraliqda aniqlangan va hosilasi mavjud bo'lsin. Agar $x \in (a; b)$ uchun $f'(x) > 0$ bo'lsa, $y = f(x)$ funksiya $(a; b)$ oraliqda o'suvchi funksiya bo'ladi (20-rasm).

2-teorema. $y = f(x)$ funksiya $(a; b)$ oraliqda aniqlangan va hosilasi mavjud bo'lsin. Agar $x \in (a; b)$ uchun $f'(x) < 0$ bo'lsa, $y = f(x)$ funksiya $(a; b)$ oraliqda kamayuvchi funksiya bo'ladi (21-rasm).



20-rasm.



21-rasm.

1, 2- teoremalarni isbotsiz qabul qilamiz.

1-misol. Funksiyaning o'sish va kamayish oraliqlarini toping: $f(x) = 2x^3 - 3x^2 - 12x + 3$.

△ Bu funksiya $(-\infty; +\infty)$ oraliqda aniqlangan. Uning hosilasi:

$$f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1).$$

$f'(x) > 0$, $f'(x) < 0$ tengsizliklarni oraliqlar usuli bilan yechib, $(-\infty; -1)$ va $(2; +\infty)$ oraliqlarda funksiyaning o'sishi hamda $(-1; 2)$ oraliqda funksiyaning kamayishini bilib olamiz.

Javob: $(-\infty; -1)$ va $(2; +\infty)$ oraliqlarida funksiya o'sadi; $(-1; 2)$ oraliqda esa funksiya kamayadi. ▲

2-misol. Funksiyaning o'sish va kamayish oraliqlarini toping: $f(x) = x + \frac{1}{x}$.

△ Bu funksiya $(-\infty;0)\cup(0;+\infty)$ oraliqda aniqlangan. Uning hosilasi: $f'(x)=1-\frac{1}{x^2}$; $f'(x)>0$, ya'ni $1-\frac{1}{x^2}>0$ tengsizlikni oraliqlar usuli bilan yechib, hosilaning $(-\infty;\ -1)$ va $(1;\ +\infty)$ oraliqlarda musbatligini topamiz. Xuddi shuningdek, $f'(x)<0$, ya'ni $1-\frac{1}{x^2}<0$ tengsizlikni oraliqlar usuli bilan yechib, bu tengsizlik $(-1;\ 0)$ va $(0;\ 1)$ oraliqlarda bajarilishini bilib olamiz.

Javob: funksiya $(-\infty;\ -1)$ va $(1;\ +\infty)$ oraliqlarda o'sadi; funksiya $(-1;\ 0)$ va $(0;\ 1)$ oraliqlarda esa kamayadi. ▲

Funksiyaning statsionar nuqtalari. $y=f(x)$ funksiya $(a;\ b)$ oraliqda aniqlangan bo'lsin.

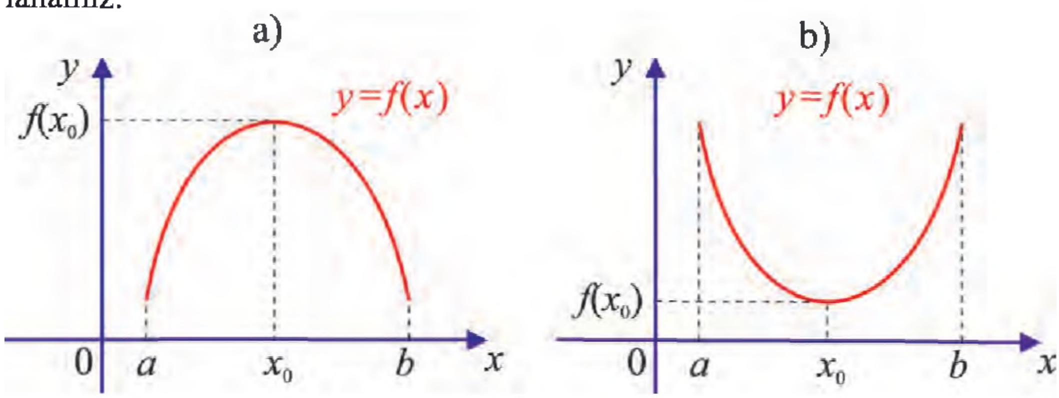
1-ta'rif. $y=f(x)$ funksiyaning hosilasi 0 ga teng bo'ladigan nuqtalar *statsionar nuqtalar* deyiladi.

3-misol. Funksiyaning statsionar nuqtalarini toping: $f(x)=2x^3-3x^2-12x+3$.

△ Funksiyaning hosilasini topib, uni nolga tenglaymiz: $f'(x)=6x^2-6x-12=0$. Bu tenglamani yechib funksiyaning statsionar nuqtalari $x_1=-1$, $x_2=2$ ekanini topamiz.

Javob: funksiyaning statsionar nuqtalari $x_1=-1$, $x_2=2$. ▲

Funksiyaning lokal maksimum va lokal minimumlari. Funksiyaning lokal maksimum va lokal minimumlarini aniqlash uchun hosiladan foydalanamiz.



22- rasm.

3-teorema. $f(x)$ funksiya $(a;\ b)$ oraliqda aniqlangan va $f'(x)$ mavjud; $(a;\ x_0)$ oraliqda $f'(x)>0$ va $(x_0;\ b)$oraliqda $f'(x)<0$ bo'lsin, $x_0\in(a;\ b)$. U holda x_0 nuqta $f(x)$ funksiyaning lokal maksimumi bo'ladi (22-*a* rasm).

4-teorema. $f(x)$ funksiya $(a;\ b)$oraliqda aniqlangan va $f'(x)$ mavjud; $(a;x_0)$ oraliqda $f'(x)<0$ va $(x_0;b)$ oraliqda $f'(x)>0$ bo'lsin, $x_0\in(a,\ b)$.

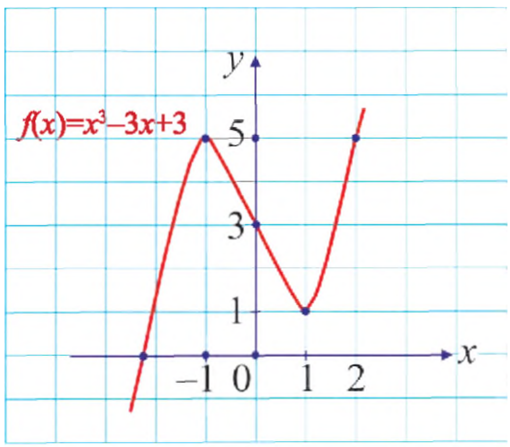
U holda x_0 nuqta $f(x)$funksiyaning lokal minimumi bo'ladi (22-*b* rasm).

3, 4-teoremalarni isbotsiz qabul qilamiz.

2-ta'rif. Funksiyaning lokal maksimum va lokal minimumlariga uning *ekstremumlari* deyiladi.

4-misol. Funksiyaning lokal maksimum va lokal minimum nuqtalarini toping: $f(x)=x^3-3x+3$.

△ Funksiyaning hosilasini topamiz: $f'(x)=3x^2-3=3(x-1)(x+1)$. Hosila barcha nuqtalarda aniqlangan va $x=\pm1$ nuqtalarda nolga aylanadi. Shuning uchun $x=\pm1$ nuqtalar funksiyaning kritik nuqtalaridir. Oraliqlar usulidan foydalanib $(-\infty;\ -1)$ va $(1;\ +\infty)$ oraliqlarda $f'(x)>0$, $(-1;\ 1)$ oraliqda esa $f'(x)<0$ ekanini aniqlaymiz. Demak, $x=-1$ lokal maksimum va $x=1$ lokal minimum nuqtalari ekan (23-rasm).



23-rasm.

Javob: $x=-1$ lokal maksimum va $x=1$ lokal minimum nuqta. ▲

Funksiyaning eng katta va eng kichik qiymatlari bilan 10-sinfdan tanishmiz.

$f(x)$ funksiya $[a;\ b]$ kesmada aniqlangan va $(a;\ b)$ da hosilasi mavjud bo'lsin. Uning eng katta qiymatini topish qoidasi shunday:

1) funksiyaning bu oraliqdagi barcha statsionar nuqtalari topiladi;

2) funksiyaning statsionar, chegaraviy a va b nuqtalardagi qiymatlari hisoblanadi;

3) bu qiymatlarning eng kattasi funksiyaning shu oraliqdagi eng katta qiymati deyiladi.

Funksiyaning eng kichik qiymati ham shu kabi topiladi.

5-misol. $f(x) = x^3 + 4{,}5x^2 - 9$ funksiyaning [–4; 2] oraliqdagi eng katta va eng kichik qiymatlarini toping.

△ Funksiyaning hosilasini topamiz: $f'(x)=3x^2+9x$. Hosilani nolga tenglab, funksiyaning statsionar nuqtalarini topamiz: $f'(x)=3x(x+3)=0$, $x_1=0$ va $x_2=-3$. Funksiyaning topilgan $x_1=0$, $x_2=-3$ hamda $a=-4$, $b=2$ nuqtalardagi qiymatlarini topamiz:

$f(0)=0^3+4{,}5\cdot 0^2-9=-9$, $f(-3)=(3)^3+4{,}5\cdot(-3)^2-9=4{,}5$,

$f(-4)=(-4)^3+4{,}5\cdot 4^2-9=-1$, $f(2)=2^3+4{,}5\cdot 2^2-9=17$.

Demak, funksiyaning eng katta qiymati 17 va eng kichik qiymati –9 ekan.

Javob: funksiyaning eng katta qiymati 17 va eng kichik qiymati –9. ▲

Hosila yordamida funksiyani tekshirish va grafigini yasash. Funksiya grafigini yasashni quyidagi ketma-ketlikda amalga oshiramiz.

Funksiyaning:

1) aniqlanish sohasini;

2) statsionar nuqtalarini;

3) o‘sish va kamayish oraliqlarini;

4) lokal maksimum va lokal minimumlarini hamda funksiyaning shu nuqtalardagi qiymatlarini topamiz;

5) topilgan ma’lumotlarga ko‘ra funksiyaning grafigini yasaymiz.

Grafikni yasashda funksiya grafigini koordinata o‘qlari bilan kesisish va boshqa ayrim nuqtalarini topish maqsadga muvofiq.

6-misol. $f(x)=x^3-3x$ funksiyani hosila yordamida tekshiring va uning grafigini yasang.

1. Funksiya $(-\infty; +\infty)$ oraliqda aniqlangan.

2. Statsionar nuqtalarini topamiz:

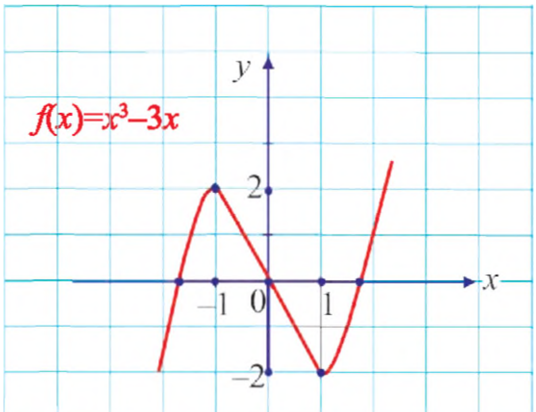
$f'(x)=(x^3-3x)'=3x^2-3=0$. $x_1=1$ va $x_2=-1$ statsionar nuqtalardir.

3. Funksiyaning o‘sish va kamayish oraliqlarini topamiz:

$(-\infty; -1) \cup (1; +\infty)$ oraliqlarda $f'(x) > 0$ bo‘lgani uchun $f(x)$ funksiya shu oraliqlarda o‘sadi va $(-1; 1)$ oraliqda $f'(x) < 0$ bo‘lgani uchun $f(x)=x^3-3x$ funksiya $(-1; 1)$ oraliqda kamayadi.

4. $x=-1$ bo'lganda funksiya lokal maksimum $f(-1)=(-1)^3-3\cdot(-1)=2$ ga va $x=1$ bo'lganda funksiya lokal minimum $f(1)=1^3-3\cdot1=-2$ ga ega.

5. Funksiyaning Ox o'qi bilan kesisish nuqtalarini topamiz: $x^3-3x=x(x^2-3)=0$. Bundan $x=0$ yoki $x^2-3=0$ tenglamani hosil qilamiz. Tenglamani yechib $x_1=0$, $x_2=\sqrt{3}$, $x_3=-\sqrt{3}$ funksiya grafigining Ox o'qi bilan kesisish nuqtalarini topamiz. Natijada 24- rasmdagi grafikni hosil qilamiz.



24-rasm.

Savol va topshiriqlar

1. Funksiyaning o'sish va kamayish oraliqlari qanday topiladi?
2. Funksiyaning statsionar nuqtasiga ta'rif bering.
3. Funksiyaning lokal maksimum va lokal minimumlari qanday topiladi?
4. Funksiyaning eng katta va eng kichik qiymatlari qanday topiladi?
5. Hosila yordamida funksiyaning grafigini yasash bosqichlarini ayting va bitta misolda tushuntiring.
6. Funksiyaning statsionar nuqtalari uning ekstremum nuqtalari bo'lishi shartmi? Misollar keltiring.
7. $f(x)=\frac{1}{4}x^4-\frac{1}{2}x^2$ funksiyani hosila yordamida tekshiring va grafigini yasang.

Mashqlar

69. Funksiyaning o‘sish va kamayish oraliqlarini toping:

1) $f(x)=2-9x$; 2) $f(x)=\frac{1}{2}x-8$; 3) $f(x)=x^3-27x$;

4) $f(x)=\frac{x-1}{x}$; 5) $f(x)=x^2-2x+4$; 6) $f(x)=x(x^2-6)$;

7) $f(x)=-x^2+2x-3$; 8) $f(x)=\frac{1}{x^2}$; 9) $f(x)=x^4-2x^2$;

10) $f(x)=3x^4-8x^3+16$; 11) $f(x)=\frac{1}{1+x^2}$; 12) $f(x)=\sin x$;

13) $f(x)=\cos x$; 14) $f(x)=\operatorname{tg} x$; 15*) $f(x)=\sin 2x+\cos 2x$.

70. Funksiyaning statsionar nuqtalarini toping:

1) $f(x)=2x^2-3x+1$; 2) $f(x)=9x-\frac{1}{3}x^3$; 3*) $f(x)=|x-1|$;

4) $f(x)=x^2$; 5) $f(x)=8x^3+5x$; 6) $f(x)=3x-4$;

7*) $f(x)=|x|+1$; 8) $f(x)=2x^3+3x^2-6$; 9) $f(x)=3+8x^2-x^4$.

71. Funksiyaning lokal maksimum va lokal minimumlarini toping:

1) $f(x)=x^2-\frac{1}{2}x^4$; 2) $f(x)=(x-4)^8$; 3) $f(x)=4-3x^2-2x^3$;

4) $f(x)=\frac{5}{x}+\frac{x}{5}$; 5) $f(x)=x^4-2x^3+x^2-3$; 6) $f(x)=3\operatorname{tg} x$;

7) $f(x) = 2\sin x + 3$; 8) $f(x) = -5\cos x-7$; 9) $f(x)=x^4-x^3+4$.

72. Funksiyaning o‘sish va kamayish oraliqlarini toping:

1) $f(x)=x^3-27x$; 2*) $f(x)=\frac{3x}{x^2+1}$; 3*) $f(x)=x+\frac{4}{x^2}$;

4) $f(x)=5\sin x+13$; 5) $f(x)=15\cos x-7$; 6) $f(x)=-3\operatorname{tg} x$.

73. Funksiyaning eng katta va eng kichik qiymatlarini toping:

1) $f(x)=x^4-8x^2+3$, $x\in[-4;1]$; 2) $f(x)=3x^5-5x^3+1$, $x\in[-2;2]$;

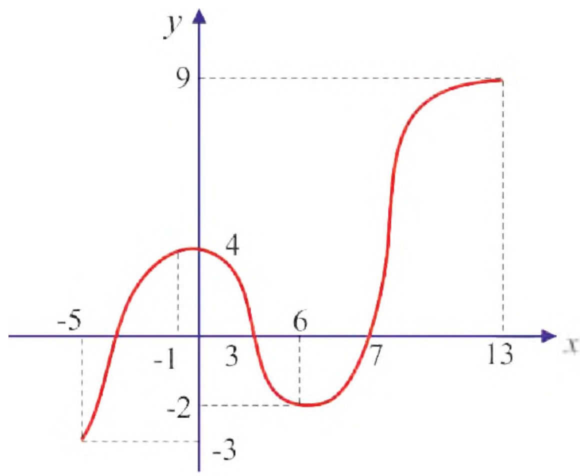
3) $f(x)=\frac{x}{x+1}$, $x\in[1;2]$; 4) $f(x)=3x^3-6x^2-5x+8$, $x\in[-1;4]$.

74. Funksiyani tekshiring va grafigini yasang:

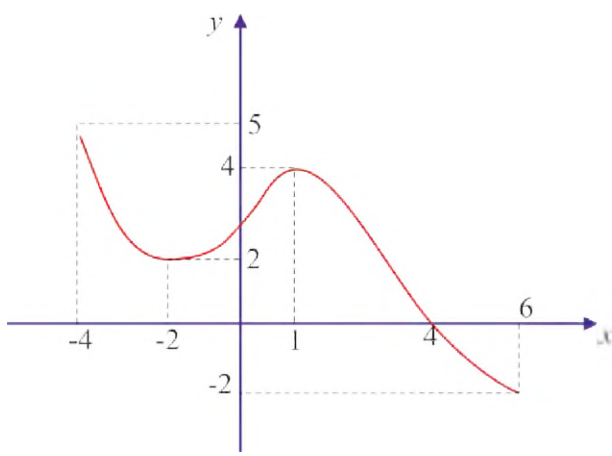
1) $y = x^3 - 6x^2 + 9x - 2;$ | 2) $y = \frac{1}{5}x^5 - \frac{2}{3}x^3 + 1;$ | 3) $y = x^4 - 4x^3 + 15.$

75*. Funksiya hosilasining grafigiga qarab (25, 26-rasmlar), quyidagilarni toping:

1) statsionar nuqtalarni; 2) o'sish oraliqlarini;
3) kamayish oraliqlarini; 4) lokal maksimumlarini;
5) lokal minimumlarni.



25-rasm.



26-rasm.

Nazorat ishi namunasi

I variant

1. Hosilani toping: $f(x) = 20x^3 + 6x^2 - 7x + 3$.

2. $f(x) = x^2 - 5x + 4$ va $g(x) = \frac{x+1}{x-2}$ bo‘lsa, $f(g(3))$ ni hisoblang.

3. $f(x) = x^3 - 5x^2 + 7x + 1$ funksiya uchun quyidagilarni toping:

1) statsionar nuqtalarni;
2) o‘sish oraliqlarini;
3) kamayish oraliqlarini;
4) lokal maksimumlarini;
5) lokal minimumlarini.

4. Hosilani toping: $(3x + 5)^3 + \sin^2 x$.

5. $f(x) = \sqrt{1-3x}$ bo‘lsa $f'\left(\frac{1}{4}\right)$ ni hisoblang.

II variant

1. Hosilani toping: $f(x)=10x^3+16x^2+7x-3$.

2. $f(x)=x^2+6x-3$ va $g(x) = \frac{x-1}{x+2}$ bo‘lsa, $f(g(3))$ ni hisoblang.

3. $f(x)=x^3+2x^2-x+3$ funksiya uchun quyidagilarni toping:

1) statsionar nuqtalarni;
2) o‘sish oraliqlarini;
3) kamayish oraliqlarini;
4) lokal maksimumlarini;
5) lokal minimumlarini.

4. Hosilani toping: $(2x - 6)^3 + \cos^2 x$.

5. $f(x) = \sqrt{1-2x}$ bo‘lsa, $f'\left(\frac{3}{8}\right)$ ni hisoblang.

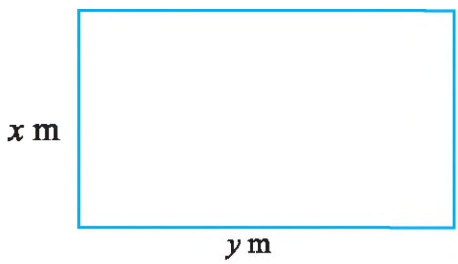
22–25 GEOMETRIK, FIZIK, IQTISODIY MAZMUNLI EKSTREMAL MASALALARNI YECHISHDA DIFFERENSIAL HISOB USULLARI

Geometrik mazmunli masalalar

1-masala. To‘g‘ri to‘rtburchak shaklidagi yer maydoni atrofini 100 m panjara bilan o‘rashmoqchi. Bu panjara eng ko‘pi bilan necha kvadrat mert yer maydonini o‘rashga yetadi?

△ Yer maydonining eni x m va bo‘yi y m bo‘lsin (27-rasm).

Masala shartiga ko‘ra yer maydonining perimetri $2x+2y=100$. Bundan $y=50-x$. Yer maydonining yuzi $S(x)=xy=x(50-x)=50x-x^2$. Masala $S(x)$ funksiyaning eng katta qiymatini topishga keltirildi. Avval $S(x)$ funksiyaning statsionar nuqtasini topamiz: $S'(x)=50-2x=0$, bundan $x=25$. $(-\infty;\ 25)$ oraliqda $S'(x)>0$ va $(25;+\infty)$ oraliqda $S'(x)<0$ bo‘lgani uchun $S(x)$ funksiya $x=25$ da eng katta qiymatga ega bo‘ladi va $S(25)=625$. Demak, 100 m panjara yordamida eng ko‘pi bilan 625 m^2 yer maydonini o‘rash mumkin. *Javob:* 625 m^2. ▲

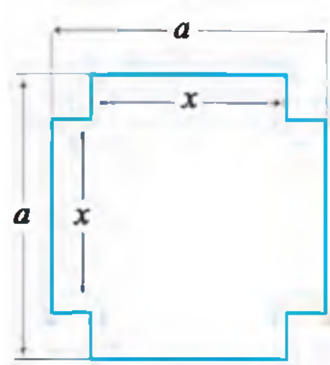


27-rasm.

Umuman, perimetri berilgan barcha to‘g‘ri to‘rtburchaklar ichida yuzi eng kattasi kvadratdir.

2-masala. Tomoni a cm bo‘lgan kvadrat shaklidagi kartondan usti ochiq quti tayyorlashmoqchi. Bunda kartonning uchlaridan bir xil kvadratchalar kesib olinadi. Qutining hajmi eng katta bo‘lishi uchun uning asos tomoni uzunligi necha santimetr bo‘lishi kerak?

△ Kartonning uchlaridan bir xil kvadratchalar qirqib olinib, asosi x cm bo‘lgan ochiq quti yasalgan, desak (28-rasm), kesib olingan kvadratchaning tomoni $\frac{a-x}{2}$ cm bo‘ladi. Shuning uchun ochiq qutining hajmi $V(x)=\frac{a-x}{2}\cdot x\cdot x=$



28-rasm.

$= -\frac{x^3}{2} + \frac{ax^2}{2}$ cm³. Demak, berilgan masala $V(x) = -\frac{x^3}{2} + \frac{ax^2}{2}$ funksiyaning [0; *a*] kesmadagi eng katta qiymatini topishga keldi. *V*(*x*) funksiyaning statsionar nuqtalarini topamiz: $V'(x) = -\frac{3}{2}x^2 + ax = 0$.

Bu yerdan $x_1 = 0$, $x_2 = \frac{2}{3}a$ statsionar nuqtalar topiladi. Ravshanki, $V\left(\frac{2}{3}a\right) = \frac{2}{27}a^3$ va $V\left(\frac{2}{3}a\right) > V(0) = V(a) = 0$. Demak, $V(x)$ ning $[0; a]$ kesmadagi eng katta qiymati $\frac{2}{27}a^3$ bo‘ladi.

Javob: ochiq qutining asos tomoni uzunligi $x = \frac{2}{3}a$ cm. ▲

Fizik mazmunli masalalar

3-masala. Moddiy nuqta $s(t) = -\frac{t^4}{12} + t^3$ qonuniyat bilan harakatlanmoqda (*s*(*t*) metrda, *t* vaqt esa sekundda o‘lchanadi). Quyidagilarni:

1) Eng katta tezlanishga erishiladigan vaqtni (t_0);

2) t_0 vaqtdagi oniy tezlikni;

3) t_0 vaqt ichida bosib o‘tilgan yo‘lni toping.

△ Moddiy nuqtaning tezligini topamiz:

$$v(t) = s'(t) = \left(-\frac{t^4}{12} + t^3\right)' = -\frac{t^3}{3} + 3t^2.$$

Fizikadan ma’lumki, tezlikdan olingan hosila tezlanishni beradi, ya’ni:

$$a(t) = v'(t) = -t^2 + 6t.$$

1) Eng katta tezlanishga ega bo‘ladigan t_0 vaqtni aniqlash uchun $a(t) = v'(t) = -t^2 + 6t$ funksiyani maksimumga tekshiramiz. Avval

$a'(t) = -2t + 6 = 0$ tenglamani yechamiz, bundan $t_0 = 3$. (0; 3) oraliqda $a'(t) > 0$ va $(3; +\infty)$ oraliqda $a'(t) < 0$ bo‘lgani uchun $t = 3$ da $a(t)$ eng katta qiymatga erishadi.

2) t_0 vaqtdagi oniy tezlikni hisoblaymiz: $v(3) = -\frac{3^3}{3} + 3 \cdot 3^2 = 18\frac{\text{m}}{\text{s}}$.

3) t_0 vaqt ichida bosib o‘tilgan yo‘l $s(t)=-\frac{t^4}{12}+t^3$ formulaga $t_0=3$ ni qo‘yib hisoblanadi: $s(3)=-\frac{3^4}{12}+3^3=-\frac{27}{4}+27=\frac{81}{4}=20,25$ m.

Javob: 1) 3 s; 2) $18\frac{\text{m}}{\text{s}}$; 3) 20,25 m. ▲

4-masala. Moddiy nuqta $s(t)=\frac{t^3}{3}-t^2+4t+50$ qonuniyat bilan harakatlanmoqda (s(t) masofa metrda, vaqt t sekundda o‘lchanadi). Quyidagilarni toping:

1) eng kichik tezlikka erishiladigan vaqtni (t_0);
2) t_0 vaqtdagi tezlanishni;
3) t_0 vaqt ichida bosib o‘tilgan yo‘lni.

△ Moddiy nuqtaning tezligi va tezlanishini topamiz:

$$v(t)=s'(t)=\left(\frac{t^3}{3}-t^2+4t+50\right)'=t^2-2t+4,$$

$$a(t)=v'(t)=\left(t^2-2t+4\right)'=2t-2.$$

1) Eng kichik tezlikga erishiladigan t_0 vaqtni aniqlaymiz:

$$v'(t)=(t^2-2t+4)'=2t-2=0, \text{ bundan } t_0=1.$$

(0; 1) oraliqda $v'(t)<0$ va $(1;+\infty)$ oraliqda $v'(t)>0$ bo‘lgani uchun $t_0=1$ da $v(t)$ eng kichik qiymatga erishadi.

2) t_0 vaqtdagi tezlanishni hisoblaymiz: $a(1)=2\cdot1-2=0$ m/s^2.

3) t_0 vaqt ichida bosib o‘tilgan yo‘lni $s(t)=\frac{t^3}{3}-t^2+4t+50$ formulaga $t_0=1$ ni qo‘yib hisoblanadi, ya‘ni $s(1)=\frac{1^3}{3}-1^2+4\cdot1+50=53\frac{1}{3}$ m.

Javob: 1) 1 s; 2) 0 m/s^2; 3) $53\frac{1}{3}$m. ▲

5-masala. Havo shariga $t\in[0;8]$ minut oralig‘ida $V(t)=2t^3-3t^2+10t+2$ (m^3) hajmda havo purkalmoqda. Quyidagilarni toping:

1) boshlang‘ich vaqtdagi havo hajmini;
2) $t=8$ minutdagi havo hajmini;

3) $t=4$ minutdagi havo purkash tezligini;

△ 1) boshlang‘ich vaqtdagi havo hajmini topish uchun $V(t)=2t^3-$ $-3t^2+10t+2$ m^3 formulaga $t=0$ qo‘yiladi, ya‘ni $V(0)=2$ m^3.

2) $t=8$ minut vaqtdagi havo hajmini topish uchun $V(t)=2t^3-3t^2+10t+2$ m^3 formulaga $t=8$ qo‘yiladi:

$$V(8)=2\cdot 8^3-3\cdot 8^2+10\cdot 8+2=1024-192+80+2=914 \text{ m}^3;$$

3) havo purkash tezligini topamiz:

$$v'(t)=\left(2t^3-3t^2+10t+2\right)'=6t^2-6t+10\left(\frac{\text{m}^3}{\text{min.}}\right).$$

Demak, $v'(4)=6\cdot 4^2-6\cdot 4+10=96-24+10=82\left(\frac{\text{m}^3}{\text{min}}\right)$.

Demak, $a(3)=12\cdot 3-6=30\left(\frac{\text{m}^3}{\text{min}^2}\right)$.

Javob: 1) 2 m^3; 2) 914 m^3; 3) $82\frac{\text{m}^3}{\text{min}}$. ▲

Iqtisodiy mazmunli masalalar

6-masala. Karima ko‘ylak tikish uchun buyurtma oldi. Bir oyda x ta ko‘ylak tiksa, $p(x)=-x^2+100x$ ming so‘m daromad qiladi. Quyidagilarni toping:

1) eng katta daromad olish uchun qancha ko‘ylak tikish kerak?

2) eng katta daromad qancha bo‘ladi?

△ 1) $p(x)=-x^2+100x$ funksiyani maksimumga tekshiramiz:

$p'(x)=(-x^2+100x)'=-2x+100=0$, bundan $x_0=50$. (0; 50) kesmada $p'(x)>0$ va $(50;+\infty)$ oraliqda $p'(x)<0$ bo‘lgani uchun $x_0=50$ bo‘lganda funksiya eng katta qiymatga ega bo‘ladi. Demak, eng katta daromad olish uchun 50 ta ko‘ylak tikish kerak ekan.

2) Eng katta daromad qanchaligini topish uchun $p(x)=-x^2+100x$ ifodaga $x_0=50$ ni qo‘yamiz:

$p(50)=-50^2+100\cdot 50=-2500+5000=2500$(ming so‘m)$=2500000$ so‘m.

Javob: 1) 50 ta ko‘ylak; 2) 2 500 000 so‘m. ▲

Savol va topshiriqlar

Hosilani tatbiq qilib yechiladigan:
1) geometrik; 2) fizik; 3) iqtisodiy mazmunli masalaga misol keltiring.

Mashqlar

76. To'g'ri to'rtburchak shaklidagi yer maydonining atrofini o'rashmoqchi. 300 m panjara yordamida eng ko'pi bilan necha kvadrat metr yer maydonini o'rash mumkin?

77. To'g'ri to'rtburchak shaklidagi yer maydonining atrofini o'rashmoqchi. 480 m panjara yordamida eng ko'pi bilan necha kvadrat metr yer maydonini o'rash mumkin?

78*. Tomoni 120 cm bo'lgan kvadrat shaklidagi kartondan usti ochiq quti tayyorlandi. Bunda kartonning uchlaridan bir xil kvadratchalar kesib olindi. Qutining hajmi eng katta bo'lishi uchun kesib olingan kvadratchaning tomoni necha santimetr bo'lishi kerak?

79*. Konserva banka silindr shaklida bo'lib, uning to'la sirti 216 π cm^2 ga teng. Bankaga eng ko'p suv sig'ishi (ketishi) uchun banka asosining radiusi va balandligi qanday bo'lishi kerak?

80. To'g'ri to'rtburchak shaklidagi maydonning yuzi 6400 m^2. Maydonning tomonlari qanday bo'lganda uni o'rash uchun eng kam panjara zarur bo'ladi?

81*. Radiusi 5m bo'lgan sharga eng kichik hajmli konus tashqi chizilgan. Konusning balandligini toping.

82*. Metalldan sig'imi 13,5 *l*, asosi kvadratdan iborat bo'lgan to'g'ri burchakli parallelepiped yasalmoqda. Idishning o'lchamlari qanday bo'lganda uni yasash uchun eng kam metall ketadi?

83. Moddiy nuqta $s(t)=-\frac{t^4}{4}+5t^3$ qonuniyat bilan harakatlanmoqda ($s(t)$ metrda, vaqt t sekundda o'lchanadi). Quyidagilarni toping:
1) eng katta tezlanishga erishiladigan t_0 vaqtni;
2) t_0 vaqtdagi oniy tezlikni;
3) t_0 vaqt ichida bosib o'tilgan yo'lni.

84. Moddiy nuqta $s(t)=-\frac{t^4}{2}+12t^3$ qonuniyat bilan harakatlanmoqda ($s(t)$ m da, vaqt t sekundda o'lchanadi).

1) eng katta tezlanishga erishiladigan t_0 vaqtni;

2) t_0 vaqtdagi oniy tezlikni;

3) t_0 vaqt ichida bosib o'tilgan yo'lni toping.

85. Moddiy nuqta $s(t)=\frac{t^3}{9}-2t^2+40t+50$ qonuniyat bilan harakatlanmoqda ($s(t)$ metrda, vaqt t sekundda o'lchanadi).

1) eng kichik tezlikka erishiladigan t_0 vaqtni;

2) t_0 vaqtdagi tezlanishni;

3) t_0 vaqt ichida bosib o'tilgan yo'lni toping.

86. Moddiy nuqta $s(t)=\frac{t^3}{2}-3t^2+8t+5$ qonuniyat bilan harakatlanmoqda ($s(t)$ metrda, vaqt t sekundda o'lchanadi). Quyidagilarni toping:

1) eng kichik tezlikka erishiladigan t_0 vaqtni;

2) t_0 vaqtdagi tezlanishni;

3) t_0 vaqt ichida bosib o'tilgan yo'lni.

87. Havo shariga $t\in[0;\ 10]$ minut oralig'ida $V(t)=5t^3+3t^2+2t+4$ (m^3) havo purkalmoqda.

1) boshlang'ich vaqtdagi havo hajmini;

2) $t=10$ minutdagi havo hajmini;

3) $t=5$ minutdagi havo purkash tezligini toping;

88. Havo shariga $t\in[0;\ 15]$ minut oralig'da $V(t)=t^3+13t^2+t+20$ (m^3) havo purkalmoqda.

1) boshlang'ich vaqtdagi havo hajmini;

2) $t=15$ minutdagi havo hajmini;

3) $t=10$ minutdagi havo purkash tezligini toping;

89. Muslima shim tikish uchun buyurtma oldi. U bir oyda x ta shim tiksa, $p(x)=-2x^2+120x$ ming so'm daromad qiladi. Quyidagilarni toping:

1) daromadni eng katta qilish uchun qancha shim tikishi kerak?

2) eng katta daromad qancha bo'ladi?

90. Muxlisa yubka tikish uchun buyurtma oldi. Bir oyda x ta yubka tiksa, $p(x)=-3x^2+96x$ (ming so'm) daromad qiladi. Quyidagilarni toping:

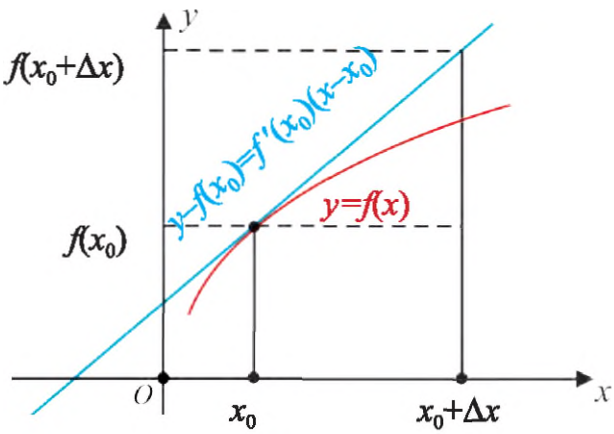
1) daromadni eng katta qilish uchun qancha yubka tikish kerak?

2) eng katta daromad qancha bo'ladi?

$y=f(x)$ funksiya x_0 nuqtada chekli $f'(x_0)$ hosilaga ega bo'lsin.

x_0 abssissali nuqtada $y=f(x)$ funksiya grafigiga o'tkazilgan urinma tenglamasi $y-f(x_0)=f'(x_0)(x-x_0)$ kabi yozilishini bilamiz.

x_0 nuqta yaqinida $y=f(x)$ funksiya grafigini urinmaning mos kesmasi bilan almashtirsa bo'ladi (29-rasmga qarang):



29-rasm.

$x-x_0$ orttirmani Δx deb belgilasak (ya'ni $x=x_0+\Delta x$ deb olsak) quyidagi taqribiy munosabatga ega bo'lamiz:

$$f(x)\approx f(x_0)+f'(x_0)(x-x_0),\ \text{yoki}$$

$$f(x_0+\Delta x)\approx f(x_0)+f'(x_0)\cdot\Delta x \quad (1)$$

(1) taqribiy formula *kichik orttirmalar formulasi* deb nomlanadi.

Izoh. x_0 nuqta sifatida $f(x_0), f'(x_0)$ qiymatlar oson hisoblanadigan nuqtani tanlab olish tavsiya etiladi. Shu bilan birga x nuqta x_0 ga qancha yaqin bo'lsa, bunday almashtirish aniqroq bo'lishini qayd etamiz.

Endi biz kichik orttirmalar formulasiga tayangan holda taqribiy hisoblashlarni bajaramiz.

1-misol. $f(x) = x^7-2x^6+3x^2-x+3$ funksiyaning $x = 2{,}02$ nuqtadagi qiymatini taqribiy hisoblang.

△ $x = 2{,}02$ nuqtaga yaqin bo'lgan $x_0 = 2$ nuqtani olsak, bu nuqtada $f(x)$ funksiya qiymati osonlikcha topiladi: $f(x_0) = f(2) = 13$.

Bu funksiyaning hosilasini topamiz: $f'(x) = 7x^6 - 12x^5 + 6x - 1$.

U holda $f'(x_0) = f'(2) = 75$, $\Delta x = x - x_0 = 2{,}02 - 2 = 0{,}02$ bo'ladi.

Demak, (1) formulaga ko'ra $f(2{,}02) = f(2 + 0{,}02) \approx 13 + 75 \cdot 0{,}02 = 14{,}5$.

Kalkulator yoki boshqa hisoblash vositasi yordamida $f(2{,}02) \approx 14{,}57995$ qiymatni hosil qilishimiz mumkin. ▲

2-misol. $\sqrt{1{,}02}$ ildizning qiymatini taqribiy hisoblang.

△ $f(x) = \sqrt{x}$ funksiyani qaraymiz. Uning hosilasini topamiz:

$$f'(x) = \left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}.$$

$x_0 = 1$ deb olsak, $f(x_0) = f(1) = \sqrt{1} = 1$,

$$f'(x_0) = f'(1) = \frac{1}{2}\frac{1}{\sqrt{1}} = \frac{1}{2}, \quad \Delta x = x - x_0 = 1{,}02 - 1 = 0{,}02 \text{ bo'ladi.}$$

Demak, (1) formulaga ko'ra

$$\sqrt{1{,}02} = \sqrt{1 + 0{,}02} \approx 1 + \frac{1}{2} \cdot 0{,}02 = 1{,}01.$$

Kalkulator yoki boshqa hisoblash vositasi yordamida $\sqrt{1{,}02} \approx 1{,}0099504938...$ qiymatni hosil qilishimiz mumkin. ▲

3-misol. $\sqrt[3]{7.997}$ ning qiymatini taqribiy hisoblang.

△ $f(x) = \sqrt[3]{x}$ funksiyani qaraymiz. Uning hosilasini topamiz:

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}.$$

$x_0 = 8$ deb olsak, $f(x_0) = f(8) = \sqrt[3]{8} = 2$,

$$f'(x_0) = f'(8) = \frac{1}{3}8^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{12},$$

$\Delta x = 7{,}997 - 8 = -0{,}003$ bo'ladi.

Demak, (1) formulaga ko'ra

$$\sqrt[3]{7,997}=\sqrt[3]{8+(-0,003)}\approx 2-\frac{0,003}{12}=1,9997.$$

Kalkulator yoki boshqa hisoblash vositasi yordamida

$\sqrt[3]{7.997}\approx 1,9997499687...$ qiymatni hosil qilishimiz mumkin. ▲

4-misol. sin 29° ning qiymatini taqribiy hisoblang.

△ $f(x)=\sin x$ funksiyani qaraymiz. Uning hosilasini topamiz: $f'(x)=\cos x$.

$$x_0=\frac{\pi}{6} \text{ deb olsak, } f(x_0)=f\left(\frac{\pi}{6}\right)=\sin\frac{\pi}{6}=\frac{1}{2},$$

$$f'(x_0)=f'\left(\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2},\ \Delta x=\frac{29\pi}{180}-\frac{\pi}{6}=-\frac{\pi}{180} \text{ bo‘ladi.}$$

Demak, (1) formulaga ko‘ra

$$\sin 29°=\sin\left(\frac{\pi}{6}+\left(-\frac{\pi}{180}\right)\right)\approx \sin\frac{\pi}{6}-\frac{\sqrt{3}}{2}\frac{\pi}{180}=\frac{1}{2}-\frac{\sqrt{3}}{2}\frac{\pi}{180}\approx 0,484\ldots .$$

Kalkulator yoki boshqa hisoblash vositasi yordamida $\sin 29^\circ\approx 0,4848096202...$ qiymatni hosil qilishimiz mumkin. ▲

5-misol. Logarifmlarni hisoblash uchun kichik orttirmalar formulasini keltiramiz.

△ $f(x)=\ln x$; $f'(x)=\frac{1}{x}$. (1) ga ko‘ra, $\ln(x_0+\Delta x)\approx \ln x_0+\frac{1}{x_0}\cdot\Delta x$ — kichik orttirmalar formulasini hosil qilamiz.

Agar $x_0=1$ va $\Delta x=t$ bo‘lsa, $\ln(1+t)\approx t$ bo‘ladi.
Bundan, masalan, ln1,3907= ln(1+0,3907) ≈ 0,3907 qiymatni olamiz.

Agar $x_0=0$, ya’ni $\Delta x=x-x_0=x$ bo‘lsa, (1) kichik orttirmalar formulasi

$$f(x)\approx f(0)+f'(0)x \tag{2}$$

ko‘rinishni oladi. ▲

Sinfda bajariladigan topshiriq. (2) formulaga asoslanib, x yetarlicha kichik bo‘lganda

$\sin x\approx x$, $\operatorname{tg} x\approx x$, $e^x\approx 1+x$, $(1+x)^m\approx 1+mx$, jumladan, $\sqrt{1+x}\approx 1+\frac{1}{2}x$, $\sqrt[3]{1+x}\approx 1+\frac{1}{3}x$ taqribiy formulalarni hosil qiling.

6-misol. $\frac{1}{0,997^{30}}$ ifodani taqribiy hisoblang.

△ $(1+x)^m \approx 1+mx$ formuladan foydalanamiz:

$$\frac{1}{0,997^{30}} = (1-0,003)^{-30} \approx 1+(-30)(-0,003)=1+0,09=1,09.$$

Kalkulator yoki boshqa hisoblash vositasi yordamida $\frac{1}{0,997^{30}} \approx 1,0943223033...$ qiymatni hosil qilishimiz mumkin. ▲

$(1+x)^m \approx 1+mx$ taqribiy formuladan foydalanib ildizlarni tezkor hisoblash usulini taklif qilish mumkin.

Chindan ham, n – natural son bo‘lib, $|B|$ soni $|A^n|$ ga nisbatan yetarlicha kichik bo‘lsin.

U holda

$$\sqrt[n]{A^n+B} = A\left(1+\frac{B}{A^n}\right)^{\frac{1}{n}} \approx A\left(1+\frac{B}{nA^n}\right) \text{ yoki}$$

$$\sqrt[n]{A^n+B} \approx A+\frac{B}{nA^{n-1}}.$$

Masalan, $\sqrt[3]{131} = \sqrt[3]{125+6} = 5+\frac{6}{3\cdot 5^2} = 5,08.$

Kalkulator yoki boshqa hisoblash vositasi yordamida $\sqrt[3]{125} = 5,0788...$ qiymatni hosil qilishimiz mumkin.

(2) formulaga asoslanib, x yetarlicha kichik bo‘lganda $\cos x$ ning qiymatini taqribiy hisoblaylik. $(\cos x)' = -\sin x$ bo‘lgani uchun $f(x) \approx f(0)+f'(0)x$ formula $\cos x \approx \cos 0-(\sin 0)x=1$, ya’ni $\cos x \approx 1$ ko‘rinishni oladi. Ravshanki, bunday “taqribiy” formula bizni qanoatlantirmaydi. Shuning uchun, boshqacha yo‘l tutamiz. Asosiy trigonometrik ayniyatdan $\cos x = \pm\sqrt{1-\sin^2 x}$ tenglikni hosil qilamiz.

Yuqorida qayd etganimizdek, x yetarlicha kichik bo‘lganda $\sin x \approx x$ bo‘ladi. Demak, $\cos x = \sqrt{1-\sin^2 x} \approx \sqrt{1-x^2}$.

Ravshanki, x yetarlicha kichik bo'lganda x^2 ham kichik bo'ladi.

Demak, $\sqrt{1+x} \approx 1+\frac{1}{2}x$ formuladan $\sqrt{1-x^2} \approx 1-\frac{x^2}{2}$ formula bevosita kelib chiqadi, ya'ni $\cos x \approx 1-\frac{x^2}{2}$ formula o'rinli bo'ladi.

7-misol. $\cos 44°$ ni taqribiy hisoblang.

△ $\cos(x-y)=\cos x\cos y+\sin x\sin y$ bo'lgani uchun

$$\cos 44° = \cos(\frac{\pi}{4}-\frac{\pi}{180}) = \cos\frac{\pi}{4}\cos\frac{\pi}{180}+\sin\frac{\pi}{4}\sin\frac{\pi}{180} =$$

$$= \frac{\sqrt{2}}{2}(\cos\frac{\pi}{180}+\sin\frac{\pi}{180}). \quad \cos\frac{\pi}{180} \approx 1-\frac{1}{2}(\frac{\pi}{180})^2 = 0{,}9998476\ldots,$$

$$\sin\frac{\pi}{180} \approx \frac{\pi}{180} = 0{,}0174532\ldots.$$

Demak, $\cos 44° \approx 0{,}7193403\ldots$.

Kalkulator yoki boshqa hisoblash vositasi yordamida $\cos 44° \approx 0{,}7193339\ldots$ qiymatni hosil qilamiz.

Savol va topshiriqlar

1. Kichik orttirmalar formulasini yozing.
2. Kichik orttirmalar formulasining tatbiqiga oid misollar keltiring.

Mashqlar

91. $f(x)$ funksiyaning x_1 va x_2 nuqtalardagi qiymatini taqribiy hisoblang:

a) $f(x)=x^4+2x$, $x_1=2{,}016$, $x_2=0{,}97$;

b) $f(x)=x^5-x^2$, $x_1=1{,}995$, $x_2=0{,}96$;

d) $f(x)=x^3-x$, $x_1=3{,}02$, $x_2=0{,}92$;

e) $f(x)=x^2+3x$, $x_1=5{,}04$, $x_2=1{,}98$.

$(1+x)^m \approx 1+mx$ taqribiy formuladan foydalanib, sonli ifoda qiymatini hisoblang (**92–93**):

92. a) $1{,}002^{100}$; b) $0{,}995^6$; d) $1{,}03^{200}$; e) $0{,}998^{20}$.

93. a) $\sqrt{1{,}004}$; b) $\sqrt{25{,}012}$; d) $\sqrt{0{,}997}$; e) $\sqrt{4{,}0016}$.

Taqribiy formulalardan foydalanib, hisoblang (**94 – 97**):

94. a) tg 44°; b) cos 61°; d) sin 31°; e) ctg 47°.

95. a) $\cos\left(\frac{\pi}{6}+0,04\right)$; b) $\sin\left(\frac{\pi}{6}-0,02\right)$;

c) $\sin\left(\frac{\pi}{6}+0,03\right)$; d) $\mathrm{tg}\left(\frac{\pi}{6}-0,05\right)$.

96. a) $\frac{1}{1,003^{20}}$; b) $\frac{1}{0,996^{40}}$; d) $\frac{1}{2,0016^{3}}$; e) $\frac{1}{0,994^{5}}$.

97. a) ln0,9; b) $e^{0,015}$; d) $\frac{1}{0,994^{5}}$.

$y=f(x)$ ning ko‘rsatilgan nuqtadagi taqribiy qiymatini hisoblang

(**98 – 106**):

98. $y=\sqrt[3]{x^3+7x}$, $x=1,012$.

99. $y=\sqrt{x^2+x+3}$, $x=1,97$.

100. $y=x^3$, $x=1,021$.

101. $y=x^4$, $x=0,998$.

102. $y=\sqrt[3]{x^2}$, $x=1,03$.

103. $y=x^6$, $x=2,01$.

104*. $y=\sqrt{1+x+\sin x}$, $x=0,01$.

105*. $y=\sqrt[3]{3x+\cos x}$, $x=0,01$.

106*. $y=\sqrt[4]{2x-\sin(\pi x/2)}$, $x=1,02$.

29–32 HOSILA YORDAMIDA MODELLASHTIRISH

10-sinfda (**79 – 81** mavzu) bakteriyalar sonining ko‘payish jarayonini o‘rgandik. Endi bu hodisaga boshqacha yondashaylik.

1-masala. Har bir bakteriya ma’lum vaqtdan (bir necha soat yoki minutlardan) so‘ng ikkiga bo‘linadi va bakteriyalar soni ikki karra ortadi. Navbatdagi vaqtdan so‘ng mazkur ikkita bakteriya ham ikkiga bo‘linadi va populatsiya miqdori (bakteriyalar umumiy soni) yana ikki karra ortadi... Bu ko‘payish jarayoni qulay (populatsiya uchun zarur resurslar, joy, oziqa, suv, energiya va hokazolar yetarli bo‘lgan) sharoitlarda davom etaveradi, deylik.

Bakteriyalarning ***ko‘payish tezligi*** bakteriyalar umumiy soniga proporsional deb faraz qilaylik.

Bakteriyalar populatsiyasining soni ixtiyoriy t vaqtga nisbatan qanday o‘zgaradi?

△ $b(t)$ deb t vaqt oralig‘idagi bakteriyalar populatsiyasining umumiy sonini belgilaylik.

Hosilaning ma’nosiga ko‘ra, bakteriyalar ko‘payish tezligi $b'(t)$ ga teng.

Farazimizga ko‘ra, ixtiyoriy t vaqtda $b'(t)$ miqdor $b(t)$ miqdorga proporsional, ya’ni

$$b'(t)=kb(t) \qquad (1)$$

munosabat o‘rinli. Bu yerda k – proporsionallik koeffitsiyenti.

$b_0=b(0)$ – boshlang‘ich $t=0$ vaqtdagi populatsiya soni bo‘lsin.

Ravshanki, $b(t)=b_0e^{kt}$ funksiya (1) ni qanoatlantiradi.

Chindan ham, $b'(t)=(b_0e^{kt})'=kb_0e^{kt}=kb(t)$.

Dastlab 10 million bakteriya bo‘lsa (b_0=10 mln), bunday bakteriyalar soni bir soatdan so‘ng $b(1)=10e^k=20$ (mln) ga teng bo‘ladi, ya’ni $e^k=2$. Bundan $k=\ln 2$ ga ega bo‘lamiz.

t vaqt oralig‘idagi bakteriyalar populatsiyasining sonini topaylik:

$$b(t)=10e^{(\ln 2)t}=10\cdot 2^t \text{ (mln)}.$$

Bu natija 10-sinfda olingan natija bilan ustma-ust tushmoqda. ▲

Tarixiy ma'lumot. 18-asrda ingliz olimi Tomas Maltus yuqoridagi fikrlarga o'xshash fikr yuritib, yer yuzidagi aholi sonining o'sishi uchun

$$N'(t)=kN(t) \tag{2}$$

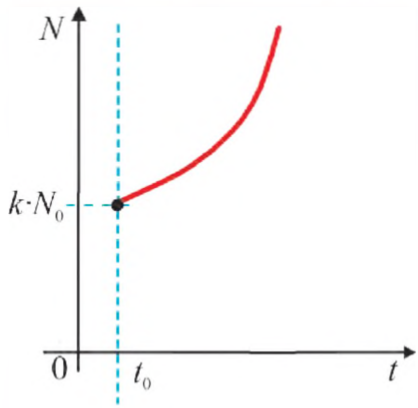
munosabatni hosil qildi, bu yerda $N(t)$ – vaqtning t momentidagi aholi soni.

$N_0=N(t_0)$ – boshlang'ich t_0 vaqtdagi aholi soni bo'lsin.

Bu holda $N(t)=N_0e^{k(t-t_0)}$ funksiya (2) tenglamani qanoatlantiradi.

Chindan ham, $N'(t)= N_0(e^{k(t-t_0)})'=kN_0e^{k(t-t_0)}=kN(t)$.

$N(t)=N_0e^{k(t-t_0)}$ qonuniyat aholining ***eksponensial o'sishini,*** ya'ni shiddatli, to'xtovsiz o'sish jarayonini ifodalashini inobatga olib, Tomas Maltus vaqt o'tishi bilan insoniyatga oziqa resurslari yetmasligini «bashorat" qilganligini qayd etamiz (30-rasmga qarang).



30-rasm.

2-masala. Ekologiya tirik organizmlarning tashqi muhit bilan o'zaro munosabatini o'rganadi. Ko'payish yoki turli sabablarga ko'ra nobud bo'lish bilan bog'liq bo'lgan populatsiyalar sonining o'zgarish tezligi vaqtga qanday bog'lanishda ekanini o'rganing.

△ $N(t)$ – vaqtning t momentidagi populatsiya soni bo'lsin, u holda agar vaqtning bir birligida populatsiyada tug'iladigan jonzotlar sonini A, nobud bo'ladiganlari sonini B desak, yetarli asos bilan aytish mumkinki, N ning vaqtga nisbatan o'zgarish tezligi

$$N'(t) =A-B \tag{3}$$

munosabatni qanoatlantiradi.

Tadqiqotchilar A va B ning N ga bog'liqligini quyidagicha tavsiflaydilar.

a) Eng sodda hol: $A=aN(t)$, $B=bN(t)$. Bu yerda a va b – vaqtning bir birligida tug'ilish va nobud bo'lish koeffitsiyentlari.

Bu holda (3) munosabatni

$$N'(t) = (a-b)N(t) \tag{4}$$

ko'rinishda yozish mumkin.

$N_0=N(t_0)$ – boshlang'ich t_0 vaqtdagi populatsiya soni bo'lsin.

Bu holda $N(t)=N_0e^{(a-b)(t-t_0)}$ funksiya (4) ni qanoatlantiradi (tekshiring).

b) $A=aN(t)$, $B=bN^2(t)$ hol ham uchraydi.

Bunda

$$N'(t)=aN(t) - bN^2(t) \tag{5}$$

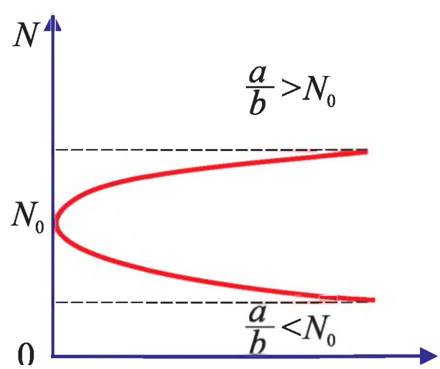
munosabat hosil bo'ladi.

Tekshirish mumkinki, $N(t)=\dfrac{N_0 a/b}{N_0+[a/b-N_0]e^{-a(t-t_0)}}$ funksiya (5) tenglamani qanoatlantiradi. ▲

(4) munosabatni 1845-yilda belgiyalik demograf-olim Ferxyulst populatsiyadagi ichki kurashni hisobga olgan holda kashf qildi. Bu natija Maltusning (2) munosabatiga nisbatan populatsiyaning rivojlanishini aniqroq tavsiflaydi.

Populatsiyaning o'sish-kamayishi a va b sonlarga qanday bog'liq bo'ladi, degan savol tug'ilishi tabiiy.

31-rasmda $\dfrac{a}{b}>N_0$ va $\dfrac{a}{b}<N_0$ hollar uchun $N(t)=\dfrac{N_0 a/b}{N_0+[a/b-N_0]e^{-a(t-t_0)}}$ ko'rinishdagi funksiya grafiklari tasvirlangan:



31-rasm.

Ko‘rinib turibdiki, vaqt kechishi bilan populatsiya soni $\frac{a}{b}$ soniga yaqinlashar ekan. Mazkur holat *to‘yinish* deb nomlangan hodisani bildiradi.

Chizmada tasvirlangan egri chiziq Maltus tomonidan *logistik egri chiziq* deb nomlanib, u inson turmushining turli sohalarida uchrab turadi.

Funksiyaning hosilasini shu funksiya bilan bog‘lovchi $y'(x)=F(x;\ y)$ ko‘rinishdagi munosabat *differensial tenglama* deyiladi.

Yuqorida keltirilgan (1) – (5) munosabatlar differensial tenglamalarga misollardir.

Differensial tenglamani qanoatlantiradigan har qanday funksiya uning yechimi deyiladi. Oliy matematikada muayyan shartlarda $y'(x)=F(x,\ y)$ ko‘rinishdagi differensial tenglamaning $y(x_0)=y_0$ boshlang‘ich shartni qanoatlantiradigan yagona $y(x)$ yechimi mavjudligi isbot qilingan.

3-masala. Vaqtning t momentida sotilayotgan mahsulot haqida xabardor bo‘lgan xaridorlar soni $x(t)$ ning vaqtga bog‘liqligini o‘rganing. (Bu masala reklama samaradorligini aniqlashda muhim.)

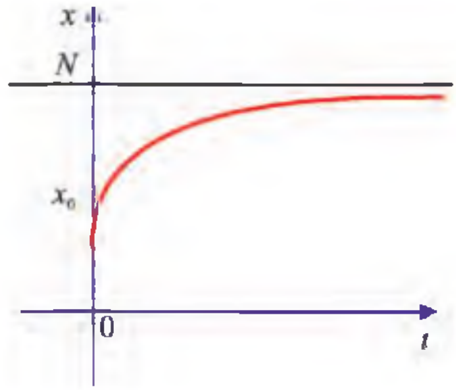
△ Barcha xaridorlar sonini N deb belgilasak, sotilayotgan mahsulotdan bexabarlar soni $N-x(t)$ bo‘ladi.

Mahsulot haqida xabardor bo‘lgan xaridorlar sonining o‘sish tezligi $x(t)$ ga va $N–x(t)$ ga proporsional deb hisoblasak, quyidagi differensial tenglamaga ega bo‘lamiz:

$$x'(t)=kx(t)(N-x(t)),$$ bu yerda $k > 0$ – proporsionallik koeffitsiyenti.

Bu tenglamaning yechimi $x(t)=\dfrac{N}{1+Pe^{-Nkt}}$ dan iborat, bunda $P=\dfrac{1}{e^{NC}}$, C – o‘zgarmas son.

Ravshanki, har qanday holatda t vaqt kechishi bilan Pe^{-Nkt} had kichiklashib boraveradi va bundan $x(t)=\dfrac{N}{1+Pe^{-Nkt}}$ ifodaning qiymati N ga yaqinlashadi (32-rasmga qarang). ▲



32-rasm.

4-masala. Massasi m, issiqlik sig'imi c o'zgarmas bo'lgan jism boshlang'ich momentda T_0 temperaturaga ega bo'lsin. Havo temperaturasi o'zgarmas va τ ($T > \tau$) ga teng. Jismning cheksiz kichik vaqt ichida bergan issiqligi jism va havo temperaturalari orasidagi farqqa, shuningdek, vaqtga proporsional ekanligini e'tiborga olgan holda, jismning sovish qonunini toping.

△ Sovish davomida jism temperaturasi T_0 dan τ gacha pasayadi. Vaqtning t momentida jism temperaturasi $T(t)$ ga teng bo'lsin. Cheksiz kichik vaqt oralig'ida jism bergan issiqlik miqdori, yuqorida aytilganiga ko'ra,

$$Q'(t) = -k(T - \tau)$$

ga teng, bu yerda k – proporsionallik koeffitsiyenti.

Ikkinchi tomondan, fizikadan ma'lumki, jism T temperaturadan τ temperaturagacha soviganda beradigan issiqlik miqdori $Q = mc(T(t) - \tau)$ ga teng. Hosilani hisoblaymiz:

$$Q'(t) = mcT'(t). \qquad (6)$$

$Q'(t)$ uchun topilgan har ikkala ifodani taqqoslab, $mcT'(t) = -k(T - \tau)$ differensial tenglamani hosil qilamiz.

$$T(t) = \tau + Ce^{-\frac{k}{mc}t}$$

funksiya (6) differensial tenglamani qanoatlantiradi (o'zingiz tekshiring!), bu yerda C – ixtiyoriy o'zgarmas son.

Boshlang'ich shart ($t=0$ da $T=T_0$) C ni topishga imkon beradi:

$$C=T_0-\tau.$$

Shuning uchun jismning sovish qonuni quyidagi ko'rinishda yoziladi:

$$T(t)=\tau+(T_0-\tau)e^{-\frac{k}{mc}}.$$

Javob: $T(t)=\tau+(T_0-\tau)e^{-\frac{k}{mc}}$ ▲.

5-masala. Tandirdan olingan (uzilgan) nonning temperaturasi 20 minut ichida 100° dan 60° gacha pasayadi. Tashqi muhit temperaturasi 25°. Nonning temperaturasi qancha vaqtda 30° gacha pasayadi?

△ Yuqoridagi masalaning yechimidan foydalanib, nonning sovish qonunini quyidagi ko'rinishda yoza olamiz:

$$T(t)=\tau+(T_0-\tau)e^{-\frac{k}{mc}}=25+(100-25)e^{at}=25+75e^{at},$$

bu yerda a – noma'lum koeffitsiyent.

a ni topish uchun $t=20$ da $T(20)=60$ tenglikdan foydalanamiz:

$$T(20)=25+75e^{20a}=60,$$

$$75e^{20a}=35,\quad (e^a)^{20}=\frac{35}{75}=\frac{7}{15},\quad e^a=\left(\frac{7}{15}\right)^{\frac{1}{20}}.$$

Demak, nonning sovushi $T=25+75\left(\frac{7}{15}\right)^{\frac{t}{20}}$ qonuniyatga bo'ysunar ekan.

Nonning temperaturasi 30° gacha pasayish vaqtini topamiz:

$$30=25+75\left(\frac{7}{15}\right)^{\frac{t}{20}},\quad \left(\frac{7}{15}\right)^{\frac{t}{20}}=\frac{5}{75}=\frac{1}{15},$$

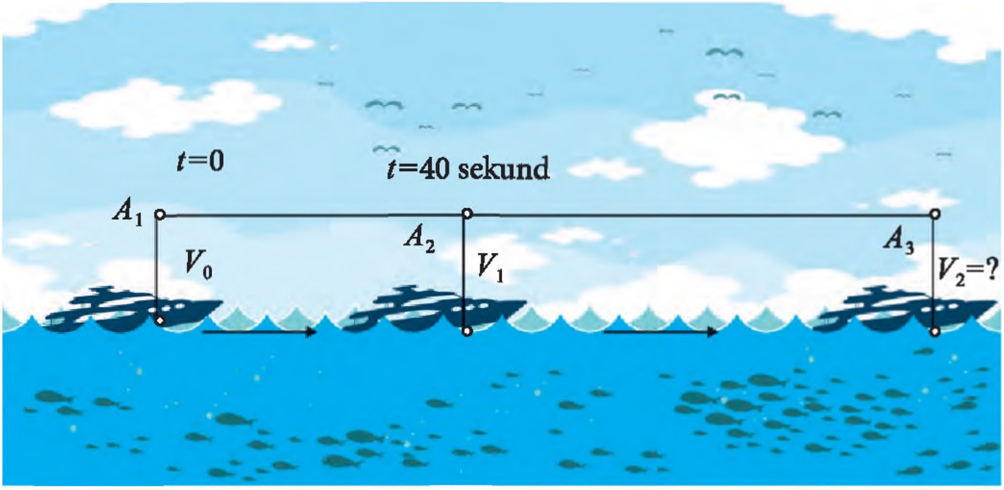
$$\ln\left(\frac{7}{15}\right)^{\frac{t}{20}}=\frac{t}{20}(\ln(7)-\ln(15))$$

bo'lgani uchun $t^*=\dfrac{-20\ln 15}{\ln 7-\ln 15}\approx\dfrac{-20\cdot 2{,}7081}{-0{,}762}\approx 71.$

Javob: 1 soat-u 11 minutda nonning temperaturasi 30° gacha pasayadi. ▲

6-masala. Motorli qayiq turg'un suvda 20 km/soat tezlik bilan harakatlanmoqda. Ma'lum vaqtdan keyin motor ishdan chiqdi. Motor to'xtagandan 40 sekund vaqt o'tgach qayiqning tezligi 8 km/soat bo'ldi.

Suvning qarshiligi tezlikka proporsional bo‘lsa, motor to‘xtagandan 2 minut vaqt o‘tgach qayiq tezligini toping.



33-rasm.

△ Qayiqqa $F = -kv$ kuch ta’sir qilmoqda. Nyuton qonuniga ko‘ra $F=mv'(t)$ Bundan $mv'(t) = -kv$.

Bu tenglamani $v(t)=Ce^{-\frac{k}{m}t}$ ko‘rinishdagi funksiya qanoatlantiradi. $t=0$ da $v=20$ shartidan $C = 20$ kelib chiqadi.

Bundan $v(t)=20e^{-\frac{r}{m}t}$. $t=40\ \text{s}=\frac{1}{90}$ soat bo‘lganda qayiqning tezligi 8 km/soatga teng, bundan $8=20e^{-\frac{r}{m}\cdot\frac{1}{90}}$ yoki $e^{\frac{r}{m}}=\left(\frac{5}{2}\right)^{90}$ hamda

$t=2\text{min}=\frac{1}{30}$ soat bo‘lganidan $v=20\left[\left(\frac{5}{2}\right)^{90}\right]^{-\frac{1}{30}}=20\left(\frac{5}{2}\right)^{-3}=\frac{32}{25}\approx 1,28$

(km/s) ekanini topamiz.

Javob: Motor to‘xtaganidan 2 minut vaqt o‘tgach, qayiqning tezligi taxminan 1,28 km/soat ga teng bo‘ladi. ▲

7-masala. Radioaktiv yemirilish natijasida radioaktiv modda massasi $m(t)$ ning vaqtga nisbatan o‘zgarish qonuniyatini toping. Bu yerda $m(t)$ gramm, t – yillarda o‘lchanadi.

△ Yemirilish tezligi massaga proporsional deb faraz qilsak,

$$m'(t) = -\alpha\, m(t) \qquad (7)$$

differensial tenglamaga ega bo‘lamiz. $m(t)=Ce^{-\alpha t}$ funksiya bu tenglamaning yechimi ekanligini tekshirish mumkin.

$m(t_0) = m_0$ boshlang‘ich shartdan $m(t) = m_0 e^{-\alpha\,(t-t_0)}$ qonuniyatga ega bo‘lamiz. *Javob:* $m(t) = m_0 e^{-\alpha\,(t-t_0)}$. ▲

Iqtisodiy modellar. Talab va taklif iqtisodiyotning fundamental (asosiy) tushunchalaridir.

Talab (tovarlar va xizmatlarga talab) – xaridor, iste’molchining bozordagi muayyan tovarlarni, ne’matlarni sotib olish istagi; bozorga chiqqan va pul imkoniyatlari bilan ta’minlangan ehtiyojlari.

Talab miqdorining o‘zgarishiga bir qancha omillar ta’sir qiladi. Ularning orasida eng muhimi narx omilidir. Tovar narxining pasayishi sotib olinadigan tovar miqdorining o‘sishi va aksincha, narxning o‘sishi xarid miqdorining kamayishiga olib keladi.

Taklif — muayyan vaqtda va muayyan narxlar bilan bozorga chiqarilgan va chiqarilishi mumkin bo‘lgan tovarlar va xizmatlar miqdori bilan ifoda etiladi; taklif – ishlab chiqaruvchi (sotuvchi)larning o‘z tovarlarini bozorda sotishga bo‘lgan istagi. Bozorda tovar narxi bilan uning taklif miqdori o‘rtasida bevosita bog‘liqlik mavjud: narx qanchalik yuqori bo‘lsa, boshqa sharoitlar o‘zgarmagan hollarda, sotish uchun shuncha ko‘proq tovar taklif etiladi, yoki aksincha, narx pasayishi bilan taklif hajmi qisqaradi.

Talab va taklifninng tub mazmuni ularning narx orqali o‘zaro aloqadorlikda mavjud bo‘lishidir. Bu aloqadorlik — talab va taklif qonuni bozor iqtisodiyotining obyektiv qonuni hisoblanadi. Talab va taklif qonuniga ko‘ra, bozordagi taklif va talab faqat miqdoran emas, balki o‘zining tarkibi jihatidan ham bir-biriga mos kelishi kerak, shundagina bozor muvozanatiga erishiladi. Bu qonun ayirboshlash qonuni bo‘lib, bozorni boshqaruvchi va tartiblovchi kuch darajasiga ko‘tariladi. Unga ko‘ra bozordagi talab o‘zgarishlari darhol ishlab chiqarishga yetkazilishi kerak. Bozordagi talab va taklif nisbatiga qarab ishlab chiqarish sur’atlari va tuzilmasi tashkil topadi.

Quyidagi *masalani* qaraylik.

Fermer uzoq muddat davomida mevalarni bozorda sotishga chiqarib keladi. Har hafta yakunida u narxning o‘zgarish tezligini kuzatib, kelgusi haftaga chiqariladigan mevalarning yangi narxini chamalaydi.

Xuddi shunday iste'molchilar ham narxning o'zgarish tezligini kuzatib, kelgusi haftaga sotib olinadigan mevalarning miqdorini belgilaydilar.

Kelgusi haftadagi mevalarning narxini p orqali, narxning o'zgarish tezligini esa p' orqali belgilaylik.

Taklif ham, talab ham tovar narxi bilan uning o'zgarish tezligiga bog'liq ekanligini ishonch bilan aytishimiz mumkin. Bu bog'lanish qanday bo'ladi?

△ Bunday bog'lanishlarning eng sodda ko'rinishi quyidagicha bo'lar ekan: $y = ap' + bp + c$, bu yerda a, b, c – haqiqiy sonlar.

Masalan, q orqali talabni, s orqali esa taklifni belgilasak, ular uchun yuqoridagi bog'lanishlar $q = 4p' - 2p + 39$, $s = 44p' + 2p - 1$ tenglamalar yordamida ifodalanishi mumkin.

Bu holda talab va taklifning o'zaro tengligi $4p' - 2p + 39 = 44p' + 2p - 1$ munosabat yordamida ifodalanadi.

Bu tenglikdan $p' = -\frac{p-10}{10}$ ko'rinishdagi differensial tenglamani hosil qilamiz.

Agar boshlang'ich narxni $p(0) = p_0$ deb belgilasak, narx $p = (p_0 - 10)e^{-\frac{t}{10}} + 10$ qonuniyat bilan o'zgarishini hosil qilamiz. ▲

Investitsiya. Faraz qilaylik, qandaydir mahsulot p narx bilan sotiladi, $Q(t)$ funksiya t vaqt mobaynida ishlab chiqarilgan mahsulot miqdori o'zgarishini bildiradi desak, u holda t vaqt davomida $pQ(t)$ ga teng daromad olinadi. Aytaylik, olingan daromadning bir qismi mahsulot ishlab chiqarish investitsiyasiga sarf bo'lsin, ya'ni

$$I(t) = mpQ(t), \tag{8}$$

m – investitsiya normasi, o'zgarmas son va $0 < m < 1$.

Agar bozor yetarlicha ta'minlangan va ishlab chiqarilgan mahsulot to'la sotilgan degan tasavvurdan kelib chiqilsa, bu holat ishlab chiqarish tezligining yana oshishiga olib keladi.

Ishlab chiqarish tezligi esa investitsiyaning o'sishiga proporsional, ya'ni

$$Q' = l \cdot I(t), \tag{9}$$

bu yerda l – proporsionallik koeffitsiyenti.

(8) formulani (9) ga qo'yib

$$Q' = kQ, \quad k = lmp \qquad (10)$$

differensial tenglamani hosil qilamiz.

C – ixtiyoriy o'zgarmas son bo'lganda $Q = Ce^{kt}$ ko'rinishdagi funksiya (10) differensial tenglamani qanoatlantiradi.

Faraz qilaylik, boshlang'ich moment $t = t_0$ da mahsulot ishlab chiqarish hajmi Q_0 berilgan. U holda bu shartdan o'zgarmas C ni topish mumkin:

$$Q_0 = Ce^{kt_0}, \text{ bundan } C = Q_0 e^{-kt_0}.$$

Natijada ishlab chiqarish hajmi $Q = Q_0 e^{k(t-t_0)}$ qonuniyat bilan o'zgarishini bilib olamiz.

? Savol va topshiriqlar

1. Bakteriyalarning ma'lum vaqtdan so'ng ikkiga bo'lina borishi jarayonini hosila yordamida modellashtiring.
2. Tomas Maltusning yer yuzidagi aholi soni o'sishiga oid masalasini tushuntiring.
3. Tomas Maltusning logistik egri chizig'ini tushuntiring.
4. Reklama samaradorligiga oid masalani hosila yordamida modellashtiring.

Mashqlar

Matndagi 4-masala yechimidan foydalanib, mashqlarni bajaring **(107–108)**:

107. Temperaturasi 25°C bo'lgan metall parchasi pechga qo'yildi. Pechning temperaturasi 25°C dan boshlab minutiga 20°C tezlik bilan tekis ravishda ko'tarila boshladi. Pech va metall temperaturasining farqi T°C bo'lganda, metall minutiga $10 \cdot T$°C tezlik bilan isitila boshlaydi. Metall parchasini 30 minutdan keyingi temperaturasini toping.

108. Jismning boshlang'ich temperaturasi 5°C. Jism N minut davomida 10°C gacha isidi. Atrof-muhit temperaturasi 25°C bo'lib turibdi. Jism qachon 20°C gacha isiydi?

Matndagi 7-masala yechimidan foydalanib, mashqlarni bajaring:

109. Tajribalarga ko'ra 1 yil davomida radiyning har bir grammidan 0,44 mg modda yemiriladi

a) necha yildan so'ng mavjud radiyning 20 foizi yemiriladi?

b) mavjud radiyning 400 yildan so'ng necha foizi qoladi?

Matndagi 6-masalani yechishdagi mulohazalardan foydalanib, mashqlarni bajaring **(110 – 111):**

110. Qayiq suvning qarshiligi ta'siri ostida o'z harakatini sekinlashtiradi. Suvning qarshiligi qayiq tezligiga proporsional. Qayiqning boshlang'ich tezligi 1,5 m/s. 4 sekunddan so'ng uning tezligi 1 m/s ni tashkil qildi.Necha sekunddan so'ng qayiqning tezligi 2 marta kamayadi?

111. 10 *l* hajmdagi idish havo bilan to'ldirilgan (80% azot, 20% kislorod). Shu idishga 1 sekundda 1 litr tezlikda azot purkalmoqda. U uzluksiz ravishda aralashib, shu tezlikda idishdan chiqmoqda. Qancha vaqtdan so'ng idishda 95% azotli aralashma hosil bo'ladi?

Ko'rsatma: $y(t)$ bilan t vaqtdagi azot ulushini belgilasak, $y(t)$ funksiya $y' \cdot V = a(1-y)$ munosabatni qanoatlantiradi deylik. Bu yerda V – isitish hajmi, a – purkash tezligi.

Nazorat ishi namunasi

1. Asosi kvadrat bo'lgan to'g'ri burchakli parallelepiped shaklidagi usti ochiq metall idish yasashmoqchi. Idish hajmi 270 *l* bo'lishi kerak. Idishning o'lchamlari qanday bo'lganda uni yasashda eng kam metall ketadi?

2. Moddiy nuqta $s(t) = -\frac{t^4}{4} + 72t^3$ qonuniyat bilan harakatlanmoqda ($s(t)$ metrda, t vaqt sekundda o'lchanadi).

1) eng katta tezlanishga erishadigan vaqtni (t_0);

2) t_0 vaqtdagi oniy tezlikni;

3) t_0 vaqt mobaynida bosib o'tilgan yo'lni toping.

3. Taqribiy hisoblash formulasidan foydalanib $\ln 0,92$ ni toping.

4. Taqribiy hisoblash formulasidan foydalanib sin(–1; 2)ni toping.

5. Mahsulot ishlab chiqaruvchi tadbirkorning kunlik daromadi quyidagi formula bilan hisoblanadi:

$P(x) = -3x^2 + 42x - 6$ (ming so'm), bu yerda x – mahsulotlar soni.

Quyidagilarni aniqlang:

1) eng katta daromad olish uchun tadbirkor nechta mahsulot ishlab chiqarishi kerak?

2) tadbirkorning eng katta daromadi necha so'mni tashkil qiladi?

MASALALAR YECHISH

112. Moddiy nuqta harakatining qonuni $s=s(t)$ ga ko‘ra uning eng katta yoki eng kichik tezligini toping:

1) $s=13t$; | 2) $s=17t-5$; | 3) $s=t^2+5t+18$; | 4) $s=t^3+2t^2+5t+8$;
5) $s=2t^3+5t^2+6t+3$; | 6) $s=13t^3+2t^2$; | 7) $s=t^3+t^2+3$.

113. Berilgan funksiya grafigiga: 1) $x_0=-1$; 2) $x_0=2,2$; 3) $x_0=0$ abssissali nuqtada o‘tkazilgan urinmani toping:

1) $f(x)=12x^2+5x+1$; | 2) $f(x)=13x+4$; | 3) $f(x)=60$; | 4) $f(x)=x^3+4x$.

114. Berilgan funksiya uchun $y=-7x+2$ to‘g‘ri chiziqqa parallel bo‘lgan urinma tenglamasini yozing:

1) $f(x)=5x^3-2x^2+16$; | 2) $f(x)=-4x^2+5x+3$; | 3) $f(x)=-8x+5$.

115. Berilgan $f(x)$ va $g(x)$ funksiyalar grafiklarining urinmalari parallel bo‘ladigan nuqtalarini toping:

1) $f(x)=2x^2-3x+4$, $g(x)=12x-8$;
2) $f(x)=18x+19$, $g(x)=-15x+18$;
3) $f(x)=2x+13$, $g(x)=4x-19$;
4) $f(x)=2x^3$, $g(x)=4x^2$;
5) $f(x)=2x^3+3x^2$, $g(x)=15x-17$;
6) $f(x)=2x^4$, $g(x)=4x^3$.

116. 1) $y=\frac{1}{x}$ funksiya grafigining $x=-\frac{1}{2}$ nuqtadan o‘tuvchi urinmasi tenglamasini tuzing.

2) $y=x^2$ parabolaning $x=1$ va $x=3$ abssissalarga mos nuqtalari tutashtirilgan. Parabolaning ushbu 2 nuqtani tutashtiruvchi kesmaga parallel bo‘lgan urinmasi qaysi nuqtadan o‘tadi?

3) Moddiy nuqta $s(t)=\frac{2}{9}\cdot\sin\frac{\pi t}{2}+3$ qonuniyat bilan harakatlanmoqda (s – santimetrda, t – sekundda). Moddiy nuqtaning 1-sekunddagi tezlanishini toping.

117. Funksiyaning ko‘rsatilgan nuqtadagi hosilasini hisoblang:

1) $f(x)=x^2-15$, $x_0=-\frac{1}{2}$; 2) $f(x)=3\cos x$, $x_0=-\pi$;

3) $f(x)=\frac{3}{x}$, $x_0=-2$; 4) $f(x)=-\sin x$, $x_0=-\frac{\pi}{3}$.

5) $f(x)=x^3-4$, $x_0=5$; 6) $f(x)=\sin x$, $x_0=\frac{\pi}{6}$;

7) $f(x)=\frac{1}{x^3}$, $x_0=-2$; 8) $f(x)=\cos 5x$, $x_0=\frac{\pi}{4}$;

9) $f(x)=-\cos 2x$, $x_0=-\frac{\pi}{8}$.

118. Ko'rsatilgan vaqtdagi tezlik va tezlanishni toping:

1) $s(t)=5t^2-t+50$ $t_0=2$; 2) $s(t)=t^3+12t^2+1$, $t_0=1$;

3) $s(t)=2t+t^3$, $t_0=5$; 4) $s(t)=8\sin t$, $t_0=\frac{\pi}{2}$.

119. Funksiyaning abssissasi ko'rsatilgan nuqtadagi hosilasini hisoblang:

1) $f(x)=x^2-15$, $x_0=\frac{1}{2}$; 2) $f(x)=3\cos x$, $x_0=\pi$;

3) $f(x)=\frac{3}{x}$, $x_0=2$; 4) $f(x)=-\sin x$, $x_0=\frac{\pi}{3}$.

5) $f(x)=x^3-4$, $x_0=-5$; 6) $f(x)=\sin x$, $x_0=-\frac{\pi}{6}$;

7) $f(x)=\frac{1}{x^3}$, $x_0=2$; 8) $f(x)=\cos 5x$, $x_0=-\frac{\pi}{4}$;

9) $f(x)=-\cos 2x$, $x_0=\frac{\pi}{8}$; 10) $f(x)=\sin 2x$, $x_0=\frac{\pi}{4}$.

120. Ko'rsatilgan vaqtdagi tezlik va tezlanishni toping:

1) $s(t)=3t^2-2t+10$, $t_0=2$; 2) $s(t)=t^3-6t^2+1$, $t_0=1$;

3) $s(t)=5t+2t^3$, $t_0=5$; 4) $s(t)=8\cos t$, $t_0=\frac{\pi}{2}$.

Berilgan funksiyaning hosilasini toping (**121–122**):

121. 1) $f(x)=-x^2+x+30$; | 2) $f(x)=\sin x-\cos x$; | 3) $f(x)=\sqrt{x}-\frac{1}{x}$;

4) $f(x)=4^x-\sin x$; | 5) $f(x)=8\cos x$; | 6) $f(x)=\ln x-10x^2+x-1$.

122. 1) $y=x^4$; | 2) $y=\frac{x-1}{x+2}$; | 3) $y=x-\frac{20}{x}$; | 4) $y=x^2\ln x$;

5) $y=x^3\sin x$; | 6) $y=e^x\sin x$; | 7) $y=\frac{x+1}{4x^2}$; | 8) $y=2(10x-1)\sin x$.

123. Berilgan funksiyalar uchun $f'(-\frac{\pi}{2})$, $f'(\frac{\pi}{4})$ sonlarni hisoblang:

1) $f(x)=e^x\cos x$; 2) $f(x)=3x+1$; 3) $f(x)=2x^2+x+3$;

4) $f(x)=\sin x+x^2$; 5) $f(x)=\sin x+\cos x$; 6) $f(x)=\sin x$;

7) $f(x)=\cos x+x^4$; 8) $f(x)=\sin 3x+\cos 3x$.

124. Moddiy nuqta $x(t)=-\frac{t^3}{6}+6t^2+15$ qonuniyat bilan harakatlanmoqda.

1) tezlanish nol boʻlgan t_0 vaqtni; 2) shu t_0 vaqtdagi tezlikni toping.

125*. $f(x)=x^2-13x+2$ funksiya Ox oʻqi bilan qanday burchak ostida kesishadi?

126. $f'(0)$ sonni toping: 1) $f(x)=x^6-4x^3+4$; | 2) $f(x)=(x+10)^6$.

127. $y'(x)$ ni toping: 1) $y(x)=\sin^2 x$; | 2) $y(x)=\cos^2 x$; | 3) $y(x)=\mathrm{tg}^2 x$.

128. Funksiyaning oʻsish va kamayish oraliqlarini toping:

1) $f(x)=3+7x$; 2) $f(x)=x^3+17x$; 3) $f(x)=\frac{1}{4}x+18$;

4) $f(x)=\frac{x+21}{x}$; 5) $f(x)=x^2+5x-14$; 6) $f(x)=x(x^2+8)$;

7) $f(x)=-x^2-4x+6$; 8) $f(x)=-\frac{1}{x^2}$;

9) $f(x)=x^3-12x^2-17x-23$; 10) $f(x)=3x^4+18x^3-6$;

11) $f(x)=x^3-5x^2+19x+22$; 12) $f(x)=x^4+7x^2$.

129. Funksiyaning statsionar nuqtalarini toping:

1) $f(x)=3x^2-7x+9$; | 2) $f(x)=19x-\frac{1}{7}x^3$; 3) $f(x)=5x^3$;

4) $f(x)=8x^2$; 5) $f(x)=7x-14$; 6) $f(x)=27-x^3$;

7) $f(x)=12x^3+13x^2-16$; 8) $f(x)=x^3-6x^2+9$.

130. Funksiyaning lokal maksimum va lokal mimimumlarini toping:

1) $f(x)=x^2-\frac{1}{4}x^4$; 2) $f(x)=14+13x^2-12x^3$;

3) $f(x)=x^4-3x^3+x^2+9$; 4) $f(x)=2x^4-x^3+7$.

131. Funksiyaning o‘sish, kamayish oraliqlari hamda lokal maksimum va minimumlarini toping:

1) $f(x)=x^3-64x$; 2) $f(x)=2x^3-24$; 3) $f(x)=4x^3-108x$.

132. Funksiyaning eng katta va eng kichik qiymatlarini toping:

1) $f(x)=x^4-3x^2+2$, $x\in[-4; 1]$; 2) $f(x)=x^5+6x^3+1$, $x\in[-1; 2]$;

3) $f(x)=\frac{x}{x+4}$, $x\in[1; 5]$; 4) $f(x)=x^3+6x^2+5x+8$, $x\in[-3; 4]$.

133. Funksiyaning grafigini yasang:

1) $y=x^3-2x^2+3x-2$; 2) $y=\frac{1}{5}x^5+\frac{2}{3}x^3$; 3) $y=x^4+4x^3$.

134. To‘g‘ri to‘rtburchak shaklidagi ekin maydonining atrofini o‘rash uchun 1000 metr panjara sotib olindi. Bu panjara yordamida eng ko‘pi bilan necha kvadrat metr maydonni o‘rab olish mumkin?

135. Tomoni 16 dm bo‘lgan kvadrat shaklidagi kartondan usti ochiq quti tayyorlandi. Bunda kartonning uchlaridan bir xil kvadratchalar kesib olindi. Qutining hajmi eng katta bo‘lishi uchun uning asosi necha santimetr bo‘lishi kerak?

136*. Konserva banka silindr shaklida bo‘lib, uning to‘la sirti 512π cm^2 ga teng. Bankaga eng ko‘p suv sig‘ishi uchun banka asosining radiusi va balandligi qanday bo‘lishi kerak?

137. To‘g‘ri to‘rtburchak shaklidagi maydoning yuzi 3600 m^2. Maydonning tomonlari qanday bo‘lganda uni o‘rash uchun eng kam panjara zarur bo‘ladi?

138*. Radiusi 8 dm bo‘lgan sharga eng kichik hajmli konus tashqi chizilgan. Shu konus balandligini toping.

139*. Asosi kvadrat bo‘lgan to‘g‘ri burchakli parallelepiped shaklidagi ochiq metall idishga 32 *l* suyuqlik ketadi. Idishning o‘lchamlari qanday bo‘lganda uni yasashga eng kam metall sarflanadi?

140. Moddiy nuqta $s(t)=-\frac{t^4}{4}+10t^3$ qonuniyat bilan harakatlanmoqda ($s(t)$ metrda, t sekundda o'lchanadi).

1) eng katta tezlanishga erishadigan (t_0) vaqtni;
2) t_0 vaqtdagi oniy tezlikni;
3) t_0 vaqtda bosib o'tilgan yo'lni toping.

141. Havo shariga $t\in[0;10]$ minut oralig'da $V(t)=t^3+3t^2+2t+4$ m^3 havo purkalmoqda.

1) boshlang'ich vaqtdagi havo hajmini;
2) $t=10$ minutdagi havo hajmini;
3) $t=5$ minutdagi havo purkash tezligini toping.

142. Akrom shim tikish uchun buyurtma oldi. Bir oyda x ta shim tiksa, $p(x)=-2x^2+240x$ (ming so'm) daromad qiladi.

1) daromadni eng katta qilish uchun qancha shim tikish kerak?
2) eng katta daromad necha so'm bo'ladi?

143. Funksiyaning hosilasini toping:

1) $y=e^{3x}$;	2) $y=e^{\sin x}$;	3) $y=\sin(3x+2)$;	4) $y=(2x+1)^4$;
5) $y=\frac{x-2}{x^2+1}$;	6) $y=\frac{\ln x}{x}$;	7) $y=\mathrm{arctg}2x$;	8) $y=x^2\cdot\cos x$.

144. $f(x)=e^{2x}$ va $g(x)=4x+2$ funksiyalar uchun $F(x)$ murakkab funksiyani tuzing:

1) $F(x)=f(g(x))$;
2) $F(x)=f(x)^{g(x)}$;
3) $F(x)=g(f(x))$;
4) $F(x)=\sqrt{g(g(x))}$.

145. Murakkab funksiyaning hosilasini toping:

1) $y=(x^2+1)^5$;
2) $y=\ln\cos x$;
3) $y=\sqrt{5x-7}$;
4) $y=\sqrt{\mathrm{tg}(2x-3)}$;
5) $y=\mathrm{arctg}(3x-4)$;
6*) $y=\sin(\mathrm{arctg}2x)$;
7) $y=\sin^3 x+\cos^3 x$;
8*) $y=e^{\sin(\cos x)}$.

146. Funksiyaning o'sish va kamayish oraliqlarini toping:

1) $y=2+x-x^2$;
2) $y=\frac{\sqrt{x}}{x+100}$ $(x\geq 0)$;
3) $y=3x-x^3$;
4) $y=2x-\sin x$;
5) $y=\frac{2x}{1+x^2}$;
6) $y=\frac{x^2}{2^x}$;
7) $y=(x-1)^3$;
8) $y=(x-1)^4$.

147. Funksiyaning statsionar nuqtalari, lokal maksimum va lokal minimumlarini toping:

1) $y=x^3-6x^2+9x-4$;
2) $y=\frac{2x}{1+x^2}$;
3) $y=x+\frac{1}{x}$;
4) $y=\sqrt{2x-x^2}$.

148. Funksiyaning ko'rsatilgan oraliqdagi eng katta va eng kichik qiymatlarini toping:

1) $f(x)=2^x$, $[-1;5]$;
2) $f(x)=x^2-4x+6$, $[-3;10]$;
3) $f(x)=x+\frac{1}{x}$, $[0{,}01;100]$;
4) $f(x)=\sqrt{5-4x}$, $[-1;1]$;
5) $f(x)=\cos x$, $\left[-\frac{\pi}{2};\pi\right]$;
6) $f(x)=|x^2-3x+2|$, $[-10;10]$;
7) $f(x)=\sin x$, $\left[\frac{\pi}{2};\pi\right]$;
8) $f(x)=|x^2+3x+2|$, $[-15;10]$.

149. Funksiyani tekshiring va grafigini yasang:

1) $y=3x-x^3$;
2) $y=1+x^2-\frac{x^4}{2}$;
3) $y=(x+1)(x-2)^2$;
4) $y=x+\frac{1}{x}$;
5) $y=\sqrt{16-x^2}$;
6) $y=\sqrt{x^2-9}$;
7) $y=x^2-5|x|+6$;
8) $y=\frac{1}{4}x^4-\frac{1}{2}x^2$.

II BOB. INTEGRAL VA UNING TATBIQLARI

37–39 BOSHLANG‘ICH FUNKSIYA VA ANIQMAS INTEGRAL TUSHUNCHALARI

Agar nuqta harakat boshlanganidan boshlab t vaqt mobaynida $s(t)$ masofani o‘tgan bo‘lsa, uning oniy tezligi $s(t)$ funksiyaning hosilasiga teng ekanini bilasiz: $v(t)=s'(t)$. Amaliyotda *teskari masala:* nuqtaning berilgan harakat tezligi $v(t)$ bo‘yicha uning bosib o‘tgan yo‘li $s(t)$ ni topish masalasi ham uchraydi. Shunday $s(t)$ funksiyani topish kerakki, uning hosilasi $v(t)$ bo‘lsin. Agar $s'(t)=v(t)$ bo‘lsa, $s(t)$ funksiya $v(t)$ funksiyaning *boshlang‘ich funksiyasi* deyiladi. Umuman, shunday ta’rif kiritish mumkin:

Agar $(a;\ b)$ ga tegishli ixtiyoriy x uchun $F'(x)=f(x)$ bo‘lsa, $F(x)$ funksiya $(a;\ b)$ oraliqda $f(x)$ ning *boshlang‘ich funksiyasi* deyiladi.

1-misol. a – berilgan biror son va $v(t)=at$ bo‘lsa, $s(t)=\frac{1}{2}at^2$ funksiya $v(t)$ funksiyaning boshlang‘ichidir, chunki $s'(t)=(\frac{at^2}{2})'=at=v(t)$.

2-misol. $f(x)=x^2$, $x\in(-\infty;\ \infty)$, bo‘lsa, $F(x)=\frac{1}{3}x^3$ funksiya $f(x)$ ning $(-\infty;\ \infty)$ dagi boshlang‘ich funksiyasi bo‘ladi, chunki

$$F'(x)=(\frac{1}{3}x^3)'=\frac{1}{3}\cdot 3x^2=x^2=f(x).$$

3-misol. $f(x)=\frac{1}{\cos^2 x}$, bunda $x\neq\frac{\pi}{2}+k\pi$, $k\in Z$, funksiya uchun $F(x)=\mathrm{tg}x$ boshlang‘ich funksiya bo‘ladi, chunki $(\mathrm{tg}x)'=\frac{1}{\cos^2 x}$.

4-misol. $f(x)=\frac{1}{x}, x>0$, bo‘lsa, $F(x)=\ln x$ funksiya $\frac{1}{x}$ ning boshlang‘ich funksiyasi bo‘ladi, chunki $F'(x)=(\ln x)' = \frac{1}{x}$.

1-masala. $F_1(x)=\frac{x^4}{4}$, $F_2(x)=\frac{x^4}{4}+17$, $F_3(x)=\frac{x^4}{4}-25$ funksiyalar ayni bir $f(x)=x^3$ funksiyaning boshlang‘ich funksiyalari ekanini isbotlang.

△ Hosilalar jadvaliga muvofiq yoza olamiz:

1) $F_1'(x)=(\frac{x^4}{4})'=4\cdot\frac{x^3}{4}=x^3=f(x);$

2) $F_2'(x)=(\frac{x^4}{4}+17)'=(\frac{x^4}{4})'+(17)'=4\cdot\frac{x^3}{4}+0+x^3=x^3=f(x);$

3) $F_3'(x)=(\frac{x^4}{4}-25)'=(\frac{x^4}{4})'-(25)'=4\cdot\frac{x^3}{4}-0=x^3=f(x).$

Bu masaladan shunday xulosaga kelish mumkin: ixtiyoriy $F(x)=\frac{x^4}{4}+C$ funksiya (C – biror o‘zgarmas son) ham $f(x)=x^3$ uchun boshlang‘ich funksiya bo‘la oladi. Chindan ham, $F'(x)=(\frac{x^4}{4}+C)'=(\frac{x^4}{4})'+C'=4\cdot\frac{x^3}{4}+0=x^3=f(x)$. ▲

Bu masaladan yana shunday xulosaga kelish mumkin: berilgan $f(x)$ funksiya uchun uning boshlang‘ich funksiyasi bir qiymatli aniqlanmaydi.

Agar $F(x)$ funksiya $f(x)$ ning biror oraliqlari boshlang‘ich funksiyasi bo‘lsa, $f(x)$ funksiyaning barcha boshlang‘ichlari $F(x)+C$ (C – ixtiyoriy o‘zgarmas son) ko‘rinishida yoziladi.
$F(x)+C$ ko‘rinishidagi barcha funksiyalar to‘plami $f(x)$ ning *aniqmas integrali* deyiladi va $\int f(x)dx$ kabi belgilanadi.

Demak, $\int f(x)dx = F(x)+C.$

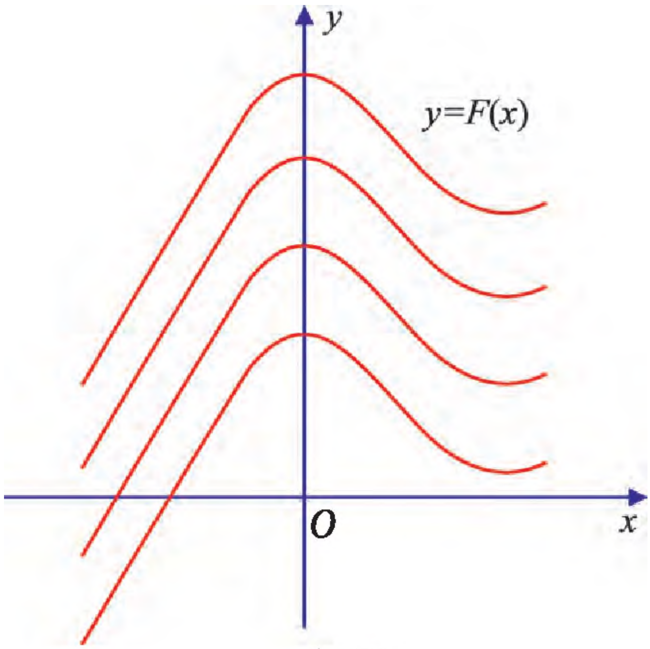
$\int$ – integral belgisi, $f(x)$ – integral ostidagi funksiya, $f(x)dx$ esa integral ostidagi ifoda deyiladi.

5-misol. $\int a^x dx=\frac{a^x}{\ln a}+C$, chunki hosilalar jadvaliga ko‘ra,

$$(\frac{a^x}{\ln a}+C)'=(a^x)'\cdot\frac{1}{\ln a}+C'=a^x\cdot\ln a\cdot\frac{1}{\ln a}+0=a^x.$$

6-misol. $\int x^k dx = \frac{x^{k+1}}{k+1} + C,\ k \neq -1,$ chunki $(\frac{x^{k+1}}{k+1} + C)' = \frac{1}{k+1} \cdot (x^{k+1})' + C' =$
$= \frac{k+1}{k+1} \cdot x^k + 0 = x^k$. $k = -1$ bo‘lsa, $x > 0$ da 4-misolga ko‘ra, $\int \frac{dx}{x} = \ln x + C$.

$y = F(x) + C$ funksiyaning grafigi $y = F(x)$ funksiya grafigini Oy o‘q bo‘ylab siljitishdan hosil qilinadi (1-rasm). O‘zgarmas son C ni tanlash hisobiga boshlang‘ich funksiya grafigining berilgan nuqta orqali o‘tishiga erishish mumkin.



1-rasm.

2-masala. $f(x) = x^2$ funksiyaning grafigi $A(3;\ 10)$ nuqtadan o‘tadigan boshlang‘ich funksiyasini toping.

△ $f(x) = x^2$ funksiyaning barcha boshlang‘ich funksiyalari $F(x) = \frac{x^3}{3} + C$ ko‘rinishda bo‘ladi, chunki $F'(x) = (\frac{x^3}{3} + C)' = \frac{1}{3} \cdot 3x^2 + C' = x^2 + 0 = x^2$.

O‘zgarmas son C ni $F(x) = \frac{x^3}{3} + C$ funksiyaning grafigi (3; 10) nuqtadan o‘tadigan qilib tanlaymiz: $x=3$ da $F(3) = 10$ bo‘lishi kerak. Bundan

$10=\frac{3^3}{3}+C$, $C=1$. Demak, izlanayotgan boshlang'ich funksiya $F(x)=\frac{x^3}{3}+1$ bo'ladi. *Javob:* $\frac{x^3}{3}+1$. ▲

3-masala. $f(x)=\sqrt[3]{x}$ funksiyaning grafigi $A(8;15)$ nuqtadan o'tadigan boshlang'ich funksiyasini toping.

△ $f(x)=\sqrt[3]{x}$ ning barcha boshlang'ich funksiyalari $F(x)=\frac{3}{4}\cdot x^{\frac{4}{3}}+C$ ko'rinishida bo'ladi, chunki

$$F'(x)=\left(\frac{3}{4}\cdot x^{\frac{4}{3}}+C\right)'=\frac{3}{4}(x^{\frac{4}{3}})'+C'=\frac{3}{4}\cdot\frac{4}{3}\cdot x^{\frac{1}{3}}+C'=x^{\frac{1}{3}}+0=\sqrt[3]{x}.$$

O'zgarmas son C ni shunday tanlaymizki, $F(x)=\frac{3}{4}x^{\frac{4}{3}}+C$ funksiyaning grafigi $A(8,\ 15)$ nuqtadan o'tsin, ya'ni $F(8)=15$ tenglik bajarilsin. $x^{\frac{4}{3}}=x\sqrt[3]{x}$ ekanidan $15=\frac{3}{4}\cdot 8\cdot\sqrt[3]{8}+C$, bundan $C=3$. Demak, izlanayotgan boshlang'ich funksiya $F(x)=\frac{3}{4}x\sqrt[3]{x}+3$ bo'ladi. *Javob:* $\frac{3}{4}x\sqrt[3]{x}+3$. ▲

4*-masala. $\int\frac{dx}{x}=\ln|x|+C$ ekanini ko'rsating.

△ $x>0$ da $\int\frac{dx}{x}=\ln x+C$, chunki $(\ln x+C)'=\frac{1}{x}+0=\frac{1}{x}$;

$x<0$ da $\int\frac{dx}{x}=\ln(-x)+C$, chunki $(\ln(-x)+C)'=\frac{(-1)}{(-x)}+0=\frac{1}{x}$. ▲

? Savol va topshiriqlar

1. Boshlang'ich funksiya nima? Misollar keltiring.
2. Berilgan $f(x)$ funksiya uchun boshlang'ich funksiya bir qiymatli topiladimi? Nima uchun?
3. Boshlang'ich funksiya $F(x)$ ning grafigini berilgan nuqtadan o'tishiga qanday qilib erishish mumkin? Misolda tushuntiring.

Mashqlar

1. Haqiqiy sonlar to'plami $R=(-\infty;\ \infty)$ da $f(x)$ funksiya uchun $F(x)$ funksiyaning boshlang'ich funksiya ekanini isbotlang:

1) $F(x)=x^2-\sin 2x+2018,\qquad f(x)=2x-2\cos 2x;$

2) $F(x)=-\cos\frac{x}{2}-x^3+28,\qquad f(x)=\frac{1}{2}\sin\frac{x}{2}-3x^2;$

3) $F(x)=2x^4+\cos^2 x+3x,\qquad f(x)=8x^3-\sin 2x+3;$

4) $F(x)=3x^5+\sin^2 x-7x,\qquad f(x)=15x^4+\sin 2x-7.$

Quyidagi funksiyalarning barcha boshlang'ich funksiyalarini, hosilalar jadvalidan foydalanib toping (**2 – 6**):

2. 1) $f(x)=x^2\cdot\sqrt{x};$ 2) $f(x)=6x^5;$ 3) $f(x)=x^{10};$ 4) $f(x)=\frac{2}{3}\cdot\sqrt{x};$

5) $f(x)=\sin x;$ | 6) $f(x)=\cos x;$ | 7) $f(x)=\sin 2x;$ | 8) $f(x)=\cos 2x;$

3. 1) $f(x)=4^x;$ 2) $f(x)=\pi^x;$ 3) $f(x)=e^x;$ 4) $f(x)=a^x;$

5) $f(x)=a^{2x};$ 6) $f(x)=e^{\pi x};$ 7) $f(x)=10^{3x};$ 8) $f(x)=e^{2x+3}.$

4. 1) $f(x)=\frac{1}{2x+3};$ 2) $f(x)=\frac{1}{4x-5};$ 3) $f(x)=\frac{1}{2x+7};$

4) $f(x)=\frac{1}{ax};$ 5) $f(x)=\frac{1}{ax+b};$ 6) $f(x)=\frac{a}{ax-b}.$

5. 1) $f(x)=\sin 3x;$ 2) $f(x)=\sin(2x+5);$ 3) $f(x)=\sin(4x+\pi);$

4) $f(x)=\cos 5x;$ 5) $f(x)=\cos(3x-2);$ 6) $f(x)=\cos(2x+\frac{\pi}{2}).$

6. 1) $f(x)=\frac{1}{x^2};$ | 2) $f(x)=\frac{1}{x^5};$ | 3) $f(x)=(3x+2)^2;$ | 4) $f(x)=(2x-1)^3.$

7. Berilgan $f(x)$ funksiya uchun uning ko'rsatilgan A nuqtadan o'tuvchi boshlang'ich funksiyasini toping:

1) $f(x)=2x+3,\quad A(1;\ 5);$ 2) $f(x)=-x^2+2x+5,\quad A(0;\ 2);$

3) $f(x)=\sin x,\quad A(0;\ 3);$ 4) $f(x)=\cos x,\quad A(\frac{\pi}{2};\ 5).$

Berilgan $f(x)$ funksiya uchun uning shunday boshlang‘ich funksiyasini topingki, bu boshlang‘ich funksiyaning grafigi y to‘g‘ri chiziq bilan faqat bitta umumiy nuqtaga ega bo‘lsin (**8 – 9**):

8. 1) $f(x)=4x+8$, $y=3$; 2) $f(x)=3-x$, $y=7$,

3) $f(x)=4{,}5x+9$, $y=6{,}8$; 4) $f(x)=2x-6$, $y=1$.

9*. $f(x)=ax+b$, $y=k$.

Ko‘rsatma: $F(x)=\frac{ax^2}{2}+bx+C$, masala shartidan va $\frac{ax^2}{2}+bx+C=k$ kvadrat tenglamadan C ni toping. $C=\frac{2ak+b^2}{2a}=k+\frac{b^2}{2a}$ bo‘ladi.

10*. $f(x)$ uchun uning shunday boshlang‘ich funksiyasini topingki, bu boshlang‘ich funksiyaning grafigi ko‘rsatilgan nuqtalardan o‘tsin:

1) $f'(x)=\frac{16}{x^3}$, $A(1;\,10)$ va $B(4;\,-2)$;

2) $f'(x)=\frac{54}{x^4}$, $A(-1;\,4)$ va $B(3;\,4)$;

3) $f'(x)=6x$, $A(1;6)$ va $B(3;30)$;

4) $f'(x)=20x^3$; $A(1;9)$ va $B(-1;7)$.

Ko‘rsatma: Berilgan $f'(x)$ bo‘yicha $f(x)+C_1$ topiladi. So‘ngra $f(x)+C_1$ uchun boshlang‘ich funksiyasi $F(x)=\int f(x)dx+C_1x+C_2$ topiladi. Berilgan nuqtalar koordinatalarini oxirgi tenglikka qo‘yib, C_1 va C_2 sonlarni topish uchun chiziqli tenglamalar sistemasiga kelinadi.

11*. Berilgan $f(x)$ funksiya uchun uning shunday boshlang‘ich funksiyasini topingki, bu boshlang‘ich funksiyaning grafigi bilan $f(x)$ hosilasining grafigi abssissasi ko‘rsatilgan nuqtada kesishsin:

1) $f(x)=(3x-2)^{\frac{1}{3}}$, $x_0=1$; 2) $f(x)=(4x+5)^{\frac{1}{4}}$, $x_0=-1$;

3) $f(x)=(7x-5)^{\frac{1}{7}}$, $x_0=1$; 4) $f(x)=(kx+b)^{\frac{1}{k}}$, $x_0=\frac{1-b}{k}$.

12. Berilgan $f(x)$ funksiya uchun ko'rsatilgan nuqtadan o'tuvchi boshlang'ich funksiyani toping:

1) $f(x)=\frac{5}{x-2}$, $A(3;\ 7)$; 2) $f(x)=\frac{3}{x+1}$, $A(0;\ 1)$;

3) $f(x)=\cos x$, $A(\frac{\pi}{2};\ 8)$; 4) $f(x)=\sin x$, $A(\pi;\ 10)$.

13. $F(x)$ funksiya son o'qida $f(x)$ funksiyaning boshlang'ich funksiyasi ekanini ko'rsating:

1) $F(x)=k\cdot e^{\frac{x}{k}}$, $f(x)=e^{\frac{x}{k}}$, $k\neq 0$;

2) $F(x)=C+\sin kx$, $f(x)=k\cdot\cos kx$, C – o'zgarmas son;

3) $F(x)=C+\cos kx$, $f(x)=-k\cdot\sin kx$, C – o'zgarmas son;

4) $F(x)=\frac{1}{5}\sin(5x+12)$, $f(x)=\cos(5x+12)$.

14. $f(x)$ funksiyaning ko'rsatilgan nuqtadan o'tuvchi boshlang'ich funksiyasini toping:

1) $f(x)=\sin 3x$, $A(\frac{\pi}{3};\ \frac{1}{3})$; 2) $f(x)=\cos 5x$, $A(\frac{\pi}{2};\ \frac{4}{5})$;

3) $f(x)=\cos\frac{x}{2}$, $A(\frac{\pi}{3};\ 1)$; 4) $f(x)=\sin\frac{x}{3}$, $A(\pi;\ \frac{9}{2})$.

15. $f(x)$ funksiya uchun uning berilgan tenglamalar sistemasining yechimi $(x_0;\ y_0)$ nuqtadan o'tuvchi boshlang'ich funksiyasini toping:

1) $f(x) = 3x^2$; $\begin{cases}\log_2 x+\log_2 y=3,\\ 4\log_2 x-\log_2 y=2;\end{cases}$

2) $f(x)=4x^3$; $\begin{cases}5^x+5^y=30,\\ 3\cdot 5^x-2\cdot 5^y=15;\end{cases}$

3) $f(x)=\cos x$; $\begin{cases}x+y=\frac{3\pi}{2},\\ 4x-3y=-\pi;\end{cases}$

4) $f(x)=\frac{1}{5x+e;}$; $\begin{cases}2^x+3^y=4,\\ 3\cdot 2^x-3^y=0.\end{cases}$

40–43 INTEGRALLAR JADVALI. INTEGRALLASHNING ENG SODDA QOIDALARI

Integrallar jadvalini hosilalar jadvali yordamida tuzish mumkin.

№	Funksiya $f(x)$	Boshlang'ich funksiya $F(x)+C$
1	$x^p, \quad p \neq -1$	$\frac{x^{p+1}}{p+1}+C$
2	$1/x$	$\ln\lvert x\rvert+C$
3	e^x	e^x+C
4	$\sin x$	$-\cos x+C$
5	$\cos x$	$\sin x+C$
6	$(kx+b)^p, \quad p \neq -1, \quad k \neq 0$	$\frac{(kx+b)^{p+1}}{k(p+1)}+C$
7	$\frac{1}{kx+b}, \quad k \neq 0$	$\frac{1}{k}\ln\lvert kx+b\rvert+C$
8	$e^{kx+b}, \ k \neq 0$	$\frac{1}{k}e^{kx+b}+C$
9	$\sin(kx+b), \ k \neq 0$	$-\frac{1}{k}\cos(kx+b)+C$
10	$\cos(kx+b), \quad k \neq 0$	$\frac{1}{k}\sin(kx+b)+C$
11	$1/\cos^2 x$	$\mathrm{tg}x+C$
12	$1/\sin^2 x$	$-\mathrm{ctg}x+C$
13	a^x	$\frac{a^x}{\ln a}+C$
14	$\frac{1}{1+x^2}$	$\mathrm{arctg}x+C$
15	$f(kx+b)$	$\frac{1}{k}F(kx+b)+C$
16	$f(g(x))g'(x)$	$F(g(x))+C$

Biror X oraliqda aniqlangan $F(x)$ funksiya $f(x)$ funksiyaning boshlang‘ich funksiyasi bo‘lishi uchun ikkala $F(x)$ va $f(x)$ funksiya ham ayni shu X oraliqda aniqlangan bo‘lishi kerak.

Masalan, $\frac{1}{5x-8}$ funksiyaning $5x-8>0$, ya’ni $x>1{,}6$ oraliqdagi integrali, jadvalga muvofiq, $\frac{1}{5}\ln(5x-8)+C$ ga teng.

Differensiyalash qoidalaridan foydalanib, *integrallash qoidalarini* bayon qilish mumkin.

$F(x)$ va $G(x)$ funksiyalar biror oraliqda, mos ravishda, $f(x)$ va $g(x)$ funksiyalarning boshlang‘ich funksiyalari bo‘lsin. Ushbu qoidalar o‘rinlidir:

1-qoida: $a\cdot F(x)$ funksiya $a\cdot f(x)$ funksiyaning boshlang‘ich funksiyasi bo‘ladi, ya’ni

$$\int a\cdot f(x)dx = a\cdot F(x)+C.$$

2-qoida: $F(x)\pm G(x)$ funksiya $f(x)\pm g(x)$ funksiyaning boshlang‘ich funksiyasi bo‘ladi, ya’ni:

$$\int(f(x)\pm g(x))dx=\int f(x)dx \pm \int g(x)dx=F(x)\pm G(x)+C.$$

1-misol. $f(x)=5\sin(3x+2)$ funksiyaning integralini toping.

△ Bu funksiyaning integralini 1-qoida va integrallar jadvalining 9-bandiga muvofiq topamiz:

$$\int f(x)dx=\int 5\sin(3x+2)dx=5\int \sin(3x+2)dx=$$

$$=5\cdot\left(-\frac{1}{3}\cos(3x+2)\right)+C=-\frac{5}{3}\cos(3x+2)+C,$$

chunki integrallar jadvaliga ko‘ra

$$\int \sin(3x+2)dx=-\frac{1}{3}\cos(3x+2)+C.$$

Javob: $-\frac{5}{3}\cos(3x+2)+C.$ ▲

2-misol. $f(x)=8x^7+2\cos 2x$ funksiyaning integralini toping.

△ Bu funksiyaning integralini 1- va 2-qoidalar hamda integrallar jadvalining 1- va 10-bandiga muvofiq topamiz:

$$\int f(x)dx=\int(8x^7+2\cos 2x)dx=8\int x^7dx+2\int \cos 2xdx$$

$$=8\cdot\frac{1}{8}x^8+2\cdot\frac{1}{2}\sin 2x+C=x^8+\sin 2x+C.$$

Javob: $x^8+\sin 2x+C.$ ▲

3-misol. $\int\frac{xdx}{x^2+8}$ integralni hisoblang.

△ Bu kabi misollarni yechishda *o'zgaruvchini almashtirish* qulay. Agar $x^2+8=u$ deyilsa, $du=2xdx$, $xdx=\frac{1}{2}du$ bo'ladi. U holda

$$\int\frac{xdx}{x^2+8}=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ln|u|+C=\frac{1}{2}\ln(x^2+8)+C.$$

Tekshirish: Topilgan boshlang'ich funksiyadan hosila olinsa, integral ostidagi funksiya $\frac{x}{x^2+8}$ hosil bo'lishi kerak. Chindan ham,

$$\left(\frac{1}{2}\ln(x^2+8)+C\right)'=\frac{1}{2}(\ln(x^2+8))'+C'=\frac{1}{2}\cdot\frac{1}{x^2+8}\cdot(x^2+8)'=\frac{1}{2}\cdot\frac{2x}{x^2+8}=\frac{x}{x^2+8}.$$

Javob: $\frac{1}{2}\cdot\ln(x^2+8)+C.$ ▲

4-misol. $\int e^{\sin x}\cos xdx$ integralni hisoblang.

△ $\sin x=t$ almashtirish bajaramiz. U holda $dt=\cos xdx$ va berilgan integral $\int e^t dt$ ko'rinishni oladi. Integrallar jadvallarining 3-bandiga ko'ra $\int e^t dt=e^t+C$ bo'ladi. Demak, $\int e^{\sin x}\cos xdx=e^{\sin x}+C.$

Tekshirish. $(e^{\sin x}+C)'=(e^{\sin x})'+C'=e^{\sin x}(\sin x)'+0=e^{\sin x}\cos x$ – berilgan integral ostidagi funksiyani hosil qildik.

Javob: $e^{\sin x}+C.$ ▲

5-misol. $\int \sin 5x \cdot \cos 3x dx$ integralni hisoblang.

△ Bunda $2\sin 5x \cdot \cos 3x = \sin 8x + \sin 2x$ ayniyat yordam beradi. U holda

$$\int \sin 5x \cos 3x dx = \frac{1}{2}\int \sin 8x dx + \frac{1}{2}\int \sin 2x dx =$$

$$= \frac{1}{16}(-\cos 8x) + \frac{1}{4}(-\cos 2x) + C = -\frac{\cos 8x}{16} - \frac{\cos 2x}{4} + C.$$

Javob: $-\frac{\cos 8x}{16} - \frac{\cos 2x}{4} + C.$ ▲

6*-misol. $\int \cos mx \cos nx dx$ integralni hisoblang.

△ $\cos mx \cos nx = \frac{1}{2}(\cos(m+n)x + \cos(m-n)x)$ ayniyatga va integrallash jadvalining 10-bandiga muvofiq:

$$\int \cos mx \cos nx dx = \frac{1}{2}\int \cos(m+n)x dx + \frac{1}{2}\int \cos(m-n)x dx =$$

$$= \frac{1}{2}\cdot\frac{\sin(m+n)x}{m+n} + \frac{1}{2}\cdot\frac{\sin(m-n)x}{m-n} + C.$$

Javob: $\frac{1}{2}\cdot\frac{\sin(m+n)x}{m+n} + \frac{1}{2}\cdot\frac{\sin(m-n)x}{m-n} + C.$ ▲

7-misol. $\int \frac{dx}{x^2 - 5x + 6}$ integralni hisoblang.

△ Integral ostidagi funksiya uchun quyidagi tengliklar o‘rinlidir:

$$\frac{1}{x^2 - 5x + 6} = \frac{1}{(x-2)(x-3)} = \frac{(x-2)-(x-3)}{(x-2)(x-3)} = \frac{1}{x-3} - \frac{1}{x-2}.$$

Bundan

$$\int \frac{dx}{x^2 - 5x + 6} = \int \left(\frac{1}{x-3} - \frac{1}{x-2}\right)dx = \int \frac{dx}{x-3} - \int \frac{dx}{x-2} =$$

$$= \ln|x-3| - \ln|x-2| + C = \ln\left|\frac{x-3}{x-2}\right| + C,$$

Javob: $\ln\left|\frac{x-3}{x-2}\right| + C.$ ▲

8-misol. $\int \frac{dx}{1+\cos x}$ integralni hisoblang.

△ Bu integralni hisoblash uchun $1+\cos x=2\cos^2\frac{x}{2}$ va $\int \frac{dx}{\cos^2 x}=\mathrm{tg}x+C$ ekanidan foydalanamiz. U holda

$$\int \frac{dx}{1+\cos x}=\int \frac{dx}{2\cos^2\frac{x}{2}}=\frac{1}{2}\cdot 2\cdot \mathrm{tg}\frac{x}{2}+C=\mathrm{tg}\frac{x}{2}+C.$$

Tekshirish: $(\mathrm{tg}\frac{x}{2}+C)'=(\mathrm{tg}\frac{x}{2})'+C'=\frac{1}{\cos^2\frac{x}{2}}\cdot(\frac{x}{2})'+0=\frac{1}{2}\cdot\frac{1}{\cos^2\frac{x}{2}}=\frac{1}{1+\cos x}$ integral ostidagi funksiya hosil bo'ldi.

Javob: $\mathrm{tg}\frac{x}{2}+C.$ ▲

9-misol. $\int \sin^2 2x dx$ integralni hisoblang.

△ Integralni hisoblash uchun $2\sin^2 2x=1-\cos 4x$ ayniyatdan foydalanamiz.

$$\int \sin^2 2x dx=\int \frac{1}{2}(1-\cos 4x)dx=\frac{1}{2}\int dx-\frac{1}{2}\int \cos 4x dx=\frac{x}{2}-\frac{1}{2}\cdot\frac{1}{4}\sin 4x+C=\frac{x}{2}-\frac{1}{8}\sin 4x+C.$$

Javob: $\frac{x}{2}-\frac{1}{8}\sin 4x+C.$ ▲

? Savol va topshiriqlar

1. Integrallar jadvalidagi o'zingiz xohlagan 4 ta misolni tanlang va uni isbotlang.
2. Integrallashning sodda qoidalarini bayon qiling. Misollarda tushuntiring.
3. O'zgaruvchi almashtirish usuli nima? $\int e^{\cos 2x}\sin 2x dx$ integralni hisoblashda shu usulni qo'llang va misolni yechish jarayonini tushuntiring.

Mashqlar

Berilgan funksiyaning boshlang‘ich funksiyalaridan birini toping (**16 – 18**):

16. 1) $3x^5-4x^3$; 2) $8x^7-5x^4$; 3) $\frac{4}{x}-\frac{4}{x^2}$; 4) $\frac{5}{x^4}+\frac{3}{x^5}$;

5) $\sqrt[3]{x}+3\sqrt[3]{x}$; 6) $7\sqrt[3]{x}-5\sqrt{x}$; 7) $5x^4+4x^3-2x^2$.

17. 1) $5\cos x-3\sin x$; 2) $7\sin x+4\cos x$; 3) $2\cos x-a^x$;

4) $5e^x+2\cos x+1$; 5) $4+2\cdot e^{-x}-7\sin x$; 6) $\frac{6}{\sqrt[3]{x}}+\frac{4}{x}-e^{-x}$.

18. 1) $(x-2)^3$; 2) $(x+5)^4$; 3) $\frac{1}{\sqrt{x-5}}$ 4) $\frac{6}{\sqrt[3]{x+7}}$;

5) $4\cos(x+5)+\frac{8}{x-7}$; 6) $2\sin(x-3)-\frac{4}{x-2}$; 7) $(3x+7)^4+\frac{1}{x^5}$.

Berilgan funksiyaning barcha boshlang‘ich funksiyalarini toping (**19 – 20**):

19. 1) $\cos(5x+3)$; 2) $\sin(7x-6)$; 3) $\cos(\frac{2x}{3}+1)$;

4) $\sin(\frac{5x}{7}-2)$; 5) $e^{\frac{2x+3}{4}}$; 6) e^{3-2x};

7) $\frac{4}{\cos^2 x}$; 8) $\frac{3}{\cos^2 4x}$; 9) $\frac{5}{\sin^2 5x}$.

20. 1) $\frac{4}{x^5}-(1-2x)^3$; 2) $(3x+2)^4-\frac{1}{x^6}$; 3) $x+\frac{2}{\cos^6 x}-1$;

4) $2x-\frac{3}{\sin^2 x}+6$; 5) $(1+3x)(x-1)$; 6) $\frac{1}{2}\cdot\sqrt[3]{x^2}+2\sin(3x-1)$.

21. Berilgan $f(x)$ funksiya uchun grafigi $A(x; y)$ nuqtadan o‘tadigan boshlang‘ich funksiyani toping:

1) $f(x)=\sin 4x$, $A(\frac{\pi}{4};\ 7)$; 2) $f(x)=\cos 5x$, $A(\frac{\pi}{4};\ 4)$;

3) $f(x)=3x^2+\frac{2}{\sqrt{x+2}}$, $A(-1;0)$; 4) $f(x)=4x^3-\frac{1}{2\sqrt{x-1}}$ $A(2;0)$;

5) $f(x)=\cos^2 3x+\sin^2 3x+\frac{1}{4}\sin 4x,\ A(\frac{\pi}{8};\ \frac{\pi}{8})$;

6) $f(x)=\text{tg}x\cdot\text{ctg}x-2\cos\frac{x}{2},\ A(2\pi;\ 2\pi)$;

7) $f(x)=\frac{2}{\sqrt{5-2x}}+4x,\ A(2;6)$; 8) $f(x)=6x^2-\frac{1}{2\sqrt{2-x}},\ A(-2;\ 4)$.

Integrallarni toping (**22 – 28**):

22. 1) $\int(x^3-\sin 2x-3)dx$; 2) $\int(x^4+\cos 3x+4)dx$;

3) $\int(x^2-\sin\frac{x}{2}+\cos\frac{x}{2})dx$; 4) $\int(4x^3+\cos\frac{x}{3}+\sin\frac{x}{3})dx$.

23*. 1) $\int(\frac{8}{\sin^2 x}+6\cos^2 x+2)dx$; 2) $\int(\frac{6}{\cos^2 x}-8\sin^2 x+3)dx$;

3) $\int\sin 2x\cos 2xdx$; 4) $\int(\sin 3x\cos x+\cos 3x\sin x)dx$;

5) $\int(\sin 2x\cdot\sin 4x+\cos 2x\cos x)dx$; 6) $\int\cos^2 5xdx$.

24*. 1) $\int\sin 5x\cos 3xdx$; | 2) $\int\cos 2x\cos 3xdx$; | 3) $\int\sin 7x\sin 3xdx$.

25*. 1) $\int\frac{x}{x+1}dx$; | 2) $\int\frac{dx}{x^2-7x+12}$; | 3) $\int\frac{(x-3)dx}{x^2-4x+3}$; | 4) $\frac{(x+4)dx}{x^2-16}$.

26. 1) $\int\frac{x^5+x^3-2}{x^2+1}dx$; 2) $\frac{x^2-1}{x^2+1}dx$; 3) $\int\frac{dx}{1+\cos 2x}$;

4) $\frac{dx}{1-\cos 2x}$; 5) $\int\frac{dx}{4(x^2-4)}$; 6) $\int(1-2\sin^2 5x)dx$.

27. 1) $\int(x^3-1)^4x^2dx$; 2) $\int\frac{xdx}{(1+x^2)^3}$; 3) $\int\frac{\text{tg}x}{\cos^3 x}dx$;

4) $\int\frac{\text{ctg}x}{\sin^2 x}dx$; 5) $\int\sin^3 xdx$; 6) $\int\cos^3 xdx$.

28*. 1) $\int\frac{xdx}{\sqrt{x-1}}$; 2) $\int x\cdot\sqrt{x-4}dx$; 3) $\int\frac{(x-1)dx}{\sqrt{x+1}}$;

4) $\int(\text{tg}^2x+\text{tg}^4x)dx$; 5) $\int(\text{ctg}^2x+\text{ctg}^4x)dx$.

Berilgan $f(x)$ funksiya uchun grafigi $A(x;\ y)$ nuqtadan o'tadigan boshlang'ich funksiyani toping **(29 – 30)**:

29. 1) $f(x)=\frac{3}{2}\cdot\cos\frac{x}{3}$, $A(\pi;\ 4)$;

2) $f(x)=\frac{3}{5}\cdot\sin 5x$, $A(\frac{\pi}{2};\ 3)$;

3) $f(x)=2\sin 5x+2\cos\frac{x}{2}$, $A(\frac{\pi}{3};\ 0)$;

30. 1) $f(x)=3x^2-2x+8$, $A(1;\ 9)$;

2) $f(x)=4x^3-3x^2+2x+1$, $A(-1;\ 4)$;

3) $f(x)=5x^4+3x^2+2$, $A(-2;\ 1)$.

31. Integralni toping:

1) $\int(x^2-1)(x+2)dx$; 2) $\int(x+2)(x^2-9)dx$; 3) $\int(x^2+1)(x^3-1)dx$;

4) $\int\frac{1-4x^2+\sqrt{1-2x}}{1-2x}dx$; 5) $\int\frac{9x^2-4-\sqrt{3x+2}}{3x+2}dx$;

6) $\int(e^{5-2x}-2^x)dx$; 7) $\int(e^{3x+2}+10^x)dx$.

32. Integralni hisoblang:

1) $\int\frac{dx}{x^2+6x+10}$; 2) $\int\frac{dx}{x^2-4x+5}$; 3) $\int\frac{dx}{x^2+10x+26}$.

Namuna: $I=\int\frac{dx}{x^2+4x+5}$ integralni hisoblang.

△ $I=\int\frac{dx}{x^2+4x+5}=\int\frac{dx}{1+(x+2)^2}$; $x+2=u$ deyilsa, $1+(x+2)^2=1+u^2$ $x'=u'$ va integrallar jadvalining 14–15 bandlariga ko'ra

$$I=\int\frac{du}{1+u^2}=\operatorname{arctg}u+C=\operatorname{arctg}(x+2)+C.$$

Tekshirish:

$$(\operatorname{arctg}(x+2)+C)'=(\operatorname{arctg}(x+2))'+C'=\frac{1}{1+(x+2)^2}+0=$$
$$=\frac{1}{1+(x+2)^2}=\frac{1}{x^2+4x+5}.$$

Javob: $\operatorname{arctg}(x+2)+C$. ▲

Integrallash qoidalaridan yana biri *bo'laklab integrallashdir*.

3-qoida*. Agar biror X oraliqda $f(x)$ va $g(x)$ funksiyalar uzluksiz $f'(x)$ va $g'(x)$ hosilaga ega bo'lsa, u holda

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx \qquad (1)$$

formula o'rinlidir. Bu formula *bo'laklab integrallash formulasi* deyiladi.

Bu formulaning isboti $f(x)$ va $g(x)$ funksiyalar ko'paytmasini differensiallash qoidasi $(f(x)g(x))'=f'(x)g(x)+f(x)g'(x)$ va $\int f'(x)dx=f(x)+C$ ekanidan kelib chiqadi.

Formuladan *foydalanish yo'rig'i*: 1) integral ostidagi ifoda $f(x)$ va $g'(x)$ lar ko'paytmasi ko'rinishida yozib olinadi; 2) $g'(x)$ va $g(x)f'(x)$ ifodalarning integrallarini oson (qulay) hisoblanadigan qilib olish nazarda tutiladi.

1-misol. $\int x\cdot e^x dx$ integralni hisoblang.

△ Bu yerda $f(x)=x$, $g'(x)=e^x$ deb olish qulay, chunki

$g(x)=\int g'(x)dx=\int e^x dx=e^x$, $f'(x)=1$. U holda (1) ga asosan,

$$\int xe^x dx = x\cdot e^x - \int e^x dx = x\cdot e^x - e^x + C.$$

Demak, $\int xe^x dx = e^x\cdot(x-1)+C.$

Javob: $e^x(x-1)+C$. ▲

2-misol. $\int \ln x dx$ integralni hisoblang.

△ Integral ostidagi $\ln x$ funksiyani $f(x)=\ln x$ va $g'(x)=1$ larning ko'paytmasi deb hisoblaymiz: $\ln x=f(x)\cdot g'(x)$.

U holda $f'(x)=\frac{1}{x}$, $g(x)=\int 1\cdot dx = x+C.$

(1) formulaga ko'ra,

$$\int \ln x dx = x\ln x - \int x\cdot\frac{1}{x}dx = x\ln x - \int dx = x\ln x - x + C =$$

$$= x(\ln x - 1) + C = x\cdot(\ln x - \ln e) + C = x\cdot\ln\frac{x}{e} + C.$$

Demak, $\int \ln x dx = x \cdot \ln \frac{x}{e} + C.$

Tekshirish:

$$(x \ln \frac{x}{e} + C)' = (x \ln \frac{x}{e})' + C' = x' \cdot \ln \frac{x}{e} + x(\ln \frac{x}{e})' + 0 =$$

$$= \ln \frac{x}{e} + x \cdot \frac{e}{x} \cdot \frac{1}{e} = \ln x - \ln e + 1 = \ln x - 1 + 1 = \ln x.$$

Javob: $x \cdot \ln \frac{x}{e} + C.$ ▲

3-misol. $\int x \cos x dx$ integralni hisoblang.

△ Integralni hisoblash uchun $f(x)=x$, $g'(x)=\cos x$ deyish qulay. U holda $f'(x)=1$, $g(x)=\int \cos x dx = \sin x$ (bu yerda boshlang'ich funksiyalardan bittasini oldik, shuning uchun o'zgarmas son C ni yozmadik). Bo'laklab integrallash formulasiga muvofiq,

$$\int x \cos x dx = x \cdot \sin x - \int \sin x dx = x \sin x + \cos x + C.$$

Javob: x sin$x$+cos$x$+C. ▲

Integrallarni hisoblang (**33 – 35**):

33*. 1) $\int x \sin x dx$; 2) $\int x^2 \cos x dx$; 3) $\int x \ln x dx$; 4) $\int 2x \ln x dx$.

34*. 1) $\int x \cos 2x dx$; 2) $\int x \sin 3x dx$; 3) $\int x \sin \frac{x}{3} dx$; 4) $\int x \cos \frac{x}{4} dx$.

35*. 1) $\int 2^x \cdot x dx$; 2) $\int 3^x \cdot x dx$; 3) $\int 5^x \cdot x dx$; 4) $\int \mathrm{tg}^2 nx dx$;

5) $\int \frac{2^{x+1} - 5^{x-1}}{10^x} dx$; 6) $\int \frac{e^{3x}+1}{e^x+1} dx$; 7) $\int (3^x + 4^x)^2 dx$;

8) $\int (e^{-x} + e^{-2x}) dx$; 9) $\int \frac{e^{4x}-1}{e^{2x}-1} dx$; 10) $\int \frac{e^x dx}{\pi + e^x}$;

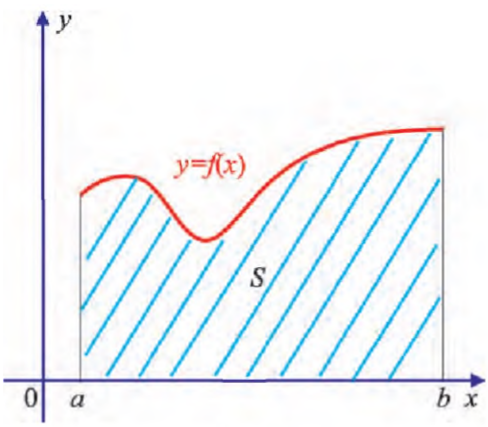
11) $\int x \cdot e^{-x^2} dx$; 12) $\int \frac{dx}{e^x + e^{-x}}$; 13) $\int \frac{\ln^2 x}{x} dx$.

ANIQ INTEGRAL. NYUTON–LEYBNIS FORMULASI

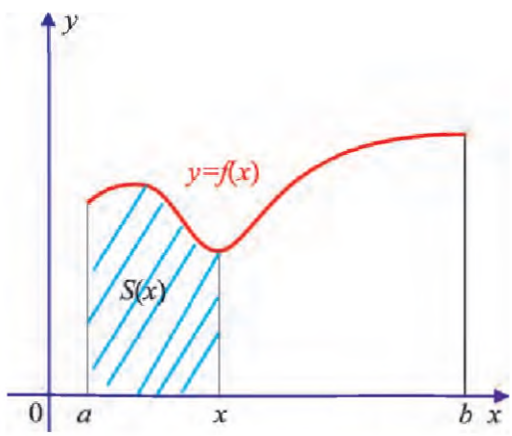
2-rasmda tasvirlangan shakl *egri chiziqli trapetsiya* deyiladi. Bu shakl yuqoridan $y = f(x)$ funksiyaning grafigi bilan, quyidan $[a, b]$ kesma bilan, yon tomonlardan esa $x=a$, $x=b$ to‘g‘ri chiziqlarning kesmalari bilan chegaralangan. $[a; b]$ kesma egri chiziqli trepetsiyaning *asosi* deyiladi.

Egri chiziqli trapetsiyaning yuzini qaysi formulaga ko‘ra hisoblaymiz, degan savol tug‘iladi.

Bu yuzni S deb belgilaylik. S yuzni $f(x)$ funksiyaning boshlang‘ich funksiyasi yordamida hisoblash mumkin ekan. Shunga oid mulohazalarni keltiramiz.



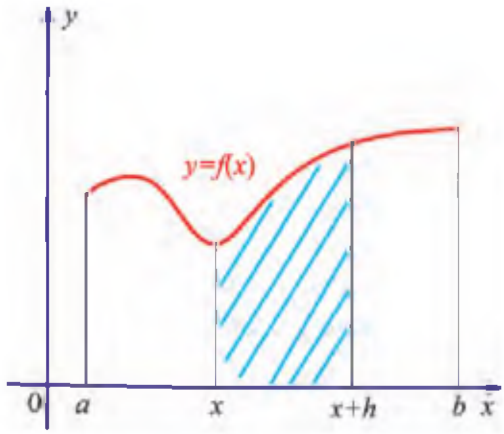
2-rasm.



3-rasm.

$[a; x]$ asosli egri chiziqli trapetsiyaning yuzini $S(x)$ deb belgilaymiz (3-rasm), bunda x shu $[a; b]$ kesmadagi istalgan nuqta: $x=a$ bo‘lganda $[a; x]$ kesma nuqtaga aylanadi, shuning uchun $S(a)=0$; $x=b$ da $S(b) = S$.

$S(x)$ ni $f(x)$ funksiyaning boshlang‘ich funksiyasi bo‘lishini, ya’ni $S'(x) = f(x)$ ekanini ko‘rsatamiz.



4-rasm.

△ $S(x+h) - S(x)$ ayirmani ko‘raylik, bunda $h>0$ ($h<0$ hol ham xuddi shunday ko‘riladi). Bu ayirma asosi $[x;\ x+h]$ bo‘lgan egri chiziqli trapetsiyaning yuziga teng (4-rasm). Agar h son kichik bo‘lsa, u holda bu yuz taqriban $f(x)\cdot h$ ga teng, ya’ni $S(x+h)-S(x)\approx f(x)\cdot h$. Demak,

$$\frac{S(x+h)-S(x)}{h}\approx f(x).$$

Bu taqribiy tenglikning chap qismi $h\to 0$ da hosilaning ta‘rifiga ko‘ra $S'(x)$ ga intiladi. Shuning uchun $h\to 0$ da $S'(x)=f(x)$ tenglik hosil bo‘ladi. Demak, $S(x)$ yuz $f(x)$ funksiya uchun boshlang‘ich funksiyasi ekan. ▲

Boshlang‘ich funksiya $S(x)$ dan ixtiyoriy boshqa boshlang‘ich $F(x)$ funksiya o‘zgarmas songa farq qiladi, ya’ni

$$F(x)=S(x)+C.$$

Bu tenglikdan $x=a$ da $F(a) = S(a)+C$ va $S(a) = 0$ bo‘lgani uchun $C=F(a)$. U holda (1) tenglikni quyidagicha yozish mumkin:

$S(x)=F(x)-F(a)$. Bundan $x=b$ da $S(b)=F(b)-F(a)$ ekanini topamiz.

Demak, *egri chiziqli trapetsiyaning yuzini* (2-rasm) quyidagi formula yordamida hisoblash mumkin:

$$S=F(b)-F(a), \qquad (2)$$

bunda $F(x)$ – berilgan $f(x)$ funksiyaning istalgan boshlang‘ich funksiyasi.

Shunday qilib, egri chiziqli trapetsiyaning yuzini hisoblash $f(x)$ funksiyaning $F(x)$ boshlang'ich funksiyasini topishga, ya'ni $f(x)$ funksiyani integrallashga keltiriladi.

$F(b) - F(a)$ ayirma $f(x)$ funksiyaning $[a; b]$ kesmadagi *aniq integrali* deyiladi va bunday belgilanadi: $\int_a^b f(x)dx$

(o'qilishi: "*a* dan *b* gacha integral ef iks de iks"), ya'ni

$$\int_a^b f(x)dx = F(b) - F(a). \tag{3}$$

(3) formula *Nyuton–Leybnis formulasi* deb ataladi.

(2) va (3) formulaga muvofiq:

$$S = \int_a^b f(x)dx. \tag{4}$$

Integralni hisoblashda, odatda, quyidagicha belgilash kiritiladi:

$F(b)-F(a)=F(x)\Big|_a^b$. U holda (3) formulani shunday yozish mumkin:

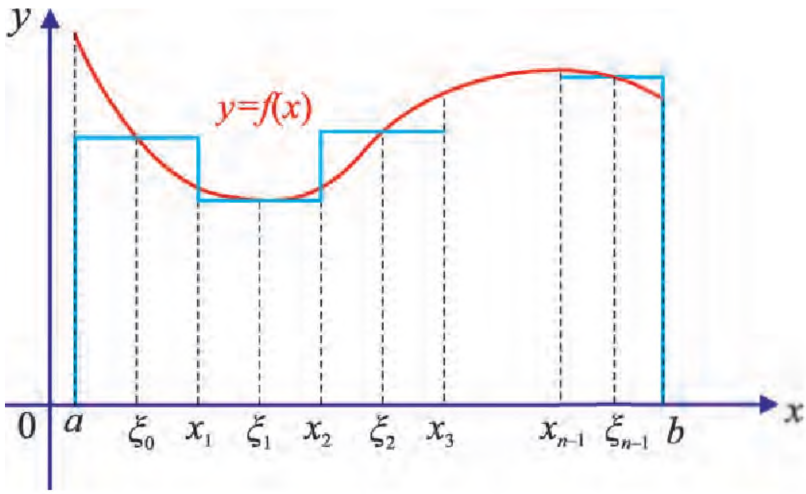
$$S = \int_a^b f(x)dx = F(x)\Big|_a^b. \tag{5}$$

Shu o'rinda qisqacha *tarixiy ma'lumotni* aytish joiz.

Egri chiziqlar bilan chegaralangan shakl yuzini hisoblash masalasi aniq integral tushunchasiga olib kelgan. Uzluksiz $f(x)$ funksiya aniqlangan $[a; b]$ kesma $a=x_0, x_1.., x_{n-1}, ... x_n= b$ nuqtalar yordamida o'zaro teng $[x_k; x_{k+1}]$ $(k=0, 1, ... n-1)$ kesmalarga bo'lingan va har bir $[x_k; x_{k+1}]$ kesmadan ixtiyoriy ξ_k nuqta olingan. $[x_k; x_{k+1}]$ kesma uzunligi $\Delta x_k=x_{k+1}-x_k$ ni berilgan $f(x)$ funksiyaning ξ_k nuqtadagi qiymati $f(\xi_k)$ ga ko'paytirilgan va ushbu

$$S_n=f(\xi_0)\Delta x_0+f(\xi_1)\Delta x_1+\ldots+f(\xi_{k-1})\Delta_{n-1} \tag{6}$$

yig'indisi tuzilgan, bunda har bir qo'shiluvchi asosi Δx_k va balandligi $f(\xi_k)$ bo'lgan to'g'ri to'rtburchakning yuzidir. S_n yig'indi egri chiziqli trapetsiyaning yuzi S ga taqriban teng: $S_n \approx S$ (5-rasm).



5-rasm.

(6) yig'indi $f(x)$ funksiyaning $[a;\ b]$ kesmadagi ***integral yig'indisi*** deyiladi. Agar n cheksizlikka intilganda ($n\to\infty$), Δx_k nolga intilsa ($\Delta x_k\to 0$), u holda S_n integral yig'indi biror songa intiladi. Ayni shu son $f(x)$ funksiyaning $[a;\ b]$ kesmadagi ***integrali*** deb ataladi.

1-misol. 6-rasmda tasvirlangan egri chiziqli trapetsiyaning yuzini toping.

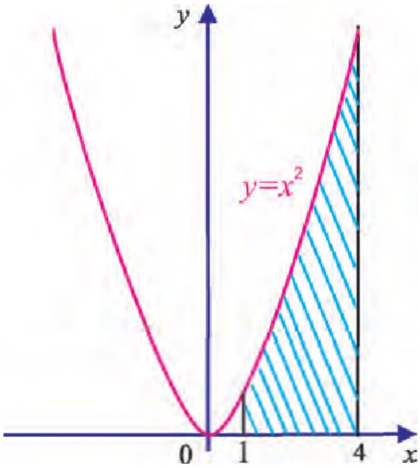
△ (4) formulaga muvofiq $S=\int_1^4 x^2dx$. Bu integralni Nyuton–Leybnis formulasi (3) yordamida hisoblaymiz. $f(x)=x^2$ funksiyaning boshlang'ich funksiyalaridan biri $F(x)=\frac{x^3}{3}$ ekani ravshan. Demak,

$$S=\int_1^4 x^2dx=\frac{x^3}{3}\bigg|_1^4=\frac{1}{3}(4^3-1^3)=\frac{1}{3}\cdot 63=21 \text{ (kv. birlik).}$$

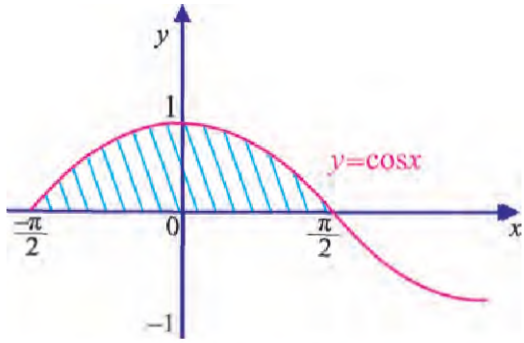
Javob: $S=21$ kv. birlik. ▲

2-misol. 7-rasmdagi shtrixlangan soha yuzini toping.

△ Shtrixlangan soha egri chiziqli trapetsiya bo'lib, u yuqoridan $y=\cos x$ funksiyaning grafigi, pastdan esa $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ kesma bilan chegaralangan. $y=\cos x$ – juft funksiya, soha Oy o'qqa nisbatan simmetrik. Shu ma'lumotlarga ko'ra, soha yuzi $S=2\int_0^{\frac{\pi}{2}}\cos x dx$ ga teng bo'ladi.



6-rasm.



7-rasm.

Nyuton–Leybnis formulasi va (5) formulaga ko‘ra:

$$S=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos x dx=\sin x\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}=\sin\frac{\pi}{2}-\sin(\frac{-\pi}{2})=1-(-1)=1+1=2 \text{ (kv. birlik).}$$

Javob. 2 kv.birlik. ▲

3-misol. $\int_0^{\pi}\cos x dx$ aniq integralni hisoblang.

△ Nyuton–Leybnis formulasi va (5) formulasiga ko‘ra:

$$\int_0^{\pi}\cos x dx=\sin x\Big|_0^{\pi}=\sin\pi-\sin 0=0.$$

Javob: 0. ▲

4-misol. $\int_{-1}^{2}(2x^2-3x+4)dx$ aniq integralni hisoblang.

△ Nyuton–Leybnis formulasi va (5) formulaga ko‘ra:

$$\int_{-1}^{2}(2x^2-3x+4)dx=(\frac{2}{3}x^3-\frac{3}{2}x^2+4x)\Big|_{-1}^{2}=\frac{22}{3}-(-\frac{37}{6})=\frac{81}{6}=13{,}5\text{(kv. birlik).}$$

Javob: 13,5 kv. birlik. ▲

5-misol. $S=\int_0^{\frac{\pi}{3}}\sin^2(3x+\frac{\pi}{6})\, dx$ aniq integralni hisoblang.

△ Avval aniqmas integralni topamiz:

$$\int \sin^2(3x+\frac{\pi}{6})dx=\frac{1}{2}\int(1-\cos(6x+\frac{\pi}{3}))dx=\frac{1}{2}\cdot(x-\frac{1}{6}\sin(6x+\frac{\pi}{3})).$$

U holda $S=\frac{1}{2}(x-\frac{1}{6}\sin(6x+\frac{\pi}{3}))\Big|_0^{\frac{\pi}{3}}=\frac{1}{2}\cdot(\frac{\pi}{3}-\frac{1}{6}\sin(2\pi+\frac{\pi}{3}))-\frac{1}{2}(0-\frac{1}{6}\sin\frac{\pi}{3})=$

$=\frac{\pi}{6}-\frac{1}{12}\sin\frac{\pi}{3}+\frac{1}{12}\sin\frac{\pi}{3}=\frac{\pi}{6}.$

Javob: $S=\frac{\pi}{6}$. ▲

6-misol. $\int_2^6\sqrt{2x-3}\,dx$ aniq integralni hisoblang.

△ Avval aniqmas integralni topamiz:

Integrallar jadvaliga ko'ra $\int\sqrt{2x-3}dx=\frac{1}{3}\cdot(2x-3)^{\frac{3}{2}}+C.$ U holda

$$\int_2^6\sqrt{2x-3}dx=\frac{1}{3}\cdot(2x-3)^{\frac{3}{2}}\Big|_2^6=\frac{1}{3}\cdot\left((2\cdot6-3)^{\frac{3}{2}}-(2\cdot2-3)^{\frac{3}{2}}\right)=\frac{1}{3}\cdot(27-1)=\frac{26}{3}=8\frac{2}{3}.$$

Javob: $8\frac{2}{3}$. ▲

Aniq integral quyidagi xossalarga ega:

1. $\int_a^a f(x)dx=0.$ Chindan ham, $\int_a^a f(x)dx=F(a)-F(a)=0.$

2. $\int_a^b f(x)dx=-\int_b^a f(x)dx.$

△ $\int_a^b f(x)dx=F(b)-F(a);\ \int_b^a f(x)dx=F(a)-F(b)=-(F(b)-F(a)).$

Demak, $-\int_b^a f(x)dx=F(b)-F(a)=\int_a^b f(x)dx.$ ▲

3. a, b, c – haqiqiy sonlar bo‘lsa, $\int_b^a f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ (aniq integralning additivlik xossasi).

4. $f(x)$, $x \in R$, juft funksiya bo‘lsa, u holda $\int_{-a}^{a} f(x)dx = 2\cdot\int_0^a f(x)dx$.

5. Agar $f(x) \geq 0$, $x \in [a,b]$ bo‘lsa, $\int_a^b f(x)dx \geq 0$ bo‘ladi.

6. $x \in [a,b]$ da $f(x) < g(x)$ bo‘lsa, u holda $\int_a^b f(x)dx \leq \int_a^b g(x)dx$ bo‘ladi.

? Savol va topshiriqlar

1. Aniq integral nima?
2. Egri chiziqli trapetsiya yuzini hisoblash masalasini ayting. Misollarda tushuntiring.
3. Nyuton–Leybnis formulasi nima? Uning mazmun-mohiyatini ayting.
4. Aniq integralning xossalarini ayting. Misollarda tushuntiring.

Mashqlar

Aniq integrallarni hisoblang (**36 – 41**):

36. 1) $\int_0^2 3x^2dx$; 2) $\int_0^2 2xdx$; 3) $\int_{-1}^4 5xdx$; 4) $\int_1^2 8\cdot x^3dx$;

5) $\int_1^e \frac{1}{x}dx$; 6) $\int_3^4 \frac{1}{x^2}dx$; 7) $\int_1^2 \frac{1}{x^4}dx$; 8) $\int_0^1 \sqrt{2x}dx$;

9) $\int_1^4 \frac{2}{\sqrt{x}}dx$; 10) $\int_8^{27} \frac{dx}{\sqrt[3]{x}}$; 11) $\int_{-1}^3 \frac{dx}{\sqrt{2x+3}}$; 12) $\int_0^3 x\sqrt{x+1}dx$.

37. 1) $\int_{\frac{\pi}{2}}^{\pi} \cos(2x+\frac{\pi}{4})dx$; 2) $\int_{-\pi}^{\pi} \sin^2 2xdx$;

3) $\int_0^{\frac{\pi}{6}} \sin 3x\cos 3xdx$; 4) $\int_0^{\frac{\pi}{8}} (\cos^2 2x - \sin^2 2x)dx$.

38. 1) $\int_0^{\ln 2} e^{2x}dx;$ 2) $\int_0^2 e^{4x}dx;$ 3) $\int_1^3 (e^{2x}-e^x)dx.$

39. 1) $\int_{-1}^1 (x^2+3x))(x-1)dx;$ 2) $\int_{-1}^0 (x+2)(x^2-3)dx;$

3) $\int_1^3 (x+\frac{1}{x})^2 dx;$ 4) $\int_{-2}^{-1} \frac{1}{x^2}(1-\frac{1}{x})dx.$

40*. 1) $\int_1^6 \frac{dx}{\sqrt{3x-2}};$ 2) $\int_0^3 \frac{dx}{\sqrt{x+1}};$ 3) $\int_0^{\frac{\pi}{8}} (\sin^4 2x+\cos^4 2x)dx.$

41*. 1) $\int_1^5 x^2\cdot\sqrt{x-1}dx;$ 2) $\int_1^5 \frac{x^2-6x+10}{x-3}dx;$ 3) $\int_0^1 \frac{x^2+2x+4}{x+1}dx.$

42*. 1) Shunday a va b sonlarni topingki, $f(x)=a\cdot 2^x+b$ funksiya $f'(1)=2,$

$\int_0^3 f(x)dx=7$ shartlarni qanoatlantirsin.

2) $\int_1^b (b-4x)dx \geq 6-5b$ tengsizlik bajariladigan barcha $b>1$ sonlarni toping.

43*. 1) $\int_1^2 (b^2+(4-4b)x+4x^3)dx \leq 12$ tengsizlik bajariladigan barcha b sonlarni toping.

2) Qanday $a>0$ sonlar uchun $\int_{-a}^a e^x dx > \frac{3}{2}$ tengsizlik bajariladi?

44. $f(x)$ funksiyani a ning ixtiyoriy qiymatida tengliklar bajariladigan qilib tanlang:

1) $\int_0^a f(x)dx = 2a^2-3a;$ 2) $\int_0^a f(x)dx=4a-a^2;$

3) $\int_0^a f(x)dx=\frac{1}{3}a^3-\frac{3}{2}a^2;$ 4) $\int_0^a f(x)dx=a^2+a+\sin a.$

Integrallarni hisoblang (**45 – 46**):

45. 1) $\int_0^1 (e^{-x}+1)^2 dx;$ 2) $\int_{-2}^{-1} 10^x \cdot 2^{-x} dx;$ 3) $\int_0^1 (e^{-x}-1)^2 dx;$

4) $\int_{-3}^{-1} 3^{-x} 6^x dx;$ 5) $\int_{\ln 2}^{\ln 3} e^{-3x} dx;$ 6) $\int_{\ln 3}^{\ln 5} e^{2x} dx.$

46*. 1) $\int_0^1 \frac{2^x+3^x}{6^{x+1}} dx;$ 2) $\int_0^1 \frac{2^{x-1}+5^{x-1}}{10^x} dx;$ 3) $\int_0^{\sqrt{e-1}} \frac{2xdx}{x^2+1};$

4) $\int_{\sqrt{3}}^{\sqrt{e+2}} \frac{2xdx}{x^2-2};$ 5) $\int_0^1 \frac{3^x+4^x}{12^x} dx;$ 6) $\int_0^2 4^{-x} \cdot 8^x dx.$

47. $x=a$, $x=b$ to‘g‘ri chiziqlar, Ox o‘qi va $y=f(x)$ funksiya grafigi bilan chegaralangan egri chiziqli trapetsiyaning yuzini toping. Mos rasm chizing:

1) $a=1$, $b=2$, $f(x)=x^3$; 2) $a=2$, $b=4$, $f(x)=x^2$;

3) $a=-2$, $b=1$, $f(x)=x^2+2$; 4) $a=1$, $b=2$, $f(x)=x^3+2$;

5) $a=\frac{\pi}{3}$, $b=\frac{2\pi}{3}$, $f(x)=\sin x$; 6) $a=\frac{\pi}{4}$, $b=\frac{\pi}{2}$, $f(x)=\cos x$.

48. Ox o‘qi va berilgan parabola bilan chegaralangan shaklning yuzini toping:

1) $y=9-x^2$; 2) $y=16-x^2$; 3) $y=-x^2+5x-6$;

4) $y=-x^2+7x-10$; 5) $y=-x^2+4x$; 6) $y=-x^2-3x$.

Quyidagi chiziqlar bilan chegaralangan shaklning yuzini toping. Mos rasm chizing (**49 – 50**):

49. 1) $y=-x^2+2x$, $y=0$; 2) $y=-x^2+3x+18$, $y=0$;

3) $y=2x^2+1$, $y=0$, $x=-1$, $x=1$; 4) $y=-x^2+2x$, $y=x$.

50. 1) $y=-2x^2+7x$, $y=3{,}5-x$; 2) $y=x^2$, $y=0$, $x=3$;

3) $y=x^2$, $y=0$, $y=-x+2$; 4) $y=2\sqrt{x}$, $y=0$, $x=1$, $x=4$.

5) $y=\frac{1}{a}\cdot x^2, y=a\cdot\sqrt{x}$; 6) $y=2^x, y=2, x=0$;

7) $y=|\lg x|, y=0, y=2, x=0.$

Nazorat ishi namunasi
I variant

1. $f(x)=\frac{x^3}{2}-\cos 3x$ funksiyaning barcha boshlang‘ich funksiyalarini toping.

2. Agar $F(\frac{3}{2})=1$ bo‘lsa, $f(x)=\frac{6}{(4-3x)^2}$ funksiyaning boshlang‘ich funksiyasi $F(x)$ ni toping.

3. Hisoblang: $\int_{-1}^{2}(x^2-6x+9)dx$.

4. Hisoblang: $\int_{0}^{\pi}\sin\frac{x}{3}dx$.

5. Ox o‘qi, $x=-1$ va $x=2$ to‘g‘ri chiziqlar va $y=9-x^2$ parabola bilan chegaralangan egri chiziqli trapetsiyaning yuzini hisoblang.

II variant

1. $f(x)=\frac{x^4}{3}+\sin 4x$ funksiyaning barcha boshlang‘ich funksiyalarini toping.

2. Agar $F\left(\frac{1}{2}\right)=2$ bo‘lsa, $f(x)=\frac{3}{(2-5x)^3}$ funksiyaning boshlang‘ich funksiyasi $F(x)$ ni toping.

3. Hisoblang: $\int_{-3}^{1}(x^2+7x-8)dx$.

4. Hisoblang: $\int_{-\pi}^{\pi}\cos\frac{x}{2}dx$

5. Ox o‘qi, $x=-2$ va $x=3$ to‘g‘ri chiziqlar va $y=x^2-1$ parabola bilan chegaralangan egri chiziqli trapetsiyaning yuzini hisoblang.

JAVOBLAR
I BOB

1. *a*) Puls chastotasi – bu yurakning bir minutda qancha urishini ko'rsatuvchi belgi. Demak, bir minutda Madinaning yuragi 67 marta uradi. *b*) 4020. **2.** *a*) $\approx 0{,}00150\,\frac{\text{xato}}{\text{so'z}}$. Sifat ortdi; *b*) $\approx 0{,}15$. **3.** Ma'ruf unumliroq ishlagan. **4.** ***a)*** $\approx 0{,}000177\ \frac{\text{mm}}{\text{km}}$. **5.** $89\frac{\text{km}}{\text{soat}}$ yoki $89\frac{\text{m}}{\text{s}}$. **6.** a) $0{,}1\ \frac{m}{s}$; b) $0{,}9\ \frac{m}{s}$; c) $0{,}5\ \frac{m}{s}$. **7.** 1) a) $3{,}1\ \frac{dona}{g}$; $4{,}22\ \frac{dona}{g}$; b) doza 2 grammdan 8 grammgacha oshirilganda hasharotlar soni tez kamayadi, keyin esa kamayishi sust bo'ladi. **8.** a) 7; b) 7; c) 11; d) 16; e) 0; f) 5. **9.** a) 5; b) 7; c) c. **10.** a) –2; b) 7; c) –1; d) 1. **11.** a) –3; b) –5; c) –1 d) 6; e) –4; f) –8; g) 1; h) 2; i) 5. **13.** a) $3x^2$; b) $-\frac{1}{x^2}$; c) $\frac{1}{2\sqrt{x}}$; d) 0. **15.** a) 2; b) $6x+5$; c) $6x^2+8x+6$. **16*.** a) $f'(x)=a$; b) $f'(x)=2ax+b$; c) $f'(x)=3ax^2+2bx+c$. **20.** 1) $4x^3$; 2) $-2x^{-3}$; 3) $-3x^{-4}$. **21.** 2) $-x^{-2}+1$; 4) $4x^3+3x^2+2x-1+x^{-2}+2x^{-3}$. **22.** 2) 1; 4) $-\frac{1}{(2\sqrt{x}(\sqrt{x}-1)^2)}$.

23. 2) 53,25. **24.** 2) –3; 4) 2. **25.** 2) $-\frac{4}{x^2}+\frac{1}{4}$; 4) $2x-\frac{2}{x^3}$. **26.** 2) $3(x+2)^2$; 4) $2x$.

27. 3) $-\frac{2x^9+4x^3}{(x^6-1)^2}$; 4) $-\frac{1}{(x-1)^2}$; 6) $4x^3-4$; 8) $7x^6+3x^2-3x^{-4}-7x^{-8}$. **28.** 2) 0; 4) $\frac{1}{\cos^2 x}$; 6) $\frac{1}{x\ln 2}$; 8) $1+\ln x$; 10) $2e^x-\frac{1}{x}-\frac{1}{x^2}$. **29.** 2) $2e^x\cos x$; 4) $\frac{1-\ln x}{x^2}$; 6) $5+\frac{1}{2\sqrt{x}}+\frac{1}{3\sqrt[3]{x^2}}$; 8) $3(2+x)^2$. **30.** 2) 11. **31.** 2) 0. **32.** 2) $-\frac{1}{\cos^2 x}$; 4) $-\frac{1}{\sin^2 x\cos^2 x}$; 6) $2x\sin x+x^2\cdot\cos x$; 8) $x\cos x$. **33.** 2) 1. **34.** 2) $n\pi$, $n\in z$; 4) 1. **35.** 1) $\frac{1}{x^2}-1$; 2) $4x^2-1$. **36.** 2) $\frac{1+x^2}{1-x^2}$; 4) $\frac{x+2}{x}$. **37.** 2) x^4; 4) x^2-1. **38.** 2) x^3+3x^2+3x+1; 4) x^6+1.

39. x^2-2x. **43.** 2) $e^{\sin x}\cos x$; 4) $\sin 2x$; 6) $\frac{4}{4x-1}$; 8) $20(2x-1)^9$. **44.** 3) $-\text{tg}x$; 8) $-30x^2\cos^{29}x\cdot\sin x+2x\cos^{30}x$; 9) $\frac{5\text{ctg}x}{x}-\frac{5\ln x}{\sin^2 x}$. **45.** 2) $y=3x-4$; $y=3x-4$; $y=3x-4$. 4) $y=-x-2$; $y=8x+16$; $y=-4x$. **46.** 2) $y=7x-6$. **47.** 4) 0 va $\frac{2}{3}$; 6) 0 va $\frac{3}{4}$. **48.** 1) $y=x-2$; $y=-17x-11$; $y=-5x+1$. **49.** 2) 0,1 ; 0,331 . **50.** 2) a) 0,2718; b) 9,06; 4) a) 0,938127;

b) 31,2709. **51.** 2) a) 0; b) 0; 4) a) 0,119401; b) 11,9401 . **52.** 1) 4; 2) –7; 3) 6; 4) 19/28; 5) 0. **53.** 2) 29; 4) $32x-3$; 6) $18-2x$; 8) $48x^2+10x-2$. **54.** 1) a) 15; b) 15; c) 15; d) 15; 4) a) –29; b) 12; c) 5; d) –1. **55.** 2) $3(x+2)^2$; 4) $1-x^{-2}$. **56.** 1) 12; 2) 3.

57. 15 m/s. **58.** 3) $\frac{1}{5\sqrt[5]{x^4}}+\text{tg}x+\frac{x}{\cos^2 x}-\frac{1}{x\ln 3}$; 10) $7^x x^7\ln 7+7^x\cdot 7x^6$; 12) $\frac{\sqrt{2}}{2}-\cos x$;

14) $8-2^x$. **59.** 2) 4; 4) 2. **60.** 2) Ø. **61.** 1 va 2 . **62.** 2) $-2x^{-3}-1$. **63.** 2) 2,75.

64. 2) $\frac{x^2+16x-24}{(x+8)^2}$; 4) $6x^2+8x+5$; 6) $14x+12$. **65.** 2) $\frac{-2x^7-4x^5-5x^4+21x^2+7}{(x^5+7)^2}$.

66. 2) $e^{5x}(4\cos x-6\sin x)$; 4) $\frac{1-2\ln x}{x^3}$. **67.** 2) –4; 4) $-\frac{1}{\sin^2 1}-\frac{1}{20}$.

68. 1) $2x\sin x+x^2\cos x$; 2) $-\frac{\text{tg}x}{\ln 15}$; 4) $\frac{35\text{tg}^{34}x}{\cos^2 x}$; 8) $(2x-10)\ln\cos x-(x^2-10x+7)\text{tg}x$.

69. 3) o‘sish: $(-\infty; -3) \cup (3; -\infty)$ kamayish: $(-3;3)$.
4) o‘sish: $(-\infty; 0) \cup (0; +\infty)$; kamayish: Ø.
6) o‘sish: $(-\infty; \sqrt{2}) \cup (\sqrt{2}; +\infty)$; kamayish: $-\sqrt{2}; \sqrt{2}$.
8) o‘sish: $(-\infty; 0)$; kamayish: $(0; +\infty)$.
9) o‘sish: $(-1; 0) \cup (1; +\infty)$; kamayish: $(-\infty; -1) \cup (1; +\infty)$.
10) o‘sish: $(2; +\infty)$; kamayish: $(-\infty; 2)$.
14) o‘sish: $(-\frac{\pi}{2}+n\pi; \frac{\pi}{2}+n\pi)$, $n\in\mathbb{Z}$; kamayish: Ø.

70. 2) –3; 3 . 4) 0. 6) Ø. 8) 0; –1.

71. 2) lokal minimum $x=4$; lokal maksimum mavjud emas.
4) lokal minimum $x=5$; lokal maksimum $x=-5$.
6) lokal minimum $x=0{,}75$; lokal maksimum mavjud emas.
8) lokal minimum $x=2n\pi$, $n\in\mathbb{Z}$; lokal maksimum $x=\pi+2n\pi$, $n\in\mathbb{Z}$.

72. 2) o‘sadi $(-1; 1)$; kamayadi: $(-\infty; -1) \cup (1; +\infty)$.
4) o‘sadi: $(-\frac{\pi}{2}+n\pi; \frac{\pi}{2}+n\pi)$, $n\in\mathbb{Z}$; kamayadi: $(-\frac{\pi}{2}+2n\pi; \frac{3\pi}{2}+2n\pi)$, $n\in\mathbb{Z}$;
6) o‘sadi: Ø ; kamayadi: $(-\frac{\pi}{2}+n\pi; \frac{\pi}{2}+n\pi)$, $n\in\mathbb{Z}$.

73. 2) eng katta qiymat: 57; eng kichik qiymat: –55.
4) eng katta qiymat: 84; eng kichik qiymat: $-\frac{28}{9}$.

76. 5625m^2. **80.** 80 m. **83.** 1) 5 s; 2) 250 m/s; 3) $\frac{1875}{4}$m.

87. 1) 4m^3; 2) 5324 m^3; 3) 407 $\frac{\text{m}^3}{\text{min}}$;

89. 1) 30 ta; 2) 1800000 so‘m .

91. d) 24,52, –0,1; e) 40,52, 9,86. **93.** g) 2,0004. **94.** e) 0,9302.

95. d) 0,526. **96.** d) 0,1247. **112.** 1) eng katta 13; eng kichik 13; 3) eng katta mavjud emas; eng kichik 5; 5) eng katta mavjud emas; eng kichik $\frac{11}{6}$.

113. 2) $y=13x+4$;$y=13x+4$; $y=13x+4$. **114.** 1) mavjud emas. **115.** 3) mavjud emas.

117. 1) -1; 2) 0; 3) $-\frac{3}{4}$; 4) $-\frac{1}{2}$; 5) 75; 6) $\frac{\sqrt{3}}{2}$; 7) $-\frac{3}{16}$; 8) $\frac{5\sqrt{2}}{2}$; 9) $-\sqrt{2}$.

118. 1) 19; 10; 2) 27;30; 3) 77; 30; 4) 0; –8.

119. 1) 1; 2) 0; 3) $-\frac{3}{4}$; 4) $-\frac{1}{2}$; 5) 75; 6) $\frac{\sqrt{3}}{2}$; 7) $-\frac{3}{16}$; 8) $\frac{5\sqrt{2}}{2}$; 9) $\sqrt{2}$; 10) 0.

120. 1) 10; 6. 2) 15; 18. 3) 225; 80.

121. 1) $-2x+1$; 2) $\cos x+\sin x$; 4) $4^x\ln 4-\cos x$; 6) $\frac{1}{x}-20x+1$. **122.** 1) $4x^3$; 3) $1+\frac{20}{x^2}$; 6) $e^x(\sin x+\cos x)$;8) $20\sin x+2(10x-1)\cos x$.

123. 1) $\frac{1}{\sqrt{e^{\pi}}}$; 0; 2) 3; 3; 3) $-2\pi+1$; $\pi+1$. 4) $-\pi$; $\frac{\pi}{2}+\frac{\sqrt{2}}{2}$; 5) 1; 0; 6) 0; $\frac{\sqrt{2}}{2}$; 7) $1-\frac{\pi^3}{2}$; $-\frac{\sqrt{2}}{2}+\frac{\pi^3}{16}$. 8) 3; $-3\sqrt{2}$.

124. 1) 12; 2)72. **126.** 1) 0; 2) 600 000. **127.** 2) $-\sin 2x$.

128. 2) o‘sish: $(-\infty; +\infty)$; kamayish: $\varnothing$.
4) o‘sish: $\varnothing$; kamayish: $(-\infty; 0) \cup (0; +\infty)$.
6) o‘sish: $(-\infty; +\infty)$; kamayish: $\varnothing$.
8) o‘sish: $(0; +\infty)$; kamayish: $(-\infty; 0)$.

129. 2) $\sqrt{\frac{133}{3}}$; $-\sqrt{\frac{133}{3}}$. 4) 0; 6) 3; –3; 8) 0; $-\frac{13}{18}$.

130. 2) lokal minimum: $x=9$. lokal maxsimum: mavjud emas.

131. 2) eng katta: 81; eng kichik: –6. **134.** 62 500 m^2.

143. 1) $3e^{3x}$; 2) $e^{\sin x}\cos x$; 3) $3\cos(3x+2)$; 4) $8(2x+1)^3$;

144. 1) e^{8x+4}; 2) e^{8x^2+4x}; 3) $4e^{2x}+2$; 4) $\sqrt{16x+10}$.

145. 1) $10x(x^2+1)^4$; 3) $\frac{5}{2\sqrt{5x-7}}$; 8) $-e^{\sin(\cos x)}\cdot\cos(\cos x)\cdot\sin x$.

146. 1) o‘sadi: $(-\infty; 0,5)$; kamayadi: $(0,5; -\infty)$;
3) o‘sadi: $(-1; 1)$; kamayadi: $(-\infty; -1)\cup(1;+\infty)$.
4) o‘sadi: $(-\infty; +\infty)$; kamayadi: $\varnothing$.
7) o‘sadi: $(-\infty; +\infty)$; kamayadi: $\varnothing$.
8) o‘sadi: $(1; +\infty)$; kamayadi: $(-\infty; 1)$.

147. 1) statsionar nuqtalari: 1 va 3; lokal maksimum: 0; lokal minimum: –4.

II BOB

2. 2) x^6+C; 4) $x^{\frac{3}{2}}+C$; 6) $\sin x+C$; 8) $\frac{1}{2}\sin 2x+C$. **3.** 2) $\frac{\pi^x}{\ln\pi}+C$; 4) $\frac{a^x}{\ln a}+C$; 6) $\frac{e^{\pi x}}{\pi}+C$. **4.** 4) $\frac{1}{a}\ln x+C$. **5.** 4) $\frac{1}{5}\sin 5x+C$; 6) $\frac{1}{2}\cos 2x+C$.

6. 4) $\frac{1}{8}(2x-1)^4+C$. **7.** 2) $-\frac{1}{3}x^3+x^2+5x+2$; 4) $\sin x+4$. **8.** 1) $2x^2+8x+11$; 2) $-\frac{x^2}{2}+3x+2{,}5$; 3) $\frac{9}{4}x^2+9x+15{,}8$; 4) $x^2-6x+10$. **10.** 1) $\frac{8}{x}-2x+4$; 2) $\frac{9}{x^2}+2x-3$; 3) x^3-x+6; 4) x^5+7x+1. **11.** 1) $\frac{1}{4}\cdot(3x-2)^{\frac{4}{3}}+\frac{3}{4}$; 2) $\frac{1}{5}\cdot(4x+5)^{\frac{5}{4}}+\frac{4}{5}$; 3) $\frac{1}{8}\cdot(7x-5)^{\frac{8}{7}}+\frac{7}{8}$; 4) $\frac{1}{k+1}\cdot(kx+b)^{\frac{k+1}{k}}+\frac{k}{k+1}$.

12. 1) $5\ln|x-2|+7$; 2) $3\ln|x+1|+1$; 3) $\sin x+7$; 4) $-\cos x+9$. **14.** 2) $\frac{1}{5}\cdot\sin 5x+\frac{3}{5}$; 4) $-3\cos\frac{x}{3}+6$. **15.** 1) x^3-4; 2) x^4-15. **16.** 2) x^8+x^5; 4) $-\frac{5}{3}\cdot\frac{1}{x^3}-\frac{3}{4}\cdot\frac{1}{x^4}$.

17. 2) $-7\cos x+4\sin x$; 4) $5e^x+2\sin x$. **18.** 2) $\frac{1}{5}(x+5)^5$; 4) $9\cdot(x+1)^{\frac{2}{3}}$; 6) $-2\cos(x-3)-4\ln|x-2|$. **19.** 2) $-\frac{1}{7}\cdot\cos(7x-6)+C$; 4) $-\frac{7}{5}\cos(\frac{5x}{7}-2)+C$; 6) $-\frac{1}{2}\cdot e^{3-2x}+C$. **20.** 2) $\frac{1}{15}\cdot(3x+2)^5+\frac{1}{5}x^{-5}+C$; 4) $x^2+3\,\mathrm{ctg}\,x+6x+C$. **21.** 2) $\frac{1}{5}\sin 5x+3\frac{4}{5}$; 4) $x^4-\sqrt{x-1}-15$. **22.** 2) $\frac{1}{5}x^5+\frac{1}{3}\sin 3x+4x+C$; 4) $x^4+3\sin\frac{x}{3}-3\cdot\cos\frac{x}{3}+C$.

23. 2) $\frac{-1}{4}\cos 4x+C$. **24.** 1) $\frac{-1}{16}\cos 8x-\frac{1}{4}\cos 4x$. **25.** 2) $\ln\left|\frac{x-4}{x-3}\right|+C$; 4) $\ln|x-4|+C$.

26. 2) $x-\mathrm{arctg}\,x+C$; 4) $-\frac{1}{2}\mathrm{ctg}\,x+C$. **27.** 2) $-\frac{1}{4(1+x^2)^2}+C$; 4) $-\frac{1}{2}\mathrm{ctg}^2x+C$.

28. 2) $\frac{8}{3}(x-4)^{\frac{3}{2}}+\frac{2}{5}(x-4)^{\frac{5}{2}}+C$; 4) $\frac{1}{3}\mathrm{tg}^3x+C$. **29.** 2) $-\frac{3}{25}\cos 5x+3$. **31.** 4) $x+x^2-\sqrt{1-2x}+C$. **33.** 1) $\sin x-x\cos x+C$; 2) $x^2\cdot\sin x-2\sin x+2x\cos x+C$;

3) $\frac{1}{2}\cdot x^2\ln x-\frac{1}{4}x^2+C$;   4) $x\cdot \mathrm{arctg}x-\frac{1}{2}\ln(1+x^2)+C$.

34. 1) $\frac{1}{2}\cdot(x\sin 2x+\frac{1}{2}\cos 2x)+C$;   3) $9\sin\frac{x}{3}-3x\cdot\cos\frac{x}{3}+C$.

36. 4) 30. **37.** 4) $\frac{1}{4}$. **38.** 2) $\frac{1}{4}\cdot(e^8-1)$. **39.** $\frac{1}{8}$. **40.** 2) 2. **41.** 1,5+ln2. **42.** 1) $a=\frac{1}{\ln 2}$, $b=\frac{7(\ln^2 2-1)}{3\ln^2 2}$;   2) $b=2$. **43.** 1) $b=3$; 2) $a>\ln 2$. **44.** 1) $f(x)=4x-3$;   2) $f(x)=4-2x$;   3) $f(x)=x^2-3x$; 4) $f(x)=1+2x+\cos x$. **45.** 2) $\frac{4}{5\ln 5}$; 6) 8. **46.** 2) $\frac{0,4}{\ln 5}+\frac{0,1}{\ln 2}$; 4) 1. **47.** 2) $\frac{56}{3}$; 4) $1-\frac{\sqrt{2}}{2}$. **48.** 2) $85\frac{1}{3}$. **49.** 1) $\frac{4}{3}$;   2) 121,5;   3) $\frac{10}{3}$;   4) $\frac{1}{6}$.

50. 1) 9;   2) 9;   3) 4,5;

Foydalanilgan va tavsiya etiladigan adabiyotlar

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MUNDARIJA

GEOMETRIYA

I BOB. FAZODA KOORDINATALAR SISTEMASI VA VEKTORLAR

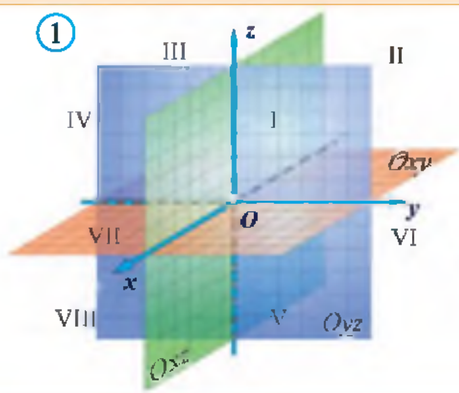
1. FAZODA KOORDINATALAR SISTEMASI

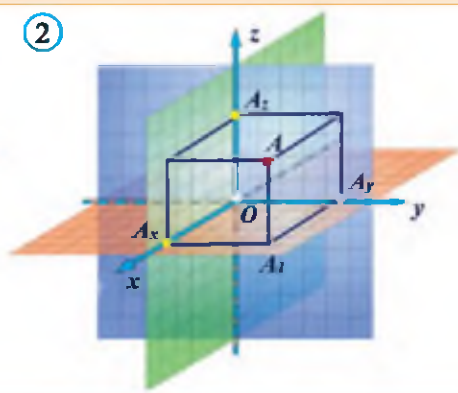
1.1. Fazoda dekart koordinatalar sistemasi

Tekislikda dekart koordinatalari sistemasi bilan quyi sinflarda tanishgansiz. Fazoda koordinatalar sistemasi ham tekislikdagiga o‘xshash kiritiladi. *O* nuqtada kesishuvchi va koordinata boshi shu nuqtada bo‘lgan o‘zaro perpendikular uchta *Ox*, *Oy* va *Oz* koordinata o‘qlarini qaraymiz.

Bu to‘g‘ri chiziqlarning har bir jufti orqali *Oxy*, *Oxz* va *Oyz* tekisliklar o‘tkazamiz (1- rasm). Fazoda to‘g‘ri burchakli dekart koordinatalari sistemasi shu tariqa kiritiladi va unda

O nuqta – *koordinatalar boshi*,
Ox, *Oy* va *Oz* to‘g‘ri chiziqlar – *koordinata o‘qlari*,
Ox – *abssissalar*, *Oy* – *ordinatalar* va *Oz* o‘qi – *applikatalar o‘qi*,
Oxy, *Oyz* va *Oxz* tekisliklar – ***koordinatalar tekisliklari*** deb ataladi.





Koordinatalar tekisliklari fazoni 8 ta *oktantaga* (*nimchorakka*) bo‘ladi (1- rasm).

Fazoda ixtiyoriy *A* nuqta berilgan bo‘lsin. Bu nuqtadan *Oxy*, *Oyz* va *Oxz* koordinata tekisliklariga perpendikular tekisliklar o‘tkazamiz (2- rasm). Bu tekisliklardan biri *Ox* o‘qini A_x nuqtada kesib o‘tadi.

A_x nuqtaning x o‘qidagi koordinatasi *A* nuqtaning x – *koordinatasi* yoki *abssissasi* deb ataladi.

A nuqtaning *y – koordinatasi* (*ordinatasi*) hamda *z – koordinatasi* (*applikatasi*) ham shu tariqa aniqlanadi.

A nuqtaning koordinatalari *A* (*x*; *y*; *z*) yoki qisqaroq (*x*; *y*; *z*) tarzda belgilanadi. 3- rasmda tasvirlangan nuqtalar quyidagi koordinatalarga ega: *A* (0; 5; 0), *B*(4; 0; 0), *M* (0; 5; 4), *K* (2; 3; 4), *P* (–2; 3; –4).

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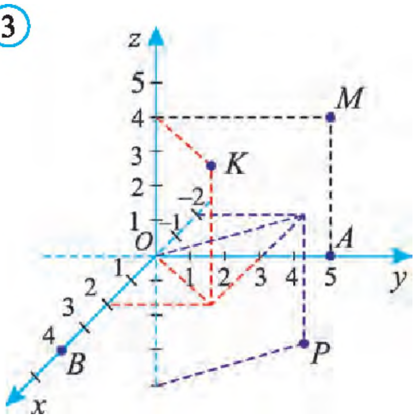

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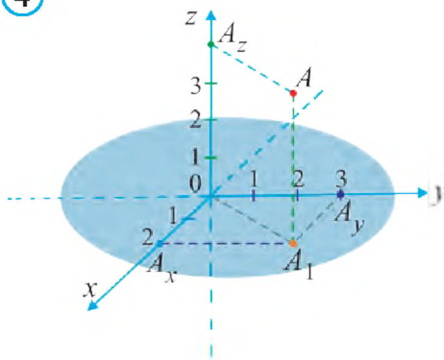

1- masala. Fazoda dekart koordinatalari sistemasi kiritilgan. Undagi *A*(2; 3; 4) nuqtaning o‘rnini aniqlang.

Yechish. Koordinata boshidan *Ox* va *Oy* o‘qlarining musbat yo‘nalishida, mos ravishda, $OA_x = 2$ va $OA_y = 3$ kesmalarni qo‘yamiz (4- rasm).

A_x nuqtadan *Oxy* tekislikda yotgan va *Oy* o‘qiga parallel to‘g‘ri chiziq o‘tkazamiz. A_y nuqtadan *Oxy* tekislikda yotgan va *Ox* o‘qiga parallel to‘g‘ri chiziq o‘tkazamiz. Bu to‘g‘ri chiziqlar kesishish nuqtasini A_1 bilan belgilaymiz. A_1 nuqtadan *Oxy* tekislikka perpendikular o‘tkazamiz va unda *Oz* o‘qining musbat yo‘nalishida $AA_1 = 4$ kesma qo‘yamiz. Hosil bo‘lgan *A* (2; 3; 4) nuqta izlanayotgan nuqta bo‘ladi. □

Zamonaviy raqamli-dasturli boshqariladigan stanoklar va avtomatlashtirilgan robotlar uchun koordinatalar sistemasidan foydalanib dasturlar tuziladi va ular asosida metallarga ishlov beriladi (5- rasm).

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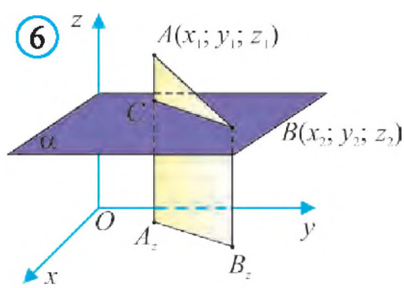

1.2. Ikki nuqta orasidagi masofa

Ikkita $A(x_1; y_1; z_1)$ va $B(x_2; y_2; z_2)$ nuqtalar berilgan bo'lsin.

1. Avval AB to'g'ri chiziq Oz o'qiga parallel bo'lmagan holni qaraymiz (6- rasm). A va B nuqtalar orqali Oz o'qiga parallel chiziqlar o'tkazamiz. Ular Oxy tekislikni A_z va B_z nuqtalarda kesib o'tsin.

Bu nuqtalarning z koordinatasi 0 ga teng bo'lib, x va y koordinatalari esa mos ravishda A, B nuqtalarning x va y koordinatalariga teng.

Endi B nuqta orqali Oxy tekislikka parallel α tekislik o'tkazamiz. U AA_z to'g'ri chiziqni biror C nuqtada kesib o'tadi.

Pifagor teoremasiga ko'ra: $AB^2 = AC^2 + CB^2$.

Lekin $CB = A_zB_z$, $A_zB_z^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ va $AC = |z_2 - z_1|$.

Shuning uchun $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$.

2. AB kesma Oz o'qiga parallel, ya'ni $AB = |z_2 - z_1|$ bo'lganda ham yuqoridagi formula o'rinli bo'ladi, chunki bu holda $x_1 = x_2$, $y_1 = y_2$.

Demak, *A va B nuqtalar orasidagi masofa*:

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2} \quad (1)$$

Izoh. (1) formula to'g'ri burchakli parallelepipedning o'lchamlari $a=|x_2-x_1|$, $b=|y_2-y_1|$, $c=|z_2-z_1|$ bo'lganda, uning diagonali uzunligini ifodalaydi.

Sfera va shar tenglamasi. Ma'lumki, $A(a; b; c)$ nuqtadan R masofada yotgan barcha $M(x; y; z)$ nuqtalar sferani tashkil qiladi (7- rasm). Unda (1) formulaga ko'ra, markazi $A(a; b; c)$ nuqtada radiusi R ga teng bo'lgan sferada yotgan barcha nuqtalar koordinatalari $(x - a)^2 + (y - b)^2 + (z - c)^2 = R^2$ tenglikni qanoatlantiradi.

Unda, ravshanki, markazi $A(a; b; c)$ nuqtada, radiusi R ga teng bo'lgan shar tenglamasi $(x - a)^2 + (y - b)^2 + (z - c)^2 \leq R^2$ tarzda ifodalanadi.

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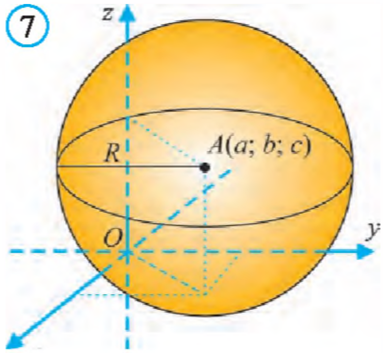

2- masala. Uchlari $A(9; 3; -5)$, $B(2; 10; -5)$, $C(2; 3; 2)$ nuqtalarda bo'lgan ABC uchburchakning perimetrini toping.

Yechish: ABC uchburchakning perimetri $P = AB + AC + BC$. Ikki nuqta orasidagi masofa formulasi $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ dan foydalanib uchburchak tomonlarini topamiz:

$$AB = \sqrt{(2-9)^2 + (10-3)^2 + (-5+5)^2} = \sqrt{49+49} = 7\sqrt{2},$$

$$AC = \sqrt{(2-9)^2 + (3-3)^2 + (2+5)^2} = \sqrt{49+49} = 7\sqrt{2},$$

$$BC = \sqrt{(2-2)^2 + (3-10)^2 + (2+5)^2} = \sqrt{49+49} = 7\sqrt{2}.$$

Demak, ABC uchburchak teng tomonli va uning perimetri:
$P = 3 \cdot 7\sqrt{2} = 21\sqrt{2}$. *Javob:* $21\sqrt{2}$. □

1.3. Kesma o'rtasining koordinatalari

$A(x_1; y_1; z_1)$ va $B(x_2; y_2; z_2)$ – ixtiyoriy nuqtalar bo'lib, AB kesmaning o'rtasi $C(x; y; z)$ bo'lsin (8- rasm).

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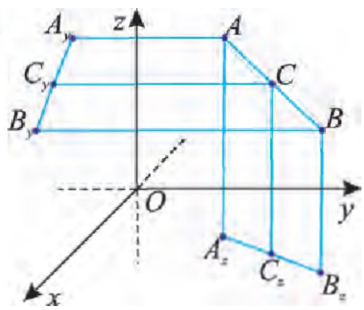

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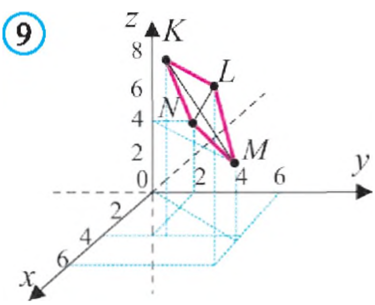

A, B va C nuqtalar orqali Oz o'qiga parallel to'g'ri chiziqlar o'tkazamiz. Ular Oxy tekislikni $A_z(x_1; y_1; 0)$, $B_z(x_2; y_2; 0)$ va $C_z(x; y; 0)$ nuqtalarda kesib o'tsin.

Fales teoremasiga ko'ra C_z nuqta A_zB_z kesmaning o'rtasi bo'ladi.

Unda tekislikda kesma o'rtasining koordinatalarini topish formulasiga ko'ra

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}.$$

z ni topish uchun Oxy tekislik o'rniga Oxz yoki Oyz tekislikni olish kifoya.

Bunda z uchun ham yuqoridagilarga o'xshash formula hosil qilinadi.

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2} \quad z = \frac{z_1 + z_2}{2}.$$

Shunga o'xshash, berilgan AB kesmani λ nisbatda ($AP : PB = \lambda$) bo'luvchi $P(x_1; y_1; z_1)$ nuqtaning koordinatalari A va B nuqtalarning koordinatalari orqali

$$x = \frac{x_1 + \lambda x_2}{1+\lambda}, \quad y = \frac{y_1 + \lambda y_2}{1+\lambda}, \quad z = \frac{z_1 + \lambda z_2}{1+\lambda}$$

formulalar yordamida topiladi. Ularning to‘g‘riligini mustaqil ko‘rsating.

3-masala. Uchlari $M(3;\ 6;\ 4)$, $N(0;\ 2;\ 4)$, $K(3;\ 2;\ 8)$, $L(6;\ 6;\ 8)$ nuqtalarda bo‘lgan *MNKL* to‘rtburchakning parallelogramm ekanligini isbotlang (9- rasm).

Isbot: Masalani yechishda diagonallari kesishish nuqtasida teng ikkiga bo‘linadigan to‘rtburchakning parallelogramm ekanligidan foydalanamiz.

MK kesma o‘rtasining koordinatalari:

$$x=\frac{3+3}{2}=3; \qquad y=\frac{6+2}{2}=4; \qquad z=\frac{4+8}{2}=6.$$

NL kesma o‘rtasining koordinatalari:

$$x=\frac{0+6}{2}=3; \qquad y=\frac{2+6}{2}=4; \qquad z=\frac{4+8}{2}=6.$$

*MK*va *NL* kesmalar o‘rtalarining koordinatalari bir xil ekanini ko‘ramiz. Bu mazkur kesmalar kesishishini va kesishish nuqtasida ular teng ikkiga bo‘linishini bildiradi.

Demak, *MNLK* to‘rtburchak – parallelogramm. □

Mavzuga oid masalalar va amaliy topshiriqlar

1. 10- rasmda tasvirlangan nuqtalarning koordinatalarini aniqlang.

2. Fazoda dekart koordinatalari sistemasi kiritilgan bo‘lib, unda $A(0;\ 3;\ 1)$, $B(-2;\ 0;\ 0)$, $C(0;\ 0;\ 8)$, $D(0;\ -9;\ 0)$, $E(5;\ -1;\ 2)$, $F(-6;\ 2;\ 1)$ nuqtalar berilgan. Bu nuqtalar qaysi *a*) koordinatalar o‘qda; *b*) koordinatalar tekisligida; *c*) oktantda yotadi?

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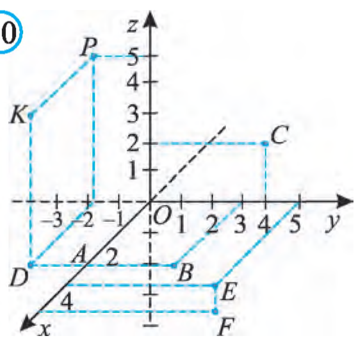

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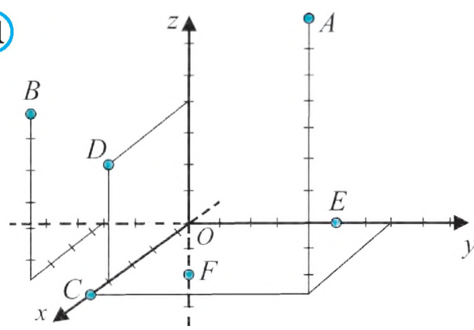
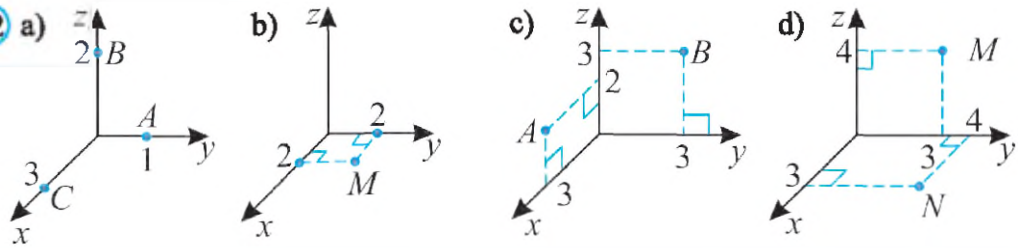


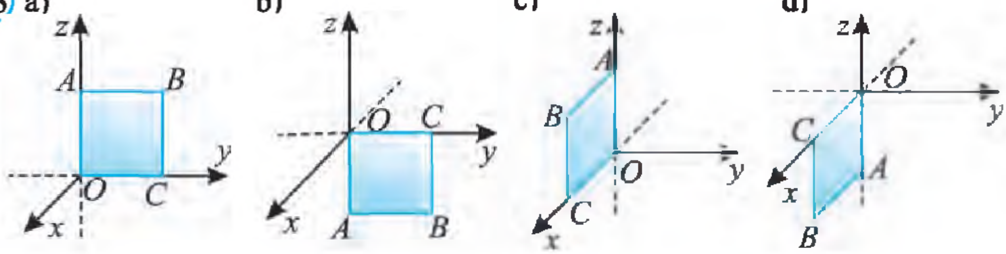
3. 11- rasmdagi nuqtalar koordinatalarini toping.

4. 12- rasmda belgilangan nuqtalarning koordinatalarini toping.

5. 13- rasmda diagonali $\sqrt{2}$ ga teng bo‘lgan kvadrat tasvirlangan. Uning uchlari koordinatalarini toping.

6. $A\ (3;\ 2;\ 4)$ nuqtaning koordinata tekisliklaridagi proyeksiyasi koordinatalarini toping.

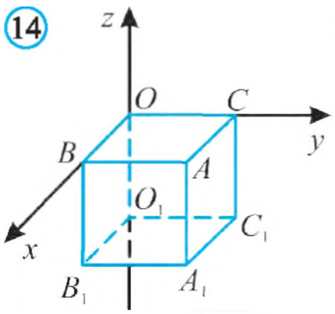
12 

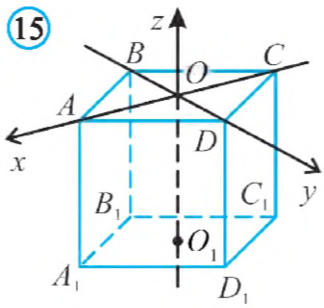
13 

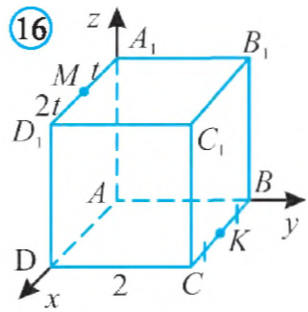
7. Fazoda **dekart** koordinatalari **sistemasi** kiritilgan **bo'lib**, unda $A(-1; 2; -3)$, $B(0; 1; 2)$, $C(0; 0; 5)$, $D(-2; 2; 0)$, $E(5; -1; 0)$, $F(0; 2; 0)$, $G(9; 0; 0)$, $H(9; 0; 2)$, $I(6; 3; 1)$, $J(-6; 3; 5)$, $K(-6; -2; 3)$, $L(6; -2; 4)$, $M(6; 3; -9)$, $N(-6; 3; -8)$, $O(-6; -3; -6)$, $P(6; -3; -2)$ nuqtalar berilgan bo'lsin. Bu nuqtalar qaysi koordinatalar o'qida, kordinatalar tekisligida va oktantda yotadi? Quyida berilgan jadvalni berilgan namunalarga ko'ra to'ldiring.

Nuqta o'rni	Nuqta koordinatalari xususiyati	Nuqta
Ox o'qi	$y=0$, $z=0$ faqat x koordinata noldan farqli	$G(9; 0; 0)$
Oy o'qi		
Oz o'qi		
Oxz tekislik	$z=0$, x va y koordinatalar noldan farqli	$D(-2; 2; 0)$
Oyz tekislik		
Oxz tekislik		
1- oktant	$x>0$, $y>0$, $z>0$	$I(6; 3; 1)$
2- oktant		
3- oktant		
4- oktant		
5- oktant		
6- oktant		
7- oktant		
8- oktant		

8. $A(2; 0; –3)$ va $B(3; 4; 0)$ nuqtalar orasidagi masofani toping.
9. $A(3; 3; 3)$ nuqtadan a) koordinata tekisliklarigacha; b) koordinata o‘qlarigacha; c) koordinata boshigacha bo‘lgan masofalarni toping.
10. $M(2; –3; 1)$ nuqtadan koordinata tekisliklarigacha bo‘lgan masofalarni toping.
11. Koordinata tekisliklarining har biridan 3 birlik masofada uzoqlashgan nuqtaning o‘rnini aniqlang.







12. Agar $OA = 2\sqrt{2}$ bo‘lsa, 14- rasmda tasvirlangan kubning uchlari koordinatalarini toping.
13. $C(2; 5; –1)$ va $D(2; 1; –6)$ nuqtalarning qaysi biri koordinata boshiga yaqin joylashgan?
14. Uchlari $A(1; 2; 3)$, $B(2; 3; 1)$, $C(3; 1; 2)$ nuqtalarda bo‘lgan uchburchakning perimetrini toping.
15. Uchlari $A(1; 2; 3)$, $B(2; 3; 4)$, $C(3; 4; 5)$ nuqtalarda bo‘lgan uchburchak mavjudmi?
16. $A(–2; 0; 5)$, $B(–1; 2; 3)$, $C(1; 1; –3)$, $D(0; –1; –1)$ nuqtalar parallelogramm uchlari ekanligini isbotlang.
17. ABC uchburchak turini aniqlang, uning perimetri va yuzini toping: a) $A(3; 0; 0)$, $B(0; 3; 0)$, $C(0; 0; 3)$; b) $A(2; 0; 5)$, $B(3; 4; 0)$, $C(2; 4; 0)$; c) $A(2; 4; –1)$, $B(–1; 1; 2)$, $C(5; 1; 2)$.
18. Oxy tekisligida yotuvchi va $A(0; 1; –1)$, $B(–1; 0; –1)$, $C(0; –1; 0)$ nuqtalardan baravar uzoqlikda yotuvchi nuqtaning koordinatalarini toping.
19. $A(1; 1; 1)$, $B(–1; 1; 1)$, $C(–1; –1; 1)$, $C_1(–1; –1; –1)$ nuqtalar $ABCDA_1B_1C_1D_1$ kubning uchlari bo‘lsa, uning qolgan uchlari koordinatalarini toping.
20. Uchlari $S(0; 0; 0)$, $A(2; 0; 0)$, $B(0; 2; 0)$, $C(0; 0; 2)$ nuqtalarda bo‘lgan $SABC$ piramidaning muntazam ekanligini isbotlang.
21. Markazi koordinatalar boshida, radiusi 5 ga teng bo‘lgan sfera va shar tenglamalarini yozing.

22. Markazi A (1; 2; 4) nuqtada, radiusi 3 ga teng bo‘lgan shar tenglamasini yozing.

23. Diametri uchlari A (–2; 1; 3), B (0; 2; 1) nuqtalarda yotgan sfera tenglamasini yozing.

24. Qalin qog‘ozdan kub modelini yasang. Uning bitta uchini koordinata boshi, undan chiquvchi qirralarni birlik ortlar sifatida olib, uning boshqa uchlari koordinatalarini toping.

25. AB kesma o‘rtasining koordinatalarini toping:
1) $A(-1; 0; 0)$, $B(1; 2; 0)$; 2) $A(0; 0; 0)$, $B(2; 2; 2)$; 3) $A(-2; 4; 2)$, $B(2; -4; 2)$,
4) $A(1{,}2; -3; 6{,}3)$, $B(-2{,}6; 3{,}2; -5{,}1)$; 5) $A(\sqrt{3}; 2; 1-\sqrt{2})$, $B(3\sqrt{3}; 1; 1+\sqrt{2})$.

26. 15- rasmda tasvirlangan kub qirralari o‘rtalarining va yoqlari markazlarining koordinatalarini toping.

27. $A(3;-1;4)$, $B(-1;1;-8)$, $C(2;1;-6)$, $D(0;1;2)$ nuqtalar berilgan. a) AB va CD; b) AC va BD kesmalar o‘rtasining koordinatalarini toping.

28. $M(1;-1;2)$ va $N(-3;2;4)$ nuqtalar AB kesmani uchta teng bo‘laklarga ajratadi. AB kesma uchlarining koordinatalarini toping.

29. $ABCD$ to‘rtburchakning tomonlari va $A_1B_1C_1D_1$ to‘g‘ri to‘rtburchakning tomonlariga mos ravishda parallel. $ABCD$ – to‘g‘ri to‘rtburchak ekanini isbotlang?

30. $ABCD$ to‘g‘ri to‘rtburchakning A uchidan uning tekisligiga perpendikular AK to‘g‘ri chiziq o‘tkazilgan. K nuqtadan to‘g‘ri to‘rtburchakning boshqa uchlarigacha bo‘lgan masofalar 6 cm, 7 cm va 9 cm. AK kesmaning uzunligini toping.

31*. Fazoda $A(3; 0; -1)$, $B(-4; 1; 0)$, $C(5; -2; -1)$ nuqtalar berilgan. Oyz tekislikda A, B, C nuqtalardan baravar uzoqlikda joylashgan nuqtani toping.

32. $ABCD$ parallelogrammning uchlari: a) $A(-2; -4; 3)$, $B(3; 1; 7)$, $C(4;2;-5)$; b) $A(4; 2; -1)$, $B(1; -3; -2)$, $C(-6; 2; 1)$; c) $A(-1; 7; 4)$, $B(1; 5; 2)$, $C(9; -3; -8)$ bo‘lsa, D uchining koordinatalarini toping.

33. CK kesmani $CK:KM = \lambda$ nisbatda bo‘luvchi $M(x; y; z)$ nuqtaning koordinatalarini toping. a) $C(-5; 4; 2)$, $K(1; 1;-1)$ va $\lambda=2$; b) $C(1; -1; 2)$, $K(2; -4; 1)$ va $\lambda=0{,}5$; c) $C(1; 0; -2)$, $K(9; -3; 6)$ va $\lambda = \frac{1}{3}$.

34. Uchlari $A(3; 2; 4)$, $B(1; 3; 2)$, $C(-3; 4; 3)$ nuqtalarda bo‘lgan uchburchak medianalari kesishish nuqtasi M ning koordinatalarini toping.

35. Uchlari $A(5; 6; 3)$, $B(3; 5; 1)$, $C(0; 1; 1)$ nuqtalarda bo‘lgan uchburchakning BL bissektrisasining L uchi koordinatalarini toping.

36*. Uchlari $A(4; 0; 1)$, $B(5; -2; 1)$, $C(4; 8; 5)$ nuqtalarda bo‘lgan uchburchakning AL bissektrisasi uzunligini toping.

37*. Uchlari $A(1; 3; -1)$, $B(3; -1; 1)$, $C(3; 1; -1)$ nuqtalar bo‘lgan uchburchak berilgan. Uning: a) katta tomoniga tushirilgan balandligini; b) burchaklarini; c) yuzini toping.

38*. 16- rasmda tasvirlangan kub haqidagi ma’lumotlardan foydalanib MK kesma uzunligini toping.

Tarixiy ma’lumotlar

Abu Rayhon Beruniy mashhur tabib va matematik Abu Ali ibn Sino bilan yozishmalarida unga quyidagi savolni beradi: „Nima uchun Aristotel va boshqa (faylasuf)lar tomonlarni oltita deb atashadi?”

Beruniy olti yoqli kubni olib, „boshqacha sondagi tomonlarga ega bo‘lgan” jismlar haqida gapiradi va „sharsimon jismning tomonlari yo‘qligi”ni qo‘shib qo‘yadi.

Ibn Sino esa „hamma hollarda ham tomonlar oltita deb hisoblamoq zarur, chunki har bir jismda, uning shaklidan qat’iy nazar uch o‘lchov — uzunlik, chuqurlik va kenglik mavjud” deb javob beradi.

Bu yerda Ibn Sino „olti tomon” deb ishoralari bilan olingan uchta koordinatani nazarda tutadi.

Beruniy „Qonuniy Mas’udiy” asarida olti tomonning aniq matematik ma’nosini keltiradi: „Tomonlar oltita, chunki ular jismlarning o‘lchovlari bo‘yicha harakatlari chegarasidir. O‘lchovlar uchta, bu uzunlik, kenglik va chuqurlik, ularning uchlari esa o‘lchovlardan ikki marta ko‘p”.

Asarning oldingi kitoblarida muallif yoritgichlarning osmondagi holatini osmon sferasiga nisbatan ikki koordinata – ekliptik kenglama va uzoqlama orqali yoki xuddi shunday koordinatalar orqali, ammo osmon ekvatori yoki gorizontga nisbatan aniqlaydi. Ammo yulduzlar va yoritgichlarning o‘zaro joylashuvini aniqlash masalasida ularning bir-birlarini to‘sib qolish hollarini ham e’tiborga olishga to‘g‘ri keladi. Mana shunday holda uchinchi sferik koordinataga ehtiyoj tug‘iladi. Ana shu ehtiyoj Abu Rayhon Beruniyni fazoviy koordinatalar g‘oyasini ilgari surishga olib kelgan.

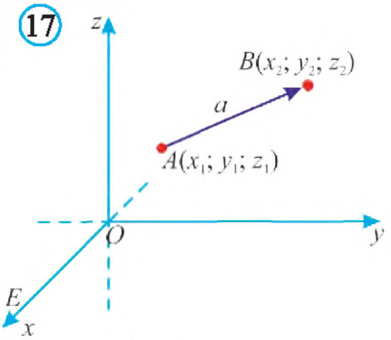
2. FAZODA VEKTORLAR VA ULAR USTIDA AMALLAR

2.1. Fazoda vektorlar

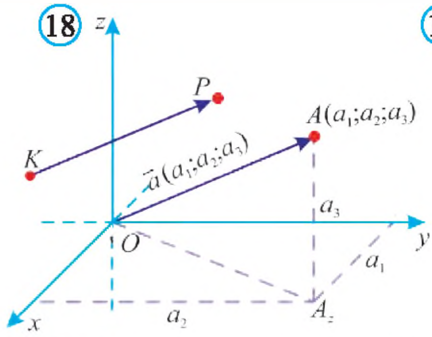
Fazoda vektor tushunchasi tekislikdagi singari kiritiladi.

Fazoda *vektor* deb yo‘naltirilgan kesmaga aytiladi.

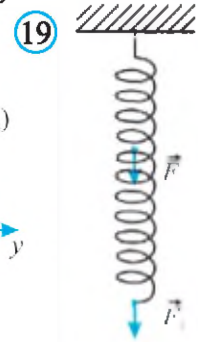
Fazoda vektorlarga oid asosiy tushunchalar: vektorning uzunligi (moduli), vektorning yo‘nalishi, vektorlarning tengligi tekislikdagi singari ta’riflanadi.



$\overline{a}(a_1;a_2;a_3)$



$\overline{AB}(x_2-x_1;y_2-y_1;z_2-z_1)$



Boshi $A\,(x_1;y_1;z_1)$ nuqtada va oxiri $B\,(x_2;y_2;z_2)$ nuqtada bo‘lgan ***vektorning koordinatalari*** deb $a_1=x_2-x_1$, $a_2=y_2-y_1$, $a_3=z_2-z_1$ sonlarga aytiladi (17- rasm).

Vektorlarning tekislikdagiga o‘xshash qator xossalari ham borki, ularni isbotsiz keltiramiz.

Xuddi tekislikdagi singari teng vektorlarning mos koordinatalari teng bo‘ladi va aksincha, mos koordinatalari teng bo‘lgan vektorlar teng bo‘ladi.

Bu vektorni uning koordinatalari bilan ifodalashga asos bo‘ladi. Vektorlar $\overline{AB}(a_1;a_2;a_3)$ yoki $\overline{a}(a_1;a_2;a_3)$ yoki qisqaroq $\overline{(a_1;\ a_2;\ a_3)}$ tarzda belgilanadi (18-rasm).

Vektor koordinatalarisiz $\overline{AB}$ (yoki qisqaroq $\overline{a}$) tarzda ham belgilanadi. Bunda uning boshi birinchi o‘rinda, oxiri esa ikkinchi o‘rinda yoziladi.

Koordinatalari nollardan iborat vektor *nol vektor* deb ataladi va $\overline{0}(0;0;0)$ yoki $\overline{0}$ tarzda belgilanadi hamda bu vektorning yo‘nalishi bo‘lmaydi.

Agar O koordinata boshi va a_1, a_2 va a_3 sonlar A nuqtaning koordinatalari, ya’ni $A(a_1;\ a_2;\ a_3)$ bo‘lsa, bu sonlar $\overline{OA}$ vektorning ham koordinatalari bo‘ladi: $\overline{OA}(a_1;\ a_2;\ a_3)$.

Lekin koordinatalar fazosida boshi $K(c_1;\ c_2;\ c_3)$ nuqtada, oxiri $P\,(c_1+a_1;\ c_2+a_2;\ c_3+a_3)$ nuqtada bo‘lgan $\overline{KP}$ vektor ham shu koordinatalar bilan ifodalanadi: $\overline{KP}(c_1+a_1-c_1;\ c_2+a_2-c_2;\ c_3+a_3-c_3)=\overline{KP}\ (a_1;\ a_2;\ a_3)$.

Shundan kelib chiqib, vektorni koordinatalar fazosida istalgan nuqtaga qo'yilgan qilib tasvirlash mumkin. Geometriyada biz shunday *erkin* vektorlar bilan ish ko'ramiz. Fizikada esa, odatda, vektorlar biror *nuqtaga qo'yilgan* bo'ladi. Masalan, 19- rasmdagi F kuch prujinaning qaysi nuqtasiga qo'yilgani bilan ahamiyatli hisoblanadi.

Vektorning uzunligi deb uni tasvirlovchi yo'naltirilgan kesmaning uzunligiga aytiladi (17- rasm). $\bar{a}$ vektorning uzunligi $|\bar{a}|$ tarzda ifodalanadi.

$\bar{a}(a_1;\ a_2;\ a_3)$ vektorning uzunligi uning koordinatalari orqali $|\bar{a}|=\sqrt{a_1^2+a_2^2+a_3^2}$ formula bilan ifodalanadi.

1- masala. $A(2; 7; -3)$, $B(1; 0; 3)$, $C(-3; -4; 5)$ va $D(-2; 3; -1)$ nuqtalar berilgan. $\overline{AB}$, $\overline{BC}$, $\overline{DC}$, $\overline{AD}$, $\overline{AD}$ va $\overline{BD}$ vektorlardan qaysilari o'zaro teng bo'ladi?

Yechish: Teng vektorlarning mos koordinatalari teng bo'ladi. Shuning uchun vektorlarning koordinatalarini topamiz:

$$\overline{AB}=(1-2,\ 0-7,\ 3-(-3))=(-1,\ -7,\ 6);$$

$$\overline{DC}=(-3-(-2),\ -4-3,\ 5-(-1))=(-1,\ -7,\ 6).$$

Demak, $\overline{AB}=\overline{DC}$. $\overline{BC}=\overline{AD}$ ekanligini mustaqil ko'rsating. □

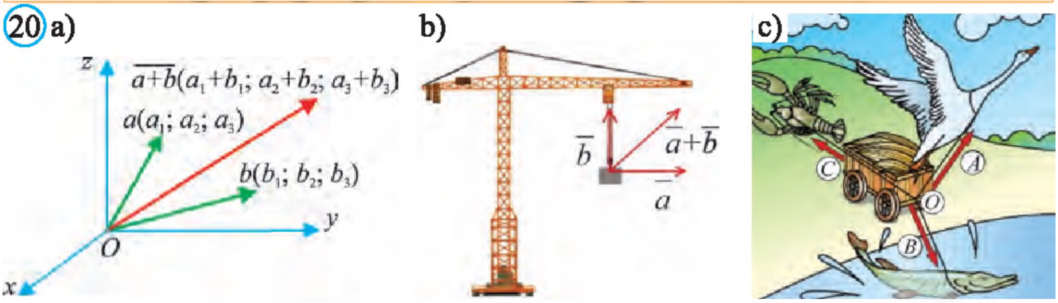
2.2. Fazoda vektorlar ustida amallar

Vektorlar ustida amallar. Vektorlarni qo'shish, songa ko'paytirish va skalar ko'paytirish amallari xuddi tekislikdagidek aniqlanadi.

$\bar{a}(a_1; a_2; a_3)$ va $\bar{b}(b_1; b_2; b_3)$ *vektorlarning yig'indisi* deb

$\bar{a}+\bar{b}=(a_1+b_1; a_2+b_2; a_3+b_3)$ vektorga aytiladi (20-rasm).

20 a) b) c)



20.*b*-rasmda kran $\bar{a}$ vektor bo'yicha, yuk esa kranga nisbatan $\bar{b}$ vektor bo'yicha harakatlanayotgan bo'lsin. Natijada yuk $\bar{a}+\bar{b}$ vektor bo'yicha harakatlanadi. Shuningdek, 20.*c*- rasmda tasvirlangan rus yozuvchisi Krilovning masali qahramonlari nima sababdan aravani joyidan qo'zg'ata olmayotganini sezgan bo'lsangiz kerak.

Vektorlar yig'indisining xossalari.

Ixtiyoriy $\bar{a}$, $\bar{b}$ va $\bar{c}$ vektorlar uchun quyidagi xossalar o'rinli:

a) $\bar{a}+\bar{b}=\bar{b}+\bar{a}$ – vektorlarni qo'shishning o'rin almashtirish qonuni;

b) $\bar{a}+(\bar{b}+\bar{c})=(\bar{a}+\bar{b})+\bar{c}$ – vektorlarni qo'shishning taqsimot qonuni.

Vektorlarni qo'shishning uchburchak qoidasi.

Ixtiyoriy A, B va C nuqtalar uchun (21-rasm): $\overline{AB}+\overline{BC}=\overline{AC}$.

Vektorlarni qo'shishning parallelogramm qoidasi.

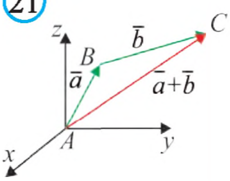
Agar $ABCD$ – parallelogramm (22- rasm) bo'lsa, $\overline{AB}+\overline{AD}=\overline{AC}$.

Vektorlarni qo'shishning ko'pburchak qoidasi.

Agar A, B, C, D va E nuqtalar ko'pburchak uchlari bo'lsa (23- rasm),

$$\overline{AB}+\overline{BC}+\overline{CD}+\overline{DE}=\overline{AE} \text{ bo'ladi.}$$

21



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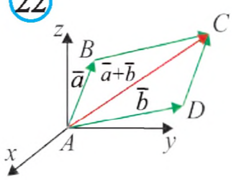

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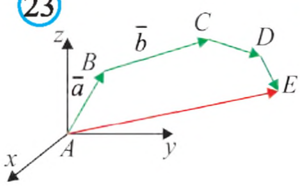

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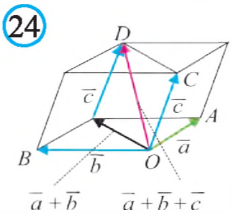


Bir tekislikda yotmagan uchta *vektorlarni qo'shishning parallelepiped* qoidasi. Agar $ABCDA_1B_1C_1D_1$ parallelepiped (24- rasm) bo'lsa,

$$\overline{AB}+\overline{AD}+\overline{AA_1}=\overline{AC} \text{ bo'ladi.}$$

$\bar{a}(a_1;a_2;a_3)$ *vektorning* λ *songa ko'paytmasi* deb $\lambda\bar{a}=(\lambda a_1;\lambda a_2;\lambda a_3)$ vektorga aytiladi (25- rasm).

Ixtiyoriy $\bar{a}$ va $\bar{b}$ vektorlar hamda λ va μ sonlar uchun

a) $\lambda(\bar{a}+\bar{b})=\lambda\bar{a}+\lambda\bar{b}$;

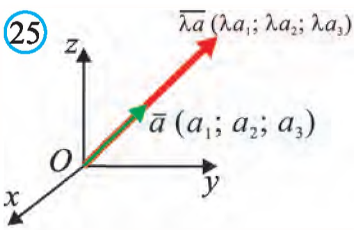
b) $(\lambda+\mu)\bar{a}=\lambda\bar{a}+\mu\bar{a}$;

c) $|\lambda\bar{a}|=|\lambda|\cdot|\bar{a}|$ va $\lambda\bar{a}$ vektorning yo'nalishi

$\lambda>0$ bo'lganda, $\bar{a}$ vektor yo'nalishi bilan bir xil va

$\lambda<0$ bo'lganda, $\bar{a}$ vektor yo'nalishiga qarama-qarshi bo'ladi.

25



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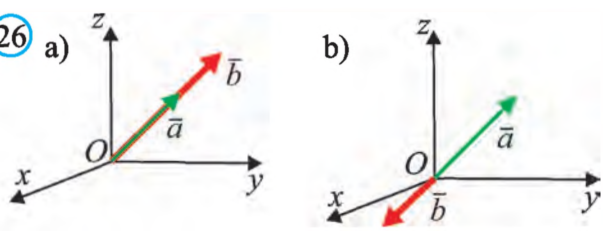


2.3. Kollinear va komplanar vektorlar

Nol vektordan farqli $\bar{a}$ va $\bar{b}$ vektorlar berilgan bo'lsin. $\bar{a}$ va $\bar{b}$ vektorlar bir xil yoki qarama-qarshi yo'nalgan bo'lsa, ular *kollinear vektorlar* deb ataladi (26- rasm).

1- xossa. $\bar{a}$ va $\bar{b}$ vektorlar uchun $\bar{a} = \lambda\bar{b}\ (\lambda \neq 0)$ tenglik o'rinli bo'lsa, ular o'zaro kollinear bo'ladi va aksincha.

Agar $\lambda > 0$ bo'lsa, $\bar{a}$ va $\bar{b}$ vektorlar bir tomonga ($\bar{a} \uparrow\uparrow \bar{b}$), agar $\lambda < 0$ bo'lsa, qarama-qarshi tomonga ($\bar{a} \uparrow\downarrow \bar{b}$) yo'nalgan bo'ladi.

2- xossa. $\bar{a}(a_1; a_2; a_3)$ va $\bar{b}(b_1; b_2; b_3)$ vektorlar o'zaro kollinear bo'lsa, ularning koordinatalari o'zaro proporsional bo'ladi: $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ va aksincha.

2- masala. Boshi A (1; 1; 1) nuqtada va oxiri Oxy tekislikdagi B nuqtada bo'lgan va $\bar{a}(1; 2; 3)$ vektorga kollinear vektorni toping.

Yechish: B nuqtaning koordinatalari $B(x; y; z)$ bo'lsin. B nuqta Oxy tekislikda yotgani uchun $z=0$. Unda $\overline{AB}(x-1; y-1; -1)$ bo'ladi.

Shartga ko'ra, $\overline{AB}(x-1; y-1; -1)$ va $\bar{a}(1, 2, 3)$ vektorlar kollinear. Demak, ularning koordinatalari o'zaro proporsional bo'ladi.

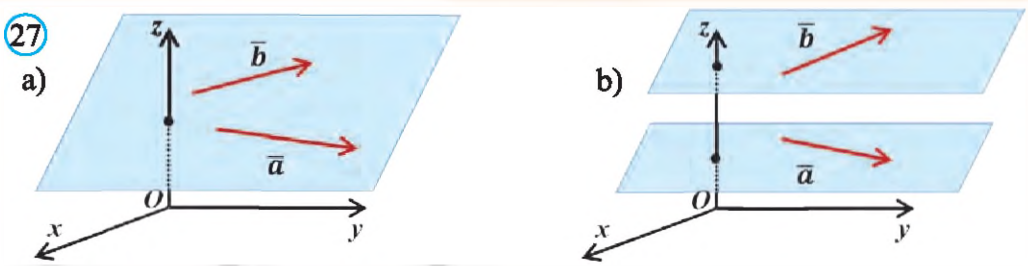
Bundan $\frac{x-1}{1} = \frac{y-1}{2} = \frac{-1}{3}$ proporsiyalarni hosil qilamiz.

Ulardan $x = \frac{2}{3}$, $y = \frac{1}{3}$ ekanligini topamiz.

Unda $\overline{AB}\left(-\frac{1}{3}; -\frac{2}{3}; -1\right)$ bo'ladi. □

Bitta tekislikda yoki parallel tekisliklarda yotuvchi vektorlar *komplanar vektorlar* deb ataladi (27- rasm).

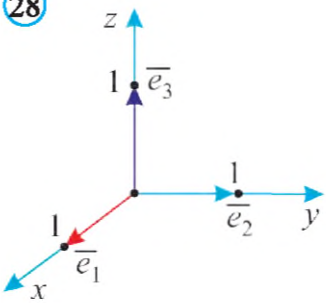
27



$\bar{e}_1$ (1; 0; 0), $\bar{e}_2$ (0; 1; 0) va $\bar{e}_3$ (0; 0; 1) vektorlar *ortlar* deb ataladi (28- rasm).

Ixtiyoriy $\bar{a}(a_1; a_2; a_3)$ vektorni $\bar{a} = a_1\bar{e}_1 + a_2\bar{e}_2 + a_3\bar{e}_3$ ko'rinishda, yagona tarzda *ortlar bo'yicha yoyish* mumkin (29- rasm).

28



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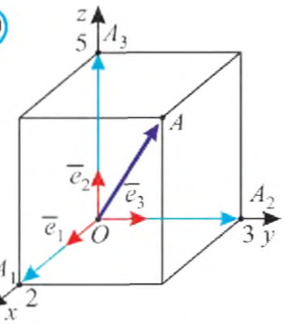


Shuningdek, uchta komplanar bo'lmagan $\overline{OA}, \overline{OB}$ va $\overline{OC}$ vektorlar berilgan bo'lsa, ixtiyoriy $\overline{OD}$ vektorni quyidagi ko'rinishda, yagona tarzda ifodalash mumkin:

$$\overline{OD} = a_1 \cdot \overline{OA} + a_2 \cdot \overline{OB} + a_3 \cdot \overline{OC}.$$

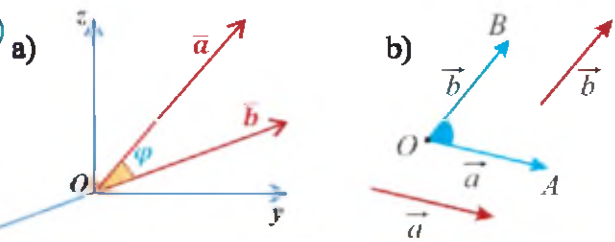
Bu yerda a_1, a_2, a_3 qandaydir haqiqiy sonlar. Bunga *vektorni berilgan vektorlar bo'yicha yoyish* deb ataladi.

2.4. Vektorlarning skalar ko'pavtmasi

Nol vektordan farqli $\overline{a}$ va $\overline{b}$ *vektorlar orasidagi burchak* deb O nuqtadan chiquvchi $\overline{OA} = \overline{a}$ va $\overline{OB} = \overline{b}$ vektorlarning yo'naltiruvchi kesmalari orasidagi burchakka aytiladi (30- rasm).

$\overline{a}$ va $\overline{b}$ vektorlar orasidagi burchak $(\widehat{\overline{a}, \overline{b}})$ tarzda ham belgilanadi.

30



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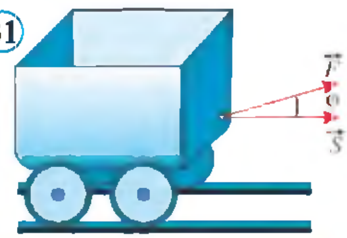


$\overline{a}$ va $\overline{b}$ *vektorlarning skalar ko'paytmasi* deb, bu vektorlar uzunliklarining ular orasidagi burchak kosinusi ko'paytmasiga aytiladi.

Agar vektorlarning biri nol vektor bo'lsa, ularning skalar ko'paytmasi nolga teng bo'ladi.

Skalar ko'paytma $\overline{a} \cdot \overline{b}$ yoki $(\overline{a}; \overline{b})$ tarzda belgilanadi. Ta'rifga ko'ra

$$(\overline{a}; \overline{b}) = |\overline{a}| \cdot |\overline{b}| \cos\varphi. \quad (1)$$

Ta'rifdan ko'rinadiki, $\overline{a}$ va $\overline{b}$ vektorlarning skalar ko'paytmasi nolga teng bo'lsa, ular *perpendikular* bo'ladi va aksincha.

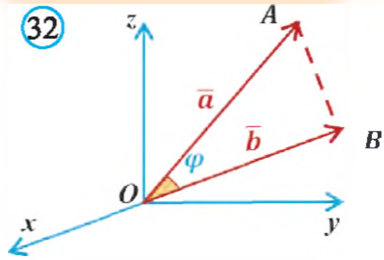
Fizikada jismni $\overline{F}$ kuch ta'siri ostida $\overline{s}$ masofaga siljitishda bajarilgan A ish (31- rasm) $\overline{F}$ va $\overline{s}$ vektorlarning skalar ko'paytmasiga teng bo'ladi:

$$A = (\overline{F}, \overline{s}) = |\overline{F}| \cdot |\overline{s}| \cos\varphi.$$

Xossa. $\bar{a}(a_1; a_2; a_3)$ va $\bar{b}(b_1; b_2; b_3)$ vektorlar uchun $(\bar{a}; \bar{b}) = a_1b_1 + a_2b_2 + a_3b_3$.

Isbot. $\bar{a}$ va $\bar{b}$ vektorlarni koordinata boshi O nuqtaga qo'yamiz (32- rasm). Unda $\overline{OA} = (a_1; a_2; a_3)$ va $\overline{OB} = (b_1; b_2; b_3)$ bo'ladi. Agar berilgan vektorlar kollinear bo'lmasa, ABO uchburchakdan iborat bo'ladi va uning uchun kosinuslar teoremasi o'rinli bo'ladi:

32



$$AB^2 = OA^2 + OB^2 - 2OA \cdot OB \cdot \cos\varphi.$$ Unda

$$OA \cdot OB \cdot \cos\varphi = \frac{1}{2}(OA^2 + OB^2 - AB^2)$$ bo'ladi. Lekin, $OA^2 = a_1^2 + a_2^2 + a_3^2$,

$OB^2 = b_1^2 + b_2^2 + b_3^2$ va $AB^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2$.

Demak, $$(\bar{a}, \bar{b}) = |\bar{a}| \cdot |\bar{b}| \cos\varphi = \frac{1}{2}(OA^2 + OB^2 - AB^2) =$$

$$= \frac{1}{2}(a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 - (b_1 - a_1)^2 - (b_2 - a_2)^2 -$$

$$-(b_3 - a_3)^2) = a_1b_1 + a_2b_2 + a_3b_3.$$

Berilgan vektorlar kollinear bo'lgan ($\varphi = 0°$, $\varphi = 180°$) holda ham bu tenglik o'rinli bo'lishini mustaqil ko'rsating. □

Vektorlarning skalar ko'paytmasining xossalari

1. $\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$ – o'rin almashtirish xossasi.
2. $(\bar{a} + \bar{b}) \cdot \bar{c} = \bar{a} \cdot \bar{c} + \bar{b} \cdot \bar{c}$ – taqsimot xossasi.
3. $\lambda \cdot (\bar{a} \cdot \bar{b}) = (\lambda \cdot \bar{a}) \cdot \bar{b} = \bar{a} \cdot (\lambda \cdot \bar{b})$ – guruhlash xossasi.
4. Agar a va b vektorlar bir xil yo'nalishdagi kollinear vektorlar bo'lsa, $\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}|$ bo'ladi, chunki $\cos 0° = 1$.
5. Agar qarama-qarshi yo'nalgan bo'lsa, $\bar{a} \cdot \bar{b} = -|\bar{a}||\bar{b}|$, chunki $\cos 180° = -1$.
6. $\bar{a} \cdot \bar{a} = |\bar{a}||\bar{a}| \cos 0° = |\bar{a}|^2 \Rightarrow \bar{a}^2 = |\bar{a}|^2$.
7. $\bar{a}$ vektor $\bar{b}$ vektorga perpendikular bo'lsa, $\bar{a} \cdot \bar{b} = 0$ bo'ladi.

Natijalar:

a) $\bar{a} = (a_1; a_2; a_3)$ vektorning uzunligi: $|\bar{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$; (1)

b) $\bar{a} = (a_1; a_2; a_3)$ va $\bar{b} = (b_1; b_2; b_3)$ vektorlar orasidagi burchak kosinusi:

$$\cos\varphi = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}; \quad (2)$$

c) $\bar{a}=(a_1;a_2;a_3)$ va $\bar{b}=(b_1;b_2;b_3)$ vektorlarning perpendikularlik sharti:

$$a_1b_1+a_2b_2+a_3b_3=0. \qquad (3)$$

3- masala. $A(0;\ 1;\ -1)$, $B(1;\ -1;\ 2)$, $C(3;\ 1;\ 0)$, $D(2;\ -3;\ 1)$ nuqtalar berilgan. $\overline{AB}$ va $\overline{CD}$ vektorlar orasidagi burchakning kosinusini toping.

Yechish. $\overline{AB}$ va $\overline{CD}$ vektorlarning koordinatalarini so'ng uzunliklarini topamiz:

$$\overline{AB}=(1-0;\ -1-1;\ 2-(-1))=(1,-2,3),$$

$$\overline{CD}=(2-3;\ -3-1;\ 1-0)=(-1,-4,1).$$

$$|\overline{AB}|=\sqrt{1^2+(-2)^2+3^2}=\sqrt{14},$$

$$|\overline{CD}|=\sqrt{(-1)^2+(-4)^2+1^2}=\sqrt{18}.$$

Demak, $\cos\varphi=\dfrac{\overline{AB}\cdot\overline{CD}}{|\overline{AB}|\cdot|\overline{CD}|}=\dfrac{1\cdot(-1)+(-2)(-4)+3\cdot1}{\sqrt{14}\cdot\sqrt{18}}=\dfrac{5}{\sqrt{63}}$. □

4- masala. $\bar{a}(1;2;0)$, $\bar{b}(1;-\frac{1}{2};0)$ vektorlar orasidagi burchakni toping.

Yechish: $\cos\varphi=\dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cdot|\bar{b}|}=\dfrac{1\cdot1+2\left(-\frac{1}{2}\right)+0\cdot0}{\sqrt{1^2+2^2+0^2}\sqrt{1^2+\left(-\frac{1}{2}\right)^2+0^2}}=\dfrac{0}{\sqrt{5}\sqrt{\frac{5}{4}}}=0.$

Demak, $\varphi=90°$. □

5- masala. $|\bar{a}|=3$, $|\bar{b}|=5$ va bu vektorlar orasidagi burchak $\frac{2\pi}{3}$ ga teng bo'lsa, $|\bar{a}+\bar{b}|$ ni toping.

Yechish: $|\bar{a}+\bar{b}|=\sqrt{(\bar{a}+\bar{b})^2}=\sqrt{\bar{a}^2+2(\bar{a}\cdot\bar{b})+\bar{b}^2}=\sqrt{|\bar{a}|^2+2|\bar{a}||\bar{b}|\cos\phi+|\bar{b}|^2}=$

$$=\sqrt{9+25+2\cdot15\cdot\left(-\frac{1}{2}\right)}=\sqrt{34-15}=\sqrt{19}$$

6- masala. Agar $\bar{a}=2\bar{i}+3\bar{j}-4\bar{k}$ va $\bar{b}=-\bar{i}-\bar{j}+2\bar{k}$ bo'lsa, 1) $\bar{c}=\bar{a}+\bar{b}$; 2) $\bar{d}=2\bar{a}-\bar{b}$ vektorning koordinatalarini va uzunligini toping.

Yechish: $\bar{a}$ va $\bar{b}$ vektorlar yoyilmalarini koordinatalari izlanayotgan vektor ifodasiga qo'yamiz: 1) $\bar{c}=\bar{a}+\bar{b}=2\bar{i}+3\bar{j}-4\bar{k}-\bar{i}-\bar{j}+2\bar{k}=\bar{i}+2\bar{j}-2\bar{k}$.

Demak, $\bar{c}=\overline{(1;2;-2)}$. Unda $|\bar{c}|=\sqrt{1^2+2^2+(-2)^2}=3$;

2) $\bar{d}=2\bar{a}-\bar{b}=2(2\bar{i}+3\bar{j}-4\bar{k})-(-\bar{i}-\bar{j}+2\bar{k})=4\bar{i}+6\bar{j}-8\bar{k}+\bar{i}+\bar{j}2\bar{k}=$
$=5\bar{i}+7\bar{j}-10\bar{k}$.

Demak, $\bar{d}=\overline{(5;7;-10)}$. Unda $|\bar{d}|=\sqrt{5^2+7^2(-10)^2}=\sqrt{174}$. □

7- masala. $\bar{a}$ va $\bar{b}$ vektorlar orasidagi burchak 30° ga teng va $|\bar{a}| = \sqrt{3}$, $|\bar{b}|=2$ bo‘lsa, $(2\bar{a} + 3\bar{b})(-2\bar{a}+\bar{b})$ ko‘paytmani hisoblang.

Yechish: Avval $\bar{a}$ va $\bar{b}$ vektorlar ko‘paytmasini hisoblaymiz:

$$(\bar{a}, \bar{b}) = |\bar{a}||\bar{b}| \cos 30° = \sqrt{3}\cdot 2\cdot\frac{\sqrt{3}}{2} = 3.$$

So‘ng vektorlar ko‘paytmasining taqsimot xossasiga ko‘ra, berilgan vektorlar ifodalarini ko‘phadni ko‘phadga ko‘paytirish kabi ko‘paytiramiz:

$(2\bar{a}+3\bar{b})(-2\bar{a}+\bar{b})=-4\bar{a}^2 +2(\bar{a}, \bar{b})-6(\bar{a}, \bar{b})+3\bar{b}^2=-4\bar{b}^2-4(\bar{a}, \bar{b})+3\bar{b}^2$.

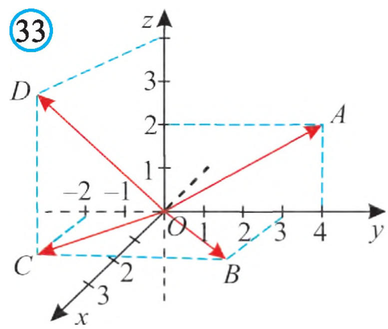
$\bar{a}^2=|\bar{a}|^2=9$, $\bar{b}=|\bar{b}|^2=4$, $(\bar{a}, \bar{b})=3$ ekanligini hisobga olsak, izlanayotgan ko‘paytma $(2\bar{a}+3\bar{b})(-2\bar{a}+\bar{b})=-4\cdot 9-4\cdot 3+3\cdot 4=-36.$ □

Mavzuga oid masalalar va amaliy topshiriqlar

39. 33- rasmdagi vektorlarning koordinatalarini aniqlang.

40. $A(1; 1; 1)$, $B(-1; 0; 1)$, $C(0; 1; 1)$ va $O(0; 0; 0)$ nuqtalar berilgan. $\overline{OA}, \overline{OB}, \overline{OC}, \overline{BO}, \overline{CO}$ va $\overline{AB}$ vektorlar koordinatalarini aniqlang.

33



41. $\overline{AB}$ $(a; b; c)$ bo‘lsa, $\overline{BA}$ vektor koordinatalarini ayting.

42. Agar a) $A(1; 2; 3)$, $B(3; 7; 6)$; b) $A(-3; 2; 1)$, $B(1; -4; 3)$ bo‘lsa, $\overline{AB}$ vektor koordinatalarini toping.

43. $\bar{a}(1; -1; 1)$, $\bar{b}(0; 2; -4)$, $\bar{c}(2; 3; -1)$, $\bar{d}(1; 2; 5)$ vektorlarning uzunligini toping.

44. Agar $\bar{a}(2; 1; 3)$ va $\bar{b}(-1; x; 2)$ vektorlar uzunligi teng bo‘lsa, x ni toping.

45. Uzunligi $\sqrt{54}$ ga teng bo‘lgan $\bar{a}(c; 2c; -c)$ vektorning koordinatalarini toping.

46. A, B, C, D, E va F nuqtalar muntazam oltiburchakning uchlari bo‘lsa, ular orqali: a) ikkita teng; b) ikkita bir xil yo‘nalgan; c) ikkita qarama-qarshi yo‘nalgan va teng; d) ikkita qarama-qarshi yo‘nalgan va teng bo‘lmagan vektorlarga misol keltiring.

47. k ning qanday qiymatida: a) $\bar{a}(4; k; 2)$; b) $\bar{a}(k-1;1;4)$; c) $\bar{a}(k; 1; k+2)$; d) $\bar{a}(k-1; k-2; k+1)$ vektorning uzunligi $\sqrt{21}$ ga teng bo‘ladi?

48. Uchta nuqta berilgan: $A(1; 1; 1)$, $B(-1; 0; 1)$, $C(0; 1; 1)$. Shunday

$D(x; y; z)$ nuqtani topingki, $\overline{AB}$ va $\overline{CD}$ vektorlar teng bo'lsin.

49. Uchta nuqta berilgan: $A(1; 0; 1)$, $B(-1; 1; 2)$, $C(0; 2; -1)$. Agar a) $\overline{AB}$ va $\overline{CD}$ vektorlar teng; b) $\overline{AB}$ va $\overline{CD}$ vektorlarning yig'indisi nol vektorga teng bo'lsa, $D(x; y; z)$ nuqtani toping.

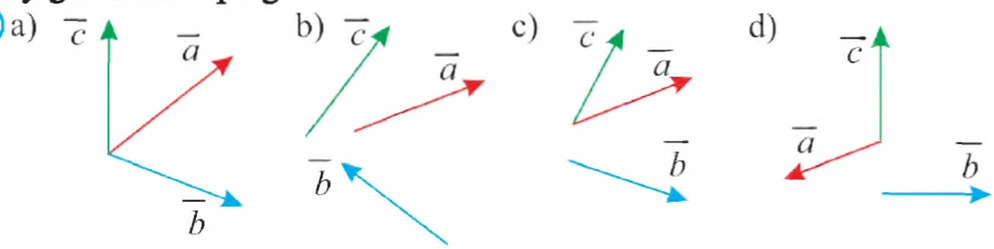
50*. (2; n; 3) va (3; 2; m) vektorlar berilgan. m va n ning qanday qiymatlarida bu vektorlar kollinear bo'ladi?

51. Boshi $A(1; 1; 1)$ nuqtada va oxiri Oxy tekislikdagi B nuqtada bo'lgan hamda $\overline{a}(1;-2; 3)$ vektorga kollinear vektorni toping.

52*. $ABCD$ parallelogrammning uchlari a) $A(-2; -4; 3)$, $B(3; 1; 7)$, $C(4; 2; -5)$; b) $A(4; 2; -1)$, $B(1; -3; -2)$, $C(-6; 2; 1)$; c) $A(-1; 7; 4)$, $B(1; 5; 2)$, $C(9; -3; -8)$; d) $A(-2; -4; 3)$, $B(3; 1; 7)$, $C(4; 2; -5)$ bo'lsa, D uchining koordinatalarini toping.

53. 34- rasmda tasvirlangan vektorlarning parallelepiped qoidasiga ko'ra yig'indisini toping.

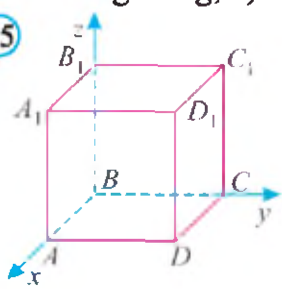
34



54. Agar $A(6;7;8)$, $B(8;2;6)$, $C(4;3;2)$, $D(2;8;4)$ va $M(3;5;2)$, $N(7;1;2)$, $P(3;-3;2)$, $K(-1;1;2)$ bo'lsa, $ABCD$ va $MNPK$ to'rtburchaklardan qaysi biri romb, qaysinisi kvadrat bo'ladi?

55. 35- rasmda tasvirlangan $ABCDA_1B_1C_1D_1$ kubda: a) $\overline{AB}$, $\overline{DD_1}$, $\overline{AC}$ vektorlarga teng; b) $\overline{A_1D_1}$, $\overline{CC_1}$, $\overline{BD}$ vektorlarga qarama-qarshi yo'nalgan; c) $\overline{BA}$, $\overline{AA_1}$ vektorlarga kollinear; d) $\overline{AB}$ va $\overline{AD}$, $\overline{AC}$ va $\overline{A_1C}$ vektorlar juftiga komplanar vektorlarni aniqlang.

35



56. Agar 1) $\overline{a}(1;-4;0)$, $\overline{b}(-4;0;8)$; 2) $\overline{a}(0;2;5)$, $\overline{b}(4; 3; 0)$ bo'lsa, $\overline{c} = \overline{a} + \overline{b}$ vektorning koordinatalarini va uzunligini toping.

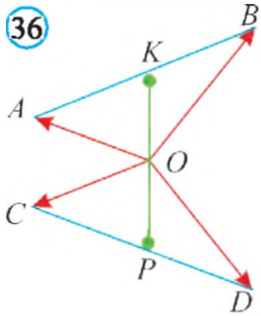
57. Agar 1) $\overline{a}(1; -4; 0)$, $\overline{b}(-4; 8; 0)$; 2) $\overline{a}(0; -2; 7)$, $\overline{b}(0; 4; -1)$ bo'lsa, $\overline{c} = \overline{a} - \overline{b}$ vektorning koordinatalarini va uzunligini toping.

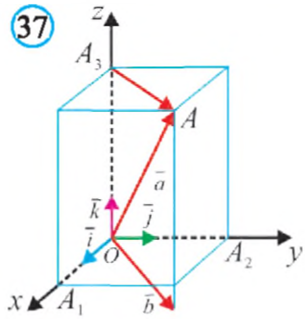
58. Agar $\overline{b}(-4; 8; 2)$ bo'lsa, a) $2\overline{b}$; b) $-3\overline{b}$; c) $-1{,}5\,\overline{c}$; d) $0 \cdot \overline{b}$ vektorning koordinatalarini va uzunligini toping.

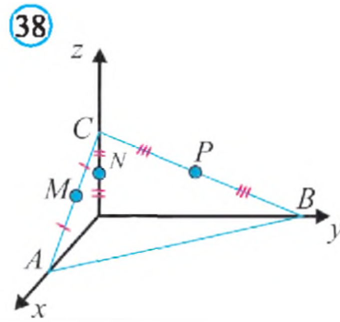
59. $\bar{a}(1; -1; 1)$, $\bar{b}(0; 2; -4)$, $\bar{c}(2; 3; -1)$, $\bar{d}(1; 2; 5)$ vektorlarni ortlar bo'yicha yoying.

60*. $\bar{a}(1; -1; 1)$, $\bar{b}(0; 2; -4)$, $\bar{c}(2; 3; -1)$, $\bar{d}(1; 2; 5)$ vektorlar berilgan. $|\bar{a} + 2\bar{b}|$, $|\bar{a} - 3\bar{b}|$, $|\bar{c} - 2\bar{d}|$, $|3\bar{a} + 4\bar{d}|$ ni toping.

61*. K va P nuqtalar ayqash to'g'ri chiziqlarda yotuvchi AB va CD kesmalarning o'rtasi hamda O nuqta KP kesmaning o'rtasi bo'lsa (36-rasm), $\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD} = \bar{0}$ ekanligini isbotlang.





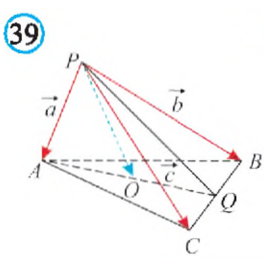


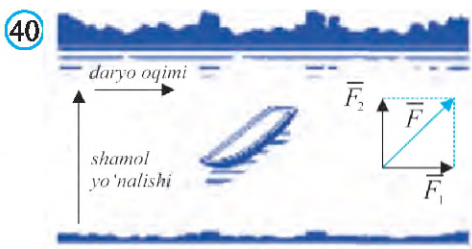
62. 37- rasmda $OA_1 = 2$, $OA_2 = 2$, $OA_3 = 3$. $\bar{a}$, $\bar{b}$ va $\overline{A_3A}$ vektorlarning koordinatalarini aniqlang.

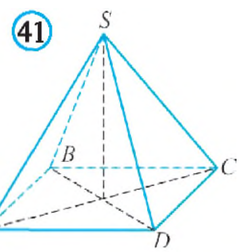
63. 38- rasmda $OA = 4$, $OB = 9$, $OC = 2$, M, N va P nuqtalar, mos ravishda, AC, OC va CB kesmalarning o'rtasi. $\overline{AC}$, $\overline{CB}$, $\overline{AB}$, $\overline{PC}$, $\overline{MC}$ va $\overline{CN}$ vektorlarning koordinatalarini toping.

64. Q nuqta $PABC$ tetraedrning BC qirrasining o'rtasi va O nuqta esa AQ kesma o'rtasi bo'lsa (39- rasm), $\overline{PO}$ vektorni $\overline{PA} = \bar{a}$, $\overline{PB} = \bar{b}$ va $\overline{PC} = \bar{c}$ vektorlar orqali ifodalang.

65*. 40- rasmda tasvirlangan qayiqqa daryo oqimi $\overline{F_1} = 120\ N$ kuch bilan va qirg'oqdan esayotgan shamol $\overline{F_2} = 100\ N$ kuch bilan ta'sir qilmoqda. Qayiqning daryoda joyidan qo'zg'almay turishi uchun uni qanday kuch bilan ushlab turish kerak?







66. Skalar ko'paytmasi: a) $\frac{1}{2}$; b) $\frac{\sqrt{3}}{2}$; c) 0; d) $-\frac{1}{2}$; e) b) $-\frac{\sqrt{2}}{2}$ ga teng bo'lgan birlik vektorlar orasidagi burchakni toping.

67. a) $\overline{a}$ (1; –1; 1), $\overline{b}$ (0; 2; –4); b) $\overline{c}$(2; 3; –1), $\overline{d}$ (1; 2; 5); c) $\overline{e}$(1; –1; 1), $\overline{f}$(0; 2; – 4); d) $\overline{g}$(2; 3; –1), $\overline{h}$(1; 2; 5) vektorlarning skalar ko'paytmasini toping.

68. *ABC* uchburchakda $\angle A = 50°$, $\angle C = 90°$. a) $\overline{BA}$ va $\overline{BC}$; b) $\overline{CA}$ va $\overline{AB}$; c) $\overline{AB}$ va $\overline{BA}$ vektorlar orasidagi burchakni toping.

69. $\overline{a}$ va $\overline{b}$ vektorlarning uzunliklari va ular orasidagi burchak mos ravishda a) 5, 12, 50°; b) 3, $\sqrt{2}$, 45°; c) 5, 6, 120°; d) 4, 7, 180° bo'lsa, ularning skalar ko'paytmasini toping.

70. *n* ning qanday qiymatida vektorlar perpendikular bo'ladi?
a) $\overline{a}(2; -1; 3)$, $\overline{b}(1; 3; n)$; b) $\overline{a}(n; -2; 1)$, $\overline{b}(n; -n; 1)$;
c) $\overline{a}(n; -2; 1)$, $\overline{b}(n; 2n; 4)$; d) $\overline{a}(4; 2n; -1)$, $\overline{b}(-1; 1; n)$.

71. $\overline{a}(1;-5;2)$, $\overline{b}(3;1;2)$ vektorlar berilgan. a) $\overline{a}+\overline{b}$ va $\overline{a}-\overline{b}$; b) $\overline{a}+2\overline{b}$ va $3\overline{a}-\overline{b}$; c) $2\overline{a}+\overline{b}$ va $3\overline{a}-2\overline{b}$ vektorlar skalar ko'paytmasini toping.

72. *A* (1; 0; 1), *B* (–1; 1; 2), *C* (0; 2; –1) nuqtalar berilgan. *Oz* koordinatalar o'qida shunday *D* nuqtani topingki, $\overline{AB}$ va $\overline{CD}$ vektorlar perpendikular bo'lsin.

73*. $(\overline{a}, \overline{b}) \le |\overline{a}| \cdot |\overline{b}|$ ekanligini asoslang. Bu vektorlar qanday bo'lganda tenglik o'rinli bo'ladi?

74*. *SABCD* piramidaning hamma qirralari o'zaro teng (41- rasm) va asosi kvadratdan iborat. a) $\overline{SA}$ va $\overline{SB}$; b) $\overline{SD}$ va $\overline{AD}$; c) $\overline{SB}$ va $\overline{SD}$; d) $\overline{AS}$ va $\overline{AC}$; e) $\overline{AC}$ va $\overline{AD}$ vektorlar orasidagi burchaklarni toping

75*. Uzunliklari birga teng $\overline{a}$, $\overline{b}$, $\overline{c}$ vektorlar juft-jufti bilan 60° li burchak tashkil etadi. a) $\overline{a}$ va $\overline{b}+\overline{a}$; b) $\overline{a}$ va $\overline{b}-\overline{c}$ vektorlar orasidagi burchakni toping.

76. *O* nuqta *ABCD* kvadratning diagonallari kesishish nuqtasi. Kvadratning *B* uchidan diagonalga parallel va *DA* to'g'ri chiziq bilan *F* nuqtada kesishadigan to'g'ri chiziq o'tkazilgan. $\overline{BF}$ vektorni $\overline{DO}$ va $\overline{DC}$ vektorlar orqali ifodalang.

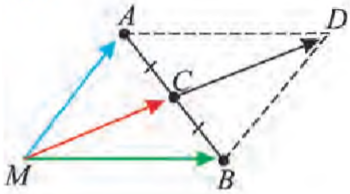
77. *O* nuqta *ABC* uchburchakning medianalari kesishish nuqtasi bo'lsa, $\overline{OC}$ vektorni $\overline{AB}$ va $\overline{AC}$ vektorlar bo'yicha yoying.

78*. *C* nuqta *AB* kesmaning o'rtasi bo'lsa (42- rasm), unda ixtiyoriy *M* nuqta uchun $\overline{MC}=\frac{1}{2}(\overline{MA}+\overline{MB})$ bo'lishini isbotlang.

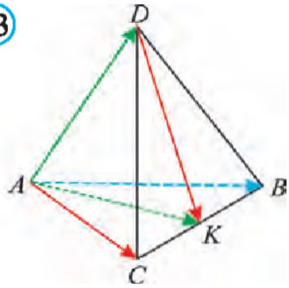
79. *K* nuqta *ABCD* tetraedr *BC* qirrasining o'rtasi bo'lsa (43- rasm), $\overline{DK}$ vektorni $\overline{AB}$, $\overline{AD}$ va $\overline{AC}$ vektorlar bo'yicha yoying.

80*. Jismning siljish yo'nalishiga nisbatan 30° li burchak ostida qo'yilgan $\overline{F}=20N$ kuch ta'sirida jism 3 m ga siljidi. Bu holatda bajarilgan ishni toping.

(42)



(43)



81*. Jismning siljish yo'nalishiga nisbatan 60° li burchak ostida qo'yilgan $\vec{F}$ =50 *N* kuch ta'sirida jism 8 m ga siljidi. Bu holatda bajarilgan ishni toping.

82*. (Koshi – Bunyakovskiy tengsizligi) Ixtiyoriy $a_1, a_2, a_3, b_1, b_2, b_3$ sonlari uchun $(a_1b_1 + a_2b_2 + a_3b_3)^2 \le (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$ tengsizlikning o'rinli bo'lishini vektorlardan foydalanib isbotlang.

3. FAZODA ALMASHTIRISHLAR VA O'XSHASHLIK

3.1. Fazoda geometrik almashtirishlar

Fazoda berilgan *F* shaklning har bir nuqtasi biror bir usulda ko'chirilsa, yangi F_1 shakl hosil bo'ladi. Agar bu ko'chirishda (akslantirishda) birinchi shaklning har xil nuqtalari ikkinchi shaklning har xil nuqtalariga ko'chsa, bu ko'chishga ***geometrik shakl almashtirish*** deb ataladi.

Butun fazoni ham geometrik shakl sifatida qarasak, fazoviy shakl almashtirish haqida ham gapirish mumkin.

Ko'rib turganingizdek, fazoda geometrik almashtirishlar tushunchasi tekislikdagi kabi kiritiladi. Shuningdek, uning quyida ko'riladigan qator turlarining xossalari va ularning isboti ham tekislikdagisiga o'xshash. Shu bois, bu xossalarning isbotiga to'xtalmaymiz va ularni mustaqil bajarishni tavsiya qilamiz.

3.2. Harakat va parallel ko'chish

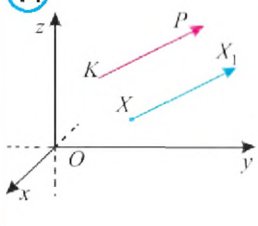
Nuqtalar orasidagi masofani saqlaydigan shakl almashtirishlar *harakat* deb ataladi. Harakatning quyidagi xossalarini keltirish mumkin.

Harakatda to'g'ri chiziq to'g'ri chiziqqa, nur-nurga, kesma unga teng kesmaga, burchak unga teng burchakka, uchburchak unga teng uchburchakka, tekislik unga teng tekislikka va tetraedr unga teng tetraedrga ko'chadi (akslanadi).

Fazoda biror harakat yordamida birini ikkinchisiga ko'chirish mumkin bo'lgan shakllar ***teng shakllar*** deyiladi.

Harakatga eng sodda misol bu parallel ko'chirishdir.

44



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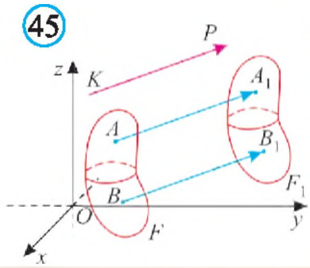

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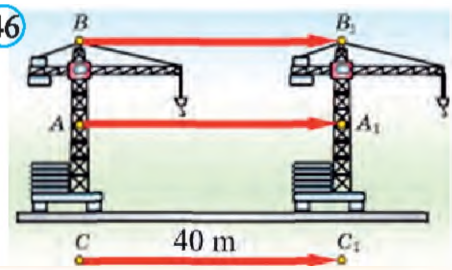


Fazoda biror $\overline{KP}$ vektor va ixtiyoriy X nuqta berilgan bo‘lsin (44- rasm). Agar X_1 nuqta $\overline{XX_1} = \overline{KP}$ shartni qanoatlantirsa, X nuqta X_1 nuqtaga $\overline{KP}$ vektor bo‘ylab *parallel ko‘chirilgan* deb ataladi.

Agar fazoda berilgan F shaklning har bir nuqtasi $\overline{KP}$ vektor bo‘ylab ko‘chirilsa (45- rasm), yangi F_1 shakl hosil bo‘ladi. Bu holda *F shakl F_1 shaklga parallel ko‘chirilgan* deyiladi. Parallel ko‘chirishda F shaklning har bir nuqtasi bir xil yo‘nalishda bir xil masofaga ko‘chirilgan bo‘ladi.

46- rasmda tasvirlangan ko‘tarma kranning har bir nuqtasi boshlang‘ich holatiga nisbatan 40 m ga parallel ko‘chgan.

Ravshanki, parallel ko‘chirish harakatdir. Shuning uchun, parallel ko‘chirishda to‘g‘ri chiziq to‘g‘ri chiziqqa, nur nurga, tekislik tekislikka, kesma unga teng kesmaga ko‘chadi va hokazo.

Aytaylik $\overline{KP} = (a; b; c)$ vektor bo‘ylab parallel ko‘chirishda F shaklning $X(x; y; z)$ nuqtasi F_1 shaklning $X_1(x_1; y_1; z_1)$ nuqtasiga o‘tsin. Unda, ta’rifga ko‘ra, quyidagilarga egamiz:

$x_1 - x = a,\ \ y_1 - y = b,\ \ z_1 - z = c$ yoki $x_1 = x + a,\ \ y_1 = y + b,\ \ z_1 = z + c.$

Bu tengliklar *parallel ko‘chirish formulalari* deb ataladi.

1- masala. $\overline{p} = (3; 2; 5)$ vektor bo‘ylab parallel ko‘chirishda $P(-2; 4; 6)$ nuqta qaysi nuqtaga ko‘chadi?

Yechish. Yuqoridagi parallel ko‘chirish formulalardan foydalanamiz:

$x_1 = -2 + 3 = 1,\ \ y_1 = 4 + 2 = 6,\ \ z_1 = 6 + 5 = 11.$ *Javob:* $P_1(1; 6; 11)$. □

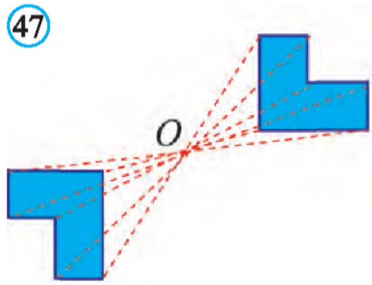
3.3. Fazoda markaziy simmetriya

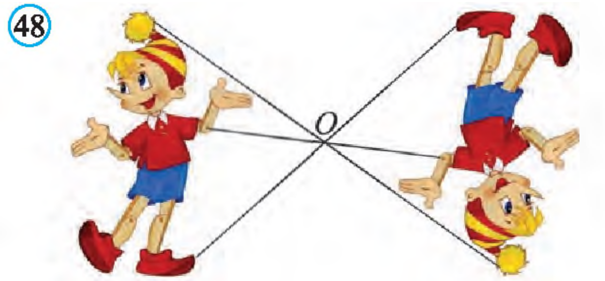
Fazoda berilgan A va A_1 nuqtalar O nuqtaga nisbatan simmetrik deyiladi, agar $\overline{AO} = \overline{OA_1}$ bo‘lsa, ya’ni O nuqta AA_1 kesmaning o‘rtasi bo‘lsa.

Agar fazoda berilgan F shaklning har bir nuqtasi O nuqtaga nisbatan simmetrik nuqtaga ko‘chsa (47- rasm), bunday almashtirishga *O nuqtaga nisbatan simmetriya* deb ataladi. 48, 49- rasmlarda O nuqtaga nisbatan simmetrik shakllar tasvirlangan.

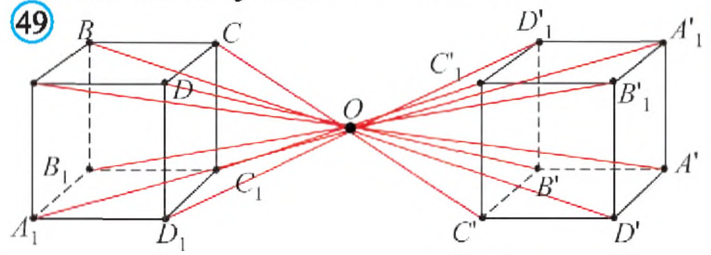
Nuqtaga nisbatan simmetriya – harakatdir.

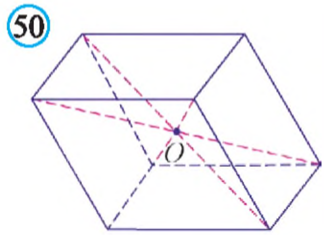
Agar F shakl O nuqtaga nisbatan simmetrik almashtirishda o‘ziga ko‘chsa, bunday shaklga *markaziy simmetrik shakl* deb ataladi.





Masalan, parallelepiped (50- rasm) diagonallari kesishish nuqtasi O ga nisbatan markaziy simmetrik shakldir.





2- masala. O (2; 4; 6) nuqtaga nisbatan markaziy simmetriyada A = (1; 2; 3) nuqta qaysi nuqtaga o‘tadi?

Yechish. $A_1 = (x; y; z)$ izlanayotgan nuqta bo‘lsin. Ta’rifga ko‘ra, O nuqta AA_1 kesmaning o‘rtasi. Demak, $2 = \frac{x+1}{2}$, $4 = \frac{y+2}{2}$, $6 = \frac{z+3}{2}$.

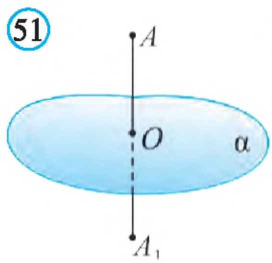
Bu tengliklardan $x = 4 - 1 = 3$, $y = 8 - 2 = 6$, $z = 12 - 3 = 9$.

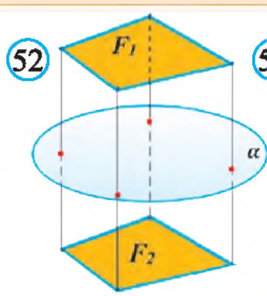
Javob: $A_1(3; 6; 9)$.□

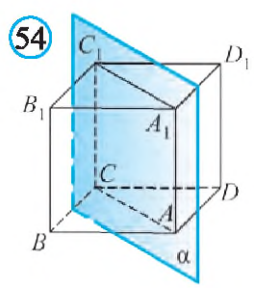
3.4. Tekislikka nisbatan simmetriya

Fazoda berilgan A va A_1 nuqtalar ***tekislikka nisbatan simmetrik*** deyiladi, agar tekislik AA_1 kesmaga perpendikular bo‘lib, uni teng ikkiga bo‘lsa (51- rasm). 52- rasmda tekislikka nisbatan simmetrik bo‘lgan F_1 va F_2 shakllar keltirilgan. Ravshanki, gavdamiz va aksimiz oyna tekisligiga nisbatan simmetrik bo‘ladi (53- rasm).

Tekislikka nisbatan simmetriya – harakatdir.







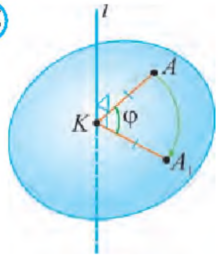
Demak, tekislikka nisbatan simmetriyada kesma o'ziga teng kesmaga, to'g'ri chiziq – to'g'ri chiziqqa va tekislik – tekislikka akslanadi.

Agar *F* shakl tekislikka nisbatan simmetrik almashtirishda o'ziga ko'chsa, bunday shaklga ***tekislikka nisbatan simmetrik shakl*** deyiladi.

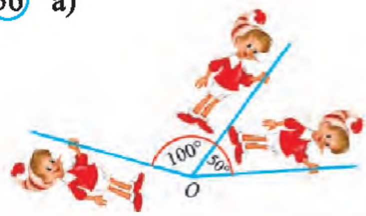
Masalan, 54- rasmda tasvirlangan kub AA_1va CC_1 qirralaridan o'tuvchi α tekislikka nisbatan simmetrik shakl bo'ladi.

3.5. Burish va o'qqa nisbatan simmetriya

55



56 a)



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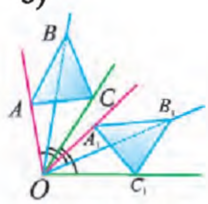

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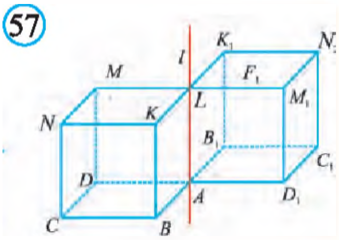


Aytaylik, fazoda *A* va A_1 nuqtalar va *l* to'g'ri chiziq berilgan bo'lsin. Agar *l* to'g'ri chiziqqa tushirilgan *AK* va A_1K perpendikularlar teng va o'zaro φ burchak tashkil qilsa, bu holda ***l to'g'ri chiziqqa nisbatan φ burchakka burish*** natijasida *A* nuqta A_1 nuqtaga o'tdi deyiladi (55- rasm).

Agar fazoda berilgan *F* shaklning har bir nuqtasi *l* to'g'ri chiziqqa nisbatan φ burchakka bursak, yangi F_1 shakl hosil bo'ladi. Bunda *F* shakl *l* to'g'ri chiziqqa nisbatan φ burchakka burishda F_1 shaklga o'tdi deyiladi. 56-rasmda shunday burishdan hosil bo'lgan shakllar ko'rsatilgan.

Masalan, 57- rasmda tasvirlangan kubni *l* to'g'ri chiziqqa nisbatan 180° burchakka burishda yangi kubni hosil qilamiz.

To'g'ri chiziqqa nisbatan burish ham harakat bo'ladi.

l to'g'ri chiziqqa nisbatan 180° burchakka burish ***l to'g'ri chiziqqa nisbatan simmetriya*** deb ataladi.

Shaklning simmetriya markazi, o'qi, tekisligi uning ***simmetriya elementlari*** deb ataladi.

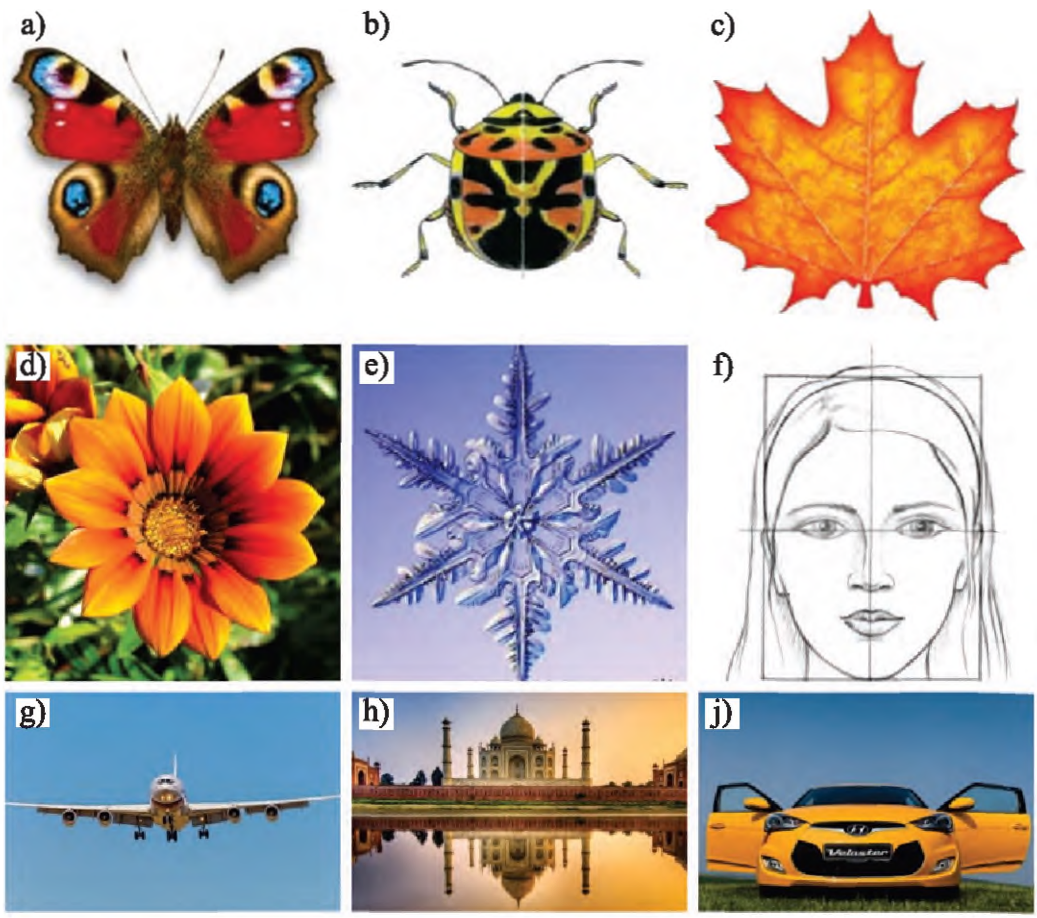
A (*x*; *y*; *z*) nuqtaga koordinata tekisliklari, koordinata o'qlari va koordinata boshiga nisbatan simmetrik nuqtalar quyidagi koordinatalarga ega bo'ladi:

Simmetriya elementi	Simmetrik nuqta koordinatalari
Oxy tekislik	$(x; y; -z)$
Oxz tekislik	$(x; -y; z)$

Oyz tekislik	$(-x; y; z)$
Ox o‘qi	$(x; -y; -z)$
Oy o‘qi	$(-x; y; -z)$
Oz o‘qi	$(-x; -y; z)$
O nuqta	$(-x; -y; -z)$

3.6. Tabiatda va texnikada simmetriya

(58)

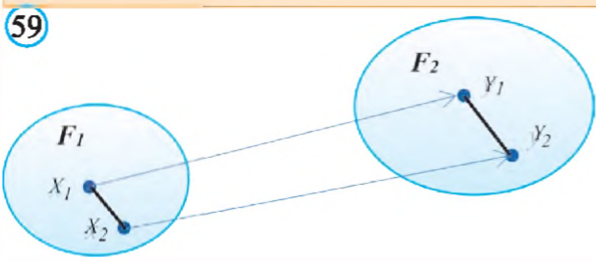


Tabiatda simmetriyani har qadamda uchratish mumkin. Masalan, jonli mavjudodlarning ko‘pchiligi, xususan, inson va hayvonlar gavdasi, o‘simliklarning barglari va gullari simmetrik tuzilgan (58- rasm). Shuningdek, jonsiz tabiat unsurlari ham borki, maslan, qor zarralari, tuz kristallari, moddalarning molekular tuzilishi ham ajoyib simmetrik shakllardan iboratdir. Bu bejiz emas, albatta, chunki simmetrik shakllar chiroyli bo‘lishi bilan birga, qaysidir ma'noda eng maqbul va mukammal hisoblanadi. Shunday ekan, tabiatdagi go'zallik va mukammallik simmetriya asosiga qurilgan, deb aytishimiz mumkin. Tabiatdagi bu go'zallik va mukammallikdan andoza olgan quruvchi, mahandis va arxitektor kabi ijodkorlar yaratgan ko'plab inshoot va binolar, qurilma va mexanizmlar, texnika va transport vositalari ham simmetrik yaratilgan. Bu ishda ularga geometriya fanining yordami beqiyosdir.

3.7. Fazoviy shakllarning o'xshashligi

Fazoda $k \neq 0$ va F_1 shaklni F_2 shaklga akslantiruvchi almashtirish berilgan bo'lsin. Bu akslantirishda F_1 shaklning ixtiyoriy X_1 va X_2 nuqtalari va ular akslangan F_2 shaklning Y_1 va Y_2 nuqtalari uchun $X_1Y_1 = k \cdot X_2Y_2$ bo'lsa, bu almashtirish *o'xshashlik almashtirishi* deb ataladi (59- rasm).

59



60

Ko'rib turganingizdek, fazoda o'xshashlik almashtirishi tushunchasi tekislikdagidek kiritiladi. Shuningdek, uning quyida ko'riladigan qator turlari ta'rifi, ularning xossalari va bu xossalarning isboti ham tekislikdagisiga o'xshash. Shu bois, bu xossalarning isbotiga to'xtalmaymiz va ularni mustaqil bajarishni tavsiya qilamiz.

Fazodagi o'xshashlik almashtirishi to'g'ri chiziqni to'g'ri chiziqqa, nurni nurga, kesmani kesmaga va burchakni burchakka akslantiradi. Shuningdek, bu almashtirish tekislikni ham tekislikka akslantiradi.

Fazoda berilgan ikki shaklning biri ikkinchisiga o'xshashlik almashtirishi orqali akslansa, ular *o'xshash shakllar* deb ataladi.

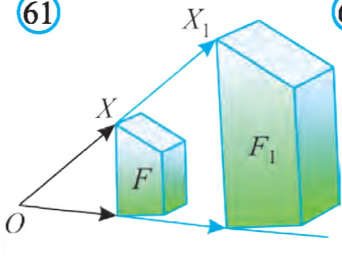
Fazoda F shakl, O nuqta va k noldan farqli ($k \neq 0$) son berilgan bo'lsin.

F shaklning ixtiyoriy X nuqtasini $\overline{OX_1} = k\,\overline{OX}$ shartni qanoatlantiruvchi X_1 nuqtaga akslantiruvchi almashtirish *O nuqtaga nisbatan k koeffitsiyentli gomotetiya* deb ataladi (61-rasm). O nuqtaga *gomotetiya markazi*, k soniga esa *gomotetiya koeffitsiyenti* deyiladi.

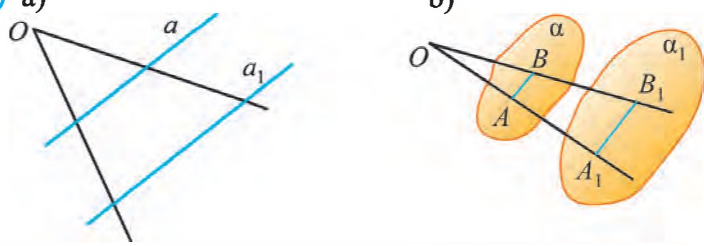
F shaklning har bir nuqtasi shu usulda akslantirilsa, natijada F_1 shakl hosil bo'ladi va bu *gomotetiyada F shakl F_1 shaklga akslanadi* deyiladi.

Ko'rib turganingizdek, fazoda gomotetiya ta'rifi tekislikdagisi bilan deyarli bir xil. Shuningdek, uning qator xossalari ham borki, ular ham, ularning isbotlari ham tekislikdagisiga o‘xshash. Shu bois, bu xossalarning isbotiga to‘xtalmaymiz va ularni mustaqil bajarishni tavsiya qilamiz.

61



62 a) b)



O nuqtaga nisbatan k koeffitsiyentli gomotetiya o‘xshashlik almashtirishidir.

Gomotetiya koeffitsiyenti k ixtiyoriy noldan farqli son bo‘lib, $k=1$ da F shakl o‘ziga o‘zi akslanadi, $k=-1$ da esa F shakl O nuqtaga nisbatan simmetrik F_1 shaklga akslanadi. Boshqa hollarda gomotetiya nuqtalar orasidagi masofani saqlamaydi, ya’ni u harakat bo‘lmaydi. Gomotetiya natijasida nuqtalar orasidagi masofa bir xil k songa ko‘payadi, ya’ni shaklning o‘lchamlari o‘zgaradi, lekin uning shakli o‘zgarmaydi.

Gomotetiyada gomotetiya markazidan o‘tmaydigan a) to‘g‘ri chiziq unga parallel to‘g‘ri chiziqqa (62.a- rasm); b) tekislik esa unga parallel tekislikka akslanadi (62.b- rasm).

Gomotetiyada gomotetiya markazidan o‘tuvchi to‘g‘ri chiziq yoki tekislik o‘ziga o‘zi akslanadi.

Mavzuga oid masalalar va amaliy topshiriqlar

83. $\bar{p} = (-2; 1; 4)$ vektor bo‘ylab parallel ko‘chirishda, a) (3; –2; 3); b) (0; 2; –3); c) (2; –5; 0) nuqta qaysi nuqtaga ko‘chadi?

84. Parallel ko‘chirishda A(4; 2; –8) nuqta (3; 7; –5) nuqtaga ko‘chdi. Parallel ko‘chirish qaysi vektor bo‘ylab amalga oshirilgan?

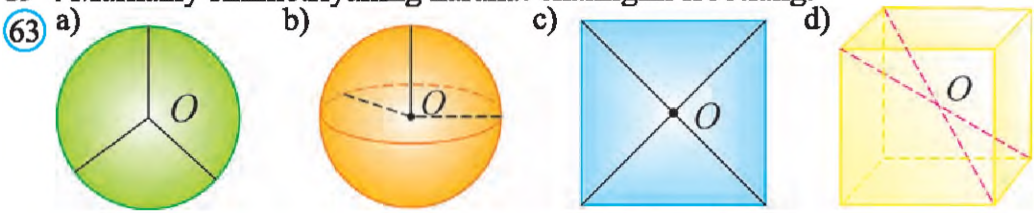
85. Parallel ko‘chirishda: a) to‘g‘ri chiziq - to‘g‘ri chiziqqa; b) nur nurga; c) tekislik tekislikka; d) kesma unga teng kesmaga ko‘chishini isbotlang.

86. O (–2; 3; –1) nuqtaga nisbatan markaziy simmetriyada A (4; 2; –3) nuqta qaysi nuqtaga o‘tadi?

87. 63- rasmda tasvirlangan shakllarda O nuqta simmetriya markazi ekanligini asoslang.

88. (–2; 5; –9), (2; 2; –7), (–6; 12; –2) nuqtalar koordinata boshiga nisbatan markaziy simmetriyada qaysi nuqtalarga o‘tadi?

89*. Markaziy simmetriyaning harakat ekanligini isbotlang.



90*. Tekislikka nisbatan simmetriyaning harakat ekanligini isbotlang.

91. Parallelepipedning (50- rasm) diagonallari kesishish nuqtasi *O* ga nisbatan markaziy simmetrik shakl ekanligini isbotlang.

92. (1; 2; –3), (0; 2; –3), (2; 2; –3) nuqtalar koordinata tekisliklariga nisbatan simmetriyalarda qaysi nuqtalarga o'tadi?

93. (2; 4; –1) nuqta koordinata tekisligiga nisbatan simmetrik akslantirishda (2; –4; –1) nuqtaga o'tdi. Akslantirish qaysi koordinata tekisligiga nisbatan amalga oshirilgan?

94. Quyidagi jadvalda berilgan 1- namuna asosida bo'sh kataklarni to'ldiring.

№	Berilgan nuqta	Simmetrik nuqta	Nimaga nisbatan simmetrik?
1	(1; 2; 3)	(1; 2; –3)	*Oxy* tekislikka nisbatan
2	(2; 4; –1)		*Oxz* tekislikka nisbatan
3		(1; 2; 3)	*Oyz* tekislik
4	(–1; –2; –3)	(–1; 2; 3)	
5	(–1; 6; 3)		*Oy* o'qi
6		(–3; 8; –2)	*Oz* o'qi
7	(4; 1; –2)		*O* nuqta

95. 49- rasmda tasvirlangan shakllarda *O* nuqta simmetriya markazi ekanligini asoslang.

96*. To'g'ri chiziqqa nisbatan burish harakat ekanligini ko'rsating.

97. *O* nuqtaga nisbatan *k* koeffitsiyentli gomotetiya o'xshashlik almashtirishi ekanligini ko'rsating.

98. *Oxy* tekislikka nisbatan simmetriyada ixtiyoriy $(x; y; z)$ nuqta $(x; y; -z)$ nuqtaga o'tishini ko'rsating.

99. *Oxz* tekislikka nisbatan simmetriyada ixtiyoriy $(x; y; z)$ nuqta $(x; -y; z)$ nuqtaga o'tishini ko'rsating,

100. Parallel ko'chirishda (1; 2; –1) nuqta (1; –1; 0) nuqtaga o'tdi. Koordinata boshi bu almashtirishda qaysi nuqtaga o'tadi?

101. Parallel ko'chirishda (3; 4; –1) nuqta (2; –4; 1) nuqtaga o'tdi. Bu almashtirishda koordinata boshi qaysi nuqtaga o'tadi?

102*. $A(2; 1; 0)$ nuqta B $(1; 0; 1)$ nuqtaga, C $(3; -2; 1)$ nuqta esa D $(2; -3; 0)$ nuqtaga oʻtadigan parallel koʻchirish mavjudmi?

103*. A $(-2; 3; 5)$ nuqta B $(1; 2; 4)$ nuqtaga, C $(4; -3; 6)$ nuqta esa $D(7; -2; 5)$ nuqtaga oʻtadigan parallel koʻchirish mavjudmi?

104. 58- rasmda tasvirlangan jonli va jonsiz obyektlar fazoviy jism sifatida qanday simmetrik shakl boʻlishi mumkinligini aniqlang.Ularning (agar mavjud boʻlsa) simmetriya markazi, simmetriya oʻqi yoki simmetriya tekisliklarini chizib koʻrsating.

105. 60- rasmda tasvirlangan ona-bolalar (matreshkalar) ning katta ona matreshkaga nisbatan oʻxshashlik koeffitsiyentlarini aniqlang.

106. Muntazam tetraedr qirrasining uzunligi 12 cm ga teng. Bu tetraedrga: a) 3; b) –4; c) $\frac{1}{2}$; d) $-\frac{1}{3}$; koeffitsiyentli gomotetik boʻlgan tetraedr qirrasining uzunligi nimaga teng?

107. Ixtiyoriy ABC uchburchak chizing va biror O nuqtani belgilang. Markazi O nuqtada va koeffitsiyenti: a) 2; b) –3; c) $-\frac{1}{2}$; d) $\frac{1}{4}$ ga teng boʻlgan gomotetiyada ABC uchburchak oʻtadigan uchburchakni quring.

108. Ixtiyoriy $SABC$ tetraedr chizing. Markazi S nuqtada va koeffitsiyenti: a) 1,5; b) –2; c) $\frac{1}{2}$; d) $\frac{1}{4}$ ga teng boʻlgan gomotetiyada $SABC$ tetraedr oʻtadigan tetraedrni quring.

109. Ixtiyoriy kub chizing. Markazi kubning biror uchida va koeffitsiyenti: a) 2; b) –2; c) $\frac{1}{2}$; d) $-\frac{1}{2}$ ga teng boʻlgan gomotetiyada bu kub oʻtadigan fazoviy geometrik shaklni quring.

110. Markazi koordinata boshida va koeffitsiyenti: a) 2,5; b) –2,5; c) $\frac{1}{4}$; d) $\frac{1}{4}$ ga teng boʻlgan gomotetiyada $A(-2; 3; 5)$ nuqta oʻtadigan nuqtaning koordinatalarini toping.

111. Markazi $O(-1; 2; 2)$ nuqtada va koeffitsiyenti: a) 0,5; b) –2; c) $\frac{1}{4}$; d) $-\frac{1}{4}$ ga teng boʻlgan gomotetiyada $A(2; 4; 0)$ nuqta oʻtadigan nuqtaning koordinatalarini toping.

112. Uchlari $O(0; 0; 0)$, $A(4; 0; 0)$, $B(0; 4; 0)$, $C(0; 0; 4)$ nuqtalarda boʻlgan tetraedr: a) markazi O nuqtada, koeffitsiyenti –1 ga teng; b) markazi A nuqtada, koeffisiyenti 2 ga teng boʻlgan gomotetiyada oʻtadigan tetraedrning uchlari koordinatalarini toping.

113*. Gomotetiyada uning markazidan oʻtmaydigan: a) toʻgʻri chiziq oʻziga parallel toʻgʻri chiziqqa, b) tekislik esa oʻziga parallel tekislikka akslanishini koʻrsating.

114*. Gomotetiyada uning markazidan oʻtuvchi toʻgʻri chiziq yoki tekislik oʻziga oʻzi akslanishini koʻrsating.

4. BOBNI TAKRORLASHGA DOIR AMALIY MASHQLAR

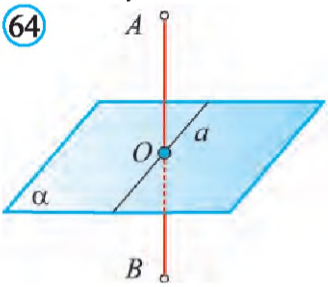
4.1. 1- test sinovi

1. $A(x_1; y_1; z_1)$ va $B(x_2; y_2; z_2)$ nuqtalar berilgan. $z_2 - z_1$ nimani anglatadi?

A) AB kesma o‘rtasining koordinatasini; B) AB kesma uzunligini;
C) $\overline{AB}$ vektor uzunligini; D) $\overline{AB}$ vektor koordinatalaridan birini.

2. 64- rasmda $AB \perp \alpha$, $a \subset \alpha$, $AO = OB$ bo‘lsa,

A) A va B nuqtalar O nuqtaga nisbatan simmetrik bo‘ladi;
B) A va B nuqtalar a to‘g‘ri chiziqqa nisbatan simmetrik bo‘ladi;
C) A va B nuqtalar α tekislikka nisbatan simmetrik bo‘ladi;
D) AB kesma a to‘g‘ri chiziqqa nisbatan simmetrik bo‘ladi.

64


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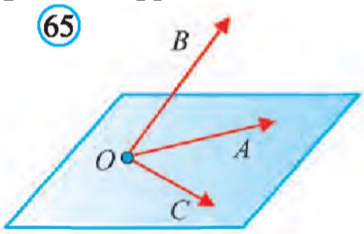

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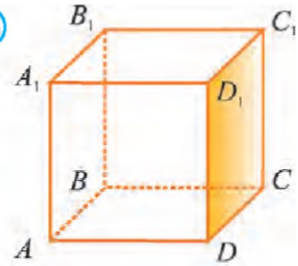


3. 65- rasmda B nuqta AOC tekislikda yotmaydi. Unda $\overline{OA}$, $\overline{OB}$ va $\overline{OC}$ vektorlar …

A) kollinear; B) komplanar;
C) bir xil yo‘nalishli; D) komplanar emas.

4. $M(-7; 1; 4)$ va $N(-1; -3; 0)$ nuqtalar berilgan. MN kesma o‘rtasining koordinatalarini toping.

A) $(-4; -1; 4)$; B) $(-4; -1; 2)$; C) $(-4; -2; 2)$; D) $(-3; 2; 2)$.

5. $A(0; -3; 2)$ va $B(4; 0; -2)$ nuqtalar berilgan. AB kesma o‘rtasi nimaga tegishli?

A) Ox o‘qiga; B) Oy o‘qiga; C) Oz o‘qiga; D) Oxy tekisligiga.

6. $A(3; 4; -3)$ nuqtadan Oz o‘qigacha bo‘lgan masofani toping.

A) 3; B) 5; C) $2\sqrt{3}$; D) $\sqrt{34}$.

7. $\overline{CD}+\overline{DE}+\overline{EF}$ vektorlar yig‘indisini toping.

A) $\overline{O}$; B) $\overline{CF}$; C) $\overline{DF}$; D) $\overline{CE}$.

8. m ning qaysi qiymatida $\overline{a}(m; 4; -3)$ va $\overline{b}(4; 8; -6)$ vektorlar kollinear bo‘ladi?

A) 2; B) 5; C) 1; D) 3.

9. O nuqta α tekislikda yotmaydi. Markazi O nuqtada bo‘lgan gomotetiyada α tekislik undan farqli bo‘lgan β tekislikka o‘tadi. Agar a to‘g‘ri chiziq α tekislikka tegishli bo‘lsa, …

A) $\alpha \parallel \beta$ bo‘ladi; B) α tekislik β tekislik bilan kesishadi;
C) $a \subset \beta$ bo‘ladi; D) $\alpha \perp \beta$ bo‘ladi.

10. AB to‘g‘ri chiziq BCD tekislikka perpendikular. Qaysi vektorlarning skalar ko‘paytmasi nolga teng bo‘ladi?

A) $\overline{CA}$ va $\overline{CB}$; B) $\overline{BD}$ va $\overline{AD}$; C) $\overline{AC}$ va $\overline{BC}$; D) $\overline{AB}$ va $\overline{CD}$.

11. Qirrasi 1 ga teng bo‘lgan $ABCDA_1B_1C_1D_1$ kub berilgan (66- rasm). $(\overline{AB}+\overline{BC})\cdot\overline{BB}$ ni toping.

A) 1; B) 0; C) –1; D) 0,5.

12. p ning qaysi qiymatida $\overline{a}(1;\ 1;\ 0)$ va $\overline{b}(0;\ 4;\ p)$ vektorlar orasidagi burchak 60° ga teng bo‘ladi?

A) 4; B) 4 yoki –4; C) 16; D) 16 yoki –16.

13. $ABCDA_1B_1C_1D_1$ kub berilgan. Parallel ko‘chirishda A_1D kesma D_1C kesmaga o‘tadi. Bu ko‘chirishda AA_1B_1 tekislik qaysi tekislikka o‘tadi?

A) DB_1B; B) DCC_1; C) AA_1C_1; D) ABC.

14. α tekislik unda yotmaydigan ABC uchburchakning simmetriya tekisligidir. Qaysi tasdiq to‘g‘ri?

A) $(ABC)\perp\alpha$; B) ABC uchburchak teng yonli;
C) ABC uchburchakning simmetriya markazi bor;
D) ABC uchburchakning simmetriya o‘qi bor.

15. $ABCDA_1B_1C_1D_1$ kub berilgan. $\overline{A_1B_1}+\overline{BC}-\overline{DD_1}$ ni toping.

A) $\overline{A_1C}$; B) $\overline{BD_1}$; c) $\overline{B_1D}$; D) $\overline{AC_1}$.

16. Qaysi geometrik almashtirish ikki ayqash to‘g‘ri chiziqlardan birini ikkinchisiga o‘tkazadi?

A) parallel ko‘chirish; B) tekislikka nisbatan simmetriya;
C) burish; D) gomotetiya.

17. $M(-1;\ 2;\ -4)$ nuqtaga Oyz tekislikka nisbatan simmetrik bo‘lgan nuqtani toping.

A) (1; –2; 4); B) (1; 2; –4); C) (–1; –2; –4); D) (–1; 2; 4).

18. Parallel ko‘chirishda $\overline{AB}$ vektor $\overline{DC}$ vektorga o‘tadi. Qaysi tasdiq noto‘g‘ri?

A) $\overline{AB}=\overline{DC}$; B) AC va BD kesma o‘rtalari ustma-ust tushadi;
C) $\overline{AB}, \overline{AC}$ va $\overline{DC}$ vektorlar komplanar; D) $ABCD$ parallelogramm.

19. $B(-3;\ 2;\ -5)$ nuqta Oxz tekislikdan qanday masofada yotibdi?

A) 2; B) 5; C) 3; D) $\sqrt{34}$.

20. $A(1;\ -2;\ 0)$, $B(1;\ -4;\ 2)$, $C(3;\ 2;\ 0)$ nuqtalar ABC uchburchakning uchlari. CM mediana uzunligini toping.

A) $2\sqrt{3}$; B) $3\sqrt{2}$; C) $\sqrt{6}$; D) 18.

21. Agar $\overline{a}(1;\ m;\ 2)$ va $\overline{b}(0{,}5m+1;\ 3;\ 1)$ vektorlar kollinear bo‘lsa, $m+n$ ni toping.

A) 3; B) 5; C) –4; D) 9.

22. $A(-1;\ -9;\ -3)$ va $B(0;\ -2;\ 1)$ nuqtalar berilgan. vektorni koordinata vektorlari (ortlar) bo‘yicha yoying.

A) $(\overline{BA}) = \overline{i} + 9\overline{j} - \overline{k}$; B) $(\overline{BA}) = \overline{i} - 9\overline{j} + \overline{k}$;
C) $(\overline{BA}) = -\overline{i} - 9\overline{j} - 4\overline{k}$; D) $(\overline{BA}) = \overline{i} + 9\overline{j} - 4\overline{k}$.

23. $A(1; -2; 2)$, $B(1; 4; 0)$, $C(-4; 1; 1)$ va $D(-5; -5; 3)$ nuqtalar berilgan. AC va BD vektorlar orasidagi burchakni toping
A) 150°; B) 30°; C) 45°; D) 90°.

24. $|\overline{a}| = 6$, $|\overline{a}+\overline{b}| = 11$, $|\overline{a}-\overline{b}| = 7$ ekanligi ma'lum bo'lsa, $|\overline{b}|$ ni toping.
A) 11; B) 18; C) 20 ; D) 7.

25. Asoslari BC va AD bo'lgan $ABCD$ trapetsiya berilgan. Agar $\overline{AB}(-7; 4; 5)$, $\overline{AC}(3; 2; -1)$, $\overline{AD}(20; -4; -12)$, M va N – mos ravishda AB va CD tomonlar o'rtasi bo'lsa, $\overline{MN}$ vektor koordinatalari yig'indisini toping.
A) 1; B) 2 ; C) 3; D) 4.

4.2. Masalalar

115. Uchlari $A(1; -2; 4)$ va $B(3; -4; 2)$ nuqtalarda bo'lgan kesma o'rtasining koordinatalarini toping.

116. $A(x; 0; 0)$ nuqta $B(1; 2; 3)$ va $C(-1; 3; 4)$ nuqtalardan teng uzoqlikdaligi ma'lum bo'lsa, x ni toping.

117. Agar kesmaning bir uchi $A(1; -5; 4)$, o'rtasi $C(4; -2; 3)$ nuqtada bo'lsa, ikkinchi uchining koordinatalari qanday bo'ladi?

118. Oxz tekisligiga nisbatan $A(1; 2; 3)$ nuqtaga simmetrik bo'lgan nuqtani toping.

119. Koordinatalar boshiga nisbatan $A(1; 2; 3)$ nuqtaga simmetrik bo'lgan nuqtani toping.

120. Oxy tekisligiga nisbatan $(1; 2; 3)$ nuqtaga simmetrik bo'lgan nuqtani toping.

121. Oy o'qqa nisbatan $(2; -3; 5)$ nuqtaga simmetrik bo'lgan nuqtani toping.

122. Quyidagi nuqtalardan qaysi biri Oyz tekislikda yotadi?
$A(2; -3; 0)$; $B(2; 0; -5)$; $C(1; 0; -4)$; $D(0; 9; -7)$; $E(1; 0; 0)$.

123. Quyidagi nuqtalardan qaysi biri Oxz tekislikda yotadi:
$A(-4; 3; 0)$; $B(0; -7; 0)$; $C(2; 0; -8)$; $D(2; -4; 6)$; $E(0; -4; 5)$?

124. $A(-3; 8; 3\sqrt{33})$ nuqtadan Ox o'qqacha bo'lgan masofani toping.

125. $A(3; -2; 5)$ va $B(-4; 5; -2)$ nuqtalar berilgan. $\overline{AB}$ vektorning koordinatalarini toping.

126. $\overline{a}(1; -2; 3)$ vektorning oxiri $B(2; 0; 4)$ nuqta bo'lsa, bu vektorning boshini toping.

127. $B(0; 4; 2)$ nuqta $\overline{a}(2; -3; 1)$ vektorning oxiri bo'lsa, bu vektor boshining koordinatalarini toping.

128. $\overline{a}(x; 1; 2)$ vektorning uzunligi 3 ga teng. x ning qiymatini toping.

129. $\overline{a}(4; -12; z)$ vektorning moduli 13 ga teng. z ning qiymatini toping.

130. Agar $\overline{a}(6; 2; 1)$ va $\overline{b}(0; -1; 2)$ bo'lsa, $\overline{c} = 2\overline{a} - \overline{b}$ vektorning uzunligini toping.

131. Agar $\overline{p}(2; 5; -1)$ va $\overline{q}(-2; 2)$ bo‘lsa, $\overline{m}= 4\overline{p}+2\overline{q}$ vektorning uzunligini toping.

132. $\overline{a}(2; -3; 4)$ va $\overline{b}(-2; -3; 1)$ vektorlarning skalar ko‘paytmasini toping.

133. $\overline{m}(-1; 5; 3)$ va $\overline{n}(2; -2; 4)$ vektorlarning skalar ko‘paytmasini toping.

134. m ning qanday qiymatida $\overline{a}(1; m; -2)$ va $\overline{b}(m; 3; -4)$ vektorlar perpendikular bo‘ladi?

135. n ning qanday qiymatida $\overline{a}(n; -2; 1)$ va $\overline{b}(n; n; 1)$ vektorlar perpendikular bo‘ladi?

136. m ning qanday qiymatida $\overline{a} = m\overline{i}+3\overline{j}+4\overline{k}$ va $\overline{b}=4\overline{i}+m\overline{j}-7\overline{k}$ vektorlar perpendikular bo‘ladi?

137. $A(1; -2; 2)$, $B(1; 4; 0)$, $C(-4; 1; 1)$ va $D(-5; -5; 3)$ nuqtalar berilgan. $\overline{AC}$ va $\overline{BD}$ vektorlar orasidagi burchakni toping.

138. n ning qanday qiymatlarida $\overline{a}(2; n; 6)$ va $\overline{b}(1; 2; 3)$ vektorlar kollinear bo‘ladi?

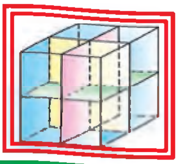
139. m ning qanday qiymatida $\overline{a}(2; 3; -4)$ va $\overline{b}(m; -6; 8)$ vektorlar parallel bo‘ladi?

140. m va n ning qanday qiymatida $\overline{a}(-1; m; 2)$ va $\overline{b}(-2; -4; n)$ vektorlar kollinear bo‘ladi?

141. $A(2; 7; -3)$ va $B(-6; -2; 1)$ nuqtalar berilgan. $\overline{BA}$ vektorni koordinatalar vektorlari (ortlari) bo‘yicha yoying.

4.3. 1- nazorat ishi namunasi

1. *Oxy* tekisligiga nisbatan (1; 2; 3) nuqtaga simmetrik bo‘lgan nuqtani toping.
2. Agar $\overline{a}(6; 3; 2)$ va $\overline{b}(-3; 1; 5)$ bo‘lsa, $\overline{c}= \overline{a}+2\overline{b}$ vektorning uzunligini toping.
3. $A(2; -1; 0)$ va $B(-2; 3; 2)$ nuqtalar berilgan. Koordinata boshidan AB kesma o‘rtasigacha bo‘lgan masofani toping
4. $A(1; -2; 2)$, $B(1; 4; 0)$, $C(-4; 1; 1)$ va $D(-5; -5; 3)$ nuqtalar berilgan. $\overline{AC}$ va $\overline{BD}$ vektorlar orasidagi burchakni toping.
5. *(Yaxshi o‘zlashtiradigan o‘quvchilar uchun qo‘shimcha masala).* Uchlari $A(4; 5; 1)$, $B(2; 3; 0)$ va $C(2; 1; -1)$ nuqtalarda bo‘lgan uchburchakning BD medianasi uzunligini toping



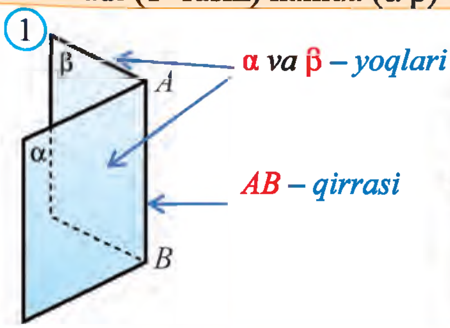
I BOB. PRIZMA VA SILINDR

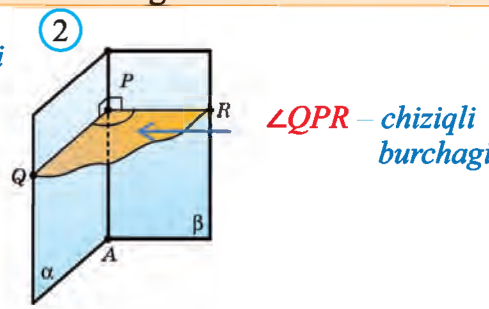
5. KO‘PYOQLI BURCHAKLAR VA KO‘PYOQLAR

5.1. Ko‘pyoqli burchaklar

Ikkiyoqli burchak bilan 10- sinfda tanishgansiz.

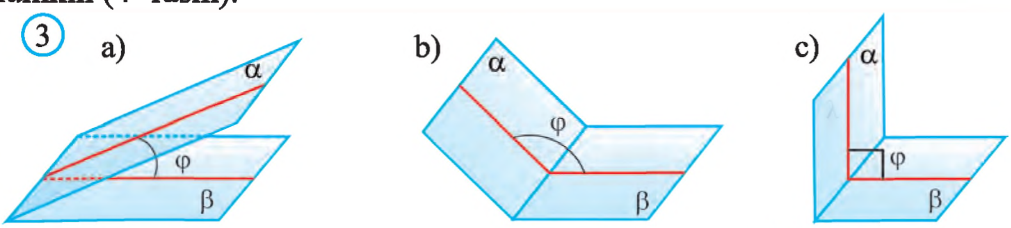
Ikkita α va β yarimtekislik (*yoqlari*) va ularni chegaralab turgan umumiy *AB* to‘g‘ri chiziq (*qirrasi*) dan iborat geometrik shakl *ikkiyoqli burchak* deb ataladi (1- rasm) hamda (α β) tarzda belgilanadi.

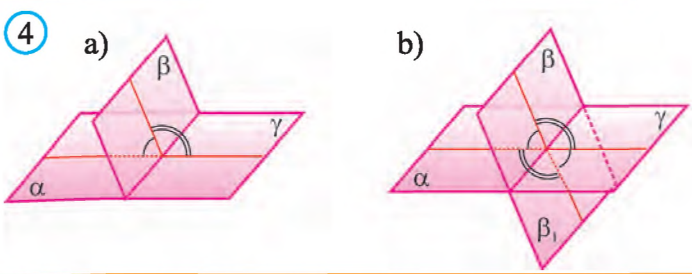


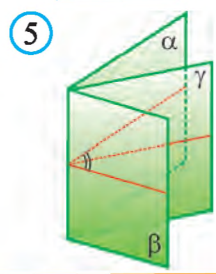


Ikkiyoqli burchak qirrasining ixtiyoriy *P* nuqtasi dan uning yoqlarida yotuvchi va bu qirraga perpendikular bo‘lgan *PR* va *PQ* nurlarni chiqaramiz. ∠*QPR* – ikkiyoqli burchakning *chiziqli burchagi* deb ataladi (2- rasm).

Ikkiyoqli burchaklar yassi burchaklar kabi chiziqli burchagining kattaligiga qarab *o‘tkir, o‘tmas, to‘g‘ri va yoyiq* bo‘ladi (3- rasm). Yassi burchaklar kabi ikkita ikkiyoqli burchaklar *qo‘shni va vertikal* bo‘lishi mumkin (4- rasm).







Ikkiyoqli burchakni teng ikkiga bo‘luvchi yarimtekislik uning *bissektori* deb ataladi (5- rasm).

1- masala. Chiziqli burchagi 60° ga teng bo'lgan ikkiyoqli burchakning yoqlarida yotgan A va B nuqtalardan (6- rasm) uning qirrasiga AA_1 va BB_1 perpendikularlar tushirilgan. Agar $AA_1 = 12$, $BB_1 = 10$ va $A_1B_1 = 13$ bo'lsa, AB kesma uzunligini toping.

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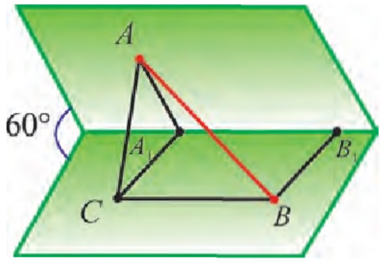


Yechish. $BB_1 || CA_1$ va $A_1B_1 || CB$ to'g'ri chiziqlarni o'tkazamiz. Hosil bo'lgan A_1B_1BC to'rtburchak parallelogramm bo'ladi. A_1B_1 to'g'ri chiziq A_1AC uchburchak tekisligiga perpendikular bo'ladi, chunki u bu tekislikda yotgan ikkita A_1A va A_1C to'g'ri chiziqlarga perpendikular. Unda BC to'g'ri chiziq ham bu tekislikka perpendikular bo'ladi.

Demak, ABC uchburchak to'g'ri burchakli uchburchak ekan.

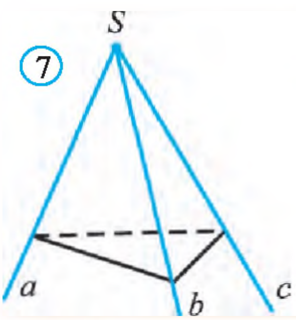
Kosinuslar teoremasiga ko'ra:

$$AC^2 = AA_1^2 + A_1C^2 - 2AA_1 \cdot A_1C \cdot \cos\alpha = 12^2 + 10^2 - 2 \cdot 12 \cdot 10 \cdot \cos 60° = 124.$$

Pifagor teoremasiga ko'ra: $AB = \sqrt{AC^2 + BC^2} = \sqrt{124 + 169} = \sqrt{293}.$

Javob: $AB = \sqrt{293}$. □

7



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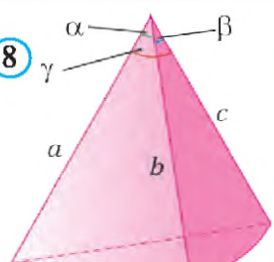

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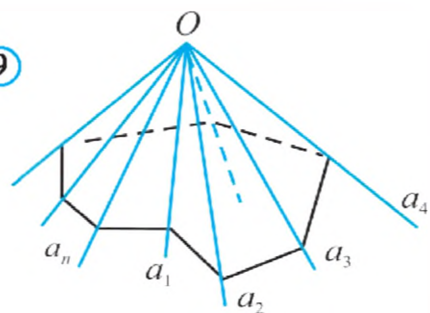


Fazoda **bir** nuqtadan **chiquvchi** ***a, b*** **va** ***c*** nurlar **uchta yassi** ***(ab), (bc)*** va ***(ac)*** **burchaklar tashkil qiladi (7- rasm).** Bu **yassi** burchaklardan **tashkil** topgan ***(abc)*** shaklga ***uchyoqli burchak*** **deyiladi. Yassi** burchaklarga **uchyoqli** burchakning ***yoqlari***, ularning tomonlariga **uchyoqli** burchakning ***qirralari***, **umumiy uchi**ga esa **uchyoqli** burchakning ***uchi*** **deyiladi**.

Uchyoqli burchakning yoqlaridan tashkil **qilgan ikkiyoqli** burchaklar **uchyoqli** burchakning ***ikkiyoqli burchaklari*** deb ataladi.

Uchta yassi ***(ab), (bc)*** **va** ***(ac)*** **burchaklar uchyoqli burchakning** ***tekis burchaklari*** **deb** ham **yuritiladi.**

Uchyoqli burchakning tekis burchaklarini, mos **ravishda,** α, β, γ **deb** belgilasak (8- rasm), **ular uchun uchburchak tengsizligi o'rinli** bo'**ladi,**

ya'ni ularning ixtiyoriysi qolgan ikkitasining yig'indisidan kichik bo'ladi: $\alpha+\beta<\gamma$, $\alpha+\gamma<\beta$, $\beta+\gamma<\alpha$ va tekis burchaklarining yig'indisi 360° dan kichik bo'ladi: $\alpha+\beta+\gamma<360^\circ$.

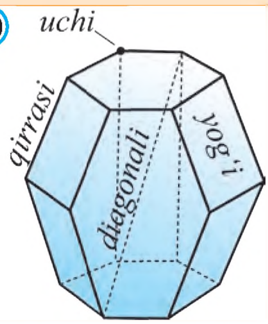
Ko'pyoqli burchak tushunchasi ham shunga o'xshash kiritiladi (9- rasm).

5.2. Ko'pyoqlar

E'tibor bergan bo'lsangiz, shu choqqacha fazoviy shakl sifatida qator jismlarning, xususan ko'pyoqlarning xossalarini o'rganib keldik. Bu fazoviy shakllarning *jism* deb atalishiga sabab, ularni fazoning biror moddiy jism egallagan va sirt bilan chegaralangan bo'lagi sifatida tasavvur etish mumkinligidir. Quyida ko'pyoqlarga tegishli ba'zi tushunchalarni eslatib o'tamiz.

Ko'pyoq deb yassi ko'pburchaklar bilan chegaralangan jismga aytiladi (10- rasm).

10



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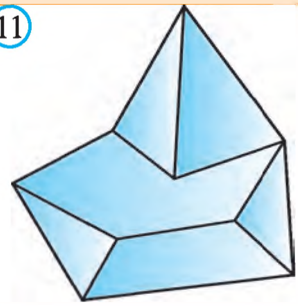
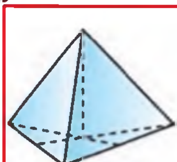
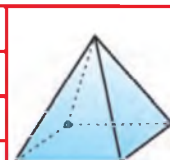
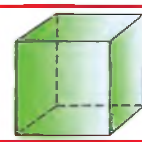


Ko'pyoq ixtiyoriy yog'i yotgan tekislikning bir tomonida yotsa, bunday ko'pyoq *qavariq ko'pyoq* deyiladi. 10- rasmda qavariq, 11- rasmda esa qavariq bo'lmagan ko'pyoqlar tasvirlangan.

Ixtiyoriy qavariq ko'pyoqning yoqlari sonini Y, uchlari sonini U va qirralari sonini Q bilan belgilaylik. Bizga ma'lum ko'pyoqlar uchun quyidagi jadvalni to'ldiraylik:

	Ko'pyoq nomi	Y	U	Q	
	Uchburchakli piramida	4	4	6	
	To'rtburchakli piramida	5	5	8	
	Uchburchakli prizma	5	6	9	
	To'rtburchakli prizma	6	8	12	
	n- burchakli piramida	$n+1$	$n+1$	$2n$	
	n- burchakli prizma	$n+2$	$2n$	$3n$	

Jadvaldan har bir ko'pyoq uchun $Y+U-Q=2$ bo'lishini payqash mumkin. Ma'lum bo'lishicha, bu munosabat barcha qavariq ko'pyoqlar uchun to'g'ri bo'lar ekan. Buni ilk bor 1752- yilga shvetsariyalik matematik Leonard Eyler aniqlagan.

Eyler teoremasi. Ixtiyoriy qavariq ko‘pyoq uchun: $Y + U - Q = 2$ munosabat o‘rinli bo‘ladi, bu yerda Y– ko‘pyoqning yoqlari, U– uchlari, Q– esa qirralari soni.

Bu teoremaning isbotiga to‘xtalmaymiz. Undan quyidagi natijalar kelib chiqadi. Ularni Eyler teoremasidan foydalanib mustaqil isbotlang.

1- natija. Ko‘pyoq tekis burchaklarining soni uning qirralari sonidan ikki marta ko‘p.

2- natija. Ko‘pyoq tekis burchaklari soni har doim juft bo‘ladi.

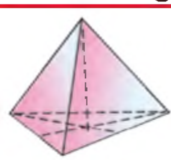
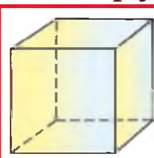
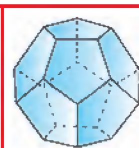
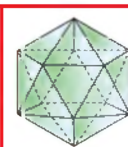
3- natija. Agar ko‘pburchakning har bir uchida bir xil k sondagi qirralar tutashsa, $U \cdot k = 2Q$ munosabat o‘rinli bo‘ladi.

4- natija. Agar ko‘pyoqning barcha yoqlari bir xil n-burchaklardan tashkil topgan bo‘lsa, $Y = 2Q$ munosabat o‘rinli bo‘ladi.

5- natija. Ko‘pyoqning tekis burchaklari yig‘indisi $360°(Y - Q)$ ga teng.

Yoqlari bir-biriga teng muntazam ko‘pburchaklardan iborat va har bir uchidan bir xil sondagi qirralar chiqadigan qavariq ko‘pyoqli *muntazam ko‘pyoqli* deb ataladi.

Ma’lum bo‘lishicha muntazam ko‘pyoqlilar besh xil bo‘lar ekan (buni mustaqil tekshirib ko‘ring). Bular quyidagilar:

Shakli					
Nomi va uning talqini	muntazam tetraedr (to‘rtyoqli)	Kub, geksaedr (oltiyoqli)	Oktaedr (sakkiz-yoqli)	Dode-kaedr (o‘n ikkiyoqli)	Ikosaedr (yigirma-yoqli)
Yoqlari	muntazam uchburchak	munta-zam to‘rt-burchak	muntazam uchburchak	munta-zam besh-burchak	munta-zam uch-burchak
Yoqlari soni	4	6	8	12	20
Qirralari soni	6	12	12	30	30
Uchlari soni	4	8	6	20	12
Har bir uchdan chiquvchi qirralar soni	3	3	4	3	5

Tarixiy ma'lumotlar

Barcha muntazam ko'pyoqlar Qadimgi Yunonistonda ma'lum edi. Yevklidning mashhur "Negizlar"ining XIII kitobi muntazam ko'pyoqlarga bag'ishlangan. Bu ko'pyoqlarni ko'pincha Platon jismlari deb ataladi. Qadimgi Yunonistonning buyuk olimi Platon (miloddan oldingi 427–347- yillar) bayon qilgan olamning idealistik tasvirida bu jismlardan to'rttasi olamning to'rt unsuriga (elementiga) o'xshatilgan: tetraedr – olov, geksaedr – Yer, ikosaedr – suv, oktaedr – havo, beshinchi ko'pyoq – dodekaedr esa butun olam tuzilishining belgisi ("beshinchi mohiyat") deb atashgan.

XVIII asrda ko'pyoqlar nazariyasiga Leonard Eyler (1707–1783) salmoqli hissa qo'shdi. 1758- yilda e'lon qilingan qavariq ko'pyoqlarning uchlari, qirralari va yoqlari soni orasidagi muosabat haqidagi Eyler teoremasi va uning isboti rang-barang ko'pyoqlar dunyosiga tartib o'rnatdi va uning go'zal geometrik zojibasini algebraik nuqtayi nazaridan bayon etdi.

Mavzuga oid masalalar va amaliy topshiriqlar

142. Ikki tekislik orasidagi burchak 47°. Bu tekisliklar kesishishidan hosil bo'lgan ikkiyoqli burchaklarning gradus o'lchovini toping.

143. Ikkiyoqli burchakning gradus o'lchovi 52° ga teng. Bu burchakka qo'shni bo'lgan ikkiyoqli burchakning gradus o'lchovi nimaga teng bo'ladi?

144. Tekis burchagi 100° bo'lgan ikkiyoqli burchakning yoqlariga perpendikular bo'lgan to'g'ri chiziqlar orasidagi burchakni toping.

145. Qo'shni ikkiyoqli burchaklarning bissektorlari orasidagi ikkiyoqli burchakning gradus o'lchovi nimaga teng?

146. A nuqta gradus o'lchovi 60° bo'lgan ikkiyoqli burchakning bissektorida yotibdi. Agar bu nuqta ikkiyoqli burchak qirrasidan 10 cm masofada yotgan bo'lsa, unda ikkiyoqli burchakning yoqlarigacha bo'lgan masofalarni toping.

147. A nuqta gradus o'lchovi 30° bo'lgan ikkiyoqli burchakning bitta yog'iga tegishli bo'lib, ikkinchi yog'idan 6 cm masofada yotibdi. Bu nuqtadan ikkiyoqli burchakning qirrasigacha bo'lgan masofani toping.

148*. A nuqta to'g'ri ikkiyoqli burchakning yoqlaridan 3 dm va 4 dm masofada yotibdi. Bu nuqtadan ikkiyoqli burchakning qirrasigacha bo'lgan masofani toping.

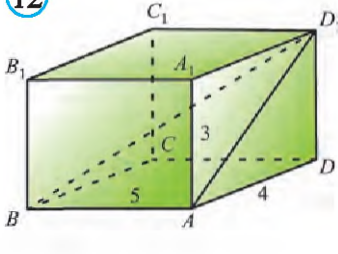
149*. Muntazam tetraedrning barcha ikkiyoqli burchaklari teng ekanligini isbotlang va ularning gradus o'lchovini toping.

150. Tekis burchaklari: a) 30°; 60°; 20°; b) 45°; 80°; 130°; c) 30°; 60°; 20°;

d) 20°; 60°; 70°; e) 76°; 34°; 110° bo‘lgan uch yoqli burchak mavjudmi?

151*. Qavariq ko‘pyoqli burchakning barcha tekis burchaklari yig‘indisi 360° dan kichik ekanligini isbotlang.

152. To‘g‘ri burchakli parallelepipedda $AB=5$, $AD=4$ va $AA_1=3$ bo‘lsa, ABD_1 burchakni toping (12- rasm).

12


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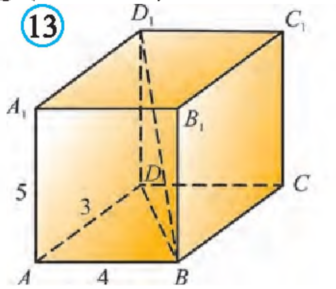

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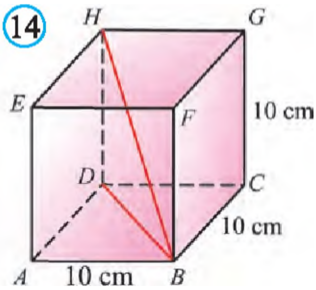


153. To‘g‘ri burchakli parallelepipedda $AB=4$, $AD=3$ va $AA_1=5$ bo‘lsa, DBD_1 burchakni toping (13- rasm).

154. 14- rasmda berilgan kubdagi DBH burchakni toping.

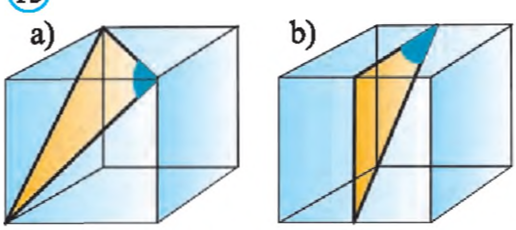
155*. n ta uchi bor qavariq ko‘pyoqning barcha tekis burchaklari yig‘indisi $360°(n-2)$ ga teng ekanligini isbotlang.

156*. Ko‘pyoq tekis burchaklarining soni uning qirralari sonidan ikki marta ko‘p bo‘lishini isbotlang.

157*. Ko‘pyoq tekis burchaklari soni har doim juft bo‘lishini isbotlang.

158*. Ko‘pyoqning tekis burchaklari yig‘indisi $360° (Y-Q)$ ga teng bo‘lishini isbotlang..

159. 15- rasmlardagi kublarda ajratib ko‘rsatilgan burchaklar kattaligini aniqlang.

15


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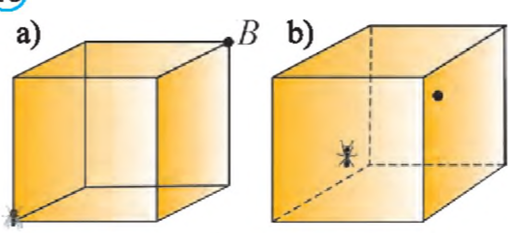
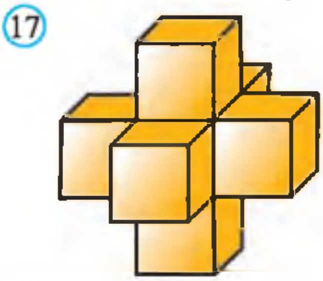


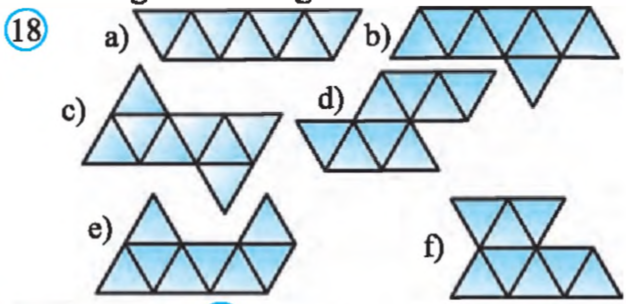
160*. 16- rasmlardagi kubning sirtidagi pashshaga: a) A uchdan B uchga; b) kub yog‘ining markazidan qarama-qarshi yog‘ining markaziga olib boradigan eng qisqa yo‘lni ko‘rsating (ko‘rsatma: kubning yoyilmasidan foydalaning).

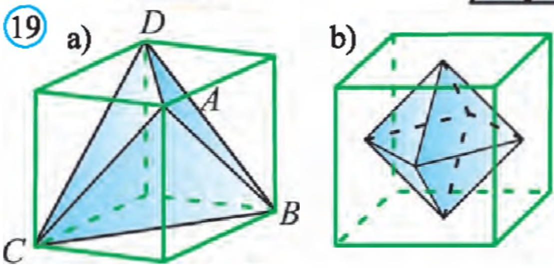
161. 17- rasmda tasvirlangan fazoviy shakl muntazam ko‘pyoqli bo‘ladimi? Uning sirti nechta kvadratdan iborat? Uning nechta uchi va qirrasi bor?

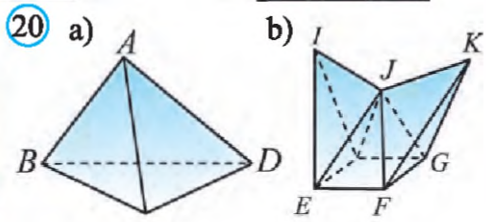
162. 18- rasmda tasvirlangan yoyilmalarning qaysi biri oktaedrga tegishli?

163. 19- rasmda tasvirlangan, kubga ichki chizilgan ko‘pyoqning: a) muntazam tetraedr; b) oktaedr ekanligini asoslang.









164. 20- rasmda tasvirlangan ko‘pyoqlarning uchlari, qirralari va yoqlari sonini aniqlab, ularni Eyler tenglamasiga qo‘yib tekshiring.

165. Qavariq ko‘pyoqning har bir uchidan uchtadan qirra chiqadi. Agar bu ko‘pyoqning qirralar soni: a) 12; b) 15 ga teng bo‘lsa, uning nechta uchi va yog‘i bor?

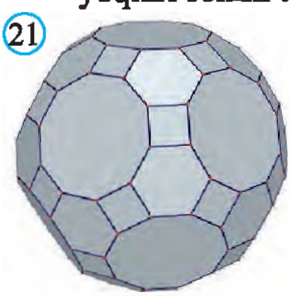
166*. 13 ta yog‘i va har bir yog‘ida 13 tadan qirrasi bo‘lgan ko‘pyoq mavjudmi?

167. Qavariq ko‘pyoqning har bir uchidan to‘rttadan qirra chiqadi. Agar bu ko‘pyoqning qirralar soni 12 ga teng bo‘lsa, uning nechta uchi va yog‘i bor?

168. a) Muntazam tetraedr; b) kub; c) oktaedr; d) dodekaedr; e) ikosaedrning uchlari, qirralari va yoqlari sonini toping va bu ko‘pyoqlar uchun Eyler tenglamasining o‘rinli bo‘lishini tekshiring.

169. Uchlari soni 8 ta, qirralari soni esa 12 ta bo‘lgan muntazam ko‘pyoqning yoqlari sonini toping va uning nomini aniqlang.

170. Uchlari soni 6 ta, qirralari soni esa 12 ta bo‘lgan muntazam ko‘pyoqning yoqlari sonini toping va uning nomini aniqlang.



171. Uchlari soni 10 ta, yoqlari soni esa 7 ta bo‘lgan ko‘pyoqning qirralari sonini toping.

172. Uchlari soni 14 ta, qirralari soni esa 21 ta bo‘lgan ko‘pyoqning yoqlari sonini toping.

173. 21- rasmdagi ko‘pyoqning 62 ta yog‘i va 120 ta uchi bor bo‘lsa, uning qirralari sonini toping.

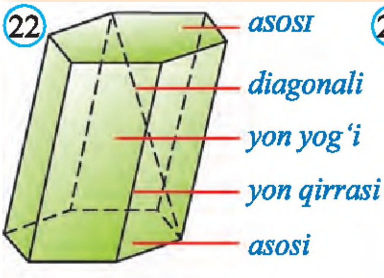
6. PRIZMA VA UNING SIRTI

6.1. Prizma va uning kesimlari

Prizmalar bilan quyi sinflardan tanishsiz. Shunday bo'lsada, ularga oid ba'zi tushuncha va xossalarni eslatib o'tamiz.

Prizma deb ikki yog'i (asosi) teng *n* burchakdan, qolgan *n* ta yoqlari esa parallelogrammlardan iborat ko'pyoqqa aytiladi (22- rasm).

22



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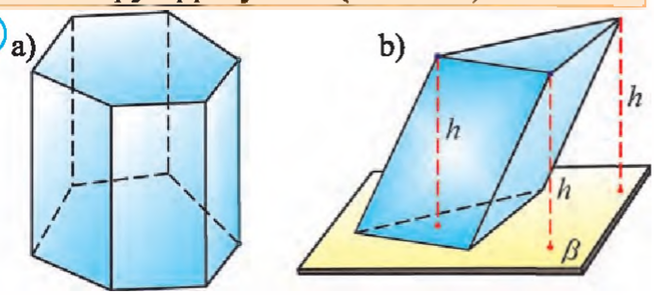
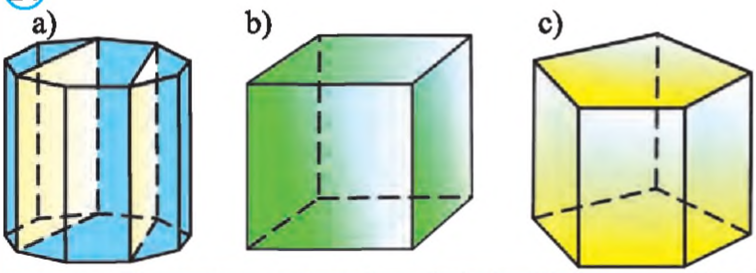


Prizma yon yoqlarining asosiga perpendikular yoki perpendikular emasligiga qarab *to'g'ri* yoki *og'ma* prizmalarga ajratiladi. 23.a- rasmda to'g'ri oltiburchakli prizma, 23.b- rasmda esa og'ma uchburchakli prizma tasvirlangan. Ravshanki, to'g'ri prizmaning yon yoqlari to'g'ri to'rtburchaklardan iborat bo'ladi.

24



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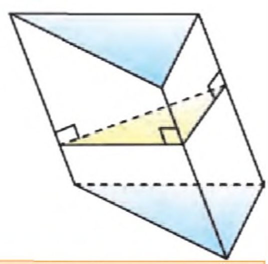


Asosi muntazam ko'pburchakdan iborat to'g'ri prizma *muntazam prizma* deb ataladi (24- rasm). Muntazam prizmaning yon yoqlari bir-biriga teng to'g'ri to'rtburchaklardan iborat bo'ladi.

Prizma asosining biror nuqtasidan ikkinchi asosiga tushirilgan perpendikular prizmaning ***balandligi*** deb ataladi (23.b- rasm).

Prizmaning ***diagonal kesimi*** deb, prizma asoslarining mos diagonallari orqali o'tkazilgan kesimga aytiladi (24.a- rasm). Prizma diagonal kesimlarining soni prizma bitta asosining diagonallari soniga teng.

Prizmaning ***perpendikular kesimi*** deb, uning barcha yon qirralariga perpendikular kesimga aytiladi (25- rasm).

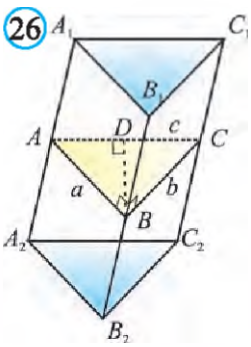
Qavariq n-burchakning $\frac{n(n-3)}{2}$ ta diagonali borligini hisobga olsak, n-burchakli prizma diagonal kesimlari soni ham $\frac{n(n-3)}{2}$ ta bo'ladi.

Har bir diagonal kesimda prizmaning ikkita diagonalini o'tkazish mumkin. Demak, n-burchakli prizmaning jami $n(n-3)$ ta diagonali bor.

1- masala. Uchburchakli og'ma prizma yon qirralari orasidagi masofalar, mos ravishda, 7 cm, 15 cm va 20 cm. Prizmaning eng katta yuzli yon yog'idan uning qarshisidagi yon qirrasigacha bo'lgan masofani toping.

Yechish. Ma'lumki, parallel to'g'ri chiziqlar orasidagi masofa bu to'g'ri chiziqlar birining biror nuqtasidan ikkinchisiga o'tkazilgan perpendikularning uzunligiga teng. Unda berilgan prizmaning ABC perpendikular kesimi tomonlarining uzunligi shu masofalarga teng bo'ladi (26- rasm). Prizmaning eng katta yuzli yog'ida eng katta AC=20 cm tomon yotadi. B_2B_1 qirradan $A_2A_1C_1C_2$ tekislikkacha bo'lgan masofa ABC uchburchakning BD balandligiga teng bo'ladi. Unda Geron formulasiga ko'ra:

26



$$S_{ABC}=\sqrt{p(p-a)(p-b)(p-c)},\quad p=\frac{a+b+c}{2},$$

$$p=\frac{a+b+c}{2}=\frac{7+15+20}{2}=21,$$

$$S_{ABC}=\sqrt{21(21-7)(21-15)(21-20)}=\sqrt{21\cdot14\cdot6\cdot1}=42.$$

Ikkinchi tomondan, $S_{ABC}=\frac{AC\cdot BD}{2}$.

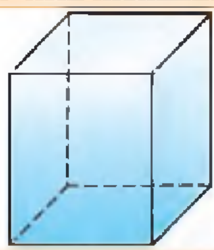
Bundan, $42=\frac{AC\cdot BD}{2}$ yoki $BD=4{,}2$ cm.

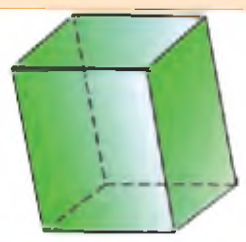
Javob: 4,2 cm.

6.2. Parallelepiped va kub

Asoslari parallelogrammdan iborat prizma *parallelepiped* deb ataladi (27- rasm). Parallelepipedlar ham prizma kabi to'g'ri (27.a- rasm) va og'ma (27.b- rasm) bo'lishi mumkin.

27 a) b)





Parallelepipedning umumiy uchga ega bo'lmagan yoqlari *qarama-qarshi yoqlari* deb ataladi.

Parallelepipedning

– 12 ta qirralari bo‘lib, ularning har to‘rttasi teng kesmalardan iborat (28.a- rasm),

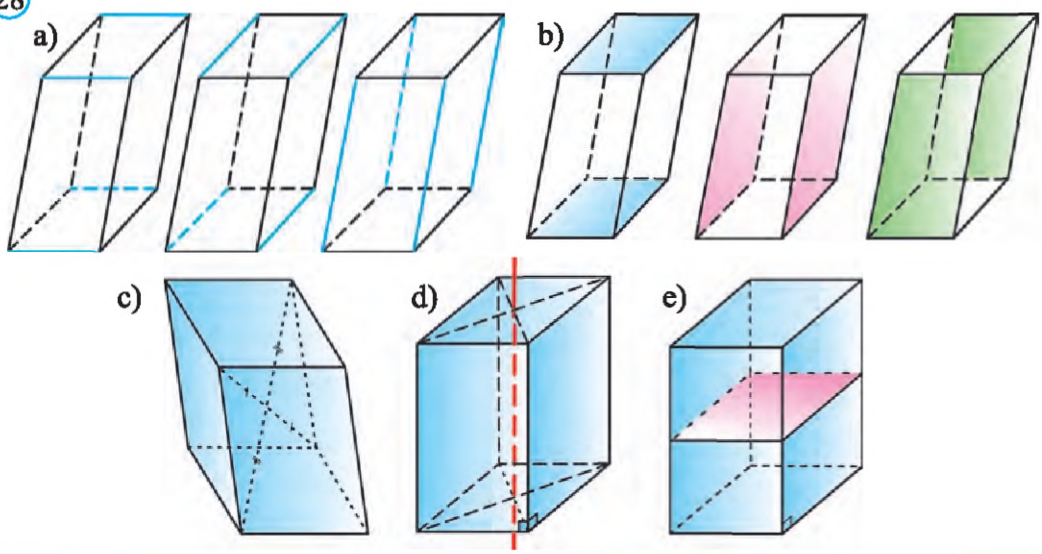
– 6 ta yoqlari bo‘lib, uning qarama-qarshi yoqlari o‘zaro parallel va teng bo‘ladi (28.b- rasm),

– 4 ta diagonali bo‘lib, ular bitta nuqtada keshishadi va kesishish nuqtasida teng ikkiga bo‘linadi (28.c- rasm),

– diagonallari keshishish nuqtasi uning simmetiya markazi bo‘ladi (28.c- rasm).

To‘g‘ri parallelepipedning simmetriya o‘qi (28.d- rasm) va simmetriya tekisligi bor (28.e -rasm).

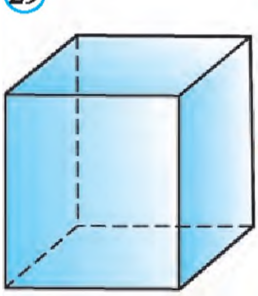
28



Asoslari to‘g‘ri to‘rtburchakdan iborat to‘g‘ri parallelepiped *to‘g‘ri burchakli parallelepiped* deb ataladi (29- rasm).

Ravshanki, to‘g‘ri burchakli parallelepipedning barcha yoqlari to‘g‘ri to‘rtburchaklardan iborat bo‘ladi.

29



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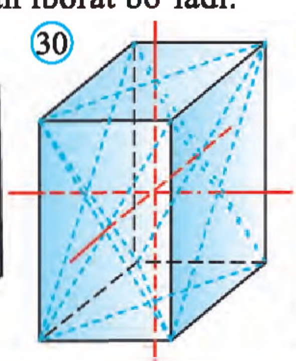

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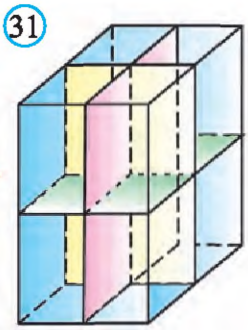
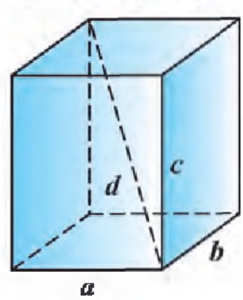


32

$$d^2 = a^2 + b^2 + c^2$$



To‘g‘ri burchakli parallelepipedning uchta simmetriya o‘qi (30- rasm) va uchta simmetriya tekisligi bor (31- rasm).

To‘g‘ri burchakli parallelepipedning bitta uchidan chiquvchi uchta qirrasi uzunliklariga uning *o‘lchamlari* deb aytiladi.

Xossa: To‘g‘ri burchakli parallelepiped d diagonalining kvadrati uning o‘lchamlari: a, b va c ning kvadratlari yig‘indisiga teng (32- rasm):

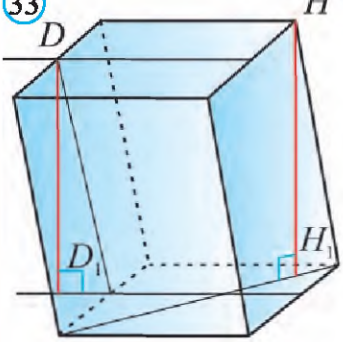
$$d^2 = a^2 + b^2 + c^2.$$

O‘lchamlari teng bo‘lgan to‘g‘ri burchakli parallelepiped *kub* deb ataladi. Ravshanki, kubning barcha yoqlari teng kvadratlardan iborat bo‘ladi. Kub bitta simmetriya markaziga, 9 ta simmetriya o‘qiga va 9 ta simmetriya tekisligiga ega.

Yuqorida prizmalarning qator xossalarini sanab o‘tdik. Ularning ba’zilarini 10- sinfda isbotlagan edik. Qolgan xossalarni isboti nisbatan sodda bo‘lganligi uchun ularni mustaqil isbotlash uchun qoldirdik.

6.3. Prizmaning yon va to‘la sirti

33- rasmda $ABCDEA_1B_1C_1D_1E_1$ prizmaning HH_1 va DD_1 balandliklari tasvirlangan. Ravshanki, muntazam prizmaning balandligi uning yon qirrasiga teng bo‘ladi.

33


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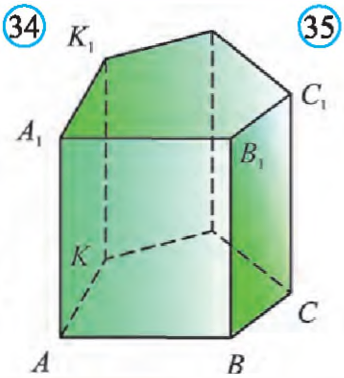

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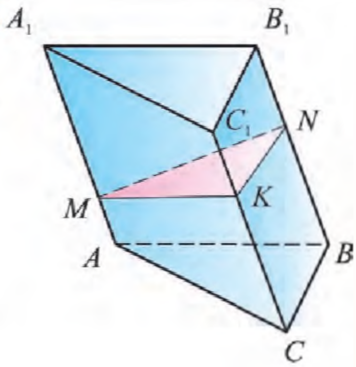


Prizma *yon sirti* (aniqrog‘i, ***yon sirtining yuzi***) uning **yon yoqlari yuzi yig‘indisiga teng, *to‘la sirti* esa yon sirti va ikkita asosining yuzi yig‘indisiga teng.**

$$S_{to'la} = S_{yon} + 2S_{asos}.$$

Teorema. **To‘g‘ri** prizmaning **yon sirti** asosining perimetri **bilan** balandligining ko‘paytmasiga **teng:**

$$S_{yon} = P_{asos} \cdot h.$$

Isbot. **Berilgan prizmaning balandligi h,** asosining perimetri $P = AB + BC + \ldots + KA$ **bo‘lsin (34-** rasm). Ravshanki, **to‘g‘ri prizmaning**

har bir yog‘i to‘g‘ri to‘rtburchakdan iborat. Bu to‘g‘ri to‘rtburchaklarning asosi prizmaning mos tomonlariga, balandligi esa prizma balandligiga teng.

Demak, $S_{yon} = AB \cdot h + BC \cdot h + ... + KA \cdot h = (AB + BC + ... KA) \cdot h = P \cdot h$. □

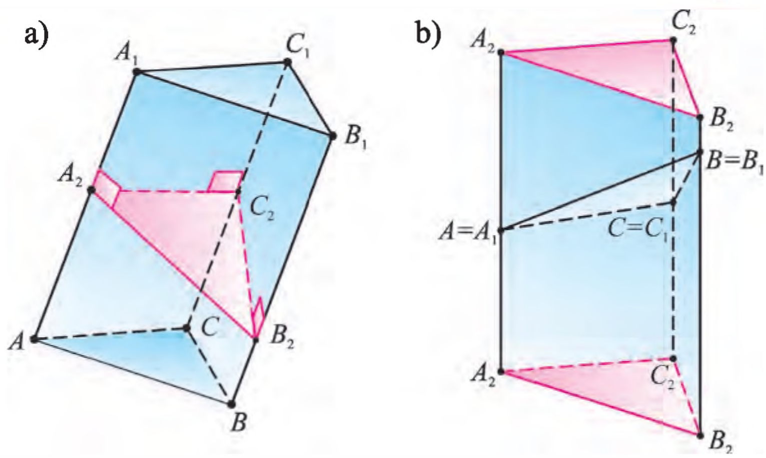
Teorema. Ihtiyority prizmaning yon sirti uning perpendikular kesimi perimetri bilan yon qirrasi uzunligining ko‘paytmasiga teng:

$$S_{yon} = P \cdot l.$$

Isbot. Perpendikular kesimning perimetri P ga teng bo‘lsin (35- rasm). Kesim prizmani ikki bo‘lakka ajratadi (36.a- rasm). Bu bo‘laklarning birini olib, prizma asoslari ustma-ust tushadigan qilib parallel ko‘chiramiz. Natijada yangi to‘g‘ri prizma hosil bo‘ladi (36.b- rasm). Ravshanki, bu prizmaning yon sirti berilgan prizma yon sirtiga teng. Uning asosi berilgan perpendikular kesimdan iborat bo‘lib, yon qirrasi l ga teng bo‘ladi.

Demak, yuqorida isbotlangan teoremaga ko‘ra: $S_{yon} = P \cdot l$ □

36



Mavzuga oid masalalar va amaliy topshiriqlar

174. Tetraedr bitta yog‘ining yuzi 6 cm² bo‘lsa, uning to‘la sirtini toping.

175. Oktaedr bitta yog‘ining yuzi 5,5 cm² bo‘lsa, uning to‘la sirtini toping.

176. Dodekaedr bitta yog‘ining yuzi 6,4 cm² bo‘lsa, uning to‘la sirtini toping.

177. Kub to‘la sirtining yuzi 105,84 cm² bo‘lsa, uning har bir yog‘i yuzini va qirrasining uzunligini toping.

178. Oktaedr to‘la sirtining yuzi $32\sqrt{3}$ cm² bo‘lsa, uning har bir yog‘i yuzini va qirrasining uzunligini toping.

179. To‘g‘riburchakli parallelepiped asosining tomonlari 7:24 nisbatda, diagonal kesimining yuzi 50 dm² ga teng. Yon sirtining yuzini toping.

180*. To‘g‘ri parallelepipedning yon qirrasi 1 m ga, asoslarining tomonlari

23 m va 11 m ga teng. Asos diagonallarining nisbati 2:3 kabi. Diagonal kesimlarining yuzini toping.

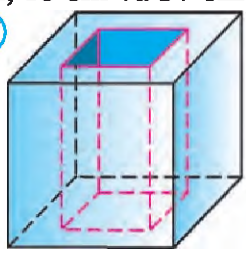
181. To'g'ri parallelepiped asosining tomonlari 3 cm va 5 cm, asosining diagonallaridan biri 4 cm ga teng. Parallelepiped kichik diagonallaridan biri asos tekisligi bilan 60° li burchak tashkil etadi. Uning diagonallari uzunligini toping.

182. To'g'ri parallelepipedning yon qirrasi 5 m, asosining tomonlari 6 m va 8 m, asosining diagonallaridan biri 12 m ga teng. Parallelepipedninng diagonallarini toping.

183*. Uchburchakli muntazam prizmaning qirrasi 3 ga teng. Asosining tomoni va o'qining o'rtasi orqali tekislik o'tkazilgan. Kesimning yuzini toping.

184. Uchburchakli to'g'ri prizmaning balandligi 50 cm, asosining tomonlari 40 cm, 13 cm va 37 cm. Prizmaning to'la sirtini toping.

37



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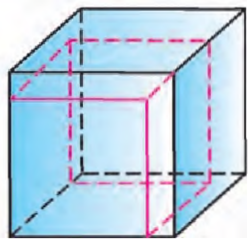

185*. 37- rasmda tasvirlangan birlik kubdan asosining tomoni 0,5 ga yon qirrasi 1 ga teng bo'lgan muntazam to'rtburchakli prizma o'yib olindi. Kubning qolgan bo'lagi to'la sirtining yuzini hisoblang.

186. Agar kubning qirrasi 1 birlik orttirilsa, uning to'la sirti 54 birlikka ortadi. Kubning qirrasini toping (38- rasm).

187. $ABCC_1B_1A_1$ og'ma prizmaning asosi ABC tengyonli uchburchak bo'lib, unda $AB=AC=10$ cm va $BC=12$ cm. A_1 uchi A, B va C uchlardan teng uzoqlikda yotadi hamda AA_1 kesma 13 cm ga teng. Prizmaning to'la sirtini toping.

188. Muntazam to'rtburchakli prizmaning yon sirti 160 ga, to'la sirti 210 ga teng. Prizma asosining diagonalini toping.

189. Uchburchakli og'ma prizmaning yon qirralari yotgan parallel to'g'ri chiziqlar orasidagi masofa 2 cm , 3 cm va 4 cm, yon qirralari esa 5 cm ga teng. Prizmaning yon sirtini toping.

190. Kubning qirralari uzunliklari yig'indisi 96 ga teng. Uning yon sirtini toping.

191. 39- rasmlarda tasvirlangan ko'pyoqlarning to'la sirtini hisoblang (hamma ikkiyoqli burchaklar to'g'ri burchak).

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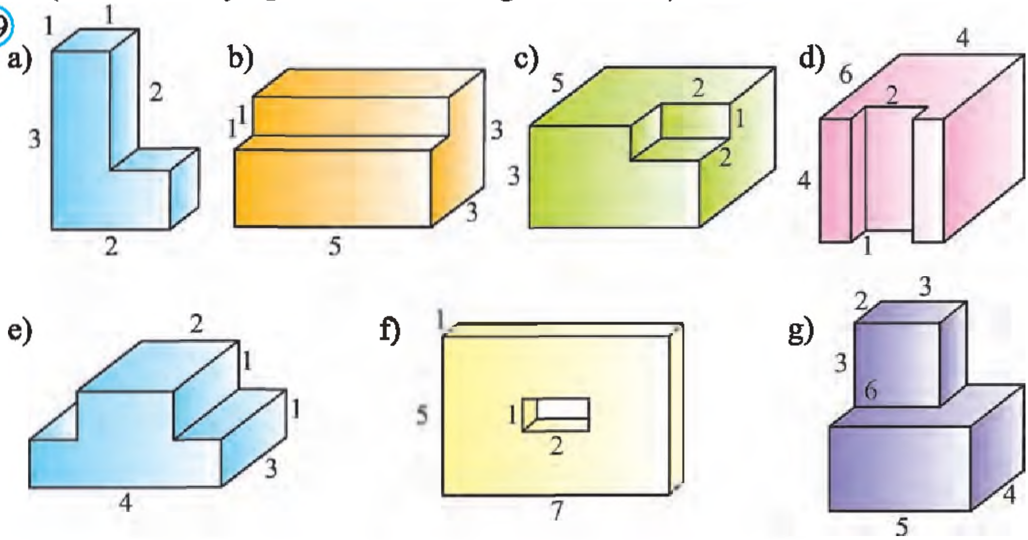

192. 40- rasmlarda tasvirlangan ko'pyoqlarning to'la sirtini hisoblang (hamma ikkiyoqli burchaklar to'g'ri burchak).

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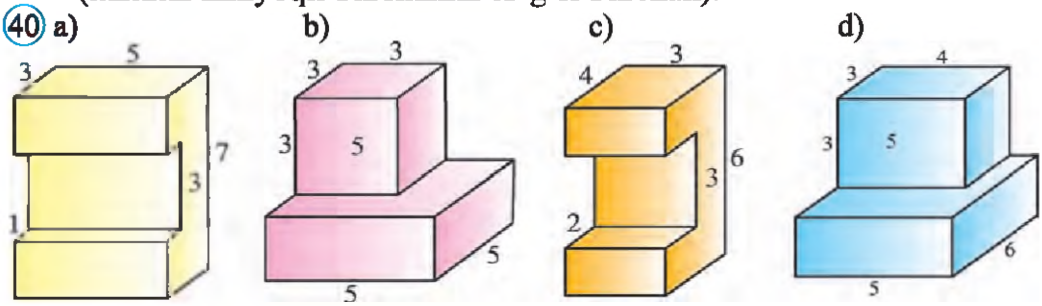


193. Oltiburchakli muntazam prizmaning **yon qirrasi 8 cm, asosining tomoni** esa **3 cm**. Prizmaning barcha qirralari **uzunliklarining yig'indisini toping**.

194. To'rtburchakli muntazam prizma **asosining** tomoni **6 cm, prizmaning** balandligi **esa 5 cm. Uning diagonal kesimi yuzini toping.**

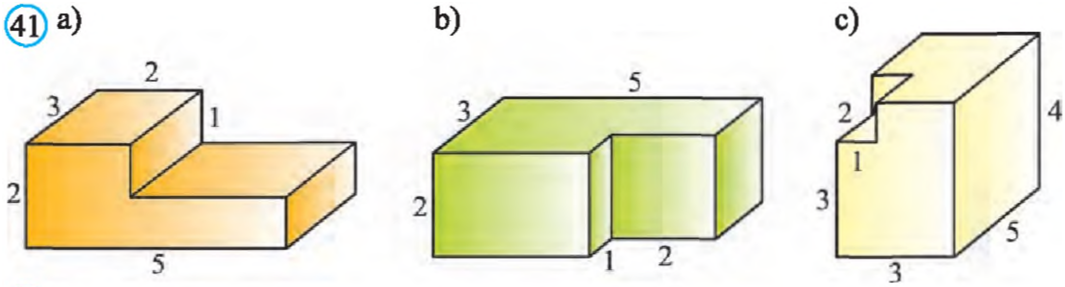
195. Uchburchakli muntazam prizma **asosining** tomoni **6 cm, yon** qirrasi **esa 12 cm**. Prizma **yon sirtining yuzini toping**.

196. 41- rasmlarda tasvirlangan **ko'pyoqlarning to'la sirtini hisoblang** (hamma **ikkiyoqli** burchaklar **to'g'ri burchak**).

197. 42- rasmlarda tasvirlangan **ko'pyoqlarning to'la sirtini hisob**lang (hamma **ikkiyoqli burchaklar to'g'ri burchak**).

198*.43- rasmdadagi **uy asosining o'lchamlari 6 m va 8 m**. **Uyning tomi asosiga 45° li burchak ostida og'gan. Tom sirti yuzini toping.**

41



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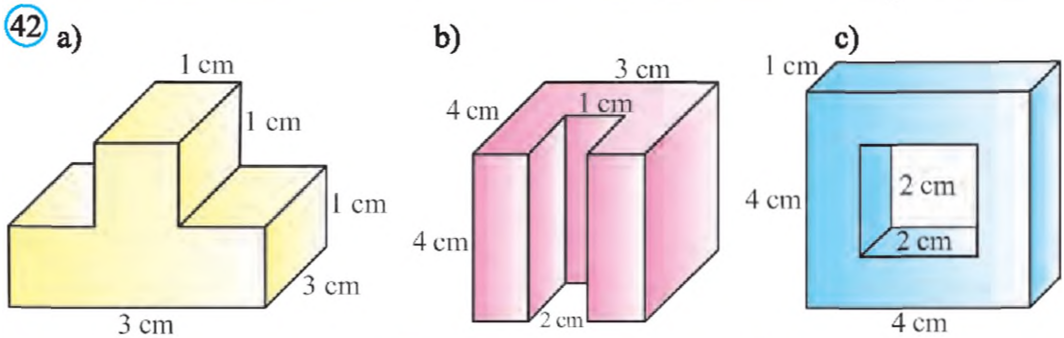

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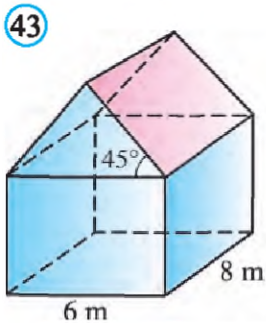

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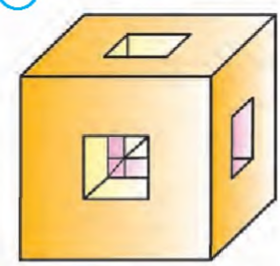

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199. Parallelepipedning bitta uchidan chiquvchi qirralari, mos ravishda, 6 cm, 8 cm va 12 cm. Parallelepiped barcha qirralari uzunliklarining yig‘indisini toping.

200. Parallelepipedning bitta umumiy uchga ega yoqlarining yuzlari 6 cm^2, 12 cm^2 va 16 cm^2. Parallelepiped to‘la sirtining yuzini toping.

201*. Qirrasi 3 cm ga teng bo‘lgan kubning har bir yog‘idan ko‘ndalang kesimi - asosi 1 cm ga teng kvadrat shaklidagi teshiklar o‘yilgan (44-rasm). Kubning qolgan qismi to‘la sirtining yuzini toping.

202*. Futbol to‘pining sirti qirralari 5 cm ga teng bo‘lgan 12 ta muntazam beshburchak va 20 ta muntazam oltiburchakdan iborat (45- rasm). Futbol to‘pining to‘la sirtini toping. To‘p kvadrat santimetri 60 so‘m turadigan charmdan ishlangan va uning 10 foizi chok va chiqitga chiqishi ma’lum bo‘lsa, to‘pga sarflangan charm narxini toping.

7. PRIZMANING HAJMI

7.1. Hajm tushunchasi

Fazoda geometrik jismga xos bo‘lgan xususiyatlardan biri bu hajm tushunchasidir. Har qanday predmet (jism) fazoning qandaydir qismini egallaydi. Masalan, g‘isht gugurt qutisiga qaraganda kattaroq joyni egallaydi. Bu qismlarni o‘zaro taqqoslash uchun hajm tushunchasi kiritiladi.

Hajm – fazoviy jismning quyidagi xossalarga ega bo‘lgan miqdoriy (sonli) ko‘rsatkichidir:

1. Har qanday jism musbat sonlarda ifodalanuvchi muayyan hajmga ega.

2. Teng jismlar hajmi ham teng.

3. Agar jism bir necha bo‘lakka bo‘lingan bo‘lsa, uning hajmi bo‘laklar hajmlari yig‘indisiga teng.

4. Qirrasi bir birlik uzunlikka teng kubning hajmi birga teng.

Hajm – uzunlik va yuz kabi sonli kattaliklardan biridir. Uzunlik o‘lchov birligining tanlanishiga qarab *birlik* (qirrasi birlik uzunlikka ega) *kub* ning hajmi 1 cm^3, 1 dm^3, 1 m^3 va hokazo hajm birliklari bilan o‘lchanadi.

Jismlar hajmini turli usullar bilan o‘lchashadi yoki hisoblashadi. Masalan, kichikroq detalning hajmini bo‘linmalarga (shkalaga) ega bo‘lgan idish (menzurka) yordamida o‘lchash mumkin (46- rasm). Chelak hajmini esa unga birlik hajmga ega bo‘lgan idish yordamida suv quyib, to‘ldirish bilan o‘lchash mumkin (47- rasm). Lekin hamma jismlarning ham hajmni bunday usullar bilan o‘lchab bo‘lmaydi. Bunday hollarda hajm turli usullar bilan hisoblanadi. Quyida shu usullar xususida to‘xtalamiz va ularning ba’zilarini isbotsiz keltiramiz.

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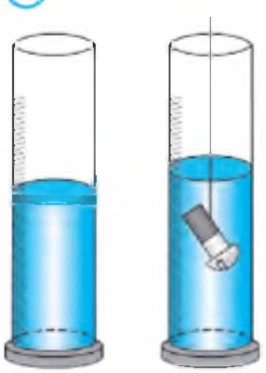

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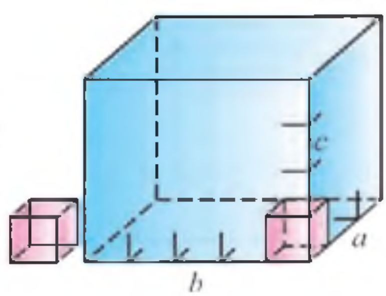

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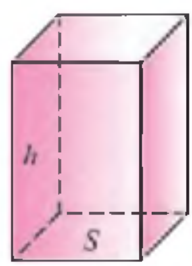


7.2. Parallelepipedning hajmi

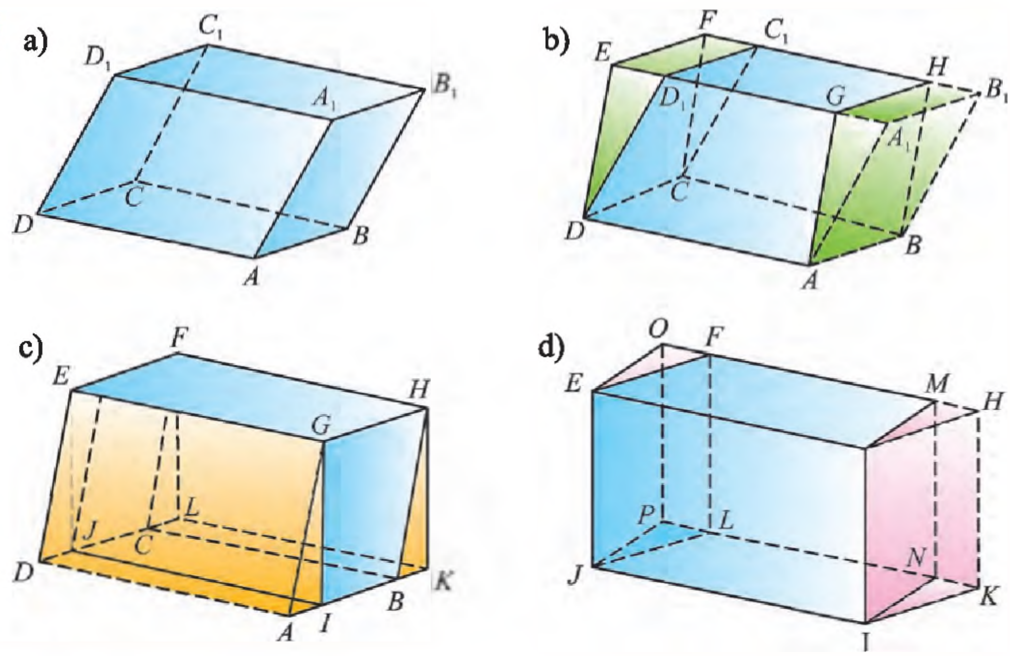
Teorema. To'g'ri burchakli parallelepipedning hajmi uning uchta o'lchamlari ko'paytmasiga teng (48- rasm): $V = a \cdot b \cdot c.$

Natija. To'g'ri burchakli parallelepipedning hajmi asosining yuzi bilan balandligining ko'paytmasiga teng (49- rasm): $V = S \cdot h.$

Teorema. Ixtiyoriy parallelepipedning hajmi asosining yuzi bilan balandligining ko'paytmasiga teng (50- rasm): $V = S \cdot h.$

Mazkur xossa yuqoridagi natijadan kelib chiqadi. Quyidagi 50- rasmlarda berilgan parallelepiped qanday qilib to'g'ri burchakli parallelepipedga to'ldirilishi tasvirlangan. Bundan foydalanib xossani mustaqil asoslang.

50

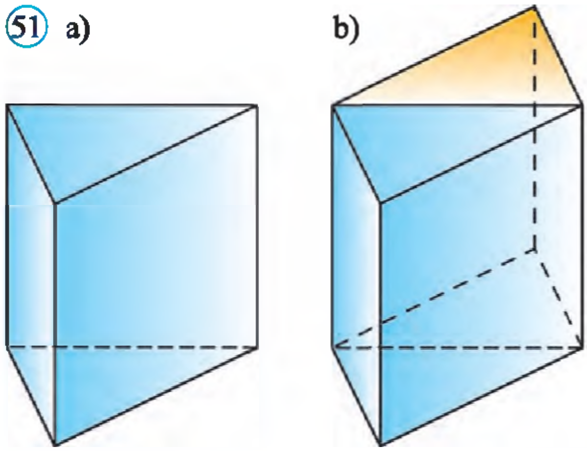


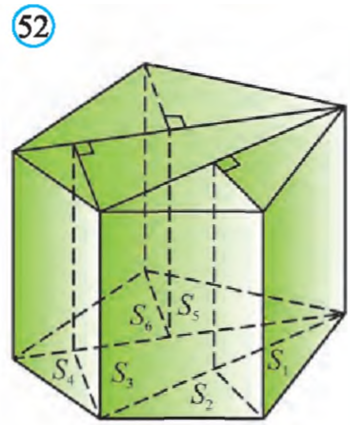
7.3. Prizmaning hajmi

Teorema. To'g'ri prizmaning hajmi asosining yuzi bilan balandligining ko'paytmasiga teng (51- rasm): $V = S \cdot h.$

Isbot. 1- hol. Asosi to'g'ri burchakli uchburchakdan iborat to'g'ri prizma berilgan bo'lsin (51.a- rasm). Bu prizmani unga teng bo'lgan prizma bilan to'g'ri burchakli paralelepipedgacha to'ldirish mumkin (51.b- rasm).

Berilgan prizmaning hajmi, asosining yuzi va balandligi, mos ravishda, V, S va h bo'lsa, hosil bo'lgan to'g'ri burchakli parallelepipedning hajmi, asosining yuzi va balandligi, mos ravishda, $2V$, $2S$ va h bo'ladi.





Demak, $2V=2S \cdot h$ yoki $V=S \cdot h$ bo‘ladi.

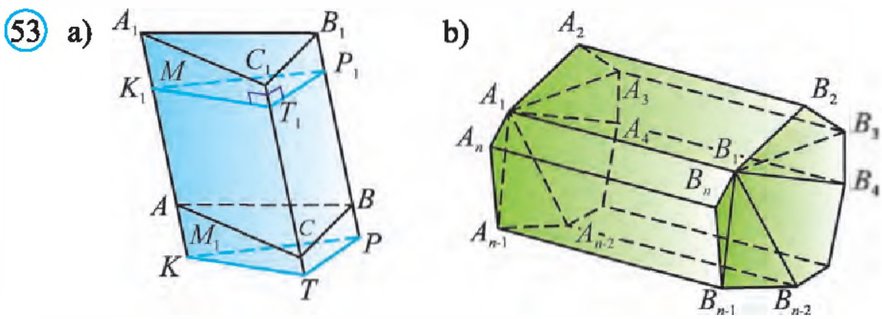
2- hol. Ixtiyoriy to‘g‘ri n-burchakli prizma berilgan bo‘lib, uning asosi yuzi S, balandligi esa h ga teng bo‘lsin. Prizmaning asosi – n-burchakni uning diagonallari bilan uchburchaklarga, uchburchaklarning har birini esa to‘g‘ri burchakli uchburchaklarga bo‘lish mumkin (52- rasm). Natijada, berilgan prizmani chekli sondagi asosi to‘g‘ri burchakli uchburchaklardan iborat to‘g‘ri prizmalarga ajratish mumkinligini aniqlaymiz. Bu prizmalarning balandligi h ga teng bo‘lib, ularning asoslari yig‘indisi berilgan prizma yuziga teng bo‘ladi: $S = S_1 + S_2 + ... + S_k$.

Berilgan prizmaning hajmi uni tashkil qiluvchi uchburchakli prizmalar hajmlari yig‘indisidan iborat bo‘ladi:

$$V = S_1h + S_2h + ... + S_kh = (S_1+S_2+...+S_k)\, h = S \cdot h, \quad \text{yoki } V = S \cdot h. \;\square$$

Teorema. Ixtiyoriy prizmaning hajmi asosining yuzi bilan balandligining ko‘paytmasiga teng: $V = S \cdot h.$

Bu teoremani 5.3- rasmdan foydalanib, oldin uchburchakli prizma uchun (5.3.a- rasm), so‘ng ixtiyoriy prizma uchun (5.3.b- rasm) mustaqil isbotlang.



1- masala. To‘g‘ri parallelepiped asosining tomonlari a va b ga teng bo‘lib, ular o‘zaro 30° li burchak tashkil qiladi. Agar parallelepipedning yon sirti S ga teng bo‘lsa, uning hajmini toping.

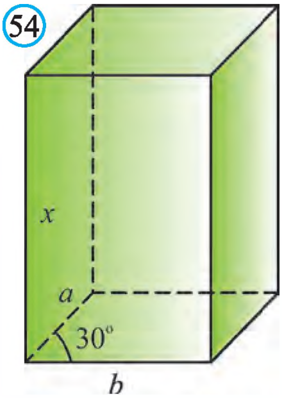
Yechish: Parallelepiped balandligini h bilan belgilaymiz (54- rasm). Unda shartga ko'ra:

$$S = (2a+2b)h \text{ yoki } h = \frac{S}{2(a+b)}.$$

$$S_{asos} = ab\sin 30° = \frac{ab}{2}.$$

$$V = S_{asos} \cdot h = \frac{ab}{2} \cdot \frac{S}{2(a+b)} = \frac{abS}{4(a+b)}.$$

54



Mavzuga oid masalalar va amaliy topshiriqlar

203. 55- rasmda tasvirlangan ko'pyoqlarning hajmini toping.

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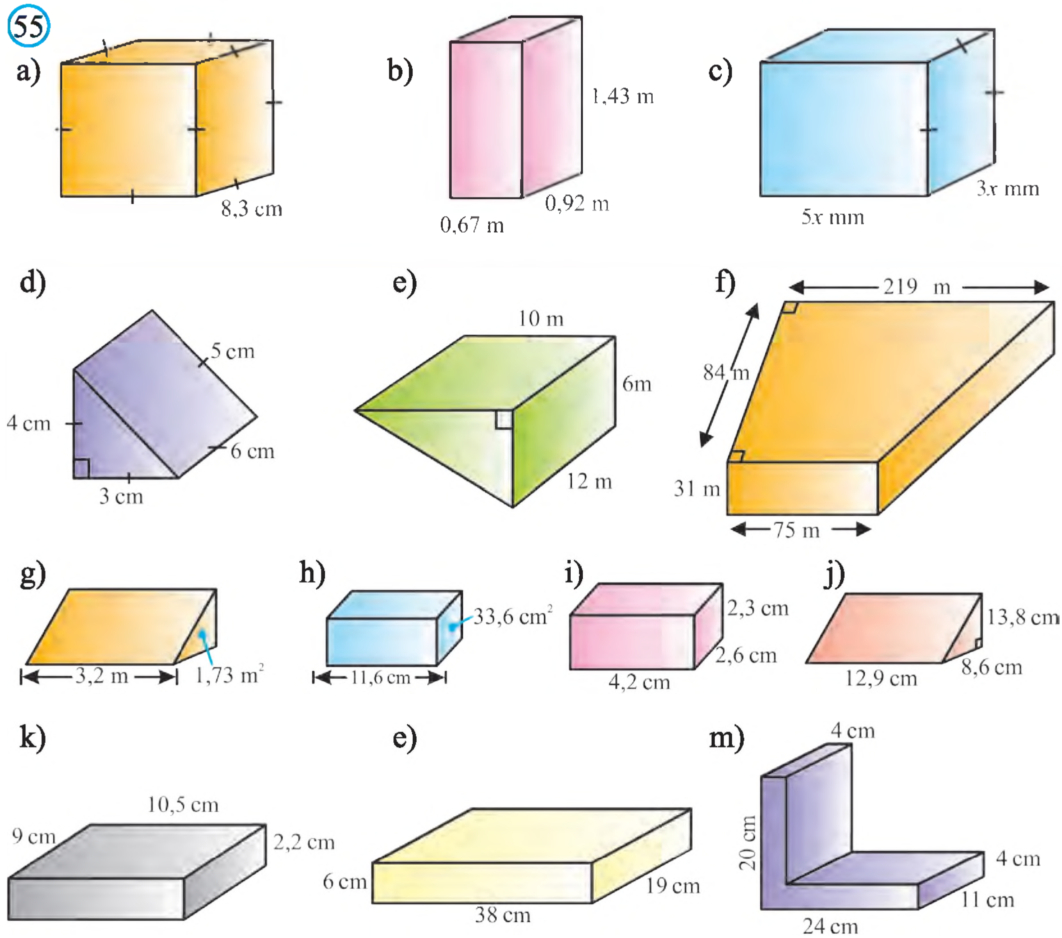
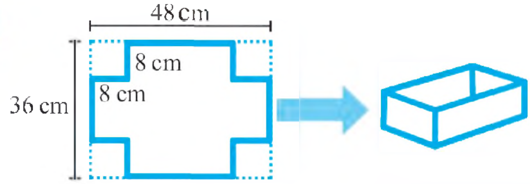


204. 56- rasmda berilgan yoyilmaga ko'ra yasalgan idishning hajmini toping.

56



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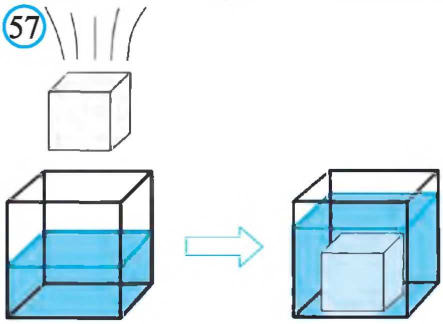

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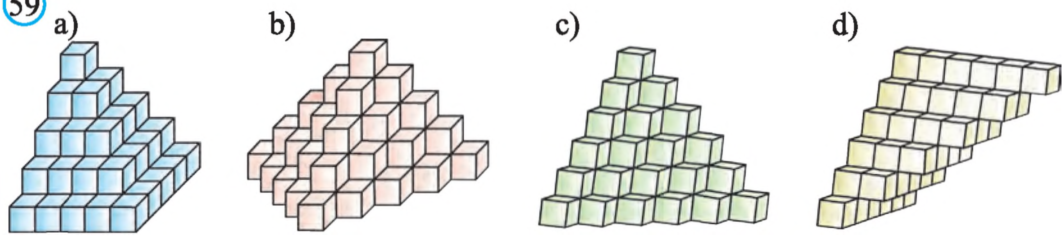
205*. 57- rasmga ko‘ra masala tuzing va uni yeching.

206. 58- rasmda keltirilgan jism 88 ta birlik kubchadan yasalgan. Jismning to‘la sirtini toping.

207. To‘g‘ri burchakli parallelepiped yog‘ining yuzi 12 ga va unga perpendikular qirra uzunligi 12 ga teng. Parallelepipedning hajmini toping.

208. 59- rasmda tasvirlangan fazoviy shakllardan qaysi birining hajmi katta, ya’ni ko‘proq kubchalardan tashkil topgan?

59



209. To‘g‘ri burchakli parallelepiped hajmi 24 ga teng va qirralaridan birining uzunligi 3 ga teng. Parallelepipedning bu qirraga perpendikular yog‘ining yuzini toping.

210. To‘g‘ri burchakli parallelepiped hajmi 60 ga teng va yoqlaridan biri-ning yuzi 12 ga teng. Parallelepipedning bu yoqqa perpendikular qirra-ning uzunligini toping.

211. To‘g‘ri burchakli parallelepiped bir uchidan chiquvchi uchta qirralari uzunliklari 4, 6 va 9 ga teng. Unga tengdosh kub qirrasini toping.

212. Kubning to‘la sirti yuzi 18 ga teng bo‘lsa, uning diagonalini toping.

213. Kubning hajmi 8 ga teng bo‘lsa, uning to‘la sirtining yuzini toping.

214. Agar kubning qirralarini 1 birlik orttirilsa, uning hajmi 19 birlikka ortadi. Kubning qirrasini toping.

215. Kubning to‘la sirtining yuzi 24 ga teng. Uning hajmini toping.

216. Kubning diagonali $\sqrt{12}$ ga teng bo‘lsa, uning hajmini toping.

217. Kubning hajmi $24\sqrt{3}$ ga teng bo‘lsa, uning diagonalini toping.

218. Birinchi kubning hajmi ikkinchisinikidan 8 marta katta. Birinchi kubning to‘la sirtining yuzi ikkinchisinikidan necha marta katta?

219. Qirrasi 30 cm bo‘lgan kub shaklidagi idishga (sisternaga) necha litr suv ketadi?

220. To‘g‘ri burchakli parallelepipedning bitta uchidan chiquvchi qirralari 2 va 6 ga teng. To‘g‘ri burchakli parallelepiped hajmi 48 ga teng. Parallelepipedning shu uchidan chiquvchi uchinchi qirrasini toping.

221. To‘g‘ri parallelepiped asosining tomonlari uzunligi $2\sqrt{2}$ cm va 5 cm, ular orasidagi burchak 45° ga teng. Agar parallelepipedning kichik diagonali 7 cm ga teng bo‘lsa, uning hajmini toping.

222*. To‘g‘ri parallelepiped asosining a va b tomonlari 30° li burchak tashkil qiladi. To‘la sirti S ga teng. Uning hajmini toping.

223. To‘g‘ri burchakli parallelepipedning o‘lchamlari 15 m, 50 m va 36 m. Unga tengdosh kubning qirrasini toping.

224. Uchburchakli to‘g‘ri prizma asosining tomonlari 29, 25 va 6 ga, qirrasi esa asosining katta balandligiga teng. Prizmaning hajmini toping.

225. 39- rasmlarda tasvirlangan ko‘pyoqlarning hajmini hisoblang (hamma ikkiyoqli burchaklar to‘g‘ri burchak).

226. 40- rasmlarda tasvirlangan ko‘pyoqlarning hajmini hisoblang (hamma ikkiyoqli burchaklar to‘g‘ri burchak).

227. To‘g‘ri parallelepipedning asosining yuzi 1 m^2 bo‘lgan rombdan iborat. Diagonal kesimlarining yuzi, mos ravishda, 3 m^2 va 6 m^2. Parallelepipedning hajmini toping.

228. 41- rasmlarda tasvirlangan ko‘pyoqlarning hajmini hisoblang (hamma ikkiyoqli burchaklar to‘g‘ri burchak).

229. 42- rasmlarda tasvirlangan ko‘pyoqlarning hajmini hisoblang (hamma ikkiyoqli burchaklar to‘g‘ri burchak).

230. Kengligi 3 m va uzunligi 20 m bo‘lgan yolakka qalinligi 10 cm bo‘lgan asfalt qatlami yotqizildi. Yo‘lak uchun qancha hajmdagi asfalt ishlatildi?

231*. Og‘ma parallelepipedning asosi – tomoni 1 m ga teng bo‘lgan kvadratdan iborat. Yon qirralaridan biri 2 m ga teng va asosining o‘ziga yopishgan har bir tomoni bilan 60° li burchak tashkil etadi. Parallelepipedning hajmini toping.

232*. Parallelepipedning yoqlari – tomoni a ga teng va o‘tkir burchagi 60° bo‘lgan teng romblardan iborat. Parallelepipedning hajmini toping.

233. Parallelepipedning har bir qirrasi 1 cm ga teng. Parallelepipedning bir uchidagi uchala yassi burchagi o‘tkir bo‘lib, har biri $2a$ ga teng. Parallelepipedning hajmini toping.

234*. Parallelepipedning bir uchidan chiquvchi uchta qirrasining uzunliklari a, b, c ga teng. a va b qirralari o‘zaro perpendikular, c qirra esa ularning har biri bilan α burchak tashkil etadi. Parallelepipedning hajmini toping (60- rasm).

235. a) Uchburchakli; b) to‘rtburchakli; c) oltiburchakli muntazam prizma asosining tomoni a va yon qirrasi b bo‘yicha hajmini toping.

236. To‘g‘ri parallelepiped asosining tomonlari a va b ga teng bo‘lib, ular o‘zaro α burchak tashkil qiladi. Parallelepipedning kichik diagonali d ga teng bo‘lsa, uning hajmini toping.

237. Uchburchakli og‘ma prizmaning yon qirralari 15 m ga, ular orasidagi masofa esa 26 m, 25 va 17 m ga teng. Prizmaning hajmini toping.

238. To‘rtburchakli muntazam prizmaning diagonali 3,5 cm ga, yon yog‘ining diagonali 2,5 cm ga teng. Prizmaning hajmini toping.

239. Uchburchakli muntazam prizma asosining tomoni a ga, yon sirti asoslari yuzlarining yig‘indisiga teng. Uning hajmini toping.

240. Oltiburchakli muntazam prizmada eng katta diagonal kesimning yuzi 4 m^2 ga, ikkita qarama-qarshi yon qirralari orasidagi masofa 2 m ga teng. Prizmaning hajmini toping.

241*. Yetti marta kir yuvishdan keyin sovunning o‘lchamlari ikki marta kamaydi (61- rasm). Agar har kir yuvganda bir xil hajmdagi sovun sarflangani ma’lum bo‘lsa, sovun yana necha marta kir yuvishga yetadi?

242*. Og‘ma prizmada yon qirralariga perpendikular va hamma yon qirralarini kesib o‘tadigan tekislik o‘tkazilgan. Hosil qilingan kesim yuzi Q, yon qirralari esa l ga teng bo‘lsa, prizmaning hajmini toping (62- rasm).

243. Uchburchakli to‘g‘ri prizma asosining tomonlari 4 cm, 5 cm,7 cm ga, yon qirrasi esa asosining katta balandligiga teng. Prizmaning hajmini toping.

244. 63- rasmlarda tasvirlangan ko‘pyoqlarning hajmini hisoblang.

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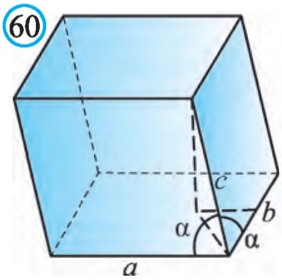

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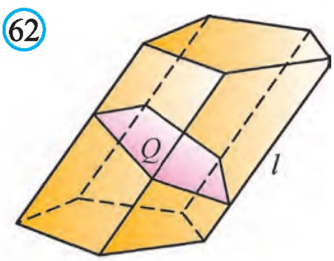
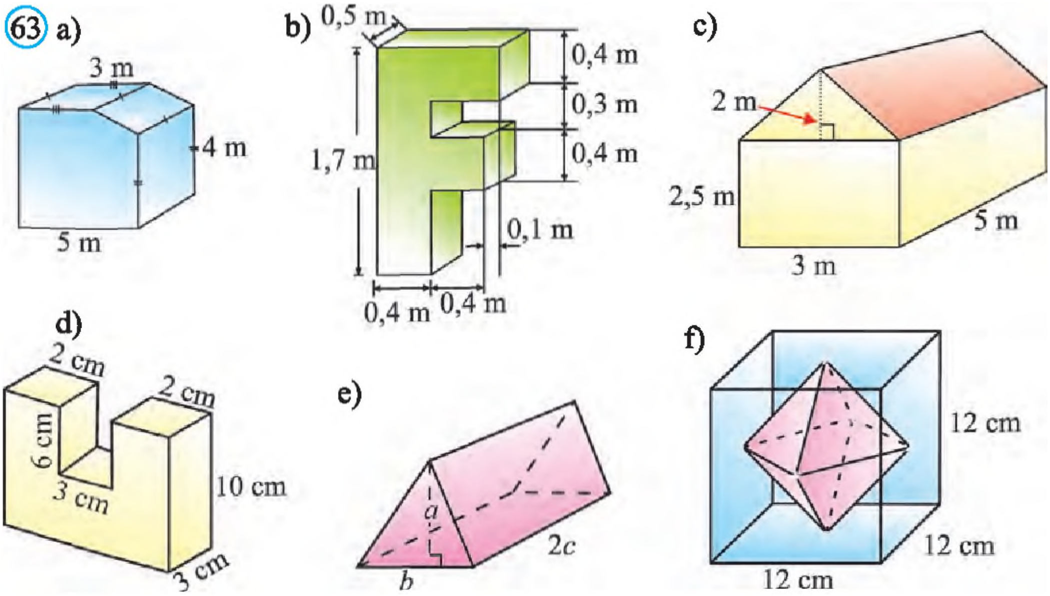




245. Uchburchakli to'g'ri prizma asosining yuzi 4 cm^2 ga, yon yoqlarining yuzlari 9 cm^2, 10 cm^2, 17 cm^2 ga teng bo'lsa, uning hajmini toping.

246*. Prizmaning asosi teng yonli uchburchak bo'lib, uning bir tomoni 2 cm, qolgan ikki tomoni 3 cm ga teng. Prizmaning yon qirrasi 4 cm ga teng va u asos tekisligi bilan 45° li burchak tashkil etadi. Bu prizmaga tengdosh kubning qirrasini toping.

247. Og'ma prizma asosining tomoni a ga teng bo'lgan teng tomonli uchburchak. Yon yoqlaridan biri asosiga perpendikular va kichik diagonali s ga teng bo'lgan rombdan iborat. Prizmaning hajmini toping.

248. Agar to'rtburchakli to'g'ri prizmaning balandligi h, diagonallari asos tekisligi bilan α va β burchaklar tashkil qiladi. Agar asosining diagonallari orasidagi burchak γ ga teng bo'lsa, prizmaning hajmini toping.

249*. Kesimi asosi 1,4 m va balandligi 1,2 m bo'lgan teng yonli uchburchak shaklidagi suv chiqaruvchi quvurning suv o'tkazish quvvatini (1 soatda oqib o'tadigan suv hajmini) hisoblang. Suvning oqish tezligi 2 m/s.

250*. Temiryo'l ko'tarmasining kesimi trapetsiya shaklida bo'lib, uning pastki asosi 14 m, yuqori asosi 8 m va balandligi 3,2 m. 1 km ko'tarmani qurish uchun qancha kub metr tuproq kerak bo'ladi?

251*. Tomoni 3,2 cm va qalinligi 0,7 cm bo'lgan muntazam sakkizburchak shaklidagi yog'och plitkaning massasi 17,3 g. Yog'ochning zichligini toping.

252. O‘lchamlari 30×40×50 (cm) bo‘lgan to‘g‘ri burchakli parallelepiped shaklidagi qutidan nechtasini o‘lchamlari 2×3×1,5 m bo‘lgan mashina kuzoviga joylashishi mumkin?

253*. O‘lchamlari 420 mm × 240 mm × 90 mm bo‘lgan to‘g‘ri burchakli parallelepiped shaklidagi, zichligi 7,8 g/cm³ bo‘lgan po‘lat plitalarning nechtasini yuk ko‘tarish quvvati 3 t bo‘lgan yuk mashinasida tashish mumkin?

254. O‘lchamlari 250 mm × 120 mm × 65 mm bo‘lgan to‘g‘ri burchakli parallelepiped shaklidagi, zichligi 1,6 g/cm³ bo‘lgan g‘ishtning nechtasini yuk ko‘tarish quvvati 3 t bo‘lgan yuk mashinasiga yuklash mumkin?

255*. O‘lchamlari 820 mm × 210 mm × 120 mm bo‘lgan to‘g‘ri burchakli parallelepiped shaklidagi, zichligi 7,3 g/cm³ bo‘lgan cho‘yan plitani yuk ko‘tarish quvvati 2 t bo‘lgan ko‘tarma kran yordamida ko‘tarish mumkinmi?

256. Bo‘yi 105 m va ko‘ndalang kesimi o‘lchamlari 30 cm × 40 cm bo‘lgan to‘g‘ri to‘rtburchakdan iborat yog‘ochdan, bo‘yi 3,5 m, eni 20 cm va qalinligi 20 mm bo‘lgan nechta taxta bo‘lagi chiqadi?

257. G‘ishtning o‘lchamlari 25 × 12 × 6,5 (cm). Agar 1 m³ hajmdagi g‘ishtning massasi 1700 kg bo‘lsa, bir dona g‘ishtning massasini grammlarda aniqlang.

258. Sanitariya me’yorlariga ko‘ra, sinfdagi har bir o‘quvchiga 7,5 m³ havo to‘g‘ri keladi. Agar sinfxonaning balandligi 3,5 m va u 28 o‘quvchiga mo‘ljallangan bo‘lsa, sinfxonaning maydonini toping.

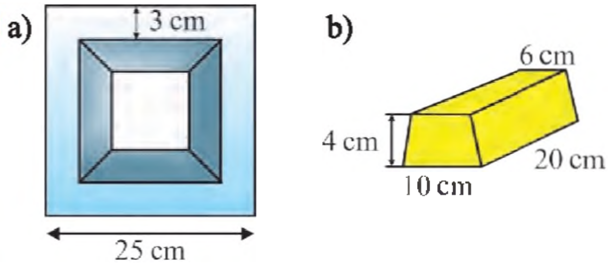
259*. Bo‘yi 100 m, eni esa 10 m bo‘lgan to‘g‘ri to‘rtburchak shaklidagi maydonni qalinligi 5 cm bo‘lgan asfalt bilan qoplash kerak. Agar 1 m³ hajmdagi asfaltning massasi 2,4 tonna va bitta yuk mashinasining yuk ko‘tarish quvvati 5 tonna bo‘lsa, bu maydonni asfaltlash uchun nechta mashina asfalt kerak bo‘ladi?

260*. O‘lchamlari 3 cm, 4 cm, 5 cm bo‘lgan, to‘g‘ri burchakli paralle-lepiped shaklidagi temir parchasiga dastgohda ishlov berildi. Bu jarayonda uning har bir qirrasi birdek kamayib, to‘la sirti 42 cm² ga kamaygani ma’lum. Bu temir parchasining hajmi ishlov berilgandan keyin qanchani tashkil qiladi?

261*. 64.a- rasmda cho‘yan quvur kesimi tasvirlangan. Rasmda berilgan ma’lumotlar asosida bir metr uzunlikdagi bunday quvurning massasini aninqlang (cho‘yanning zichligi – 7,3 g/cm³).

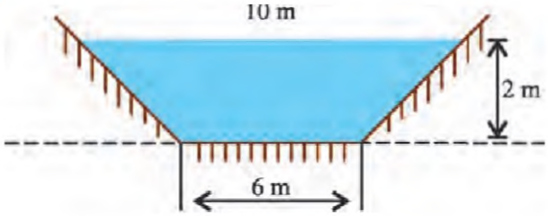
262. O‘lchamlari 64.b- rasmda berilgan oltin plitka (yombi) ning massasi 12,36 kg bo‘lsa, uning zichligini aniqlang.

(64)



263*. **Kanalning ko'ndalang kesimi asoslari 10 m, 6 m va balandligi 2 m bo'lgan teng yonli trapetsiyadan iborat** (65- rasm). **Suv oqimi tezligi 1 m/s bo'lsa, bir** minutda **bu** kanaldan **qancha** hajmdagi **suv oqib o'tadi?**

(65)



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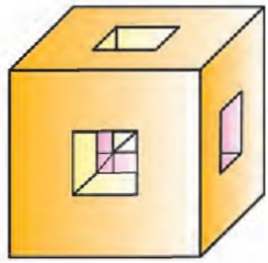
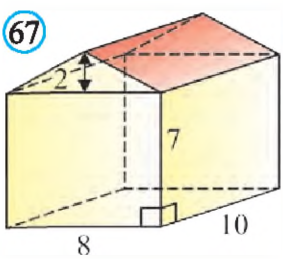


264*. **Qirrasi 6 cm** ga teng bo'lgan, **misdan** ishlangan kubning har **bir yog'idan ko'ndalang kesimi – asosi 2 cm** ga teng kvadrat shaklidagi **teshiklar o'yilgan (66- rasm). Agar misning solishtirma zichligi** 0,9 g/cm^3 **bo'lsa, kubning qolgan qismining massasini toping.**

265. To'g'ri **burchakli parallelepiped** shaklidagi **metall blok asosining o'lchamlari 7 cm va 5 cm. Blokning massasi 1285 g va metalning zichligi 7,5 g/cm^3 bo'lsa, blokning** balandligini **toping.**

266. 67- rasmda berilgan ma'lumotlar **asosida** garajning **hajmini toping.**

(67)



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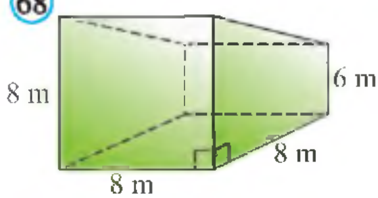

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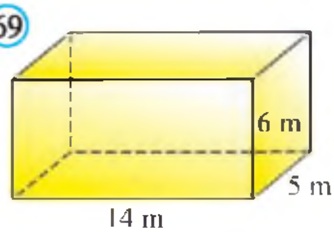

267. Gul o'stiriladigan katta **tuvak chuqurligi 2 fut, kengligi 12 fut va uzunligi 15 fut** bo'lgan **to'g'ri burchakli** parallelepiped shaklida. Tuvakning hajmini toping **va kub** metrlarda ifodalang **(1 fut = 30,48 cm).**

268. Yuk ombori **68-** rasmda tasvirlangan **trapetsiyali** prizma shaklida. Rasmda berilgan ma'**lumotlar asosida** omborning **sig'imini aniqlang.**

269*. 69- rasmda qutining o'lchamlari berilgan. Qutining asoslari 1 kvadrat metri 1000 so'm, yon yoqlari esa 1 kvadrat metri 2000 so'm bo'lgan materialdan ishlangan. Qutini yasashga necha so'mlik material ketgan?

270. Kubning hajmi V ga teng bo'lsa, uning diagonalini toping.

271. Katta to'g'ri burchakli parallelepipeddan 70- rasmda ko'rsatilgandek qilib kichik to'g'ri burchakli parallelepiped qirqib olingan. Berilgan ma'lumotlar asosida hosil bo'lgan jismning hajmini toping.

272. 71- rasmda tasvirlangan piramida hajmini toping.

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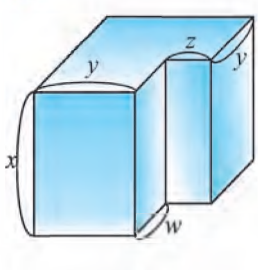

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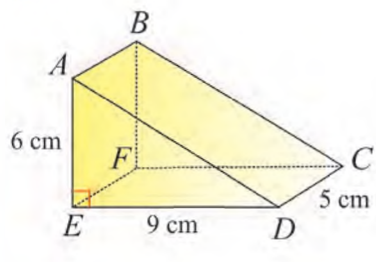

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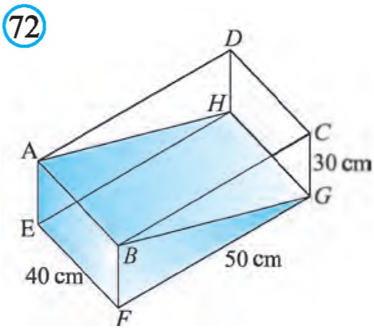
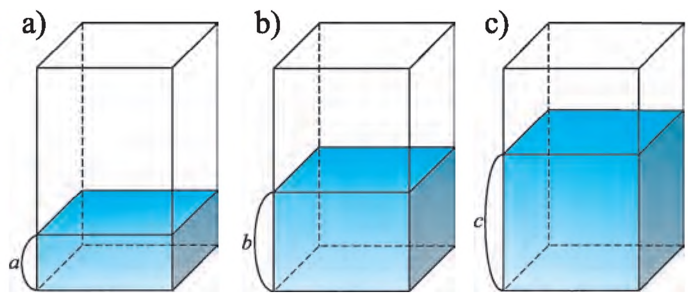


273*. 72- rasmda tasvirlangan to'g'ri burchakli parallelepiped shaklidagi akvariumda qancha suv bor?

274*. To'g'ri burchakli parallelipiped shaklidagi bir xil akvariumlarga 73- rasmda ko'rsatilgandek, turli sathdagi suv quyilgan. Bu akvariumlarga quyilgan suv hajmlarining nisbati qanday bo'ladi?

73



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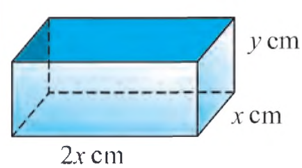


275*. Tadqiqot. Korxona sig'imi 1 litr, asosining o'lchamlari nisbati 1:2 bo'lgan to'g'ri burchakli parallelepiped shaklidagi usti ochiq qutilarni ishlab chiqarmoqchi (74- rasm). Qutini tejamli ishlab chiqarish, ya'ni unga ketadigan material eng kam bo'lishi uchun uning o'lchamlari qanday bo'lishi kerak? (x ga turli qiymatlar berib, qutining hajmini toping va ularni taqqoslash bilan yechishga urinib ko'ring yoki differensial hisob imkoniyatlaridan foydalaning.)

276*. **Muammoli vaziyat.** Geologlar tosh topib oldilar va uning hajmini taxminan bo'lsada aniqlashmoqchi. Ular ko'l yonida turishibdi va ularning ixtiyorida tosh sig'adigan katta metall bak, bir nechta sig'imi noma'lum chelaklar va sig'imi 1 litr bo'lgan butilka bor. Geologlar bu ishni qanday uddalay olishadi?

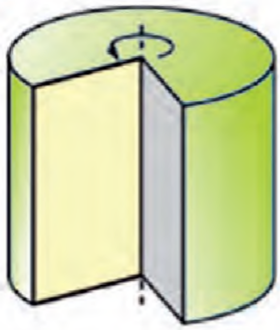
8. SILINDRNING SIRTI VA HAJMI

8.1. Silindrning sirti

Fazoviy shakllarning yana muhim sinflaridan biri – bu aylanish jismlaridir. Silindr aylanish jismlardan biri bo'lib, u bilan quyi sinflarda tanishgansiz. Silindr xossalari prizmaning xossalariga o'xshagani uchun ularni ketma-ket o'rganamiz.

Tog'ri to'rtburchakni bir tomoni atrofida aylantirishdan hosil bo'lgan jismga *silindr* (aniqrog'i, to'g'ri doiraviy silindr) deb aytiladi (75- rasm). Bu aylanishda tog'ri to'rtburchakning bir tomoni qo'zg'alishsiz qoladi. Uni *silindrning o'qi* deb ataymiz. To'rtburchakning bu tomonga qarama-qarshi yotgan tomoni aylanishidan hosil bo'lgan sirt – *silindrning yon sirti*, tomonning o'zi esa *silindrning yasovchisi* deb ataladi. Tog'ri to'rtburchakni qolgan tomonlari bu aylanishda ikkita teng doira hosil qiladi, ularni *silindrning asoslari* deb ataymiz (76- rasm).

75



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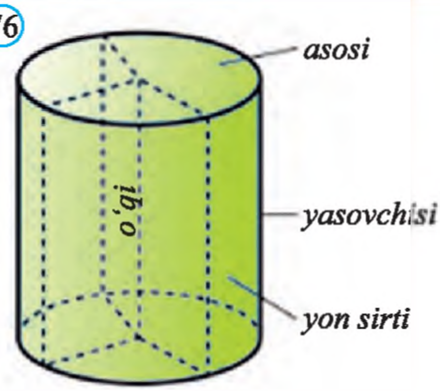
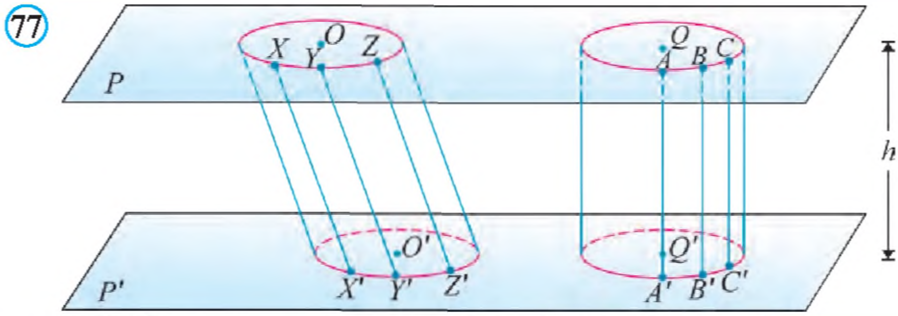


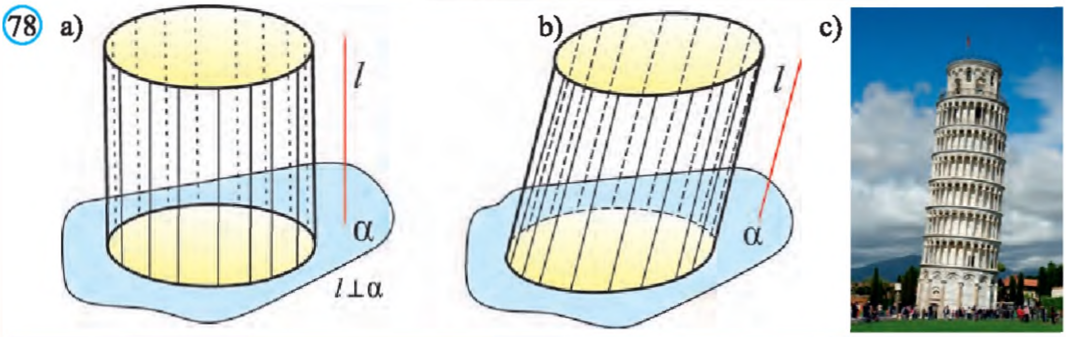
Eslatma. To'g'ri to'rtburchakni bir tomoni atrofida aylantirishdan hosil bo'lgan jism aslida *to'g'ri doiraviy silindr* deb yuritiladi. Silindr tushunchasi esa keng ma'noda quyidagicha kiritiladi.

Aytaylik, fazoda yassi F_1 shakl biror parallel ko'chirishda F_2 shaklga o'tsin. Bu ikki shakl va mazkur parallel ko'chirishda bir-biriga o'tgan nuqtalarni tutashtiruvchi kesmalardan iborat jismga *silindr* deb ataladi (77- rasm).

Agar parallel ko'chirish vektori yassi F_1 shakl tekisligiga perpendikular **bo'lsa, silindr *to'g'ri silindr*** (78.a- rasm) **deb,** aks holda ***og'ma silindr*** **(78.b- rasm) deb yuritiladi.**

78.c- rasmda tasvirlangan **Piza minorasi og'ma silindr shaklida.**





Agar F_1 shakl doiradan iborat bo'lsa, silindr ***doiraviy silindr*** deb ataladi.

To'g'ri doiraviy silindrgina aylanma jism bo'ladi. Kelgusida to'g'ri doiraviy silindrlar bilan ish ko'ramiz va ularni qisqalik uchun silindrlar deb ataymiz.

Silindrning asoslari o'zaro teng doiralardan iborat bo'lib, ular parallel tekisliklarda yotadi. Silindrning bir asosi nuqtasidan ikkinchi asosi tekisligiga tushirilgan perpendikular uning ***balandligi*** deb ataladi.

Bu parallel tekisliklar orasidagi masofa silindrning balandligiga teng bo'ladi. Silindrning o'qi uning balandligi hamdir.

Silindrning yasovchilari esa o'zaro parallel va teng bo'ladi. Shuningdek, silindr o'qi, yasovchilari va balandligi uzunliklari o'zaro teng bo'ladi.

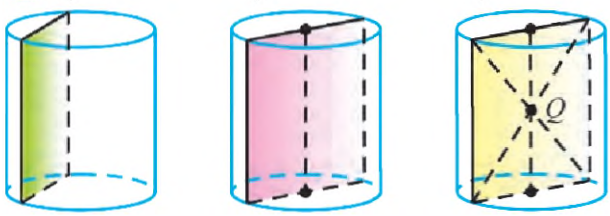
Silindrni uning o'qiga parallel tekislik bilan kesganda hosil bo'lgan kesim to'g'ri to'rtburchakdan iborat bo'ladi (79.a- rasm). Uning ikki tomoni silindrning yasovchilari, qolgan ikki tomoni esa mos ravishda asoslarning parallel vatarlaridir.

Xususan, *o'q kesim* ham to'g'ri to'rtburchak bo'ladi. U silindrning o'qi orqali o'tgan tekislik bilan kesganda hosil bo'lgan kesimdir (79.b- rasm).

O'q kesimlarning diagonallari asos markazlarini tutashtiruvchi kesmaning o'rtasi Q nuqtadan o'tadi. Shuning uchun, bu Q nuqta silindrning simmetriya markazidan iborat bo'ladi (79.c- rasm).

Q nuqtadan o'tuvchi va silindr o'qiga perpendikular bo'lgan tekislik silindrning simmetriya tekisligidan iborat bo'ladi (80- rasm). Silindrning o'qidan o'tuvchi tekisliklar ham uning simmetriya tekisliklari bo'ladi (81- rasm).

79



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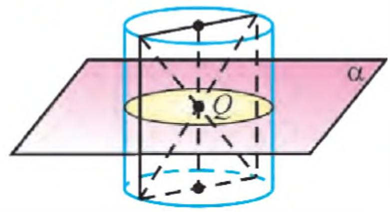


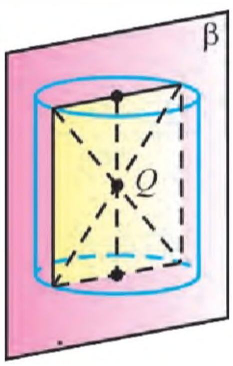
1- masala. Silindr o'q kesimining **yuzi** Q ga teng kvadratdan **iborat**. Silindr asosining yuzini toping.

Yechish. Kvadratning tomoni $\sqrt{Q}$ ga teng. U silindr asosining diametriga teng. Unda silindr asosining yuzi: $S = \pi r^2 = \pi\left(\frac{\sqrt{Q}}{2}\right)^2 = \frac{\pi Q}{4}$ ga teng. □

Teorema. Silindrning yon sirti asosining aylana uzunligi bilan yasovchisi ko'paytmasiga teng: $S_{yon} = 2\pi rl.$

Mazkur teoremani quyidagi 82- rasm asosida mustaqil isbotlang.

81



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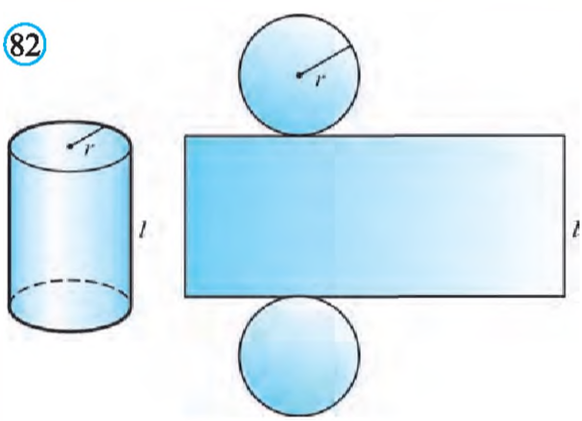


Natija. Silindr to‘la sirti uning yon sirti bilan ikkita asosining yuzi yig‘indisiga teng: $S_{to'la} = S_{yon} + 2S_{asos}$ yoki

$$S_{to'la} = 2\pi rl + 2\pi r^2 = 2\pi r\,(l + r).$$

Ixtiyoriy silindr berilgan bo‘lsin. Uning asoslaridan biriga ichki $A_1A_2...A_{n-1}A_n$ ko‘pburchakni chizamiz (83- rasm). Ko‘pburchakning $A_1, A_2, ..., A_{n-1}$ va A_n uchlari orqali, silindrning A_1B_1, A_2B_2, ..., $A_{n-1}B_{n-1}$ va A_nB_n yasovchilarni o‘tkazamiz hamda yasovchining boshqa $B_1, B_2, ..., B_{n-1}$ va B_n uchlarini ketma-ket kesmalar bilan tutashtirib chiqamiz. Natijada $A_1A_2...A_{n-1}A_n\,B_1B_2...B_{n-1}B_n$ prizmani hosil qilamiz. Bu prizma berilgan *silindrga ichki chizilgan prizma* deb ataladi. Silindr esa *prizmaga tashqi chizilgan silindr* deb yuritiladi. Agar prizma silindrga ichki chizilgan bo‘lsa, unda prizmaning asosi silindr asosiga ichki chizilgan bo‘ladi va prizmaning yon qirralari silindr yon sirtida yotadi.

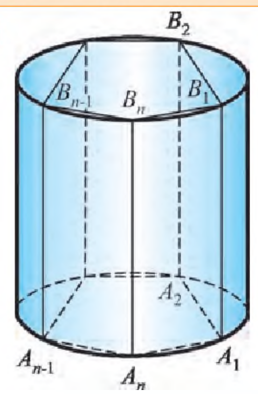
Ravshanki, agar prizma asosiga tashqi aylana chizish mumkin bo‘lsa, prizmaga tashqi silindr ham chizish mumkin.

Shunga o‘xshash *silindrga tashqi chizilgan prizma* va *prizmaga ichki chizilgan silindr* tushunchalari ham kiritiladi (84- rasm). Agar prizma silindrga tashqi chizilgan bo‘lsa, unda prizmaning asosi silindr asosiga tashqi chizilgan bo‘ladi va prizmaning yon yoqlari silindr yon sirtiga urinadi.

Ravshanki, agar prizma asosiga tashqi aylana chizish mumkin bo‘lsa, prizmaga tashqi silindr ham chizish mumkin.

8.2. Silindrning hajmi

Teorema. Silindrning hajmi asosining yuzi bilan yasovchisi ko‘paytmasiga teng: $V = S_{asos} \cdot l$.

83


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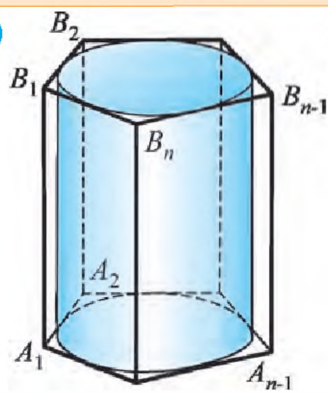

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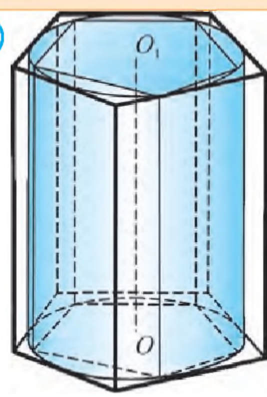


Isbot. O‘qi OO_1 bo‘lgan silindr berilgan bo‘lsin (85- rasm).
Unga ichki $A_1A_2 ... A_{n-1}A_n\,B_1B_2 ... B_{n-1}B_n$ va tashqi $C_1C_2 ... C_{n-1}C_n$

$D_1 D_2 \dots D_{n-1} D_n$ prizmalarni chizamiz. Silindr hajmini V, ichki va tashqi chizilgan prizmalar hajmini V_1 va V_2 bilan belgilasak, unda $V_1 < V < V_2$ qo'shtengsizlik o'rinli bo'ladi. Prizmalar hajmi quyidagi formulalardan topiladi:

$$V_1 = S_{A_1A_2\dots A_{n-1}A_n} \cdot l \qquad \text{va} \qquad V_2 = S_{C_1C_2\dots C_{n-1}C_n} \cdot l$$

Prizmalar asosi tomonlari soni n ni borgan sari oshirib boramiz. Unda ichki chizilgan prizma hajmi oshib boradi, tashqi chizilgan prizmaning hajmi esa kamayib boradi. Agar tomonlar soni n cheksiz kattalashib borsa, bu hajmlar orasidagi farq nolga intiladi. Silindrga ichki va tashqi chizilgan prizmalar hajmi yaqinlashgan son berilgan silindrning hajmi sifatida olinadi.

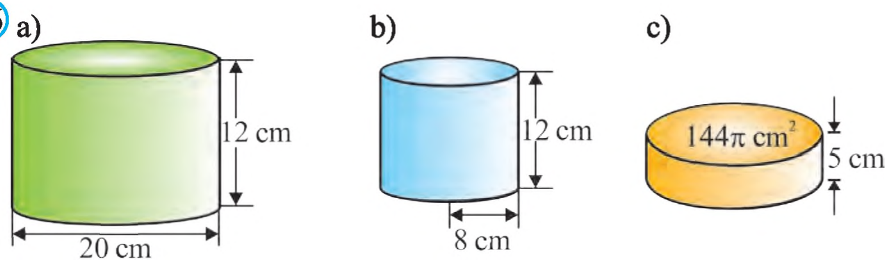
Bu jarayonda $A_1A_2\dots A_{n-1}A_n$ va $C_1C_2\dots C_{n-1}C_n$ ko'pburchaklar yuzi silindr asosida yotgan doira yuzi S ga yaqinlashadi.

Demak, $V = S_{asos} \cdot l$. □

Mavzuga oid masalalar va amaliy topshiriqlar

277. 86- rasmda keltirilgan silindrlarning yon va to'la sirtini toping.

86



278. Silindr asosining radiusi 6 cm, uning balandligi 4 cm. Silindr o'q kesimining yuzini hisoblang.

279. Silindr asosining radiusi 2 m, balandligi 3 m. O'q kesimining diagonalini toping.

280. Silindr asosining yuzi 64 π cm^2, uning balandligi 8 cm. Silindr o'q kesimining yuzini hisoblang.

281. Silindrning o'q kesimi – yuzi Q ga teng kvadrat. Silindr asosining yuzini toping.

282. Silindrning o'q kesimi yuzi 36 cm^2 bo'lgan kvadratdan iborat. Silindr yon sirtining yuzini hisoblang.

283. Silindr o'q kesimining yuzi 4 ga teng. Uning yon sirti yuzini toping.

284. Silindrning balandligi 6 cm, asosining radiusi 5 cm. Silindrning o'qiga parallel ravishda undan 4 cm masofada o'tkazilgan kesimning yuzini toping.

285. Silindr asosining radiusi 2 ga, balandligi 3 ga teng. Silindr yon sirtining yuzini toping.

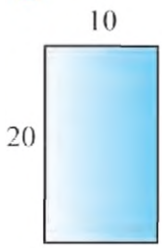
286. Silindr asosining aylana uzunligi 3π ga, balandligi 2 ga teng. Silindrning yon sirti yuzini toping.

287. Silindr yoyilmasining yuzi 24π dm^2, silindrning balandligi 4 dm. Uning asosi radiusini toping.

288. Silindr asosining radiusi 5 cm, uning balandligi 6 cm. Silindr o‘q kesimining diagonalini toping.

289. Silindrning balandligi 8 dm, asosining radiusi 5 dm. Silindr tekislik bilan shunday kesilganki, kesimda kvadrat hosil bo‘lgan. Bu kesimdan silindr o‘qigacha bo‘lgan masofani toping.

87



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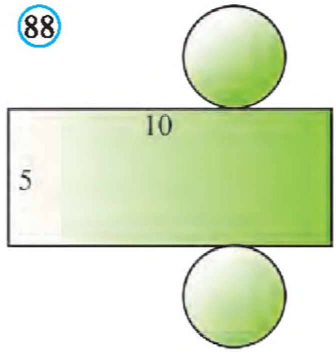


290*. 87- rasmda berilgan silindrning o‘q kesimiga ko‘ra, uning yon va to‘la sirti yuzini toping.

291*. 88- rasmda berilgan silindrning yoyilmasiga ko‘ra, uning yon va to‘la sirtining yuzini toping.

292. Silindr asosining radiusi 3 cm, balandligi esa asos radiusidan 2 cm ortiq. Silindirning hajmini hisoblang.

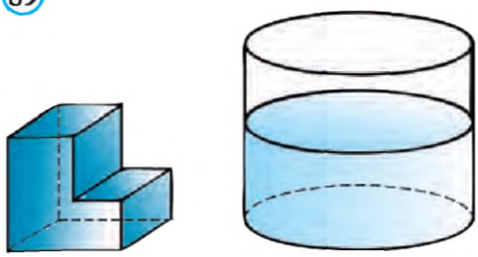
293. Silindrning hajmi 64 π cm^3, balandligi 4 cm. Silindr asosining yuzini hisoblang.

294*. Silindr shaklidagi idishga 2000 cm^3 suv solinganda suvning sathi 12 cm ni tashkil qildi. Idishga detal botirilganda esa suv sathi yana 9 cm ga ko‘tarildi. Detal hajmini aniqlang va javobni cm^3 larda ifodalang.

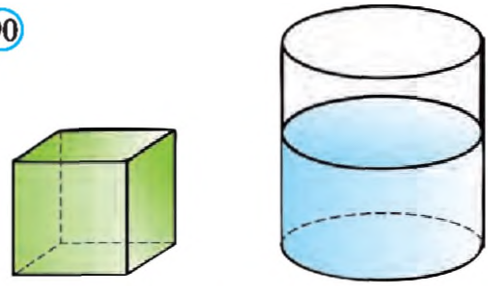
295. Silindr shaklidagi idishga 3 litr suv solinganda suvning sathi 15 cm ni tashkil qildi (89-rasm). Idishga detal botirilganda esa suv sathi yana 4 cm ga ko‘tarildi. Detal hajmini aniqlang va javobni cm^3 larda ifodalang.

296*. Silindr shaklidagi idishga 4 litr suv solinganda suvning sathi 20 cm ni tashkil qildi (90-rasm). Idishga detal botirilganda esa suv sathi yana 5 cm ga ko‘tarildi. Detal hajmini aniqlang va javobni cm^3 larda ifodalang.

89



90

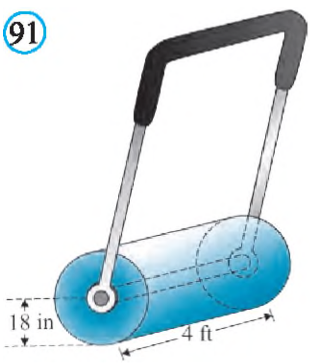


297*.91- rasmda silindr shaklidagi yo'l tekislagich moslamasi tasvirlangan. Rasmda berilganlardan foydalanib, u bir marta **aylanganda qancha maydondagi yo'lni tekislashini aniqlang.**

(Eslatma: 1 ft (fut) = 12 in. (dyuym) = 30,48 cm).

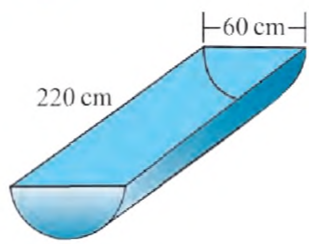
298*.92- rasmdagi suv sepishga mo'ljallangan rezina **quvurning ichki diametri 3 cm, tashqi diamerti 3,5 cm, uzunligi** esa **20 m bo'lsa,** unga **necha lirt suv ketishini toping. Agar** rezinaning **zichligi 7 g/cm^3** ekan-**ligi ma'lum bo'lsa, bu rezina quvur o'**ramining **massasini toping.**

91



92

93



299*.93- rasmda **yon sirti yarim silindr shaklida bo'lgan idish berilgan. Agar 1 cm^2 yuzali sirtni bo'yash uchun 6** g **bo'yoq talab etilsa, bu idishning ham ichki,** ham **tashqi qismini bo'yash uchun qancha** bo'**yoq kerak bo'ladi? Idishga necha litr suv ketadi?**

94

95

96

300*. Silindr shaklidagi idishlardan biri ikkinchisidan ikki marta kengroq, lekin uch marta pastroq (94- rasm). Bu idishlarning qaysi birining sig'imi katta?

301*. Asosining radiusi 5 cm, balandligi esa 20 cm bo'lgan silindr shaklidagi apelsin sharbati idishining asoslari metaldan, yon sirti esa kartondan ishlangan (95- rasm). Agar 1 cm^2 metall narxi 5 so'm, 1 cm^2 karton narxi esa 2 so'm bo'lsa, bu idishni tayyorlash uchun necha so'mlik material kerak bo'ladi? Idishga qancha apelsin sharbati ketadi?

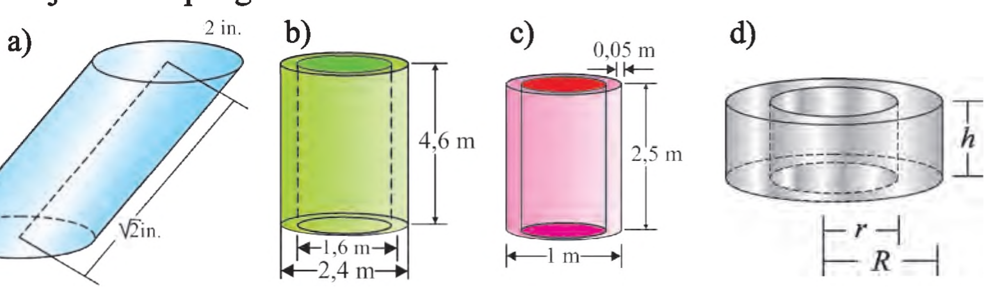
302*. Asosining radiusi 1,5 dyuym, balandligi esa 4,25 dyuym bo'lgan silindr shaklidagi konserva bankasi berilgan (96- rasm). Bankaning to'la sirti va hajmini toping. Agar 1 cm^2 metall narxi 5 so'm bo'lsa, bu idishni tayyorlash uchun necha so'mlik material kerak bo'ladi? (Eslatma: 1 in. (dyuym) = 2,54 cm.)

303*. Neft saqlanadigan idish (sisterna) balandligi 16 fut, asosining radiusi 10 fut bo'lgan silindr shaklida. Agar 1 kub fut 7,5 gallonga teng bo'lsa, bu sisternaning gallonlardagi sig'imini aniqlang. (Eslatma: 1 amerika galloni = 3,785 litr. 1 amerika barelli = 42 amerika galloni = 159 litr.)

304*. Fermerning yoqilg'i baki silindr shaklida. Bakning balandligi 6 fut, asosining radiusi 1,5 fut. Bakning gallonlardagi sig'imini aniqlang.

305. 97- rasmdagi ma'lumotlardan foydalanib, tasvirlangan fazoviy jismlar hajmini aniqlang.

97

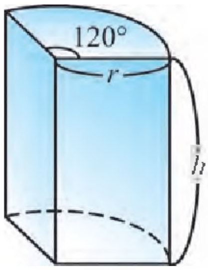


306*. Silindr shaklidagi idishga 6 cm^3 suv solindi. Idishga detal to'liq cho'ktirilganda, suv sathi 1,5 marta ko'tariladi. Detall hajmini aniqlang va javobni cm^3 larda ifodalang.

307*. Silindr shaklidagi idishdagi suvning sathi 16 cm. Idishga asosining diametri bu idishga qaraganda 2 marta kichik bo'lgan silindr shaklidagi ikkinchi idish botirilganda undagi suvning sathi qancha bo'ladi?

308. Birinchi silindr hajmi 12 m^3. Ikkinchi silindrning balandligi birinchi

98



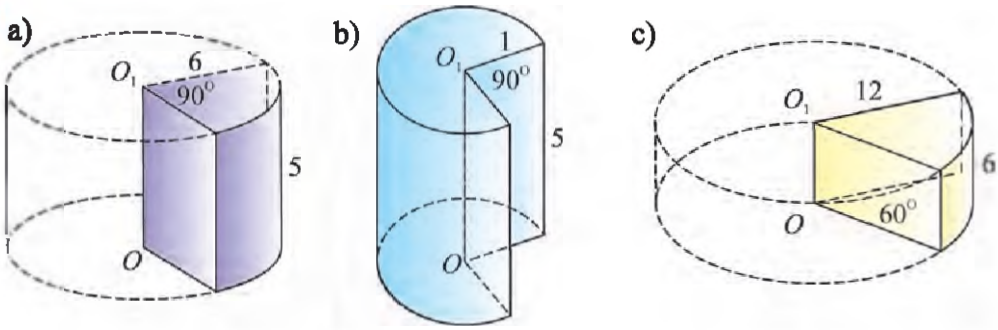
silindrga qaraganda 3 marta katta, asosining radiusi esa 2 marta kichik. Ikkinchi silindr hajmini toping.

309*. Silindr shaklidagi idish ikkinchisidan 2 marta baland, lekin 1,5 marta kengroq. Bu idishlar hajmlarining nisbatini hisoblang.

310. 98- rasmda tasvirlangan fazoviy jism hajmini toping.

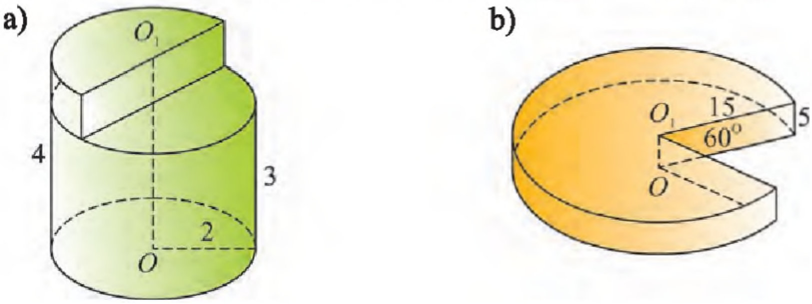
311. 99- rasmda tasvirlangan silindr bo‘lagining hajmini toping.

99



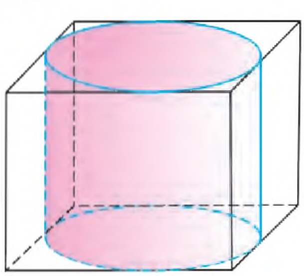
312. 100- rasmda tasvirlangan silindr bo‘lagining hajmini toping.

100



313. To‘g‘ri burchakli parallelepiped asosining radiusi va balandligi 1 ga teng bo‘lgan silindrga tashqi chizilgan (101- rasm). Parallelepiped hajmini toping.

101



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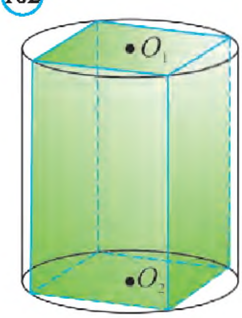

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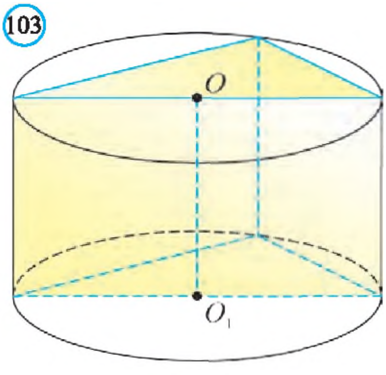


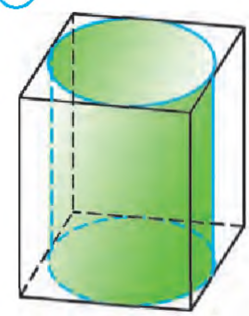
314. To'g'ri burchakli parallelepiped asosining radiusi 4 ga teng bo'lgan silindrga tashqi chizilgan (102- rasm). Parallelepiped hajmi 16 ga teng bo'lsa, silindrning balandligini toping.

315. To'g'ri prizmaning asosi katetlari 6 va 8 bo'lgan to'g'ri burchakli uchburchakdan iborat, yon qirralari esa 5 ga teng (103- rasm). Bu prizmaga tashqi chizilgan silindr hajmini toping.

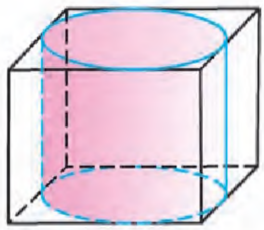
316. To'g'ri prizmaning asosi – tomoni 2 ga teng bo'lgan kvadratdan iborat, yon qirralari esa 2 ga teng. Bu prizmaga tashqi chizilgan silindr hajmini toping.

317. To'rtburchakli to'g'ri prizma asosining radiusi 2 ga teng bo'lgan silindrga tashqi chizilgan (104- rasm). Prizma yon sirtining yuzi 48 ga teng bo'lsa, silindrning balandligini toping.

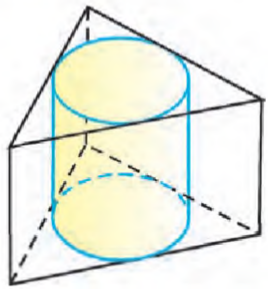
104



105



106

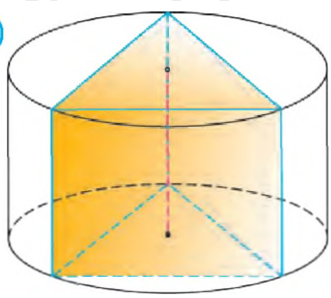


318. Muntazam to'rtburchakli prizma asosining radiusi va balandligi 1 ga teng bo'lgan silindrga tashqi chizilgan (105- rasm). Prizma yon sirtining yuzini toping.

319. Uchburchakli muntazam prizma asosining radiusi $\sqrt{3}$ ga va balandligi 2 ga teng bo'lgan silindrga tashqi chizilgan (106- rasm). Prizma yon sirtining yuzini toping.

320. Uchburchakli muntazam prizma asosining radiusi $2\sqrt{3}$ ga va balandligi 2 ga teng bo'lgan silindrga ichki chizilgan (107- rasm). Prizma yon sirtining yuzini toping.

107



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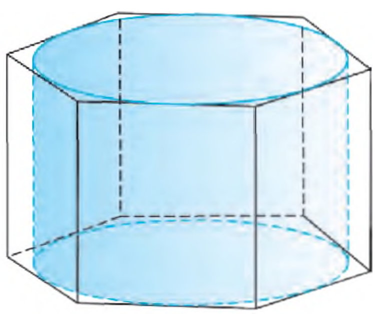


321. Oltiburchakli muntazam prizma asosining radiusi $\sqrt{3}$ ga va balandligi 2 ga teng bo‘lgan silindrga tashqi chizilgan (108- rasm). Prizma yon sirtining yuzini toping.

322*. 109- rasmda tasvirlangan detalning hajmini toping.

323*. Uzunligi 10 m, asosining diametri 1 m bo‘lgan silindr shaklidagi quvurning tashqi sirtini 1 mm qalinlikdagi bo‘yoq bilan bo‘yash uchun qancha bo‘yoq kerak bo‘ladi?

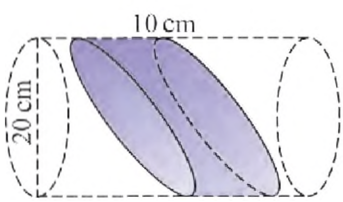
324*. 110- rasmda tasvirlangan tirsakli quvurning: a) yon sirtining yuzini; b) hajmini toping ($\pi \approx 3$ deb oling).

325*. Cho‘yan quvurning uzunligi 2 m, tashqi diametri 20 cm. Quvur devorining qalinligi 2 cm va cho‘yannning solishtirma zichligi 7,5 g/cm^3 bo‘lsa, uning massasini toping.

326*. 111- rasmdan foydalanib, og‘ma silindr uchun $S \cdot h = Q \cdot l$ tenglik o‘rinli bo‘lishini asoslang.

327*. 112- rasmda tasvirlangan silindr sirtidan A nuqtadan B nuqtaga olib boradigan eng qisqa yo‘lning uzunligini toping. (Ko‘rsatma: silindr yoyilmasidan foydalaning.)

109



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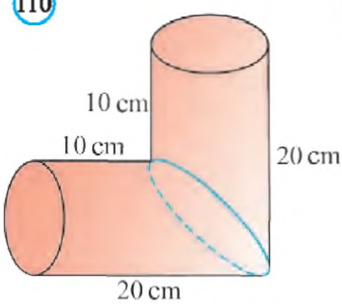
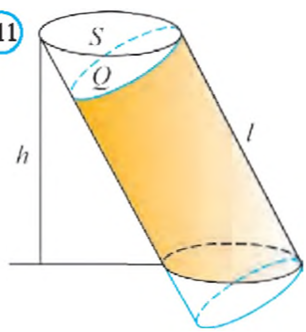


111



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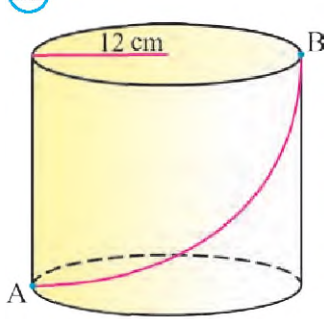


Tarixiv ma'lumotlar

Abu Ali ibn Sino

G'iyosiddin al Koshiy

Abu Rayhon Beruniyning "Astronomiya san'atidan boshlang'ich ma'lumot beruvchi kitob" (qisqacha "Astronomiya") nomli asarining geometriyaga tegishli qismida stereometriyaga kirish sifatida fazoviy shakllarning quyidagi ta'riflari keltiriladi.

Kub – jismiy shakl bo'lib, nardning soqqasiga o'xshaydi, oltita tomonidan oltita kvadrat bilan chegaralangan.

Prizma – mujassam shakl bo'lib, yon tomonidan kvadrat yoki to'g'ri to'rtburchak shaklidagi tekisliklar bilan, osti va ustidan ikkita uchburchak bilan chegaralangan.

Beruniy bergan bu ta'rifda prizmaning xususiy holi, ya'ni uchburchakli prizmaning ta'rifi keltirilgan.

Abu Rayhon Beruniyning "Qonuni Ma'sudiy" kitobi 1037- yilda yozilgan bo'lib, unda parallelepiped, prizmaning hajmlarini topish qoidalari: "Agar jism to'rtburchakli bo'lmasdan yoki boshqa xil bo'lsa, uning o'lchami quyidagicha: uning yuzini bilgin, uni chuqurlikka ko'paytirgin, natijada hajm hosil bo'ladi" tarzda berilgan.

Abu Ali ibn Sino "Donishnoma" nomli asarining "Geometrik jismlarga oid negizlar" bobida jismning va uchburchakli prizmaning ta'rifini beradi hamda ikki prizmaning o'zaro teng bo'lish shartlarini bayon qiladi. Ibn Sino prizmani quyidagicha ta'riflaydi: "Prizma ikkita uchburchakli tekis shakllar va tomonlari o'zaro parallel uchta tekis shakllar bilan chegaralangan jismdir".

G'iyosiddin Jamshid ibn Ma'sud al- Koshiyning "Hisob kitobi" nomli asarida sirtlar yuzlarini va jismlarning hajmlarini hisoblashning ko'plab qoidalari keltirilgan. U matematika, geometriya, trigonometriya, mexanika va astronomiya kabi fanlarni chuqur bilganligi uchun Ulug'bekning e'tibori va hurmatiga sazovor bo'lgan. Al- Koshiy ko'pburchaklar bilan bir qatorda prizmalar, piramidalar, silindrlar, konuslar, kesik konuslarni ham tadqiq qilgan.

9. BOBNI TAKRORLASHGA DOIR AMALIY MASHQLAR

9.1. 2- test sinovi

1. Kubning nechta simmetriya tekisligi mavjud?

A) 8; B) 9; C) 7; D) 10.

2. Agar kub diagonal kesimining yuzi $2\sqrt{2}$ ga teng bo‘lsa, uning hajmini toping.

A) $2\sqrt{2}$; B) $\sqrt{7}$; C) $4\sqrt{2}$; D) $5\sqrt{2}$.

3. To‘g‘ri burchakli parallelepiped asosining tomonlari 7 va 24. Parallelepipedning balandligi 8. Diagonal kesimining yuzini toping.

A) 168; B) 1344; C) 100; D) 200.

4. Muntazam to‘rtburchakli prizmaning diagonali 4 ga teng bo‘lib, yon yog‘i bilan 30 li burchak tashkil qiladi. Prizmaning yon sirtini toping.

A) $16\sqrt{2}$; B) 16; C) 18; D) $18\sqrt{2}$.

5. Muntazam to‘rtburchakli prizma asosining tomoni $\sqrt{2}$ ga, diagonali bilan yon yog‘i orasidagi burchak esa 30 ga teng. Prizmaning hajmini toping.

A) $8\sqrt{2}$; B) 4; C) 16; D) $4\sqrt{2}$.

6. Prizmaning jami qirralari 36 ta bo‘lsa, uning nechta yon yog‘i bor?

A) 12; B) 16; C) 9; D) 10.

7. Og‘ma prizmaning yon qirrasi 20 ga teng va asos tekisligi bilan $30°$ li burchak hosil qiladi. Prizmaning balandligini toping.

A) 12; B) $10\sqrt{3}$; C) 10; D) $10\sqrt{2}$.

8. Uchburchakli to‘g‘ri prizma asosining tomonlari 15, 20 va 25 ga, yon qirrasi asosining balandligiga teng. Prizmaning hajmini toping.

A) 600; B) 750; C) 1800; D) 1200.

9. Muntazam oltiburchakli prizmaning eng katta diagonali 8 ga teng va u yon qirrasi bilan $300°$ li burchak hosil qiladi. Prizmaning hajmini toping.

A) 72; B) 64; C) 76; D) 80.

10. O‘q kesimining yuzi 10 ga teng bo‘lgan silindr yon sirtining yuzini toping.

A) 10π; B) 20π; C) 30π; D) 15π.

11. Silindrning balandligi 8 ga, yon sirti yoyilmasining diagonali 10 ga teng. Silindr yon sirtining yuzini toping.

A) 48; B) 48π; C) 24; D) 48π.

12. Tomonlari 2 va 4 ga teng bo‘lgan to‘g‘ri to‘rtburchak o‘zining katta tomoni atrofida aylandi. Hosil bo‘lgan jismning to‘la sirtini toping.

A) 22π; B) 23π; C) 24π; D) 20π.

13. Silindrning yon sirti yuzi 72π ga teng va u yoyilganda hosil bo‘lgan to‘g‘ri to‘rtburchak diagonali asosi bilan 45° burchak tashkil qiladi. Silindr asosining radiusini toping.

A) 5; B) 4; C) 6; D) 8.

14. Silindr asosining radiusi ikki marta orttirilsa, uning hajmi necha marta ortadi?

A) 4; B) 2; C) 3; D) 6.

15. Silindrning hajmi 120π ga, yon sirti 60π ga teng. Silindr asosining radiusini toping.

A) 4; B) 5; C) 6; D) 4; 2.

16. Silindrning balandligi 5 ga, asosiga ichki chizilgan muntazam uchburchakning tomoni $3\sqrt{3}$ ga teng. Silindrning hajmini toping.

A) 25π; B) 35π; C) 45π; D) 40π.

17. Silindrning o‘q kesimi diagonali 12 ga teng bo‘lgan kvadratdan iborat. Uning hajmini toping.

A) $108\sqrt{2}\pi$; B) $54\sqrt{2}\pi$; C) $36\sqrt{2}\pi$; D) $216\sqrt{2}\pi$.

18. Silindrning to‘la sirti 24π ga, yon sirti esa 6π ga teng. Shu silindrning hajmini toping.

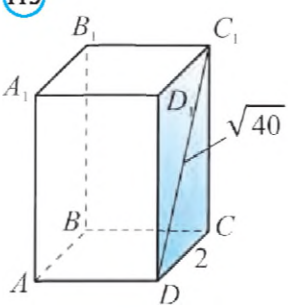
A) 7π; B) 11π; C) 8π; D) 9π.

9.2. Masalalar

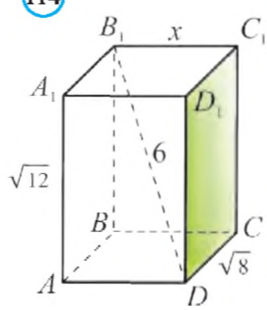
328. $ABCDA_1B_1C_1D_1$ to‘g‘ri burchakli parallelepipedda (113- rasm) $DC_1=\sqrt{40}$, $DC=2$, $P_{ABCD}=10$. Parallelepipedning diagonalini toping.

329. $ABCDA_1B_1C_1D_1$ to‘g‘ri burchakli parallelepiped. 114- rasmda berilgan ma’lumotlarga ko‘ra B_1C_1 qirraning uzunligini toping.

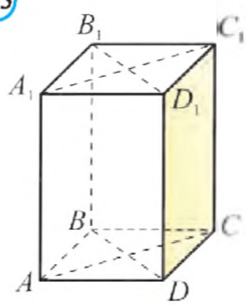
113



114

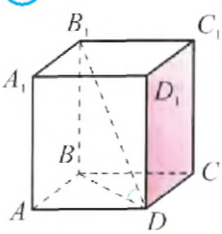


115



330. To‘g‘ri prizmaning asosi $ABCD$ romb (115- rasm). Prizmaning diagonal kesimlari yuzi 60 va 80 ga, balandligi esa 10 ga teng. Prizmaning yon sirtini toping.

331. To‘g‘ri prizmaning asosi $ABCD$ romb. Prizmaning diagonal kesimlari yuzi 24 va 32 ga, balandligi esa 4 ga teng. Prizmaning yon sirtini toping.

116


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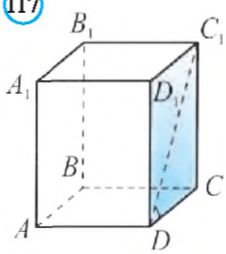
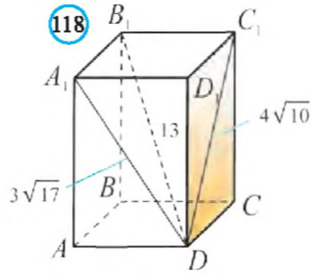
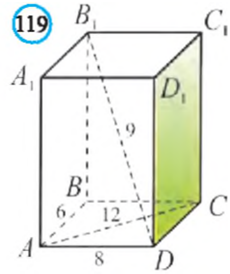


118


119


332. $ABCDA_1B_1C_1D_1$ muntazam prizma (116-rasm) $\angle B_1DB = 45°$, $S_{\text{to'la}} = 32(2\sqrt{2}+1)$. AD ni toping.

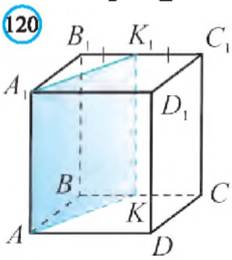
333. $ABCDA_1B_1C_1D_1$ muntazam prizma (117-rasm) $\angle C_1DC = 60°$, $S_{\text{to'la}} = 128(2\sqrt{3}+1)$. AD ni toping.

334. $ABCDA_1B_1C_1D_1$ to'g'ri burchakli parallelepiped (118- rasm) $DB_1 = 13$, $DA_1 = 3\sqrt{17}$, $DC_1 = 4\sqrt{10}$. Parallelepiped yon sirtining yuzini toping.

335. $ABCDA_1B_1C_1D_1$ to'g'ri burchakli parallelepiped (119- rasm) $AB = 6$, $AD = 8$, $DB_1 = 9$. Parallelepiped yon sirtining yuzini toping.

336. K nuqta BC qirraning o'rtasi (120- rasm). $ABKA_1B_1K_1$ prizma hajmining $ABCDA_1B_1C_1D_1$ parallelepiped hajmiga nisbatini toping.

337. N va M nuqtalar parallelepiped qirralarining o'rtalari (121- rasm). $AA_1B_1NDD_1C_1M$ prizma hajmining $ABCDA_1B_1C_1D_1$ parallelepiped hajmiga nisbatini toping.

120


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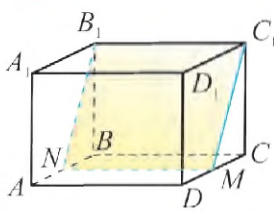

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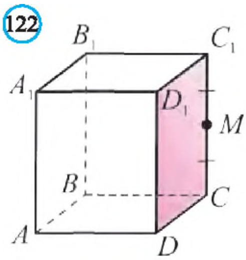
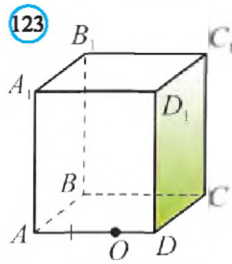


123


338. To'rtburchakli muntazam prizma yon sirtining yuzi 72 cm^2 ga, asosining yuzi esa 64 cm^2 ga teng. Prizmaning hajmini toping.

339. To'rtburchakli muntazam prizma asosining perimetri 12 cm, yon yog'ining perimetri esa 18 cm ga teng. Prizmaning hajmini toping.

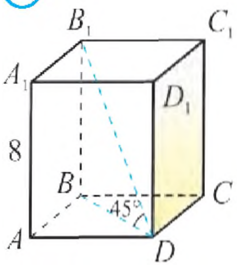
340. Kub berilgan (122- rasm). $CM = MC_1$ va ADM tekislik kubni ikki bo'lakka ajratadi. Kubning katta bo'lagi hajmining kichik bo'lagi hajmiga nisbatini toping.

341*. Kub berilgan (123- rasm). $AO : OD = 2 : 1$ va BB_1O tekislik kubni ikki bo'lakka ajratadi. Agar kubning kichik bo'lagi hajmi 6 ga teng bo'lsa, kubning hajmini toping.

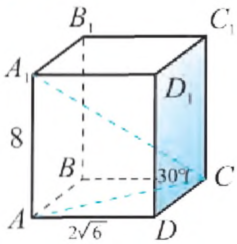
342*. To'rtburchakli muntazam prizmaning balandligi 8 ga, diagonalining asos tekisligiga qiyaligi 45° ga teng (124- rasm). Prizmaning hajmini toping.

343*. To'rtburchakli muntazam prizmada asosining tomoni $2\sqrt{6}$ ga, diagonali asos tekisligi bilan 30° li burchak tashkil qiladi (125- rasm). Prizmaning hajmini toping.

124

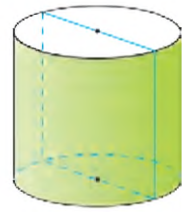


125



126

127



344. Silindr yon sirtining yuzi 91π ga teng (126- rasm). Silindr o'q kesimining yizini toping.

345. Silindr o'q kesimi yuzi 173 ga teng bo'lgan kvadrat (127- rasm). Silindr yon sirtining yuzini toping.

346. Silindr balandligi 24 ga, o'q kesim diagonali 26 ga teng. Silindr hajmini toping.

347. Silindr o'q kesimi yuzi 10 ga teng. Asos aylanasining uzunligi 8 ga teng. Silindr hajmini toping.

348. Silindr radiusi 3 ga, yon sirtining yuzi 200 ga teng. Silindr hajmini toping.

9.3. 2-nazorat ishi namunasi

1. Ikkiyoqli burchakning *A* nuqtasi uning qirrasidan 10 cm, yog'idan 5 cm uzoqlikda joylashgan. Ikkiyoqli burchakning gradus o'lchovini toping.

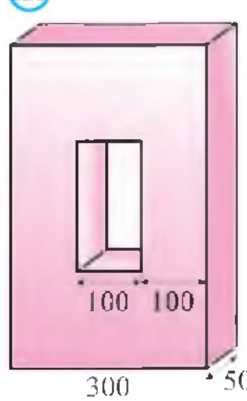
2. Oltiburchakli muntazam prizmaning barcha qirralari 2 ga teng bo'lsa, uning to'la sirtining yuzini toping.

3. Asosining diamerti 18 m va balandligi 7 m bo'lgan silindr shaklidagi sisterna neft bilan to'ldirilgan. Agar neftning zichligi 0,85 g/cm³ bo'lsa, bu sisternadagi neftning massasi necha tonna?

4. Har bir qirrasi uzunligi 4 cm ga teng bo'lgan muntazam oltiburchakli prizmaga ichki chizilgan silindr hajmini toping.

5. (*Yaxshi o'zlashtiradigan o'quvchilar uchun qo'shimcha masala.*) 128- rasmda o'lchamlar mm larda berilgan detalning to'la sirti va hajmini toping.

128



Trigonometrik funksiyalarning taqribiy qiymatlari jadvali

A	sin *A*	tg *A*	*A*	sin *A*	tg *A*	*A*	sin *A*	tg *A*
0°	0	0	30°	0,50	0,58	60°	0,87	1,73
1°	0,0175	0,0175	31°	0,52	0,60	61°	0,87	1,80
2°	0,035	0,035	32°	0,53	0,62	62°	0,88	1,88
3°	0,05	0,05	33°	0,54	0,65	63°	0,89	1,96
4°	0,07	0,07	34°	0,56	0,68	64°	0,90	2,02
5°	0,09	0,09	35°	0,57	0,70	65°	0,91	2,15
6°	0,10	0,11	36°	0,59	0,73	66°	0,91	2,25
7°	0,12	0,12	37°	0,60	0,75	67°	0,92	2,36
8°	0,14	0,14	38°	0,62	0,78	68°	0,93	2,48
9°	0,16	0,16	39°	0,63	0,81	69°	0,93	2,61
10°	0,17	0,18	40°	0,64	0,84	70°	0,94	2,78
11°	0,19	0,19	41°	0,66	0,87	71°	0,95	2,90
12°	0,21	0,21	42°	0,67	0,9	72°	0,95	3,08
13°	0,23	0,23	43°	0,68	0,93	73°	0,96	3,27
14°	0,24	0,25	44°	0,69	0,97	74°	0,96	3,49
15°	0,26	0,27	45°	0,71	1,00	75°	0,97	3,73
16°	0,28	0,29	46°	0,72	1,04	76°	0,97	4,01
17°	0,29	0,31	47°	0,73	1,07	77°	0,97	4,33
18°	0,31	0,32	48°	0,74	1,11	78°	0,98	4,71
19°	0,33	0,34	49°	0,75	1,15	79°	0,98	5,15
20°	0,34	0,36	50°	0,77	1,19	80°	0,98	5,67
21°	0,36	0,38	51°	0,78	1,23	81°	0,99	6,31
22°	0,37	0,40	52°	0,79	1,28	82°	0,99	7,12
23°	0,39	0,42	53°	0,80	1,33	83°	0,992	8,14
24°	0,41	0,45	54°	0,81	1,38	84°	0,994	9,51
25°	0,42	0,47	55°	0,82	1,43	85°	0,996	11,43
26°	0,44	0,49	56°	0,83	1,48	86°	0,998	14,30
27°	0,45	0,51	57°	0,84	1,54	87°	0,999	19,08
28°	0,47	0,53	58°	0,85	1,60	88°	1,00	28,64
29°	0,48	0,55	59°	0,86	1,66	89°	1,00	57,29

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3. $A(5; 7; 10)$, $B(4; -3; 6)$, $C(5; 0; 0)$, $D(4; 0; 4)$, $E(0; 5; 0)$, $F(0; 0; -2)$. **6.** $(3; 2; 0)$, $(3; 0; 4)$, $(0; 2; 4)$. **8.** $\sqrt{26}$. **9.** a) 3, 3, 3; b) $3\sqrt{2}, 3\sqrt{2}, 3\sqrt{2}$; c) $3\sqrt{2}$. **10.** 2, 3, 1. **11.** $(3; 3; 3)$, $(-3; 3; 3)$, $(3; -3; 3)$, $(3; 3; -3)$, $(-3; -3; 3)$, $(-3; 3; -3)$, $(3; -3; -3)$, $(-3; -3; -3)$. **12.** $O(0; 0; 0)$, $B(2; 0; 0)$, $A(2; 2; 0)$, $C(0; 2; 0)$, $O_1(0; 0; -2)$, $B_1(2; 0; -2)$, $A_1(2; 2; -2)$, $C_1(0; 2; -2)$. **13.** D nuqta. **14.** $3\sqrt{6}$. **15.** Yo‘q. **17.** *c)* teng yonli, $P=6\ (1+\sqrt{3})$, $S = 9\sqrt{2}$. **18.** $(-0{,}25; 0{,}25; 0)$. **19.** $D_1(1; -1; 1)$, $A_1(1; 1; -1)$, $B_1(-1; 1; -1)$, $D_1(1; -1; -1)$. **21.** $x^2+y^2+z^2=25$, $x^2+y^2+z^2\le 25$. **22.** $(x-1)^2+(y-2)^2+(z-4)^2=9$; $(x-1)^2+(y-2)^2+(z-4)^2\le 9$. **23.** $(x+2)^2+(y-3)^2+(z-4)^2=9$. **25.** 1)$(0; 1; 0)$; 2) $(1; 1; 1)$; 3) $(0; 0; 2)$, 4) $(-0{,}7; 0{,}1; 0{,}6)$; 5) $(2\sqrt{3}; 1{,}5; 1)$. **28.** $A(5;-4;0)$, $B(-7;5;6)$, **31**. $K\left(0; -5; \dfrac{17}{2}\right)$. **32.** a) $D(-1; -3; -9)$. **33.** *a)* $M(-1; 2; 0)$; *c)* $M(3; \frac{3}{4}; 0)$. **35.** $L(\frac{25}{8}; \frac{33}{8}; \frac{9}{4})$. **36.** $\frac{4\sqrt{2}}{5}$. **37**. a) $\sqrt{2}$; b) 30°; 30°; 120°; c) $2\sqrt{3}$. **38.** $MK=\frac{\sqrt{73}}{3}$. **39.** $A(5; 4; 10)$, $B(4; -3; 6)$, $C(5; 0; 0)$, $D(4; 0; 4)$. **40.** $\overline{OA}=(1; 1; 1)$, $\overline{OB}=(-1; 0; 1)$, $\overline{OC}=(0; 1; 1)$, $\overline{BO}=(1; 0; -1)$, $\overline{CO}=(0; -1; -1)$, $\overline{AB}=(-2; -1; 0)$. **42.** a) $\overline{AB}=(2; 5; 3)$, b) $\overline{AB}=(4; -6; 2)$. **43.** $|\bar{a}|=\sqrt{3}$; $|\bar{b}|=2\sqrt{5}$, $|\bar{c}|=\sqrt{14}$, $|\bar{d}|=\sqrt{30}$. **44.** ± 3. **45.** a) $\bar{a}(3; 6; -3)$, b) $\bar{a}(-3; -6; 3)$. **46.** a) 1 yoki -1; b) 3 yoki -1; c) 2 yoki -4; d) 3 yoki 5/3. **48.** $D(-2; 0; 1)$. **50**. $n=\frac{4}{3}$; $m=\frac{3}{2}$. **52.** a) $D(3; 0; 0)$. **56.** $\bar{c}(-3; -4; 8)$, $|\bar{c}|=\sqrt{89}$; 2) $\bar{c}(4; 5; 5)$, $|\bar{c}|=\sqrt{66}$. **57.** $\bar{c}(-3; 4; 0)$, $|\bar{c}|=5$; 2) $\bar{c}(0; 2; 6)$, $|\bar{c}|=2\sqrt{10}$. **59.** $\bar{a}=\bar{i}-\bar{j}+\bar{k}$, $\bar{b}=2\bar{j}-4\bar{k}$, $\bar{c}=2\bar{i}+3\bar{j}-\bar{k}$, $\bar{d}=\bar{i}+2\bar{j}+5\bar{k}$. **60.** $\sqrt{59}$, $\sqrt{219}$, $\sqrt{122}$, $\sqrt{918}$. **63.** $AC = AO + OC = 4i + 2k$, $AC(-4; 0; 2)$; $CB = CO + OB = 2k + 9j$, $CB\ (0; 9; 2)$; $AB = AO + OB = -4i + 9j$, $AB(-4; 7; 0)$. **65.** $\approx 180N$. **66.** a) 60°; b) 30°; c) 90°; d) 60°; e) 45°. **67.** a) -6; b) 3; c) -6; d) 3. **68.** a) 40°; b) 140°; c) 150°. **69.** a) 30; b) 3; c) 15; d) -28. **70.** a) 1/3; b) -1; c) 2; d) 4. **71.** a) 16. **75.** a) 1; b) 0. **76.** $\overline{BF}=2(\overline{DO}-\overline{DC})$. **77.** $\frac{1}{3}(2\overline{AC}-\overline{AB})$. **78.** $\frac{1}{3}(\overline{AB}+\overline{AC})-\overline{AD}$. **83.** a) $(1; -1; 7)$; b) $(-2; 3; 1)$; c) $(0; -4; 4)$. **84.** $\bar{p}(-1; 5; 3)$. **86.** $B(-8; 4; 1)$. **88.** $(2; -5; 9)$; $(-2; -2; 7)$; $(6; -12; 2)$. **93.** Oxz tekislikka nisbatan. **100.** $(0; -3; 1)$. **106.** a) 36 cm; b) 48 cm; c) 6 cm; d) 4 cm. **110.** a) $B(-5; 7{,}5; 12{,}5)$; b) $B(5; -7{,}5; -12{,}5)$; c) $B(-0{,}5; 0{,}75; 1{,}25)$; d) $B(0{,}5; -0{,}75; -1{,}25)$. **111.** a) $B(-2{,}5; 1; 3)$; b) $B(-7; 2; 6)$. **112.** a) $O_1(0; 0; 0)$, $A_1(-4; 0; 0)$, $B_1(0; -4; 0)$, $C_1\ (0; 0; -4)$; b) $O_1(-4; 0; 0)$, $A_1(4; 0; 0)$, $B_1(-4; 8; 0)$, $C_1\ (-4; 0; 8)$. **115.** $(2; -3; 3)$. **116.** -3. **117.** $(7; 1; 2)$. **118.** $(1; -2; 3)$. **119.** $(-1; -2; -3)$. **120.** $(1; 2; -3)$. **121.** $(-2; -3; -5)$. **122**. $D(0; 9; -7)$. **123.** $C(2; 0; -8)$. **124.** 19. **125.** $(-7; 7; -7)$. **126.** $(1; 2; 1)$. **127.** $(-2; 7; 1)$. **128.** ± 2. **129.** ± 3. **130.** 13. **131.** 10. **132.** 9. **133.** 0. **134.** -2. **135.** 1. **136.** 4. **137.** 90°. **138.** 4. **139.** -4. **140.** -2; 4. **141.** $8\vec{i}+9\vec{j}-4\vec{k}$.

1- test sinovi javoblari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
C	D	D	B	D	B	B	A	A	D	B	B	B	C	A	C	B	D	A	C	B	C	D	D	C

1- nazorat ishi javobi

1) $(1; 2; -3)$; 2) 13; 3) $\sqrt{2}$; 4) 90°; 5) 1.

2- bob javoblari

142. 47°, 133°, 47°, 133°. **143.** 128°. **144.** 80°. **145.** 90°. **146.** 5 cm, 5 cm. **147.** 12 cm. **148.** 5 cm. **152.** 45°. **153.** 45°. **154.** 80°. **159.** 60°, 45°. **165.** a) 4, 10; b) 5, 12. **166.** Yo'q. **170.** 6, kub. **171.** 15 ta. **172.** 9 ta. **173.** 180 ta. **174.** 24 cm^2. **175.** 44 cm^2. **176.** 76,8 cm^2. **177.** 17,64 cm. **178.** $4\sqrt{3}cm^2$, 4 cm. **179.** 124 dm^2. **180.** 20 m^2, 30 m^2. **181.** 8 cm, 8 cm. **182.** 13 cm, 9 cm. **184.** 4500 cm^2. **185.** 7,5. **186.** 4. **187.** 480 cm^2. **188.** $5\sqrt{2}$. **189.** 45 cm^2. **190.** 144. **191.** a)18; b)76; c) 110; d) 132; e) 48; f) 96; g) 124. **192.** a) 146; b) 126; c) 108; d) 146. **193.** 84 cm. **194.**$3\sqrt{2}$ cm^2. **195.** 216 cm^2. **196.** a) 58; b) 62; c) 94. **197.** a) 38; b) 92; c) 48. **198.** ≈68 m^2. **199.** 104 cm. **200.** 68 cm^2. **201.** 78 cm^2. **204.** 5120 cm^3. **207.** 144. **209.** 8. **210.** 5. **211.** 6. **212.** 3. **213.** 24. **214.** 2. **215.** 8. **216.** 8. **217.** 72. **218.** 4. **219.** 27 litr. **220.** 4. **221.** 60 cm^2. **222.** $\frac{(S-ab)ab}{4(a+b)}$. **223.** 30 m. **224.** 1200. **225.** a) 4; b) 40; c) 71; d) 88; e) 18; f) 33; g) 78. **226.** a) 90; b) 77; c) 54; d) 96. **227.** 6 m^3. **228.** a) 21; b) 26; c) 58. **230.** 6 m^3. **231.** $\sqrt{2}$ m^3. **232.** $\frac{a^3\sqrt{2}}{2}$. **233.** $2\sqrt{\sin 3\alpha \sin^3\alpha}$. **234.** $abc\sqrt{-\cos 2\alpha}$. **235.** a) $\frac{a^2b\sqrt{3}}{4}$; b) a^2b; c) $\frac{3a^2b\sqrt{3}}{4}$. **237.** 3060 m^3. **238.** 3 cm^3. **239.** $\frac{a^3}{8}$. **240.** $3\sqrt{3}$ m^3. **241.** 1 marta. **243.** 24 cm^3. **245.** 12 cm^3. **246.** 2 cm. **247.** $\frac{ac\sqrt{12a^2-3c^2}}{8}$. **248.** $\frac{h^3\sin\gamma}{2\mathrm{tg}\alpha\mathrm{tg}\beta}$. **249.** 6048 m^3/soat. **250.** 35200 m^3. **251.** 0,5 g/cm^3. **252.** 150 ta. **253.** 42 ta. **254.** 961 ta. **255.** 13 ta. **256.** 90 ta. **257.** 3315 g. **258.** 60 m^2. **259.** 24 ta. **260.** 24 cm^3. **261.** 1927,2 g. **262.** 1927,2 g. **263.** 960 m^3. **264.** 144 g. **265.** 19,3125 g/cm^3. **266.** 440 m^3. **267.** 0,0127 m^3. **271.** $(y+w+z)yx$. **274.** $a:b:c$. **277.** 240π cm^2 , 280π cm^2. **278.** 48 cm^2. **279.** 5 cm. **280.** 128 cm^2. **281.** $\pi Q/4$. **282.** 36π cm^2. **283.** 4π. **284.** 36 cm^2. **285.** 12π. **286.** 64. 6. **287.** 3 dm. **288.** $2\sqrt{34}$ cm. **289.** 3 dm. **290.** 200π, 250π. **291.** 50, 50 +50/π. **292.** 45π cm^3. **293.** 16π cm^2. **294.** 1500 cm^3. **295.** 800 cm^2. **296.** 1000 cm^2. **297.** 5574 cm^2, 1824 cm^2. **298.** 1375π cm^3, 11,375 kg. **299.** 141900 g, 310860 cm^2. **300.** Birinchisining. **301.** 2041 so'm, 15700 cm^2. **302.** 349,45 cm^2, 492 cm^3, 1747 so'm. **303.** 37680 gallon. **304.** 318 gallon. **306.** 3 cm^3. **307.** 4 cm. **308.** 9 m^3. **309.** 1,125. **311.** a) 45π; b) 3,75π; c) 144π. **312.** a) 14π; b) 937,5π. **313.** 4. **314.** 0,25. **315.** 125π. **316.** 4π. **317.** 3. **318.** 8. **319.** 36. **320.** 36. **321.** 24. **322.** ≈30 m^3. **323.** ≈3000 cm^3. **324.** a) ≈1050 cm^2; b) ≈2250 cm^3. **325.** ≈162 kg. **328.** 7. **329.** 4. **330.** 200. **331.** 160. **332.** 4. **333.** 8. **334.** 168. **336.** 1/3. **337.** 1/3. **338.** 144 m^3. **339.** 56 cm^3. **340.** 6. **341.** 2. **342.** 256. **343.** 96. **344.** 91. **345.** 173 π. **346.** 600π. **347.** 20. **348.** 300.

2- test sinovi javoblari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
B	A	D	A	B	A	C	C	A	A	A	C	C	A	A	C	A	D

2- nazorat ishi javoblari

1) 30^0; 2) $2\sqrt{3}$ +24; 3) 1513 *l*; 4) 64π cm^3; 5) 35 dm^2, 6,5 dm^3.

Eslatma. *Geometriyaga doir qiyinroq masalalar tartib raqami yulduzcha bilan, uyda bajarish tavsiya qilinayotgan masalalar qizil rangda berilgan.*

Darslikni tuzishda foydalanilgan va qo'shimcha o'rganishga tavsiya etilayotgan o'quv-uslubiy adabiyotlari va elektron resurslar

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MATEMATIKA 11

ALGEBRA VA ANALIZ ASOSLARI, GEOMETRIYA

I QISM

O‘rta ta’lim muassasalarining 11-sinfi va o‘rta maxsus, kasb-hunar ta’limi muassasalari o‘quvchilari uchun darslik

1- nashr

Muharrirlar:	N. Gayipov M. Raemov
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